Renormalization of Soft Symmetry Improved 2PIEA gap equations in the Hartree-Fock approximation

Supplement to thesis Chapter 5 "Soft Symmetry Improvement"

Mathematica notebook to compute couter-terms for the Hartree-Fock truncation of the SSI-2PIEA

Author: Michael Brown

Hartree-Fock

In[41]:=
```
ClearAll[veom, geom, neom, regularisedtadpoles, mg2soln, mn2soln,
  cteq, cteq2, cteqs, ctsolns, cts, ctegs3, rnveom, veomCtEqs, δm, δλ];
```

Hartree-Fock gap equations with counterterms

Goldstone equation of motion. Quantities in reference to the paper are:
p is the four-momentum flowing through the propagators $\Delta_G{}^{-1}$ and $\Delta_N{}^{-1}$,
mg2 is the Goldstone mass squared $m_G{}^2$,
mn2 is the Higgs mass squared $m_H{}^2$,
Z and ZΔ are the wavefunction a propagator renormalization constants,
m^2 is the (renormalized) Lagrangian mass parameter, $\delta m_0{}^2$, $\delta m_1{}^2$ are its counter-terms,
λ is the (renormalized) four point coupling,
$\delta\lambda_0$, $\delta\lambda_{1\,a}$, $\delta\lambda_{1\,b}$, $\delta\lambda_{2\,a}$, $\delta\lambda_{2\,b}$ are the independent coupling counter-terms,
v is the scalar field vacuum expectation value,
$\hbar$ is the reduced Planck constant,
n is the number of fields in the O(n) symmetry group,
ξ is the stiffness parameter,
ϵ is the solution of the Goldstone zero mode equation,
ssi = $\frac{1}{V\beta m_G{}^2}\left(\frac{1}{\epsilon}-1\right)$ is the soft symmetry improvement term in the propagator eoms,
ssi2 = $\frac{1}{\xi}(n-1)\,2\left(m_G{}^2\,\epsilon\right)^2$ is the other soft symmetry improvement term in the vev eom,
t∞g, t∞n are the divergent tadpole integrals for the Goldstone, Higgs resp.,
tfing, tfinn are the finite parts of the tadpoles for the Goldstone, Higgs resp.

Vev equation of motion

In[42]:=
```
veom = ZΔ^-1 (m^2 + δm0^2) v + (λ + δλ0)/6 v^3 + ℏ/6 ZΔ (n - 1) (λ + δλ1a) v (t∞g + tfing + ssi) +
  ℏ/6 ZΔ (3 λ + δλ1a + 2 δλ1b) v (t∞n + tfinn) + v ssi2
```

Goldstone equation of motion

In[43]:=
```
geom = p^2 - mg2 == Z ZΔ p^2 - m^2 - δm1^2 - ZΔ (λ + δλ1a)/6 v^2 -
  ℏ/6 ((n + 1) λ + (n - 1) δλ2a + 2 δλ2b) ZΔ^2 (t∞g + tfing + ssi) - ℏ/6 (λ + δλ2a) ZΔ^2 (t∞n + tfinn)
```

Higgs equation of motion

In[44]:=
```
neom = p^2 - mn2 == Z ZΔ p^2 - m^2 - δm1^2 - ZΔ v^2 (3 λ + δλ1a + 2 δλ1b)/6 -
  ℏ/6 (λ + δλ2a) (n - 1) ZΔ^2 (t∞g + tfing + ssi) - ℏ/6 (3 λ + δλ2a + 2 δλ2b) ZΔ^2 (t∞n + tfinn)
```

Infinite parts of tadpoles in MSbar

MSbar rules for 4 - 2 ϵ dimensions

In[45]:=
```
regularisedtadpoles = {t∞g → c0 Λ^2 + c1 mg2 Log[Λ^2 / μ^2], t∞n → c0 Λ^2 + c1 mn2 Log[Λ^2 / μ^2]}
```

Sub in tadpole expressions, eliminate mn2 and solve for mg2

In[46]:=
```
mg2soln = mg2 /. (geom /. regularisedtadpoles /.
      Solve[neom /. regularisedtadpoles, mn2][[1]] // Solve[#, mg2][[1]] &)
```

In[47]:=
```
mn2soln = mn2 /. (neom /. regularisedtadpoles /. mg2 → mg2soln // Solve[#, mn2][[1]] &)
```

Gather divergences proportional v, tfing and tfinn and set independently to zero

First we subtract the finite equation of motion, then gather coefficients of the remainder into a list and set each to zero (after some trimming and simplifying).

In[48]:=
```
cteq =
  ((CoefficientList[mg2soln + (-m^2 - λ/6 v^2 - ℏ/6 ((n + 1) λ) (tfing + ssi) - ℏ/6 (λ) (tfinn)),
        {p, v, tfing, tfinn}] // Flatten) //
     DeleteDuplicates // Simplify // FullSimplify) == 0 // Thread
```

```
In[49]:= cteq2 =
  ((CoefficientList[mn2soln + (-m^2 - λ/2 v^2 - ℏ/6 ((n - 1) λ) (tfing + ssi) - ℏ/2 (λ) (tfinn)),
         {p, v, tfing, tfinn}] // Flatten) //
       DeleteDuplicates // Simplify // FullSimplify) == 0 // Thread
```

Solve for counterterms

```
In[50]:= cteqs = {cteq, cteq2} // Flatten // FullSimplify // DeleteDuplicates

In[51]:= ctsolns =
   Solve[cteqs, {δm₁, δλ₁ₐ, δλ₂ₐ, δλ₁ᵦ, δλ₂ᵦ, Z, ZΔ}] // FullSimplify // DeleteDuplicates

In[52]:= cts = {δm₁^2, δλ₁ₐ, δλ₂ₐ, δλ₁ᵦ, δλ₂ᵦ, Z, ZΔ} /. ctsolns // FullSimplify // DeleteDuplicates
```

ZΔ is redundant in this truncation, can remove it :

```
In[53]:= cts /. ZΔ → 1 // FullSimplify

In[54]:= mg2soln /. ctsolns /. ZΔ → 1 // FullSimplify // DeleteDuplicates

In[55]:= mn2 /.
    ((neom /. regularisedtadpoles /. mg2 → mg2soln /. ctsolns /. ZΔ → 1 // FullSimplify //
        DeleteDuplicates) // Solve[#, mn2] &) // FullSimplify

In[56]:= rnveom =
   veom /. {mg2 → m^2 + λ/6 v^2 + ℏ/6 ((n + 1) λ) (tfing + ssi) + ℏ/6 (λ) (tfinn), mn2 → m^2 + λ/2 v^2 +
        ℏ/6 ((n - 1) λ) (tfing + ssi) + ℏ/2 (λ) (tfinn)} // Simplify // DeleteDuplicates

In[57]:= veomCtEqs =
  (((CoefficientList[(1/v rnveom - (m^2 + λ/6 v^2 + ℏ/6 ((n - 1) λ) (tfing + ssi) + ℏ/2 (λ) (tfinn) +
                ssi2)) /. regularisedtadpoles /.
             {mg2 → m^2 + λ/6 v^2 + ℏ/6 ((n + 1) λ) (tfing + ssi) + ℏ/6 (λ) (tfinn),
              mn2 → m^2 + λ/2 v^2 + ℏ/6 ((n - 1) λ) (tfing + ssi) + ℏ/2 (λ) (tfinn)} //
            Simplify // Expand // FullSimplify, {v, tfing, tfinn}] //
         Simplify // Flatten) // DeleteDuplicates // Simplify //
      FullSimplify // DeleteDuplicates) == 0 // Thread)

In[58]:= ctegs3 = (veomCtEqs /. ctsolns // Simplify // DeleteDuplicates // FullSimplify)[[1]]

In[59]:= {δm₀^2, δλ₀} /. Solve[ctegs3, {δm₀, δλ₀}] /. ZΔ → 1 // DeleteDuplicates // Simplify
```

In[60]:= $\{\delta m_1{}^2 == \delta m_0{}^2, \delta\lambda_{1\,a} == \delta\lambda_{2\,a}, \delta\lambda_{1\,b} == \delta\lambda_{2\,b}\}$ /. ctsolns /. Solve[ctegs3, {δm_0, $\delta\lambda_0$}] /. ZΔ → 1 // FullSimplify // Flatten // DeleteDuplicates

In[61]:= $\left\{\frac{\delta\lambda_{1\,a}}{\delta\lambda_{1\,b}}\right\}$ /. ctsolns /. Solve[ctegs3, {δm_0, $\delta\lambda_0$}] /. ZΔ → 1 // FullSimplify // Flatten // DeleteDuplicates

In[62]:= $\delta\lambda_{1\,b} == \delta\lambda_{2\,b}$ /. ctsolns /. ZΔ → 1 // FullSimplify // DeleteDuplicates

In[63]:= $\delta\lambda_{1\,b}$ /. ctsolns /. ZΔ → 1 // FullSimplify // DeleteDuplicates

In[64]:= $\{\delta\lambda_0 == 1\,\delta\lambda_{1\,a} + 2\,\delta\lambda_{1\,b}\}$ /. ctsolns /. Solve[ctegs3, {δm_0, $\delta\lambda_0$}] /. ZΔ → 1 // FullSimplify // Flatten // DeleteDuplicates

In[65]:= $\left\{\delta m_0{}^2 == -\frac{\left(c0\,\Lambda^2 + c1\,m^2\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\right)\lambda\,\hbar}{3}\left(\frac{\delta\lambda_{1\,a}}{\delta\lambda_{1\,b}} - 1\right)\right\}$ /. ctsolns /. Solve[ctegs3, {δm_0, $\delta\lambda_0$}] /. ZΔ → 1 // FullSimplify // Flatten // DeleteDuplicates