An alternative quantification of redshift

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ABSTRACT

Cosmological redshifts are fundamental to the study of the universe at large scales. It is shown that by inverting the traditional equations of redshift, the infinite range of quantifiable redshifts becomes finite. This implies an alternative redshift-distance relationship that correlates closely to the predictions of the concordance model of cosmology.

Key words: galaxies: distances and redshifts – cosmology: miscellaneous

1 INVERTING REDSHIFT

Redshift (z) can be quantified by comparing the observed energy (E_{obs}) of a photon to its original energy (E_{emit}):

$$1 + z = \frac{E_{emit}}{E_{obs}} \tag{1}$$

If the photon is observed to have less energy than when it was emitted ($E_{obs} < E_{emit}$), then $z > 0$ and the photon is redshifted. When the photon gains energy ($E_{obs} > E_{emit}$), then $z < 0$ and the photon is blueshifted. Since the photon will always have a positive energy, $\frac{E_{emit}}{E_{obs}}$ is always greater than zero, so z is always greater than -1. Thus, the range of redshifts is infinite ($z > 0$), and the range of blueshifts is finite ($-1 < z < 0$).

As an alternative, consider what happens when equation (1) is inverted and set equal to $1 + b$:

$$1 + b = \frac{1}{1+z} = \frac{E_{obs}}{E_{emit}} \tag{2}$$

When $b > 0$, that means the photon has gained energy, which is blueshift. When $b < 0$, the photon has less energy than when it was emitted, so the photon is redshifted. Since the photon's energy will never be negative or zero, b is always greater than -1. Quantified this way, the range of redshifts is finite ($-1 < b < 0$).

2 DISTANCE

The traditional redshift-distance relationship is:

$$d = z\frac{c}{H_0} \tag{3}$$

Where d is distance, c is the speed of light and H_0 is Hubble's constant. This equation is an approximation that is only valid for very small redshifts ($z \ll 1$).

Substituting z with the alternative quantification, $-b$:

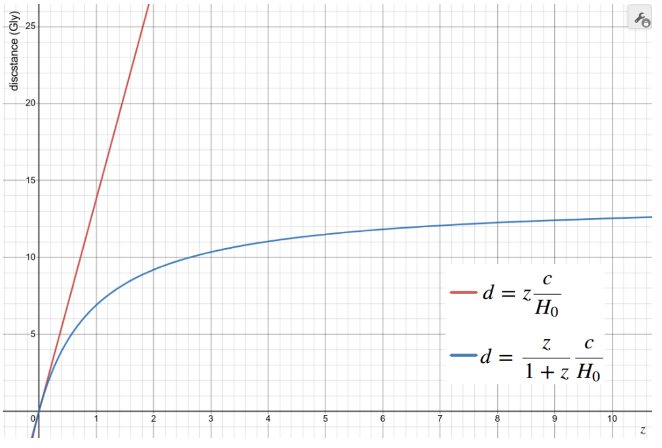

Figure 1. The traditional and alternative redshift-distance relationships.

$$d = -b\frac{c}{H_0} \tag{4}$$

Solving equation (2) for b and simplifying:

$$b = -\frac{z}{1+z} \tag{5}$$

Substituting (5) into (4) gives the alternative redshift-distance relationship:

$$d = \frac{z}{1+z}\frac{c}{H_0} \tag{6}$$

Comparing the two distance relationships (Fig. 1) shows that the distance predicted by the traditional equation climbs to infinity while the alternative equation converges on $\frac{c}{H_0}$. This is due to the range of redshifts being infinite as z and finite as $-b$ (i.e. $\frac{z}{1+z}$).

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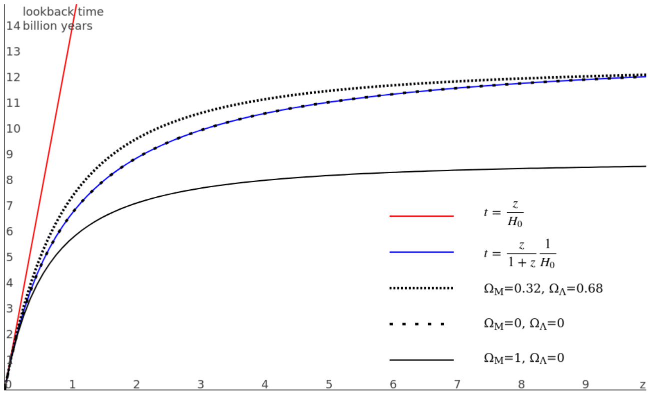

Figure 2. The redshift-time relationships compared to FLRW lookback times.

3 LOOKBACK TIME

Dividing both sides of equation (6) by the speed of light, c, gives the alternative redshift-time equation:

$$t = \frac{z}{1+z}\frac{1}{H_0} \tag{7}$$

This equation is also the solution to the lookback time integral for what's known as an 'empty' Friedmann-Lemaître-Robertson-Walker (FLRW) model. (Krisciunas 1993)

In Fig. 2, the alternative redshift-time equation is compared to the lookback times of three FLRW models: an empty model ($\Omega_M = 0, \Omega_\Lambda = 0$), a matter-only model ($\Omega_M = 1, \Omega_\Lambda = 0$), and the concordance model, ($\Omega_M = 0.32, \Omega_\Lambda = 0.68$).

The empty model and the concordance model have similar predictions while the matter-only model predicts a far younger universe. The lookback times predicted by the empty FLRW model are identical to the times given by the alternative redshift-time equation.

4 CONCLUSIONS

When quantified as z the range of redshifts is infinite. When quantified as $-b$, however, the range is finite. The traditional and alternative quantifications of redshift lead to two distance relationships. The traditional relationship is linear, and only a valid approximation for very small values of z. The alternative relationship is identical to an empty FLRW model, and a reasonable approximation to the concordance model.

While the choice between quantifications of redshifts initially seems arbitrary and inconsequential, upon further investigation, the inverted, alternative quantification is a more accurate representation of the universe.

DATA AVAILABILITY STATEMENT

The data underlying this article are available in the article.

REFERENCES

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