ML_HW2_mj2276

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1)

1a)

$$\sum_{i=1}^{n} ln\, p(y_i|\pi)$$

$$= \sum_{i=1}^{n} ln\, \pi^{y_i}\, (1-\pi)^{1-y_i}$$

$$= \sum_{i=1}^{n} y_i\, ln\, \pi + (1-y_i)\, ln(1-\pi)$$

$$\frac{\partial}{\partial \pi}(\sum_{i=1}^{n} y_i\, ln\, \pi + (1-y_i)\, ln(1-\pi)) = 0$$

$$0 = \frac{\sum_{i=1}^{n} y_i}{\pi} - \frac{\sum_{i=1}^{n}(1-y_i)}{1-\pi}$$

$$\frac{\sum_{i=1}^{n} y_i}{\pi} = \frac{\sum_{i=1}^{n}(1-y_i)}{1-\pi}$$

$$(1-\pi)\sum_{i=1}^{n} y_i = \pi \sum_{i=1}^{n}(1-y_i)$$

$$(1-\pi)\sum_{i=1}^{n} y_i = \pi\,(n - \sum_{i=1}^{n} y_i)$$

$$\sum_{i=1}^{n} y_i - \pi \sum_{i=1}^{n} y_i = \pi\, n - \pi \sum_{i=1}^{n} y_i$$

$$\sum_{i=1}^{n} y_i = \pi\, n$$

$$\hat{\pi} = \frac{\sum_{i=1}^{n} y_i}{n}$$

1b)

$$\sum_{i=1}^{n} ln\, p(x_{i,1}|\theta_y^1)$$

$$= \sum_{i=1}^{n} ln\, (\theta_y^1)^{x_{i,1}}\, (1-\theta_y^1)^{1-x_{i,1}}$$

$$= \sum_{i=1}^{n} x_{i,1}\, ln\, \theta_y^1 + (1-x_{i,1})\, ln(1-\theta_y^1)$$

$$\frac{\partial}{\partial \theta_y^1}(\sum_{i=1}^{n} x_{i,1}\, ln\, \theta_y^1 + (1-x_{i,1})\, ln(1-\theta_{y_i}^1)) = 0$$

$$0 = \frac{\sum_{i=1}^{n} x_{i1}}{\theta_y^1} - \frac{\sum_{i=1}^{n}(1-x_{i,1})}{1-\theta_y^1}$$

$$\frac{\sum_{i=1}^{n} x_{i1}}{\theta_y^1} = \frac{\sum_{i=1}^{n}(1-x_{i,1})}{1-\theta_y^1}$$

$$(1-\theta_y^1)\sum_{i=1}^{n} x_{i,1} = \theta_y^1 \sum_{i=1}^{n}(1-x_{i,1})$$

$$(1-\theta_y^1)\sum_{i=1}^{n} x_{i,1} = \theta_y^1\,(n - \sum_{i=1}^{n} x_{i,1})v \sum_{i=1}^{n} x_{i,1} - \theta_y^1 \sum_{i=1}^{n} x_{i,1} = \theta_y^1\, n - \theta_y^1 \sum_{i=1}^{n} x_{i,1}$$

$$\sum_{i=1}^{n} x_{i,1} = \theta_y^1\, n$$

$$\hat{\theta_y^1} = \frac{\sum_{i=1}^{n} x_{i,1}}{n}$$

$$=> \sum_{d=0}^{1}(\hat{\theta_{y_d}^1} = \frac{\sum_{i=1}^{n} x_{i,1}}{n})$$

1c)

$$\sum_{i=1}^{n} ln\, p(x_{i,2}|\theta_y^2)$$

$$= \sum_{i=1}^{n} ln\, \theta_y^2 (x_{i,2})^{-(\theta_y^2+1)}$$

$$= ln\, \theta_y^2 + \sum_{i=1}^{n} ln(x_{i,2})^{-(\theta_y^2+1)}$$

$$= ln\, \theta_y^2 - (\theta_y^2+1)\, \sum_{i=1}^{n} ln\,(x_{i,2})$$

$$\frac{\partial}{\partial \theta_y^2}(ln\, \theta_y^2 - (\theta_y^2+1)\, \sum_{i=1}^{n} ln\,(x_{i,2})) = 0$$

$$0 = \frac{n}{\theta_y^2} - \sum_{i=1}^{n} ln(x_{i,2})$$

$$\frac{n}{\theta_y^2} = \sum_{i=1}^{n} ln(x_{i,2})$$

$$n = \sum_{i=1}^{n} ln(x_{i,2})\theta_y^2$$

$$\hat{\theta_y^2} = \frac{n}{\sum_{i=1}^{n} ln(x_{i,2})}$$

$$=> \sum_{d=0}^{1}(\hat{\theta_{y_d}^2} = \frac{n}{\sum_{i=1}^{n} ln(x_{i,2})})$$

2)

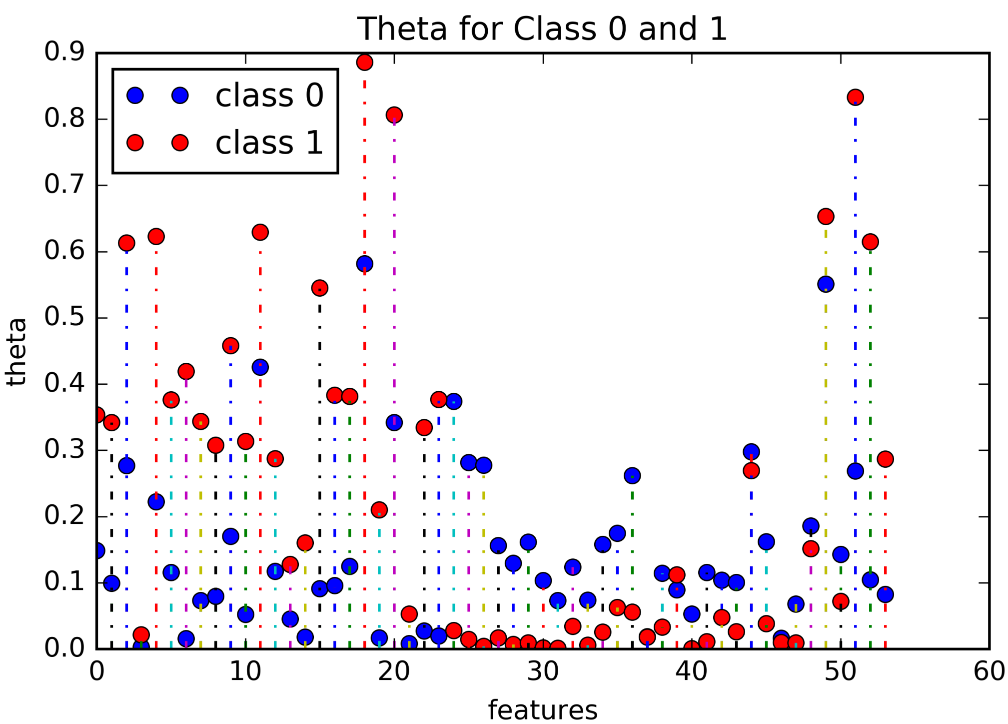
2a)

		y'	
		0	1
y	0	54	2
	1	5	32

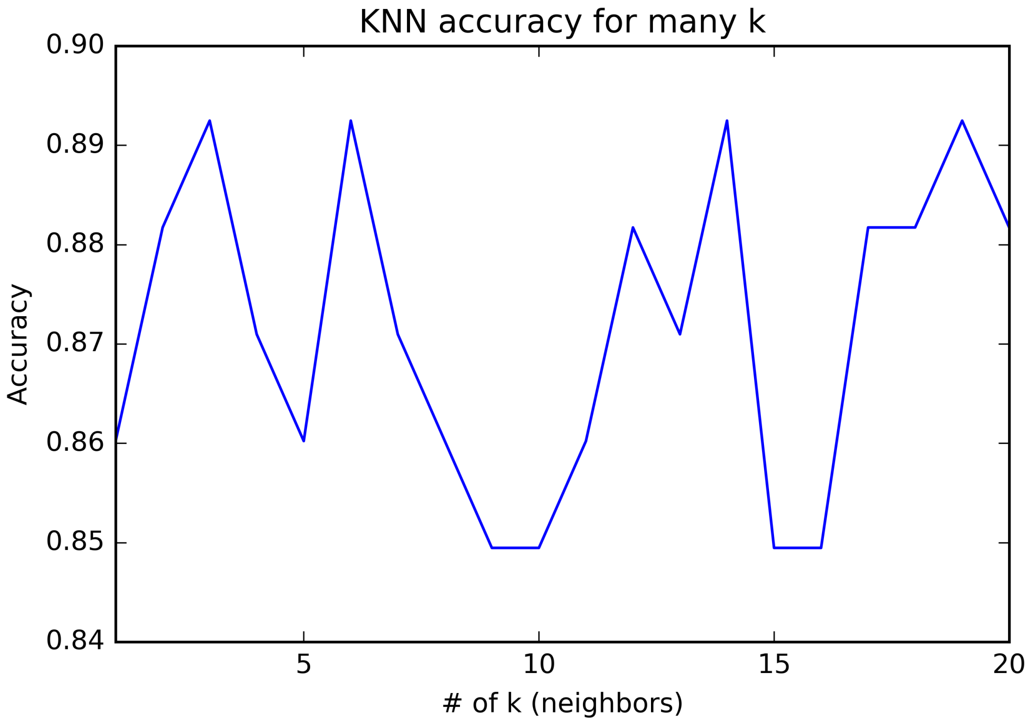
Accuracy = 92.5%

2b)

For the 16th feature, the word “free”, and the 52nd feature, the “!”, both have a higher probability to be in spam email than not. For the word free it has a probablilty of about 12% to be in a non spam email and about 39% to be in a spam email. For the “!” there is a probability of about 25% to be in a non spam email and about 84% to be in a spam email.



2c)



KNN accuracy for many k
0.90
0.89
0.88
0.87
0.86
0.85
0.84
Accuracy
5
10
15
20
# of k (neighbors)

Figure 1: