Math: Big-O/Θ/Ω Notation

Definition 1. Let f, g be functions from $\mathbb{R}^+ \to \mathbb{R}^+$. Then,

1. If $\lim\limits_{x \to \infty} \dfrac{f(x)}{g(x)} < \infty$, then we say $f = O(g)$.

2. If $\lim\limits_{x \to \infty} \dfrac{f(x)}{g(x)} > 0$, then we say $f = \Omega(g)$.

3. We say that $f = \Theta(g)$ if both $f = O(g)$ and $f = \Omega(g)$.

Intuitively, you should think of O as $\leq$, Ω as $\geq$, and Θ as $=$.

Example 1.

1. $f(x) = x$
   $g(x) = x^2$

2. $f(x) = x^2$
   $g(x) = x$

3. $f(x) = x^2 + 2x + 5$
   $g(x) = x$
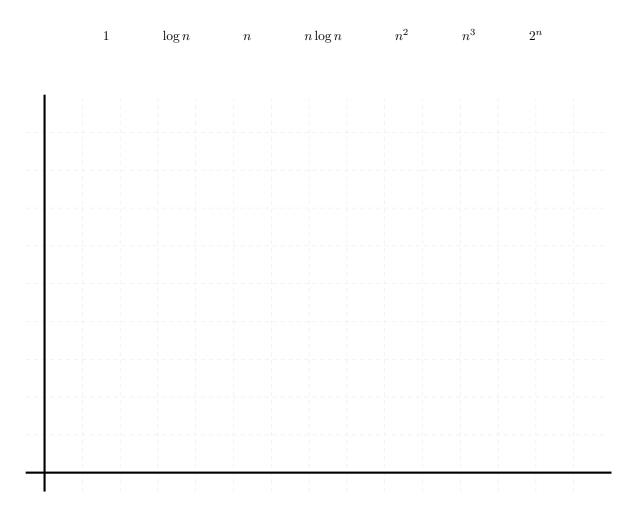
4. $f(x) = x^2 + 2x + 5$
   $g(x) = x^2$

5. $f(x) = x^2 + 2x + 5$
   $g(x) = x^3$

Example 2. You should memorize the relationship between the following functions:

$$1 \qquad \log n \qquad n \qquad n \log n \qquad n^2 \qquad n^3 \qquad 2^n$$

Example 3. Complete each equation below by adding the symbol O if $f = O(g)$, Ω if $f = \Omega(g)$, or Θ if $f = \Theta(g)$. The first row is completed for you as an example.

f(n)		g(n)
1	$=$	$O(n)$
$3n \log n$	$=$	n^2
1	$=$	$1/n$
$\log_2 n$	$=$	$\log_3 n$
$\log n$	$=$	$\frac{1}{\log n}$
$5 \cdot 10^{30}$	$=$	$\log n$
$\log n$	$=$	$\log(n^2)$
2^n	$=$	3^n
$\frac{1}{n}$	$=$	$\sqrt{\frac{1}{n}}$
$\log n$	$=$	$(\log n)^2$

Example 4. Simplify the following expressions:

1. $O\left(8n^3 + n^2\right)$

2. $O\left((7n)^3 + 5n^2 \log n + \log n\right)$

3. $O\left(100000000000\right)$

4. $O\left(\log n + 100000000000\right)$

5. $O\left(\frac{1}{n} + \frac{1}{n^2}\right)$