Renormalization of SI-2PIEA gap equations in the Hartree-Fock approximation

Author: Michael Brown
Supplement to thesis Chapter 3

Mathematica notebook to compute couter-terms for two loop truncations of the two particle irreducible effective action

```
ClearAll[veom, geom, neom, divergentpartrules, mg2soln, cteq, cts, δm, δλ];
```

Hartree-Fock gap equations with counterterms

Goldstone equation of motion. Quantities in reference to the thesis are:
p is the four-momentum flowing through the propagators $\Delta_G{}^{-1}$ and $\Delta_N{}^{-1}$,
mg2 is the Goldstone mass squared $m_G{}^2$,
mn2 is the Higgs mass squared $m_H{}^2$,
Z and ZΔ are the wavefunction a propagator renormalization constants,
m^2 is the (renormalized) Lagrangian mass parameter, $\delta m_0{}^2$, $\delta m_1{}^2$ are its counter-terms,
λ is the (renormalized) four point coupling,
$\delta\lambda_0$, $\delta\lambda_{1a}$, $\delta\lambda_{1b}$, $\delta\lambda_{2a}$, $\delta\lambda_{2b}$ are the independent coupling counter-terms,
v is the scalar field vacuum expectation value,
$\hbar$ is the reduced Planck constant,
n is the number of fields in the O(n) symmetry group,
t∞g, t∞n are the divergent tadpole integrals for the Goldstone, Higgs resp.,
tfing, tfinn are the finite parts of the tadpoles for the Goldstone, Higgs resp.

Equations of motion

Vev equation of motion

(*veom=
$$Z\Delta^{-1}\left(m^2+\delta m_0{}^2\right)v+\frac{\lambda+\delta\lambda_0}{6}v^3+\frac{\hbar}{6}Z\Delta\left(n-1\right)(\lambda+\delta\lambda_{1a})\,v\,(t\infty g+tfing)+\frac{\hbar}{6}Z\Delta\,\left(3\lambda+\delta\lambda_{1a}+2\delta\lambda_{1b}\right)v\,(t\infty n+tfinn)$$
$$finveom=m^2v+\frac{\lambda}{6}v^3+\frac{\hbar}{6}\left(n-1\right)\lambda\ v\ tfing+\frac{\hbar}{2}\lambda\ v\ tfinn$$ *)

```
veom = v mg2
```

```
mg2 v
```

Goldstone equation of motion

```
geom = p^2 - mg2 == Z ZΔ p^2 - m^2 - δm1^2 - ZΔ (λ + δλ1a)/6 v^2 -
   ħ/6 ((n + 1) λ + (n - 1) δλ2a + 2 δλ2b) ZΔ^2 (t∞g + tfing) - ħ/6 (λ + δλ2a) ZΔ^2 (t∞n + tfinn)
finmg2 = mg2 /. Solve[p^2 - mg2 == p^2 - m^2 - λ/6 v^2 - ħ/6 (n + 1) λ tfing - ħ/6 λ tfinn, mg2][[1]]
```

$$-\mathrm{mg2}+p^2 == -m^2 + p^2\, Z\, Z\Delta - \delta m_1^2 - \frac{1}{6} v^2\, Z\Delta\, (\lambda + \delta\lambda_a) - \frac{1}{6} (\mathrm{tfinn} + \mathrm{t\infty n})\, Z\Delta^2\, \hbar\, (\lambda + \delta\lambda_{2\,a}) - \frac{1}{6} (\mathrm{tfing} + \mathrm{t\infty g})\, Z\Delta^2\, \hbar\, ((1+n)\,\lambda + (-1+n)\,\delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b})$$

$$\frac{1}{6}\left(6\, m^2 + v^2\, \lambda + \mathrm{tfing}\, \lambda\, \hbar + n\, \mathrm{tfing}\, \lambda\, \hbar + \mathrm{tfinn}\, \lambda\, \hbar\right)$$

Higgs equation of motion

```
neom = p^2 - mn2 == Z ZΔ p^2 - m^2 - δm1^2 - ZΔ v^2 (3 λ + δλ1a + 2 δλ1b)/6 -
   ħ/6 (λ + δλ2a) (n - 1) ZΔ^2 (t∞g + tfing) - ħ/6 (3 λ + δλ2a + 2 δλ2b) ZΔ^2 (t∞n + tfinn)
finmn2 = mn2 /. Solve[p^2 - mn2 == p^2 - m^2 - v^2 λ/2 - ħ/6 λ (n - 1) tfing - ħ/2 λ tfinn, mn2][[1]]
```

$$-\mathrm{mn2}+p^2 == -m^2 + p^2\, Z\, Z\Delta - \delta m_1^2 - \frac{1}{6}(-1+n)\,(\mathrm{tfing} + \mathrm{t\infty g})\, Z\Delta^2\, \hbar\, (\lambda + \delta\lambda_{2\,a}) - \frac{1}{6} v^2\, Z\Delta\, (3\,\lambda + \delta\lambda_a + 2\,\delta\lambda_b) - \frac{1}{6} (\mathrm{tfinn} + \mathrm{t\infty n})\, Z\Delta^2\, \hbar\, (3\,\lambda + \delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b})$$

$$\frac{1}{6}\left(6\, m^2 + 3\, v^2\, \lambda - \mathrm{tfing}\, \lambda\, \hbar + n\, \mathrm{tfing}\, \lambda\, \hbar + 3\, \mathrm{tfinn}\, \lambda\, \hbar\right)$$

Infinite parts of tadpoles

c0, c1, Λ and μ are regularisation/renormalisation scheme dependent quantities

```
divergentpartrules = {t∞g → c0 Λ^2 + c1 mg2 Log[Λ^2 / μ^2], t∞n → c0 Λ^2 + c1 mn2 Log[Λ^2 / μ^2]}
```

$$\left\{\mathrm{t\infty g} \to c0\, \Lambda^2 + c1\, \mathrm{mg2}\, \mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right],\ \mathrm{t\infty n} \to c0\, \Lambda^2 + c1\, \mathrm{mn2}\, \mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\right\}$$

Sub in tadpole expressions, eliminate mn2 and solve for mg2

```
mn2fromneom = Solve[neom /. divergentpartrules, mn2][[1]]
```

$$\left\{mn2 \to \left(-m^2 - p^2 + p^2\, Z\, Z\Delta - \delta m_1^2 - \frac{1}{6}(-1+n)\, Z\Delta^2\, \hbar \left(tfing + c0\, \Lambda^2 + c1\, mg2\, \mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\right)(\lambda + \delta\lambda_{2\,a}) - \frac{1}{6} v^2\, Z\Delta\, (3\,\lambda + \delta\lambda_a + 2\,\delta\lambda_b) - \frac{1}{6} tfinn\, Z\Delta^2\, \hbar\, (3\,\lambda + \delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b}) - \frac{1}{6} c0\, Z\Delta^2\, \Lambda^2\, \hbar\, (3\,\lambda + \delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b})\right) \Big/ \left(-1 + \frac{1}{6} c1\, Z\Delta^2\, \hbar\, \mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right] (3\,\lambda + \delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b})\right)\right\}$$

```
mg2soln = mg2 /. (geom /. divergentpartrules /. mn2fromneom // Solve[#, mg2][[1]] &)
```

$$\left(-m^2 - p^2 + p^2\, Z\, Z\Delta - \delta m_1^2 - \frac{1}{6} v^2\, Z\Delta\, (\lambda + \delta\lambda_a) - \frac{1}{6}\, \text{tfinn}\, Z\Delta^2\, \hbar\, (\lambda + \delta\lambda_{2\,a}) - \frac{1}{6}\, c0\, Z\Delta^2\, \Lambda^2\, \hbar\, (\lambda + \delta\lambda_{2\,a}) - \frac{1}{6}\, \text{tfing}\, Z\Delta^2\, \hbar\, ((1+n)\,\lambda + (-1+n)\,\delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b}) - \frac{1}{6}\, c0\, Z\Delta^2\, \Lambda^2\, \hbar\, ((1+n)\,\lambda + (-1+n)\,\delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b}) + \frac{c1\, m^2\, Z\Delta^2\, \hbar\, \text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] (\lambda + \delta\lambda_{2\,a})}{6\left(-1 + \frac{1}{6}\, c1\, Z\Delta^2\, \hbar\, \text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] (3\,\lambda + \delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b})\right)} + \frac{c1\, p^2\, Z\Delta^2\, \hbar\, \text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] (\lambda + \delta\lambda_{2\,a})}{6\left(-1 + \frac{1}{6}\, c1\, Z\Delta^2\, \hbar\, \text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] (3\,\lambda + \delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b})\right)} - \frac{c1\, p^2\, Z\, Z\Delta^3\, \hbar\, \text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] (\lambda + \delta\lambda_{2\,a})}{6\left(-1 + \frac{1}{6}\, c1\, Z\Delta^2\, \hbar\, \text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] (3\,\lambda + \delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b})\right)} + \frac{c1\, Z\Delta^2\, \hbar\, \text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] \delta m_1^2\, (\lambda + \delta\lambda_{2\,a})}{6\left(-1 + \frac{1}{6}\, c1\, Z\Delta^2\, \hbar\, \text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] (3\,\lambda + \delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b})\right)} + \frac{c1\, (-1+n)\, \text{tfing}\, Z\Delta^4\, \hbar^2\, \text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] (\lambda + \delta\lambda_{2\,a})^2}{36\left(-1 + \frac{1}{6}\, c1\, Z\Delta^2\, \hbar\, \text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] (3\,\lambda + \delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b})\right)} + \frac{c0\, c1\, (-1+n)\, Z\Delta^4\, \Lambda^2\, \hbar^2\, \text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] (\lambda + \delta\lambda_{2\,a})^2}{36\left(-1 + \frac{1}{6}\, c1\, Z\Delta^2\, \hbar\, \text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] (3\,\lambda + \delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b})\right)} + \frac{c1\, v^2\, Z\Delta^3\, \hbar\, \text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] (\lambda + \delta\lambda_{2\,a})\, (3\,\lambda + \delta\lambda_a + 2\,\delta\lambda_b)}{36\left(-1 + \frac{1}{6}\, c1\, Z\Delta^2\, \hbar\, \text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] (3\,\lambda + \delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b})\right)} + \frac{c1\, \text{tfinn}\, Z\Delta^4\, \hbar^2\, \text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] (\lambda + \delta\lambda_{2\,a})\, (3\,\lambda + \delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b})}{36\left(-1 + \frac{1}{6}\, c1\, Z\Delta^2\, \hbar\, \text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] (3\,\lambda + \delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b})\right)} + \frac{c0\, c1\, Z\Delta^4\, \Lambda^2\, \hbar^2\, \text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] (\lambda + \delta\lambda_{2\,a})\, (3\,\lambda + \delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b})}{36\left(-1 + \frac{1}{6}\, c1\, Z\Delta^2\, \hbar\, \text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] (3\,\lambda + \delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b})\right)}\right) \Big/ \left(-1 + \frac{1}{6}\, c1\, Z\Delta^2\, \hbar\, \text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] ((1+n)\,\lambda + (-1+n)\,\delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b}) - \frac{c1^2\, (-1+n)\, Z\Delta^4\, \hbar^2\, \text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]^2 (\lambda + \delta\lambda_{2\,a})^2}{36\left(-1 + \frac{1}{6}\, c1\, Z\Delta^2\, \hbar\, \text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] (3\,\lambda + \delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b})\right)}\right)$$

```
mn2soln = mn2 /. mn2fromneom /. mg2 → mg2soln // Simplify
```

$$
-\Bigg(\Bigg(6\,m^2 + 6\,p^2 - 6\,p^2\,Z\,Z\Delta + 6\,\delta m_1^2 + v^2\,Z\Delta\,(3\,\lambda + \delta\lambda_a + 2\,\delta\lambda_b) + \text{tfinn}\,Z\Delta^2\,\hbar\,(3\,\lambda + \delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b}) + \text{c0}\,Z\Delta^2\,\Lambda^2\,\hbar\,(3\,\lambda + \delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b}) +
\Big((-1+n)\,Z\Delta^2\,\hbar\,(\lambda + \delta\lambda_{2\,a})\,\Big(18\,\text{tfing} + 18\,\text{c0}\,\Lambda^2 + 18\,\text{c1}\,m^2\,\text{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big] + 18\,\text{c1}\,p^2\,\text{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big] -
18\,\text{c1}\,p^2\,Z\,Z\Delta\,\text{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big] + 3\,\text{c1}\,v^2\,Z\Delta\,\lambda\,\text{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big] - 9\,\text{c1}\,\text{tfing}\,Z\Delta^2\,\lambda\,\hbar\,\text{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big] +
3\,\text{c1}\,\text{tfinn}\,Z\Delta^2\,\lambda\,\hbar\,\text{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big] - 6\,\text{c0}\,\text{c1}\,Z\Delta^2\,\lambda\,\Lambda^2\,\hbar\,\text{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big] - 6\,\text{c1}^2\,m^2\,Z\Delta^2\,\lambda\,\hbar\,\text{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big]^2 -
6\,\text{c1}^2\,p^2\,Z\Delta^2\,\lambda\,\hbar\,\text{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big]^2 + 6\,\text{c1}^2\,p^2\,Z\,Z\Delta^3\,\lambda\,\hbar\,\text{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big]^2 - 3\,\text{c1}\,\text{tfing}\,Z\Delta^2\,\hbar\,
\text{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big]\,\delta\lambda_{2\,a} + 3\,\text{c1}\,\text{tfinn}\,Z\Delta^2\,\hbar\,\text{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big]\,\delta\lambda_{2\,a} + \text{c1}^2\,v^2\,Z\Delta^3\,\lambda\,\hbar\,\text{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big]^2\,\delta\lambda_{2\,a} +
\text{c1}^2\,v^2\,Z\Delta^3\,\lambda\,\hbar\,\text{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big]^2\,\delta\lambda_b + \text{c1}^2\,v^2\,Z\Delta^3\,\hbar\,\text{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big]^2\,\delta\lambda_{2\,a}\,\delta\lambda_b -
6\,\text{c1}\,\text{tfing}\,Z\Delta^2\,\hbar\,\text{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big]\,\delta\lambda_{2\,b} - 6\,\text{c0}\,\text{c1}\,Z\Delta^2\,\Lambda^2\,\hbar\,\text{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big]\,\delta\lambda_{2\,b} -
6\,\text{c1}^2\,m^2\,Z\Delta^2\,\hbar\,\text{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big]^2\,\delta\lambda_{2\,b} - 6\,\text{c1}^2\,p^2\,Z\Delta^2\,\hbar\,\text{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big]^2\,\delta\lambda_{2\,b} +
6\,\text{c1}^2\,p^2\,Z\,Z\Delta^3\,\hbar\,\text{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big]^2\,\delta\lambda_{2\,b} - \text{c1}^2\,v^2\,Z\Delta^3\,\lambda\,\hbar\,\text{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big]^2\,\delta\lambda_{2\,b} -
6\,\text{c1}\,\text{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big]\,\delta m_1^2\,\Big(-3 + \text{c1}\,Z\Delta^2\,\lambda\,\hbar\,\text{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big] + \text{c1}\,Z\Delta^2\,\hbar\,\text{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big]\,\delta\lambda_{2\,b}\Big) -
\text{c1}\,v^2\,Z\Delta\,\text{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big]\,\delta\lambda_a\,\Big(-3 + \text{c1}\,Z\Delta^2\,\lambda\,\hbar\,\text{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big] + \text{c1}\,Z\Delta^2\,\hbar\,\text{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big]\,\delta\lambda_{2\,b}\Big)\Big)\Big)\Big/
\Big(\Big(-3 + \text{c1}\,Z\Delta^2\,\lambda\,\hbar\,\text{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big] + \text{c1}\,Z\Delta^2\,\hbar\,\text{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big]\,\delta\lambda_{2\,b}\Big)\,\Big(-6 + 2\,\text{c1}\,Z\Delta^2\,\lambda\,\hbar\,\text{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big] +
\text{c1}\,n\,Z\Delta^2\,\lambda\,\hbar\,\text{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big] + \text{c1}\,n\,Z\Delta^2\,\hbar\,\text{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big]\,\delta\lambda_{2\,a} + 2\,\text{c1}\,Z\Delta^2\,\hbar\,\text{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big]\,\delta\lambda_{2\,b}\Big)\Big)\Bigg)\Bigg/
\Bigg(6\,\Big(-1 + \frac{1}{6}\,\text{c1}\,Z\Delta^2\,\hbar\,\text{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big]\,(3\,\lambda + \delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b})\Big)\Bigg)
$$

Gather divergences proportional v, tfing and tfinn and set independently to zero

First we subtract the finite equation of motion, then gather coefficients of the remainder into a list and set each to zero (after some trimming and simplifying).

```
cteq = ((CoefficientList[mg2soln - finmg2, {p, v, tfing, tfinn}] // Flatten) //
    DeleteDuplicates // Simplify // FullSimplify) == 0 // Thread
```

$$\Bigg\{-\Bigg(\Big(6\,\delta m_1^2 + Z\Delta^2\,\hbar\,\Big(c0\,\Lambda^2 + c1\,m^2\,\mathrm{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big]\Big)\,\big((2+n)\,\lambda + n\,\delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b}\big)\Big)\Big/\Big(-6 + c1\,(2+n)\,Z\Delta^2\,\lambda\,\hbar\,\mathrm{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big] + c1\,Z\Delta^2\,\hbar\,\mathrm{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big]\,(n\,\delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b})\Big)\Bigg) == 0,$$

$$-\frac{\lambda\,\hbar}{6} + \big(3\,Z\Delta^2\,\hbar\,(\lambda + \delta\lambda_{2\,a})\big)\Big/\Bigg(\Big(-3 + c1\,Z\Delta^2\,\lambda\,\hbar\,\mathrm{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big] + c1\,Z\Delta^2\,\hbar\,\mathrm{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big]\,\delta\lambda_{2\,b}\Big)\Big(-6 + c1\,(2+n)\,Z\Delta^2\,\lambda\,\hbar\,\mathrm{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big] + c1\,Z\Delta^2\,\hbar\,\mathrm{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big]\,(n\,\delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b})\Big)\Bigg) == 0,$$

$$-\frac{1}{6\,c1\,\mathrm{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big]}\Bigg(6 + c1\,(1+n)\,\lambda\,\hbar\,\mathrm{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big] + \frac{18}{n\,\Big(-3 + c1\,Z\Delta^2\,\lambda\,\hbar\,\mathrm{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big] + c1\,Z\Delta^2\,\hbar\,\mathrm{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big]\,\delta\lambda_{2\,b}\Big)} + (36\,(-1+n))\Big/\Big(n\,\Big(-6 + c1\,(2+n)\,Z\Delta^2\,\lambda\,\hbar\,\mathrm{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big] + c1\,Z\Delta^2\,\hbar\,\mathrm{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big]\,(n\,\delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b})\Big)\Big)\Bigg) == 0,\ \mathrm{True},$$

$$-\frac{\lambda}{6} + \frac{Z\Delta\,(\lambda + \delta\lambda_b)}{n\,\Big(-3 + c1\,Z\Delta^2\,\lambda\,\hbar\,\mathrm{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big] + c1\,Z\Delta^2\,\hbar\,\mathrm{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big]\,\delta\lambda_{2\,b}\Big)} - \big(Z\Delta\,((2+n)\,\lambda + n\,\delta\lambda_a + 2\,\delta\lambda_b)\big)\Big/\Big(n\,\Big(-6 + c1\,(2+n)\,Z\Delta^2\,\lambda\,\hbar\,\mathrm{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big] + c1\,Z\Delta^2\,\hbar\,\mathrm{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big]\,(n\,\delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b})\Big)\Big) == 0,$$

$$(-6 + 6\,Z\,Z\Delta)\Big/\Big(-6 + c1\,(2+n)\,Z\Delta^2\,\lambda\,\hbar\,\mathrm{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big] + c1\,Z\Delta^2\,\hbar\,\mathrm{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big]\,(n\,\delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b})\Big) == 0\Bigg\}$$

```
cteq2 = ((CoefficientList[mn2soln - finmn2, {p, v, tfing, tfinn}] // Flatten) //
    DeleteDuplicates // Simplify // FullSimplify) == 0 // Thread
```

$$\Big\{-\frac{6\,\delta m_1^2+Z\Delta^2\,\hbar\left(\mathrm{c0}\,\Lambda^2+\mathrm{c1}\,m^2\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\right)\left((2+n)\,\lambda+n\,\delta\lambda_{2\,a}+2\,\delta\lambda_{2\,b}\right)}{-6+\mathrm{c1}\,(2+n)\,Z\Delta^2\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]+\mathrm{c1}\,Z\Delta^2\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\left(n\,\delta\lambda_{2\,a}+2\,\delta\lambda_{2\,b}\right)}==0,$$

$$-\frac{1}{2\,\mathrm{c1}\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]}\left(2+\mathrm{c1}\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]+\frac{6\,(-1+n)}{n\left(-3+\mathrm{c1}\,Z\Delta^2\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]+\mathrm{c1}\,Z\Delta^2\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\delta\lambda_{2\,b}\right)}+12\Big/\left(n\left(-6+\mathrm{c1}\,(2+n)\,Z\Delta^2\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]+\mathrm{c1}\,Z\Delta^2\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\left(n\,\delta\lambda_{2\,a}+2\,\delta\lambda_{2\,b}\right)\right)\right)\right)==0,$$

$$\Bigg((-1+n)\,\hbar\left(-Z\Delta^2\,\delta\lambda_{2\,a}\left(-18+\mathrm{c1}\,n\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\right)\left(-3+\mathrm{c1}\,Z\Delta^2\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\right)+\mathrm{c1}^2\,n\,Z\Delta^2\,\lambda\,\hbar^2\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]^2\delta\lambda_{2\,b}\right)+\lambda\left(18\left(-1+Z\Delta^2\right)+\mathrm{c1}\,Z\Delta^2\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\left(3\,(4+n)-\mathrm{c1}\,(2+n)\,Z\Delta^2\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\right)-\mathrm{c1}\,Z\Delta^2\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\delta\lambda_{2\,b}\left(-12+\mathrm{c1}\,(4+n)\,Z\Delta^2\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]+2\,\mathrm{c1}\,Z\Delta^2\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\delta\lambda_{2\,b}\right)\right)\Bigg)\Bigg/\left(6\left(-3+\mathrm{c1}\,Z\Delta^2\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]+\mathrm{c1}\,Z\Delta^2\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\delta\lambda_{2\,b}\right)\left(-6+\mathrm{c1}\,(2+n)\,Z\Delta^2\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]+\mathrm{c1}\,Z\Delta^2\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\left(n\,\delta\lambda_{2\,a}+2\,\delta\lambda_{2\,b}\right)\right)\right)==0,$$

$$\mathrm{True},\ -\frac{\lambda}{2}-\frac{(-1+n)\,Z\Delta\,(\lambda+\delta\lambda_b)}{n\left(-3+\mathrm{c1}\,Z\Delta^2\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]+\mathrm{c1}\,Z\Delta^2\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\delta\lambda_{2\,b}\right)}-\left(Z\Delta\left((2+n)\,\lambda+n\,\delta\lambda_a+2\,\delta\lambda_b\right)\right)\Big/\left(n\left(-6+\mathrm{c1}\,(2+n)\,Z\Delta^2\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]+\mathrm{c1}\,Z\Delta^2\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\left(n\,\delta\lambda_{2\,a}+2\,\delta\lambda_{2\,b}\right)\right)\right)==0,$$

$$(-6+6\,Z\,Z\Delta)\Big/\left(-6+\mathrm{c1}\,(2+n)\,Z\Delta^2\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]+\mathrm{c1}\,Z\Delta^2\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\left(n\,\delta\lambda_{2\,a}+2\,\delta\lambda_{2\,b}\right)\right)==0\Big\}$$

Solve for counterterms

Find counter-terms from the gap equations

```
cteqs = {cteq, cteq2} // Flatten // FullSimplify // DeleteDuplicates
```

$$\Big\{\left(6\,\delta m_1^2 + Z\Delta^2\,\hbar\left(c0\,\Lambda^2 + c1\,m^2\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\right)\left((2+n)\,\lambda + n\,\delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b}\right)\right)\Big/ \left(-6 + c1\,(2+n)\,Z\Delta^2\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] + c1\,Z\Delta^2\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\left(n\,\delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b}\right)\right) == 0,$$

$$\hbar\left(\lambda - \left(18\,Z\Delta^2\,(\lambda + \delta\lambda_{2\,a})\right)\Big/\left(\left(-3 + c1\,Z\Delta^2\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] + c1\,Z\Delta^2\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\delta\lambda_{2\,b}\right)\left(-6 + c1\,(2+n)\,Z\Delta^2\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] + c1\,Z\Delta^2\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\left(n\,\delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b}\right)\right)\right)\right) == 0,$$

$$\frac{1}{c1\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]}\left(6 + c1\,(1+n)\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] + \frac{18}{n\left(-3 + c1\,Z\Delta^2\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] + c1\,Z\Delta^2\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\delta\lambda_{2\,b}\right)} + (36\,(-1+n))\Big/\left(n\left(-6 + c1\,(2+n)\,Z\Delta^2\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] + c1\,Z\Delta^2\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\left(n\,\delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b}\right)\right)\right)\right) == 0,$$

$$\text{True},\ \lambda + \left(6\,Z\Delta\left((2+n)\,\lambda + n\,\delta\lambda_a + 2\,\delta\lambda_b\right)\right)\Big/\left(n\left(-6 + c1\,(2+n)\,Z\Delta^2\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] + c1\,Z\Delta^2\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\left(n\,\delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b}\right)\right)\right) == \frac{6\,Z\Delta\,(\lambda + \delta\lambda_b)}{n\left(-3 + c1\,Z\Delta^2\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] + c1\,Z\Delta^2\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\delta\lambda_{2\,b}\right)},$$

$$(-1 + Z\,Z\Delta)\Big/\left(-6 + c1\,(2+n)\,Z\Delta^2\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] + c1\,Z\Delta^2\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\left(n\,\delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b}\right)\right) == 0,$$

$$\frac{1}{c1\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]}\left(2 + c1\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] + \frac{6\,(-1+n)}{n\left(-3 + c1\,Z\Delta^2\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] + c1\,Z\Delta^2\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\delta\lambda_{2\,b}\right)} + 12\Big/\left(n\left(-6 + c1\,(2+n)\,Z\Delta^2\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] + c1\,Z\Delta^2\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\left(n\,\delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b}\right)\right)\right)\right) == 0,$$

$$(-1+n)\,\hbar\left(\lambda - \left(18\,Z\Delta^2\,(\lambda + \delta\lambda_{2\,a})\right)\Big/\left(\left(-3 + c1\,Z\Delta^2\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] + c1\,Z\Delta^2\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\delta\lambda_{2\,b}\right)\left(-6 + c1\,(2+n)\,Z\Delta^2\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] + c1\,Z\Delta^2\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\left(n\,\delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b}\right)\right)\right)\right) == 0,$$

$$\lambda + \frac{2\,(-1+n)\,Z\Delta\,(\lambda + \delta\lambda_b)}{n\left(-3 + c1\,Z\Delta^2\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] + c1\,Z\Delta^2\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\delta\lambda_{2\,b}\right)} + \left(2\,Z\Delta\left((2+n)\,\lambda + n\,\delta\lambda_a + 2\,\delta\lambda_b\right)\right)\Big/\left(n\left(-6 + c1\,(2+n)\,Z\Delta^2\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] + c1\,Z\Delta^2\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\left(n\,\delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b}\right)\right)\right) == 0\Big\}$$

```
cts = {δm1^2, δλ1a, δλ2a, δλ1b, δλ2b, Z, ZΔ} /. Solve[cteqs,
    {δm1, δλ1a, δλ2a, δλ1b, δλ2b, Z, ZΔ}] // FullSimplify // DeleteDuplicates
```

Solve::svars : Equations may not give solutions for all "solve" variables. ≫

$$\left\{\left\{-\frac{(2+n)\,\lambda\,\hbar\left(c0\,\Lambda^2+c1\,m^2\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\right)}{6+c1\,(2+n)\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]},\right.\right.$$
$$\lambda\left(-1+\frac{6\,(2+n)}{n\,Z\Delta\left(6+c1\,(2+n)\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\right)}-\frac{6}{3\,n\,Z\Delta+c1\,n\,Z\Delta\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]}\right),$$
$$\lambda\left(-1+\frac{18}{Z\Delta^2\left(3+c1\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\right)\left(6+c1\,(2+n)\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\right)}\right),$$
$$\left.\left.\lambda\left(-1+\frac{3}{3\,Z\Delta+c1\,Z\Delta\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]}\right),\ \lambda\left(-1+\frac{3}{Z\Delta^2\left(3+c1\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\right)}\right),\ \frac{1}{Z\Delta},\ Z\Delta\right\}\right\}$$

ZΔ is redundant in this truncation, can remove it :

```
cts /. ZΔ → 1 // FullSimplify
```

$$\left\{\left\{-\frac{(2+n)\,\lambda\,\hbar\left(c0\,\Lambda^2+c1\,m^2\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\right)}{6+c1\,(2+n)\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]},\right.\right.$$
$$\lambda\left(-1-\frac{6}{3\,n+c1\,n\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]}+\frac{6\,(2+n)}{n\left(6+c1\,(2+n)\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\right)}\right),$$
$$\lambda\left(-1+\frac{18}{\left(3+c1\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\right)\left(6+c1\,(2+n)\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\right)}\right),$$
$$\left.\left.\lambda\left(-1+\frac{3}{3+c1\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]}\right),\ \lambda\left(-1+\frac{3}{3+c1\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]}\right),\ 1,\ 1\right\}\right\}$$

Verify that the finite gap equations come out right

```
finmg2 ==
  (mg2soln /. Solve[cteqs, {δm1, δλ1a, δλ2a, δλ2b, Z, ZΔ}] /. ZΔ → 1 // FullSimplify //
    DeleteDuplicates)[[2]] // Simplify
```

Solve::svars : Equations may not give solutions for all "solve" variables. >>

FullSimplify::infd : Expression $\left(-m^2-\frac{-c0\,\Lambda^2-c1\,m^2\,\text{Log}[\text{Power}[\ll 2\gg]\,\text{Power}[\ll 2\gg]]}{c1\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]}-\frac{\ll 1\gg}{\ll 1\gg}-\frac{3\,\text{tfing}\ll 1\gg\hbar(\ll 1\gg)}{2\ll 1\gg^2(\ll 1\gg)^2}-\frac{3\,c0\,\lambda^2\,\Lambda^2\,\hbar\left(-(-1+n)\,\lambda+(1+n)\,\lambda+\frac{2\,(9\,\text{Power}[\ll 2\gg]+\ll 7\gg+\ll 1\gg)}{3\,c1\,\lambda^2\,\hbar\,\text{Log}[\text{Times}[\ll 2\gg]]}\right)}{2\,(3+c1\,\lambda\,\hbar\,\text{Log}[\text{Times}[\ll 2\gg]])^2\,(\lambda+\delta\lambda_b)^2}\right)\Big/\left(-1+\left(3\,c1\,\lambda^2\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\left(-(-1+n)\,\lambda+(1+n)\,\lambda+\frac{2\,(\text{Times}[\ll 2\gg]+\ll 7\gg+\text{Times}[\ll 5\gg])}{3\,c1\,\lambda^2\,\hbar\,\text{Log}[\ll 1\gg]}\right)\right)\Big/\left(2\,(3+c1\,\lambda\,\hbar\,\text{Log}[\ll 1\gg])^2\,(\lambda+\delta\lambda_b)^2\right)\right)$ simplified to ComplexInfinity. >>

```
True
```

```
finmn2 == mn2 /.
  ((neom /. divergentpartrules /. mg2 → mg2soln /. Solve[cteqs, {δm1, δλ1a,
        δλ2a, δλ1b, δλ2b, Z, ZΔ}] /. ZΔ → 1 // FullSimplify //
     DeleteDuplicates) // Solve[#, mn2] &) // FullSimplify
```

Solve::svars : Equations may not give solutions for all "solve" variables. >>

```
{True}
```

Verify counter-term expressions in text

$$\left\{\delta m_1{}^2 == \frac{-\hbar\,\lambda\,(n+2)}{6}\left(c0\,\Lambda^2+c1\,m^2\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\right)\frac{\delta\lambda_{1\,a}+\lambda}{\delta\lambda_{1\,b}+\lambda}\right\}\ /.$$

```
    Solve[cteqs, {δm1, δλ1a, δλ2a, δλ1b, δλ2b, Z, ZΔ}] /. ZΔ → 1 //
   FullSimplify // Flatten // DeleteDuplicates
```

Solve::svars : Equations may not give solutions for all "solve" variables. >>

```
{True}
```

```
{δλ1a == δλ2a, δλ1b == δλ2b} /. Solve[cteqs, {δm1, δλ1a, δλ2a, δλ1b, δλ2b, Z, ZΔ}] /.
    ZΔ → 1 // FullSimplify // Flatten // DeleteDuplicates
```

Solve::svars : Equations may not give solutions for all "solve" variables. >>

```
{True}
```

```
{δλ1 a / δλ1 b} /. Solve[cteqs, {δm1, δλ1 a, δλ2 a, δλ1 b, δλ2 b, Z, ZΔ}] /. ZΔ → 1 //
    FullSimplify // Flatten // DeleteDuplicates
```

Solve::svars : Equations may not give solutions for all "solve" variables. >>

$$\left\{1+\frac{3\ (2+n)}{6+c1\ (2+n)\ \lambda\,\hbar\,\mathrm{Log}\left[\frac{\Delta^2}{\mu^2}\right]}\right\}$$

```
δλ1 b /. Solve[cteqs, {δm1, δλ1 a, δλ2 a, δλ1 b, δλ2 b, Z, ZΔ}] /. ZΔ → 1 // FullSimplify //
  DeleteDuplicates
```

Solve::svars : Equations may not give solutions for all "solve" variables. >>

$$\left\{\lambda\left(-1+\frac{3}{3+c1\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Delta^2}{\mu^2}\right]}\right)\right\}$$

Total number of independent counter-term equations

```
Length[{cteqs} // Flatten // FullSimplify // DeleteDuplicates] -
  1 (* -1 because one of the "equations" is identically "True" *)
```

8