Renormalization of Soft Symmetry Improved 2PIEA gap equations in the Hartree-Fock approximation

Supplement to thesis Chapter 5 "Soft Symmetry Improvement"

Mathematica notebook to compute couter-terms for the Hartree-Fock truncation of the SSI-2PIEA

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Hartree-Fock

In[1]:=
```
ClearAll[geom, neom, intrules, msbarrules, mg2soln, cteq, cts, δm, δλ];
```

Hartree-Fock gap equations with counterterms

Goldstone equation of motion. Quantities in reference to the paper are:
p is the four-momentum flowing through the propagators $\Delta_G{}^{-1}$ and $\Delta_N{}^{-1}$,
mg2 is the Goldstone mass squared $m_G{}^2$,
mn2 is the Higgs mass squared $m_H{}^2$,
Z and ZΔ are the wavefunction a propagator renormalization constants,
m^2 is the (renormalized) Lagrangian mass parameter, $\delta\mathbf{m_0}^2$, $\delta\mathbf{m_1}^2$ are its counter-terms,
λ is the (renormalized) four point coupling,
$\delta\lambda_0$, $\delta\lambda_{1\,a}$, $\delta\lambda_{1\,b}$, $\delta\lambda_{2\,a}$, $\delta\lambda_{2\,b}$ are the independent coupling counter-terms,
v is the scalar field vacuum expectation value,
$\hbar$ is the reduced Planck constant,
n is the number of fields in the O(n) symmetry group,
ξ is the stiffness parameter,
ϵ is the solution of the Goldstone zero mode equation,
ssi = $\frac{1}{V \beta m_G{}^2}\left(\frac{1}{\epsilon} - 1\right)$ is the soft symmetry improvement term in the propagator eoms,
ssi2 = $\frac{1}{\xi}(n-1)\,2\left(m_G{}^2\,\epsilon\right)^2$ is the other soft symmetry improvement term in the vev eom,
t∞g, t∞n are the divergent tadpole integrals for the Goldstone, Higgs resp.,
tfing, tfinn are the finite parts of the tadpoles for the Goldstone, Higgs resp.

Vev equation of motion

In[2]:=
$$\text{veom} = Z\Delta^{-1}\left(m^2+\delta m_0{}^2\right) v + \frac{\lambda+\delta\lambda_0}{6} v^3 + \frac{\hbar}{6} Z\Delta\left(n-1\right)\left(\lambda+\delta\lambda_{1\,a}\right) v\,(t\infty g + tfing + ssi) + \frac{\hbar}{6} Z\Delta\left(3\lambda+\delta\lambda_{1\,a}+2\,\delta\lambda_{1\,b}\right) v\,(t\infty n + tfinn) + v\,ssi2$$

Goldstone equation of motion

In[3]:=
$$\text{geom} = p^2 - mg2 == Z\,Z\Delta\,p^2 - m^2 - \delta m_1{}^2 - Z\Delta\frac{\lambda+\delta\lambda_{1\,a}}{6} v^2 - \frac{\hbar}{6}\left(\left(n+1\right)\lambda + \left(n-1\right)\delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b}\right) Z\Delta^2\,(t\infty g + tfing + ssi) - \frac{\hbar}{6}\left(\lambda+\delta\lambda_{2\,a}\right) Z\Delta^2\,(t\infty n + tfinn)$$

Higgs equation of motion

In[4]:=
$$\text{neom} = p^2 - mn2 == Z\,Z\Delta\,p^2 - m^2 - \delta m_1{}^2 - Z\Delta\,v^2\frac{\left(3\lambda+\delta\lambda_{1\,a}+2\,\delta\lambda_{1\,b}\right)}{6} - \frac{\hbar}{6}\left(\lambda+\delta\lambda_{2\,a}\right)\left(n-1\right) Z\Delta^2\,(t\infty g + tfing + ssi) - \frac{\hbar}{6}\left(3\lambda+\delta\lambda_{2\,a}+2\,\delta\lambda_{2\,b}\right) Z\Delta^2\,(t\infty n + tfinn)$$

Infinite parts of tadpoles in MSbar

MSbar rules for 4 - 2 ϵ dimensions

In[5]:=
$$\text{msbarrules} = \left\{t\infty g \to c0\,\Lambda^2 + c1\,mg2\,\text{Log}\left[\Lambda^2/\mu^2\right],\ t\infty n \to c0\,\Lambda^2 + c1\,mn2\,\text{Log}\left[\Lambda^2/\mu^2\right]\right\}$$

Sub in tadpole expressions, eliminate mn2 and solve for mg2

In[6]:=
```
mg2soln = mg2 /.
   (geom /. msbarrules /. Solve[neom /. msbarrules, mn2][[1]] // Solve[#, mg2][[1]] &)
```

In[7]:=
```
mn2soln = mn2 /. (neom /. msbarrules /. mg2 → mg2soln // Solve[#, mn2][[1]] &)
```

Gather divergences proportional v, tfing and tfinn and set independently to zero

First we subtract the finite equation of motion, then gather coefficients of the remainder into a list and set each to zero (after some trimming and simplifying).

In[8]:=
$$\text{cteq} = \left(\left(\text{CoefficientList}\left[mg2soln + \left(-m^2 - \frac{\lambda}{6}v^2 - \frac{\hbar}{6}\left(\left(n+1\right)\lambda\right)(tfing+ssi) - \frac{\hbar}{6}(\lambda)\,(tfinn)\right), \{p, v, tfing, tfinn\}\right] // \text{Flatten}\right) // \text{DeleteDuplicates} // \text{Simplify} // \text{FullSimplify}\right) == 0 // \text{Thread}$$

In[9]:=
```
cteq2 =
 ((CoefficientList[mn2soln + (-m^2 - λ/2 v^2 - ℏ/6 ((n - 1) λ) (tfing + ssi) - ℏ/2 (λ) (tfinn)),
        {p, v, tfing, tfinn}] // Flatten) //
     DeleteDuplicates // Simplify // FullSimplify) == 0 // Thread
```

Solve for counterterms

In[10]:=
```
cteqs = {cteq, cteq2} // Flatten // FullSimplify // DeleteDuplicates
```

In[11]:=
```
ctsolns =
  Solve[cteqs, {δm₁, δλ₁ₐ, δλ₂ₐ, δλ₁ᵦ, δλ₂ᵦ, Z, ZΔ}] // FullSimplify // DeleteDuplicates
```

In[12]:=
```
cts = {δm₁^2, δλ₁ₐ, δλ₂ₐ, δλ₁ᵦ, δλ₂ᵦ, Z, ZΔ} /. ctsolns // FullSimplify // DeleteDuplicates
```

ZΔ is redundant in this truncation, can remove it :

In[13]:=
```
cts /. ZΔ → 1 // FullSimplify
```

In[14]:=
```
mg2soln /. ctsolns /. ZΔ → 1 // FullSimplify // DeleteDuplicates
```

In[15]:=
```
mn2 /. ((neom /. msbarrules /. mg2 → mg2soln /. ctsolns /. ZΔ → 1 // FullSimplify //
      DeleteDuplicates) // Solve[#, mn2] &) // FullSimplify
```

In[16]:=
```
rnveom =
  veom /. {mg2 → m^2 + λ/6 v^2 + ℏ/6 ((n + 1) λ) (tfing + ssi) + ℏ/6 (λ) (tfinn), mn2 → m^2 + λ/2 v^2 +
       ℏ/6 ((n - 1) λ) (tfing + ssi) + ℏ/2 (λ) (tfinn)} // Simplify // DeleteDuplicates
```

In[17]:=
```
veomCtEqs =
  (((CoefficientList[(1/v rnveom - (m^2 + λ/6 v^2 + ℏ/6 ((n - 1) λ) (tfing + ssi) + ℏ/2 (λ) (tfinn) +
                 ssi2)) /. msbarrules /. {mg2 → m^2 + λ/6 v^2 + ℏ/6 ((n + 1) λ) (tfing +
                 ssi) + ℏ/6 (λ) (tfinn), mn2 → m^2 + λ/2 v^2 + ℏ/6 ((n - 1) λ) (tfing +
                 ssi) + ℏ/2 (λ) (tfinn)} // Simplify // Expand // FullSimplify,
         {v, tfing, tfinn}] // Simplify // Flatten) // DeleteDuplicates //
      Simplify // FullSimplify // DeleteDuplicates) == 0 // Thread)
```

In[18]:=
```
ctegs3 = (veomCtEqs /. ctsolns // Simplify // DeleteDuplicates // FullSimplify)[[1]]
```

In[19]:=
```
{δm₀^2, δλ₀} /. Solve[ctegs3, {δm₀, δλ₀}] /. ZΔ → 1 // DeleteDuplicates // Simplify
```

In[39]:=
```
{δm₁^2 == δm₀^2, δλ₁ₐ == δλ₂ₐ, δλ₁ᵦ == δλ₂ᵦ} /. ctsolns /. Solve[ctegs3, {δm₀, δλ₀}] /.
    ZΔ → 1 // FullSimplify // Flatten // DeleteDuplicates
```

In[35]:= $\left\{\frac{\delta\lambda_{1\,a}}{\delta\lambda_{1\,b}}\right\}$ /. ctsolns /. Solve[ctegs3, {δm$_0$, $\delta\lambda_0$}] /. ZΔ → 1 // FullSimplify // Flatten // DeleteDuplicates

In[28]:= $\delta\lambda_{1\,b}$ == $\delta\lambda_{2\,b}$ /. ctsolns /. ZΔ → 1 // FullSimplify // DeleteDuplicates

In[31]:= $\delta\lambda_{1\,b}$ /. ctsolns /. ZΔ → 1 // FullSimplify // DeleteDuplicates

In[36]:= {$\delta\lambda_0$ == 1 $\delta\lambda_{1\,a}$ + 2 $\delta\lambda_{1\,b}$} /. ctsolns /. Solve[ctegs3, {δm$_0$, $\delta\lambda_0$}] /. ZΔ → 1 // FullSimplify // Flatten // DeleteDuplicates

In[38]:= $\left\{\delta m_0{}^2 == -\frac{\left(c0\,\Lambda^2 + c1\,m^2\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\right)\lambda\,\hbar}{3}\left(\frac{\delta\lambda_{1\,a}}{\delta\lambda_{1\,b}} - 1\right)\right\}$ /. ctsolns /. Solve[ctegs3, {δm$_0$, $\delta\lambda_0$}] /. ZΔ → 1 // FullSimplify // Flatten // DeleteDuplicates