On the Observation of a Time Dilated Past

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Abstract

If the observed time dilation of cosmic events is considered to be real, rather than an effect of the past moving away from an observer in expanding space, an interesting metric form of spacetime and some accompanying coordinate transformations present themselves. These transformations are shown to produce cosmological redshifts by time dilating an electromagnetic wave, increasing its period and decreasing its frequency. The relevant equations for distance and lookback time are shown to be analogous to those of the FLRW metric.

1 Introduction

It has been observed that cosmic events, such as distant supernovae, are time dilated.[1] That means the duration of an event that occurred in the distant past will be seen today as longer than the duration of the event had it been observed locally in "real time." The time dilation of cosmic events is understood as a consequence of the expansion of space, which itself is evidenced by, among other things, cosmological redshift.

This work proposes an alternative to the expanding universe called the time dilated past (TDP) hypothesis:

- **Hypothesis:** *the past is time dilated*

The hypothesis may be expressed as the coordinate transformation from the familiar 4-D Cartesian coordinates in Minkowski spacetime (t', x', y', z') to new coordinates (t, x, y, z) called TDP coordinates. The metric form of TDP coordinates can be expressed by the line element:

$$ds^2 = -(e^{H_0 t})^2 c^2 dt^2 + dx^2 + dy^2 + dz^2 \tag{1}$$

where e is Euler's number, and H_0 is Hubble's constant.

The transformation of the TDP t coordinate to the Minkowski t' coordinate is:

$$t' = H_0^{-1}(e^{H_0 t} - 1) \tag{2}$$

The transformation to the TDP t coordinate from the Minkowski t' coordinate is:

$$t = H_0^{-1} log(H_0 t' + 1) \tag{3}$$

And a change in time in Minkowski coordinates $\Delta t'$ is transformed to a change in time in TDP coordinates Δt (for $|\Delta t| \ll 100$ million years) as:

$$\Delta t' = e^{H_0 t} \Delta t \tag{4}$$

Examples of how to use these equations are provided in the sections below. The hypothesis is compared to observational data, followed by a discussion on its cosmological implications.

2 Time dilation and redshift

Consider a stationary metronome in Minkowski spacetime (always denoted by the prime $'$ symbol) that began clicking 7 billion years (Gy) in the past, and it clicks once a second. This produces a series of events starting at $t'_{start} = -7\ Gy$, and repeating every $\Delta t' = 1\ s$, so:

- $Event'_0 = (t'_{start} + 0\Delta t', 0, 0, 0)$
- $Event'_1 = (t'_{start} + 1\Delta t', 0, 0, 0)$
- $Event'_n = (t'_{start} + n\Delta t', 0, 0, 0)$

To transform the Minkowski t'_{start} coordinate to a TDP t_{start} coordinate, plug t'_{start} into equation (3), which gives:

$$t_{start} = H_0^{-1} ln(H_0(-7\ Gy) + 1) \approx -10\ Gy \tag{5}$$

In TDP coordinates, the metronome starts at $t_{start} \approx -10\ Gy$, which is $\approx 3\ Gy$ further in the past than in the Minkowski coordinates.

Next plug $\Delta t'$ and t_{start} into equation (4) and rearrange to get:

$$\Delta t = \frac{1\ s}{e^{H_0(-10\ Gy)}} = 2\ s \tag{6}$$

The time between metronome clicks after the TDP transformation is now double. This means the frequency of clicks has fallen from 1 Hz to 0.5 Hz. The metronome has slowed down.

If the clicks of the metronome are taken as the ticks of a clock, it could be said that the clock runs slower in the past than in the present, and that events in the past appear to take longer to unfold.

If the events are considered to be the oscillations of an electromagnetic wave, then that electromagnetic wave's frequency would be reduced by the transformation to TDP coordinates, which is measured as redshift.

Cosmic time dilation and redshift are both observed phenomena, and they are often considered to be separate consequences of the expansion of space. According to the time dilated past hypothesis, an electromagnetic wave in the past is a time dilated electromagnetic wave, which means it is a redshifted electromagnetic wave. The two phenomena are fundamentally the same and occur without expanding space.

3 The present rest frame

Consider the scenario where Alice observes a supernova in her galaxy, and it takes 14 days for the supernova's brightness to fall a certain amount. Her spacetime diagram has the events at coordinates (t, x):

$$Event_{Astart} = (0, 0), Event_{Aend} = (14\ days, 0) \tag{7}$$

Bob observes this supernova from his galaxy which is seven billion light years (Gly) away. He notes that the time it takes for the supernova to fall to the same level is 28 days. He knows the event took place at some time in the past t_{past} and some place in the distance d, so his spacetime diagram has the coordinates:

$$Event_{Bstart} = (t_{past}, d), Event_{Bend} = (28\ days, d) \tag{8}$$

Bob and Alice should agree on the time of the events, if we assume they're at rest with respect to each other and the coordinates. But they don't agree.

In the expanding universe, this isn't a problem, because they actually are moving away from each other. The distance between them will have increased between the start and end of the supernova. The size of the universe itself will have doubled in between the time of Alice's observation and Bob's observation.

According to the TDP hypothesis, the observers are at rest spatially, but they are not moving through time equally. Put another way, the passage of time has caused their frames of reference to differ. This can be rectified in three steps:

Step 1. Convert events to Minkowski coordinates

Use equation (2) to transform Alice's events to Minkowski coordinates (t', x'). They don't actually change, because they're so close to the origin, but it's important to realize this step is here.

Step 2. Transport Alice to the "present rest frame"

Alice's present time, $t' = 0$, is far in the past compared to Bob's present time. To even things out, transport Alice to the future, so they are both at rest and both in the present. In Minkowski coordinates, the light will take 7 Gy to travel 7 Gly, so move Alice's present time to 7 Gy in the future. That changes Alice's coordinates to:

$$Event'_{Astart} = (-7\ Gy, 0), Event'_{Aend} = (-7\ Gy + 14\ days, 0) \tag{9}$$

Step 3. Reverse the transformation to Minkowski coordinates

$$Event_{Astart} = (-10\ Gy, 0), Event_{Aend} = (-10\ Gy + 28\ days, 0) \tag{10}$$

This means in Bob's spacetime diagram $t_{past} = -10\ Gy$ and both observers agree on the timing of events.

In TDP coordinates, the measurements observers make will agree if the observers are at rest in the coordinate system, and if the measurements are all made at the same time, $t = 0$. As keeping track of the value of t for a frame of reference in TDP coordinates is important, the term "present rest frame" can be used in these circumstances.

4 Compared to FLRW

4.1 The metric forms

The expansion of space, which is of course the commonly accepted explanation of cosmic redshift and time dilation, is described by the Friedmann-Lemaître-Robertson-Walker (FLRW) metric, which can be expressed generally in Cartesian coordinates by the line element:

$$ds^2 = -c^2dt^2 + a(t)^2(dx^2 + dy^2 + dz^2) \tag{11}$$

In the FLRW metric, the spatial dimensions have the coefficient $a(t)^2$ where $a(t)$ is known as the scale factor. The TDP metric (equation [1]) doesn't have this scale factor, but it does have the time dilation factor, $e^{H_0 t}$, as a coefficient for the time dimension, which serves a similar purpose.

Consider a spacetime diagram in two dimensions (t, x), where the axes are elastic bands with markings on them. An electromagnetic (EM) wave is placed along the null geodesic $ct = x$. When the elastic bands are stretched, the markings stretch too, and the peaks of the EM wave move along with them.

When the x-axis is stretched, the wave is stretched in space, increasing its wavelength, and causing redshift. When the t-axis is stretched, the wave is stretched in time, increasing its period, decreasing its frequency, and causing redshift.

Notice that when one property of the wave changes (e.g., wavelength) the other property (e.g., frequency) does not. It could be argued that since the frequency isn't changing in FLRW coordinates, and the wavelength isn't changing in TDP coordinates, that no redshifts are occuring. The decision should be intentionally made to determine redshifts based on the property that is changing.

4.2 Coordinate speed of light

In FLRW coordinates (more commonly referred to as comoving coordinates) it was noted that the wavelength of an EM wave increases during expansion, but the frequency stays the same. This results in a wave with a velocity greater than c, since $c = f_{emit}\lambda_{emit}$.

The coordinate speed of light in FLRW comoving coordinates is $c(1 + z)$. Let's say $c(1 + z)$ is equal to some unknown velocity, v, which is greater than c if $z > 0$.

$$c(1+z) = v \tag{12}$$

$$1+z = \frac{v}{c} \tag{13}$$

$$\frac{\lambda_{obs}}{\lambda_{emit}} = \frac{v}{c} \tag{14}$$

$$\frac{\lambda_{obs}}{\lambda_{emit}} = \frac{v}{f_{emit}\lambda_{emit}} \tag{15}$$

$$\lambda_{obs} = \frac{v}{f_{emit}} \tag{16}$$

We can determine the coordinate speed of light in TDP coordinates by working backwards. We know that stretching the time dimension will increase the wave's period, and lower its frequency. This will result in a wave with a velocity v:

$$f_{obs}\lambda_{emit} = v \tag{17}$$

$$f_{obs} = \frac{v}{\lambda_{emit}} \tag{18}$$

If we divide both sides by f_{emit} we get:

$$\frac{f_{obs}}{f_{emit}} = \frac{v}{f_{emit}\lambda_{emit}} \tag{19}$$

$$\frac{1}{1+z} = \frac{v}{c} \tag{20}$$

$$v = c(1+z)^{-1} \tag{21}$$

This tells us the TDP coordinate speed of light is $c(1+z)^{-1}$, which is a sensible result, given that stretching space or time causes redshifts, time has an inverse relationship with frequency, and the FLRW coordinate speed of light is $c(1+z)$.

4.3 Scale factor vs time dilation factor

According to FLRW, the size of the universe and distances in it change over time by the scale factor. Expansion means a distance $d(t)$ at some time in the past t will be smaller than that distance is now, d_0, and can be related by the scale factor:

$$d(t) = a(t)d_0 \tag{22}$$

If we consider $d(t)$ to be a wavelength in the past, λ_{emit}, and d_0 to be the same wavelength now, λ_{obs}, then:

$$a(t) = \frac{\lambda_{emit}}{\lambda_{obs}} \tag{23}$$

$$a(t) = \frac{1}{1+z} \tag{24}$$

A similar relationship to redshift and the time dilation factor can be found in TDP coordinates. Consider two events in TDP coordinates (t, x), that happen near the origin, one second apart:

$$Event_0 = (-1, 0), Event_1 = (0, 0) \tag{25}$$

This is what the events will look like to an observer nearby watching them in real-time, so let's call the time between them $\Delta t_{emit} = 1\ s$.

We can transform to Minkowski coordinates (t', x') by using equation (4) to get:

$$Event'_0 = (-1, 0), Event'_1 = (0, 0) \tag{26}$$

The coordinates are the same because the time dilation factor at $t = 0$ is $e^{H_0 t} = 1$.

Now, let's go 7 Gy into the future, and make that the new present $t = 0$. So our events are now:

$$Event'_0 = (-7\ Gy - 1\ s, 0), Event'_1 = (-7\ Gy, 0) \tag{27}$$

And apply the transformation to TDP coordinates on these, using equations (3) and (4):

$$Event'_0 = (-10\ Gy - 2\ s, 0), Event'_1 = (-10\ Gy, 0) \tag{28}$$

These are the coordinates when the present rest frame is 7 Gy in the future from the time the events occurred. The events will appear this way to a future observer, so let's say $\Delta t_{obs} = 2s$. So, from equation (4):

$$e^{H_0 t} = \frac{\Delta t_{emit}}{\Delta t_{obs}} \tag{29}$$

Since $f = 1/t$ then:

$$e^{H_0 t} = \frac{f_{obs}}{f_{emit}} \tag{30}$$

$$e^{H_0 t} = \frac{1}{1+z} \tag{31}$$

So the scale factor and the time dilation factor both equal $(1+z)^{-1}$. But the scale factor $a(t)$ only equals $e^{H_0 t}$ in special circumstances.

4.4 FLRW $\Omega_\Lambda = 1$, $\Omega_M = \Omega_r = 0$

The FLRW model obtained using the parameters $\Omega_\Lambda = 1$ and $\Omega_M = \Omega_r = 0$ is a special case of FLRW models that does not follow expansion back to a big bang singularity, and thus does not predict an age of the universe, or have a limit to the lookback times that can be calculated. The Hubble parameter in this model never changes, and the scale factor is purely driven by dark energy. The result is that $a(t) = e^{H_0 t}$.

In this case, the FLRW scale factor does equal the TDP time dilation factor, which makes the models analogous in some respects, such as redshift relationships to lookback times and distances. This will be the model referred to by the term "FLRW $\Omega_\Lambda = 1$."

4.5 Lookback Time

The lookback time t_L in the FLRW metric is:[2]

$$t_L = \frac{1}{H_0} \int_0^z \frac{dz'}{(1+z')E(z')} \tag{32}$$

Where:

$$E(z) \equiv [\Omega_M (1+z)^3 + \Omega_\Lambda + \Omega_r (1+z)^4]^{1/2} \tag{33}$$

Using the case where $\Omega_\Lambda = 1$ and $\Omega_M = \Omega_r = 0$, makes $E(z) = 1$, equation (32) can be solved as:

$$t_L = \frac{1}{H_0} log(1+z) \tag{34}$$

In the TDP metric, the lookback time is given by $-t$, because $t < 0$ for values in the past. Equation (3) rearranged tells us:

$$1 + H_0 t' = e^{H_0 t} \tag{35}$$

$$1 + H_0 t' = \frac{1}{1+z} \tag{36}$$

$$H_0 t' = \frac{1}{1+z} - 1 \tag{37}$$

Plugging this into equation (3) gives:

$$t = \frac{1}{H_0} log(\frac{1}{1+z}) \tag{38}$$

$$-t = \frac{1}{H_0} log(1+z) = t_L \tag{39}$$

Showing that the TDP transformation to t gives the same result as the (negative) lookback time, $-t_L$, for FLRW $\Omega_\Lambda = 1$.

4.6 Distances

The present distance to a source of light in FLRW coordinates is called the comoving distance, d_C, and it is greater than the distance of the source when the light was emitted, d_{emit}, due to expanding space. So applying the scale factor gets:

$$d_C = (1+z)d_{emit} \tag{40}$$

Referring back to equation (13) regarding the speed of light in comoving coordinates, the velocity v is unknown, but if $z > 0$ then it's known that $v > c$. If we say $v - c = \Delta v$ then:

$$1 + z = \frac{c + \Delta v}{c} \tag{41}$$

We don't know what v or Δv is exactly, but we know that divided by H_0, the result is some unknown distance.

$$1 + z = \frac{c + H_0 d}{c} \tag{42}$$

$$1 + z = 1 + \frac{H_0 d}{c} \tag{43}$$

$$d = z\frac{c}{H_0} \tag{44}$$

This equation would only be valid for FLRW models where the Hubble parameter is always equal to H_0, which is only the case for FLRW $\Omega_\Lambda = 1$. Using a cosmological calculator to find both d_C and d_{emit} over a range of z for the parameters of FLRW $\Omega_\Lambda = 1$ verifies that d_C does equal equation (44) and d_{emit} equals what you get by combining equations (44) and (40):

$$d_{emit} = \frac{z}{1+z}\frac{c}{H_0} \tag{45}$$

In TDP coordinates, the distances to light sources are static, and they don't change during the transformations, so only one distance equation will be needed. The distance should be ct', so if we multiply both sides of equation (2) by c and substitute $(1+z)^{-1}$ for $e^{H_0 t}$ we get:

$$ct' = (\frac{1}{1+z} - 1)\frac{c}{H_0} \tag{46}$$

$$d = -\frac{z}{1+z}\frac{c}{H_0} \tag{47}$$

Which is the distance for d_{emit} in FLRW $\Omega_\Lambda = 1$, but with a minus sign, which doesn't affect anything.

We could also go back to equation (20), the coordinate speed of light in TDP coordinates, and say the same thing as before about v, except now that the frequency is reduced, $v < c$, and $c - v = \Delta v$, and $\Delta v = H_0 d$, where d is some unknown distance.

$$\frac{1}{1+z} = \frac{c - H_0 d}{c} \tag{48}$$

$$d = \frac{z}{1+z}\frac{c}{H_0} \tag{49}$$

It may be of interest that equation (47) can be attained by first order approximation. It is only traditionally and somewhat arbitrarily that redshift is defined by a change in wavelength over the original. Had redshift been defined as a change in frequency over the original, it could be:

$$1 + b = \frac{1}{1+z} = \frac{f_{obs}}{f_{emit}} \tag{50}$$

$$b = \frac{1}{1+z} - 1 \tag{51}$$

In this quantification, the range of redshifts is $-1 < b < 0$. Approximating distance as $d \approx bcH_0^{-1}$ instead of $d \approx zcH_0^{-1}$ as is traditionally done, combined with equation (51) gives equation (47), including the minus sign.

5 Supernovae data

The Pantheon+SH0ES data[3] has values for z and distance modulus, from which a comoving distance can be calculated. But there is no comoving distance in the TDP hypothesis to compare it to. Equation (39) however does provide a travel time for light that reflects the effects of time dilation. Multiply both sides by c to get the light travel time distance, d_{ltt}:

$$d_{ltt} = \frac{c}{H_0} log(1+z) \tag{52}$$

Comparing the light travel time distance to the data's comoving distance, using the value of Hubble's constant, $H_0 = 70\ km\ s^{-1}\ Mpc^{-1}$, is shown in Fig. (1).

6 Cosmology

The theory of expanding space is a very successful theory that answers a lot more questions about our universe than just "what are cosmic redshift and time dilation?" An abbreviated list of such answers are:

- the age of the universe
- what it was like at the beginning
- where the light elements come from

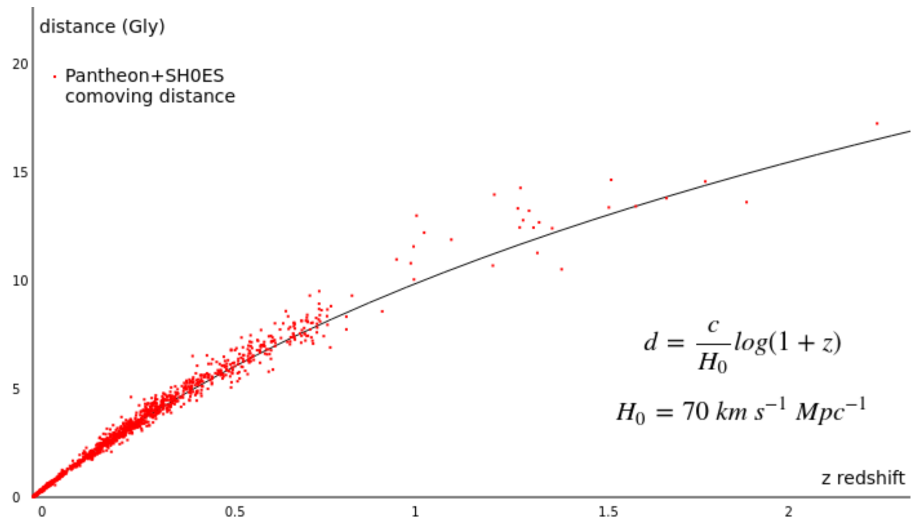

Figure 1: The Pantheon+SH0ES SNe comoving distance against the hypothesis' light travel time distance.

- what the CMB is
- when stars and galaxies began to form

The time dilated past hypothesis, on the other hand, simply describes the observed phenomena of cosmic redshift and time dilation, and there is no obvious connection between it and those questions.

The expanding universe explains why distant clocks are time dilated, and why ancient electromagnetic waves are redshifted. The TDP hypothesis only acknowledges that this happens, and provides some equations to describe the phenomena. Those equations are similar or analogous to the more flexible equations already available from the FLRW metric.

6.1 The CMB

Of the questions the TDP hypothesis fails to address, the most startling omission is that of the CMB. Unless it were discovered that the CMB is something other than relic light from the beginning of the universe, the path to a viable TDP cosmology is blocked.

There are some indications that the CMB may not be what we think it is. The so-called "axis-of-evil" is an apparent correlation between the CMB and our solar system.[4] An anomalous cold spot and a very slight temperature difference between hemispheres have also eluded explanations.[5] The CMB is also observed to have a very high entropy, higher than the universe today, which is called the initial entropy problem.[6]

Although speculative, an alternative explanation of the CMB is a possibility. In the case that it is not actually the cosmic background, the CMB would seem to be unrelated to cosmic time dilation and redshifts.

7 Conclusion

The time dilated past hypothesis is motivated by the observation of a past that is time dilated. The hypothesis can be described by a relativistic metric with many similarities to the FLRW metric, but stretching time instead of space. The stretching of a wave's period by definition reduces its frequency, which is observed as cosmological redshift.

The cosmological implications of the hypothesis, however, leave the most significant and recognizable questions that cosmology sets out to answer unaddressed. Whether such a radical departure from the currently accepted cosmological model is necessary remains to be seen.

References

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