irreversible effects is avoided, and we may be sure that these relations are the rigorous result of thermodynamic principles, with no assumptions involving the neglect or irreversible aspects of the phenomena.

The formal thermo-magnetic analogy of the Thomson transverse effect in crystals is an absorption of heat by a heat current flowing transversely in a bar carrying a longitudinal electric current of density i in a perpendicular magnetic field in amount equal to the fraction PHi/T of itself per unit length measured transversely. This may be proved at once from the equation of energy balance.

[1] P. W. Bridgman, *Phys. Rev.*, Dec., 1924, 644–651. Report of the Fourth Solvay Congress, "Conductibilité Électrique des Métaux," 352–354.

[2] H. A. Lorentz, *Fourth Solvay Congress*, 354–360.

ON THE RED SHIFT OF SPECTRAL LINES THROUGH INTERSTELLAR SPACE

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A. *Introduction.*—It is known that very distant nebulae, probably galactic systems like our own, show remarkably high receding velocities whose magnitude increases with the distance. This curious phenomenon promises to provide some important clues for the future development of our cosmological views. It may be of advantage, therefore, to point out some of the principal facts which any cosmological theory will have to account for. Then a brief discussion will be given of different theoretical suggestions related to the above effect. Finally, a new effect of masses upon light will be suggested which is a sort of gravitational analogue of the Compton effect.

B. *Discussion of the Observational Facts.*—(1) E. Hubble[1] has shown recently that the correlation between the apparent velocity of recession and the distance is roughly linear, corresponding to 500 km./sec. per 10^6 parsecs. Large deviations occur for the nearest nebulae, which may be attributed to their peculiar motions. The most recent observations by M. Humason[2] seem to indicate that for very large distances (50×10^6 light years) the individual deviations become so great (3000 km./sec. out of 8000 km./sec.) that they hardly can be due to peculiar motions and must, therefore, be accounted for in some other way.

(2) The relative shift of frequency $\frac{\Delta\nu}{\nu}$ representing the velocity of recession is apparently independent of the frequency. The available range

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in the spectrum is not very large, however. Some exceptions have been found, suggesting that $\frac{\Delta\nu}{\nu}$ for H_β is somewhat greater than for H_γ.

(3) No appreciable absorption or scattering of light can be related to the above shift of spectral lines.

(4) The optical image of an extragalactic nebula seems to be as well defined as can be expected from the resolving power of the telescopes. The distance apparently is only geometrically involved and no additional blurring of the images occurs due to some such process as multiple scattering and superposition of incoherent light beams.

(5) The spectral (absorption) lines obtained from these nebulae are not very well defined, but no systematic investigation of their shape has been carried out. According to recent observations by M. Humason the width of the lines ranges between 4 Å and 7 Å for M_{32} and M_{31} the two Andromeda nebulae.

(6) Extrapolating from Hubble's relation to objects in our own galactic system, the velocity of recession would become so small (5 km./sec. for 10,000 parsecs) that it would escape observation. The theoretical considerations proposed by the author in the following made it probable that an appreciable effect should also be observed in our galaxy. This suggestion was tested by Dr. ten Bruggencate, whose work will be published shortly. His essential result is that the velocity of recession of the globular clusters is a function of the galactic latitude, increasing with decreasing latitude.[4]

We proceed now in discussing different theoretical possibilities of accounting for the phenomenon described above.

C. de Sitter's Universe.—It has been pointed out by de Sitter that the special type of a space proposed by himself as representing our universe would imply on the average a velocity of recession of the far distant nebulae. But the linear relation of Hubble's can only be obtained by making some additional assumptions about the distribution of the nebulae. For more detailed information, we refer to a recent paper by R. C. Tolman.[3] Admitting that de Sitter's explanation accounts for the facts listed above in the sections B_1 to B_5, a correlation of the type B_6 for our own galaxy would present an almost unsurmountable obstacle for any theory based on geometry only.

D. *The Compton-Doppler Effect on Free Electrons.*—We know from different sources, that there exist very dilute gaseous masses distributed all over the interstellar spaces. The observations of the steady Ca^+ and Na absorption lines provide one of the most direct proofs of this fact. These observations also show that some of the atoms occur as ions. It may be concluded, therefore, that an adequate number of free electrons be present. One then might expect that the light coming from distant

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nebulae would undergo a shift to the red by Compton effect on those free electrons. Now the admissible deflection in one single process is very small, the angular size of the nebulae being indeed less than one degree of arc. For the change in wave-length $\Delta\lambda$ by a single Compton scattering within the above angle, one obtains then $\Delta\lambda \leq 3 \times 10^{-13}$ cm., so that a great number of collisions between the light quanta and the electrons are necessary in order to produce a change $\Delta\lambda \sim 1$ to 100 Å. But then the light scattered in all directions would make the interstellar space intolerably opaque which disposes of the above explanation.

It is possible, of course, that a great number of the electrons possess very high speed. This is suggested by the existence of the cosmic radiation. In this case, an appreciable shift of frequency may be produced by one collision. But still the difficulty of obtaining too much scattered light in all directions can hardly be avoided, at least if use is made of our present knowledge of the intensity distribution due to Compton effect. Also, it is evident that any explanation based on a scattering process like the Compton effect or the Raman effect, etc., will be in a hopeless position regarding the good definition of the images as mentioned under B_4.

E. *The Usual Gravitational Shift of Spectral Lines.*—One might expect a shift of spectral lines due to the difference of the static gravitational potential at different distances from the center of a galaxy. This effect, of course, has no relation to the distance of the observed galaxy from our own system and, therefore, cannot provide any explanation of the phenomenon discussed in this paper. But it might have some bearing on the width of the observed spectral lines as light coming from different points of the distant galaxy will show varying shifts. To get an estimate, we assume the nebulae to be a sphere of the radius $R = 2 \times 10^4$ light years and of a uniform density 10^{-20} gr./cm.$^3 > \rho > 10^{-24}$ gr./cm.^{3} Then we have for the gravitational potential $\Phi(r)$ this relation: $\Delta\Phi = \Phi(R) - \Phi(0) = \frac{2\pi}{3} f\rho R^2$ where $f = 6.68 \times 10^{-8}$ is the universal gravitational constant. And for the above limits of ρ $8 \times 10^{-8} < \frac{\Delta\nu}{\nu} = \frac{\phi}{c^2} < 8 \times 10^{-4}$ (240 km/sec.) where c is the velocity of light. This effect also might cause a violet shift of the light traveling from the outer regions of our galaxy toward the center.

F. *The Gravitational "Drag" of Light.*—According to the relativity theory, a light quantum $h\nu$ has an inertial and a gravitational mass $\frac{h\nu}{c^2}$. It should be expected, therefore, that a quantum $h\nu$ passing a mass M will not only be deflected but it will also transfer momentum and energy to the mass M and make it recoil. During this process, the light quantum will change its energy and, therefore, its frequency. It is hardly possible

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to give a completely satisfactory theory of this gravitational analogue of the Compton effect, without making use of the general theory of relativity. But a rough idea of the nature and the magnitude of the effect may be obtained in the following way.

Suppose a mass m to be traveling on a straight line (x-axis) with a uniform velocity v. The mass M is located at the point $P(x,y)$. If v is sufficiently large, then the actual path will differ only little from the straight line. The force F acting between m and M can then be obtained in the first approximation by assuming that m is traveling along the x-axis with the constant initial velocity v. If m for $t = 0$ at $x = 0$, then

$$F_x = -\frac{\partial\Phi}{\partial x} M \qquad \Phi(x,y,t) = -fm/\sqrt{y^2 + (x - vt)^2}$$

(Φ = gravitational potential at P.)

The x component of the momentum ΔG transferred to M during the time T is

$$\Delta G_x = +\int_0^T F_x dt = -M\int_0^T \frac{\partial\Phi}{\partial x} dt$$
$$= \frac{M}{v}\int_0^T \frac{\partial\Phi}{\partial t} dt = \frac{M}{v}[\Phi(T) - \Phi(0)].$$

If from $t = o$ to $t = T$ the particle has traveled the distance $L = vT$, then

$$\Delta G_x = -\frac{fmM}{v}\left[\frac{1}{\sqrt{(L-x)^2 + y^2}} - \frac{1}{\sqrt{x^2 + y^2}}\right] = \frac{fmM}{v} g_o(x,y).$$

Suppose that matter is distributed all over space with a uniform density ρ and put $M = \rho . 2\pi\, ydydx$, then the x-component of the total momentum lost by m will be

$$G_x = \lim_{D=\infty} \frac{fm}{v}\int_{-D}^{D+L}\int_0^D 2\pi y\rho\, g_o(x,y)\, dx\, dy = 0$$

An exchange of momentum in the x-direction results in this case only if we consider the actually occurring deflection of m from the straight path. This is a second order effect.

In the above calculation, we have assumed that the gravitational interaction is transmitted instantaneously. Let us consider now the case when gravity waves travel with the velocity of light c. Then we have according to the theory of the retarded potentials,

$$\Phi(x,y,t) = -\frac{fm}{r(1 - v_r/c)}\Bigg|_{t'=t-r/c}$$

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where the expression on the right has to be taken at the time of emission t' of an action reaching P at the time t. For the distance r of m and M at t' we have,

$$r^2 = y^2 + [x - v(t - r/c)]^2$$

and

$$v_r = [x - v(t - r/c)]v/r.$$

We notice again

$$\frac{\partial \Phi}{\partial x} = -\frac{1}{v}\frac{\partial \Phi}{\partial t}$$

and

$$\Delta G_x = -M\int_{t_0}^{t_T}\frac{\partial \Phi}{\partial x}\,dt = \frac{M}{v}\int_{t_0}^{t_T}\frac{\partial \Phi}{\partial t}\,dt = \frac{M}{v}[\Phi(t_T) - \Phi(t_o)].$$

The disturbance caused by the motion of m from $x = 0$, $t = 0$ to $t = T$, $x = L = vT$ is acting on M from $t_o = \sqrt{x^2 + y^2/c}$ to $t_T = T + 1/c\sqrt{(L - x)^2 + y^2}$. We obtain therefore,

$$\Delta G_x = -\frac{fmM}{v}\left[\frac{1}{\sqrt{(L - x)^2 + y^2} + (L - x)v/c} - \frac{1}{\sqrt{x^2 + y^2} - xv/c}\right].$$

Developing for $v/c \ll 1$

$$\Delta G_x = \frac{fmM}{v}[g_o(x,y) + g_1(x,y)\,v/c + \dots]$$

$g_o(x,y)$ is the same function as above

$$g_1(x,y) = \frac{L - x}{(L - x)^2 + y^2} + \frac{x}{x^2 + y^2}.$$

Assuming again a uniform distribution of matter in space, we put $M = 2\pi\, ydydx, \rho$. The integration over g_o gives zero as before and (for $D \gg L$)

$$2\pi\int_{-D}^{D+L}\int_0^D g_1(x,y)ydxdy = 2\pi lg2LD.$$

ΔG_x corresponding to the matter in a region $-D < x < D + L$ and $0 < y < D$ is therefore

$$\Delta G_x = 1.4\pi\, fm\rho LD/c.$$

In regard to D, it must be remarked that it should be as large as the dimension of the space over which masses are distributed, if those masses are regarded as independent from each other. But the masses are in reality coupled by gravitational forces and the effect of an external perturbation upon them must be computed by considering the system of the far distant masses as a whole. The correct theory will probably have to

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be worked out in terms of absorption of gravitational waves. But I think it may be safely assumed that the distance D in which the perturbing effect of the moving mass m begins to fade out is very large compared with the mutual distances of the single masses M in which matter is essentially concentrated.

Going over to the case of light, we have $v = c$ and $m = h\nu/c^2$. We conclude by analogy that a relation of the above type still is valid, especially as it can be derived by simply using dimensional reasoning. Light traveling a distance L then would lose the momentum

$$\Delta\left(\frac{h\nu}{c}\right) = \frac{1.4\pi f\rho DL}{c} \times \frac{h\nu}{c^2} \text{ and } \frac{\Delta\nu}{\nu} = 1.4\pi f\rho DL/c^2.$$

Let us compare this result with the observations.

For the total space investigated, the possible limits for ρ are according to E. Hubble 10^{-26} gr./cm.$^3 > \rho > 10^{-31}$. The mutual distance l of the galactic systems being of the order $l = 10^6$ parsecs, we may assume D for instance of the order $1000\ l = 3 \times 10^{27}$ cm. Then $\Delta\nu/\nu$ for $L = 10^6$ parsecs according to our formula will be in the limits $3 \times 10^{-2} > \Delta\nu/\nu > 3 \times 10^{-7}$. From Hubble's linear relation, we have $\Delta\nu/\nu \sim 1/600$ for the same L. In view of this agreement in order of magnitude, a further elaboration of the theory seems to be worthwhile.

Applying the above theory to globular clusters of our own galaxy it would be essential to take into account the actual mass distribution. We will, however, obtain an estimate of the order of magnitude by taking $L = 15000$ parsecs, $D = 1000\ l$ with $l = 1$ parsec for the mutual average distance of the stars. The limits for the density are 10^{-20} gr./cm.$^3 > \rho_g > 10^{-24}$ gr./cm. The redshift therefore would be $4.2 \times 10^{-4} > \frac{\Delta\nu}{\nu} > 4.2 \times 10^{-8}$.

Dr. ten Bruggencate has, in fact, been able to establish a relation between the redshift and the distribution of matter in space. He finds $\frac{\Delta\nu}{\nu} \sim 1/1000$ for light traveling through a distance of 15,000 parsecs in the galactic plane. It would be very important to measure the radial velocities of as many globular clusters as possible in order to decide definitely between the different theories. It is especially desirable to determine the redshift independent of the proper velocities of the objects observed. This might, for instance, be done with help of the steady calcium lines. It is easy to see that the above redshift should broaden these absorption lines asymmetrically toward the red. If these lines can be photographed with a high enough dispersion, the displacement of the center of gravity of the line will give the redshift independent of the velocity of the system from which the light is emitted.

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The explanation of the apparent velocity of recession of distant nebulae proposed in this paper is in qualitative accordance with all of the observational facts known so far. It is therefore desirable, in the first place, to place the computations on a sound theoretical basis involving the general theory of relativity. In the second place, the transfer of momentum from the light to the surrounding masses should be determined taking into account all of the mutual gravitational interactions. Thirdly, it is evident that the proper motions of these masses will play some rôle. Shifts of the spectral lines to the violet should indeed be expected for thermodynamic reasons if light is traveling through systems of masses with very high average velocities. Finally, it might be interesting to study the gravitational drag exerted by light upon light.

I wish to thank Dr. ten Bruggencate who kindly set out on the difficult task of testing some of the suggestions presented in this paper.

[1] E. Hubble, *Proc. Nat. Acad. Sci.*, 1929, **15**, 168.

[2] I am indebted to Mr. M. Humason and to Dr. E. Hubble for private information.

[3] R. C. Tolman, *Astrophys. J.*, **49**, 245, 1929.

[4] A paper by E. von der Pahlen and E. Freundlich in the *Publikationen des Astrophysikalischen Observatoriums zu Potsdam*, **86**, Bd. 26, Heft 3, should be mentioned in this connection. Fig. 6 on page 44 of this paper also shows a correlation between the radial velocity of globular clusters and the galactic latitude. The authors, however, interpret it as being due to real motions.

RELATIVE INTENSITIES IN NUCLEAR SPIN MULTIPLETS

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Introduction.—The discovery and resolution in certain of the atomic lines of Bi, Cs, and Tl, of a fine structure[1,2,3] much smaller than that ordinarily attributed to the electron spin has led to the notion that the nucleus of an atom may be possessed of a spin moment which is capable of interacting with the outer electrons. The order of magnitude of this interaction is very small, as the magnetic moment associated with the nuclear spin is much smaller than the corresponding moment associated with the electron spin. Very recently a paper has appeared by Hargreaves[4] in which the consequences of this picture are worked out in detail for the particular case of a single electron in the Coulomb field of a nucleus which is also possessed of a spin moment $^1/_2(h/2\pi)$. The method used is that of Pauli with multiple wave functions, the effect of the nuclear spin being regarded as a small perturbation of the multiplet states which are

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