Renormalization of Soft Symmetry Improved 2PIEA gap equations in the Hartree-Fock approximation

Supplement to thesis Chapter 5 "Soft Symmetry Improvement"

Mathematica notebook to compute couter-terms for the Hartree-Fock truncation of the SSI-2PIEA

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Hartree-Fock

In[1]:=
```
ClearAll[geom, neom, intrules, msbarrules, mg2soln, cteq, cts, δm, δλ];
```

Hartree-Fock gap equations with counterterms

Goldstone equation of motion. Quantities in reference to the paper are:
p is the four-momentum flowing through the propagators $\Delta_G{}^{-1}$ and $\Delta_N{}^{-1}$,
mg2 is the Goldstone mass squared $m_G{}^2$,
mn2 is the Higgs mass squared $m_H{}^2$,
Z and ZΔ are the wavefunction a propagator renormalization constants,
m^2 is the (renormalized) Lagrangian mass parameter, $\delta m_0{}^2$, $\delta m_1{}^2$ are its counter-terms,
λ is the (renormalized) four point coupling,
$\delta\lambda_0$, $\delta\lambda_{1a}$, $\delta\lambda_{1b}$, $\delta\lambda_{2a}$, $\delta\lambda_{2b}$ are the independent coupling counter-terms,
v is the scalar field vacuum expectation value,
$\hbar$ is the reduced Planck constant,
n is the number of fields in the O(n) symmetry group,
ξ is the stiffness parameter,
ϵ is the solution of the Goldstone zero mode equation,
ssi = $\frac{1}{V\beta m_G{}^2}\left(\frac{1}{\epsilon}-1\right)$ is the soft symmetry improvement term in the propagator eoms,
ssi2 = $\frac{1}{\xi}(n-1)\,2\left(m_G{}^2\,\epsilon\right)^2$ is the other soft symmetry improvement term in the vev eom,
t∞g, t∞n are the divergent tadpole integrals for the Goldstone, Higgs resp.,
tfing, tfinn are the finite parts of the tadpoles for the Goldstone, Higgs resp.

Vev equation of motion

In[2]:=
```
veom = ZΔ^-1 (m^2 + δm0^2) v + (λ + δλ0)/6 v^3 + ℏ/6 ZΔ (n - 1) (λ + δλ1a) v (t∞g + tfing + ssi) +
  ℏ/6 ZΔ (3 λ + δλ1a + 2 δλ1b) v (t∞n + tfinn) + v ssi2
```

Out[2]= $\text{ssi2 v} + \frac{\text{v}\,(\text{m}^2 + \delta\text{m}_0^2)}{\text{Z}\Delta} + \frac{1}{6}\,\text{v}^3\,(\lambda + \delta\lambda_0) + \frac{1}{6}\,(-1+\text{n})\,(\text{ssi} + \text{tfing} + \text{t}\infty\text{g})\,\text{v Z}\Delta\,\hbar\,(\lambda + \delta\lambda_a) + \frac{1}{6}\,(\text{tfinn} + \text{t}\infty\text{n})\,\text{v Z}\Delta\,\hbar\,(3\,\lambda + \delta\lambda_a + 2\,\delta\lambda_b)$

Goldstone equation of motion

In[3]:=
```
geom = p^2 - mg2 == Z ZΔ p^2 - m^2 - δm1^2 - ZΔ (λ + δλ1a)/6 v^2 -
  ℏ/6 ((n + 1) λ + (n - 1) δλ2a + 2 δλ2b) ZΔ^2 (t∞g + tfing + ssi) - ℏ/6 (λ + δλ2a) ZΔ^2 (t∞n + tfinn)
```

Out[3]= $-\text{mg2} + \text{p}^2 == -\text{m}^2 + \text{p}^2\,\text{Z Z}\Delta - \delta\text{m}_1^2 - \frac{1}{6}\,\text{v}^2\,\text{Z}\Delta\,(\lambda + \delta\lambda_a) - \frac{1}{6}\,(\text{tfinn} + \text{t}\infty\text{n})\,\text{Z}\Delta^2\,\hbar\,(\lambda + \delta\lambda_{2\,a}) - \frac{1}{6}\,(\text{ssi} + \text{tfing} + \text{t}\infty\text{g})\,\text{Z}\Delta^2\,\hbar\,((1+\text{n})\,\lambda + (-1+\text{n})\,\delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b})$

Higgs equation of motion

In[4]:=
```
neom = p^2 - mn2 == Z ZΔ p^2 - m^2 - δm1^2 - ZΔ v^2 (3 λ + δλ1a + 2 δλ1b)/6 -
  ℏ/6 (λ + δλ2a) (n - 1) ZΔ^2 (t∞g + tfing + ssi) - ℏ/6 (3 λ + δλ2a + 2 δλ2b) ZΔ^2 (t∞n + tfinn)
```

Out[4]= $-\text{mn2} + \text{p}^2 == -\text{m}^2 + \text{p}^2\,\text{Z Z}\Delta - \delta\text{m}_1^2 - \frac{1}{6}\,(-1+\text{n})\,(\text{ssi} + \text{tfing} + \text{t}\infty\text{g})\,\text{Z}\Delta^2\,\hbar\,(\lambda + \delta\lambda_{2\,a}) - \frac{1}{6}\,\text{v}^2\,\text{Z}\Delta\,(3\,\lambda + \delta\lambda_a + 2\,\delta\lambda_b) - \frac{1}{6}\,(\text{tfinn} + \text{t}\infty\text{n})\,\text{Z}\Delta^2\,\hbar\,(3\,\lambda + \delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b})$

Infinite parts of tadpoles in MSbar

MSbar rules for 4 - 2 ϵ dimensions

In[5]:=
```
msbarrules = {t∞g → c0 Λ^2 + c1 mg2 Log[Λ^2 / μ^2], t∞n → c0 Λ^2 + c1 mn2 Log[Λ^2 / μ^2]}
```

Out[5]= $\{\text{t}\infty\text{g} \to \text{c0}\,\Lambda^2 + \text{c1 mg2 Log}[\frac{\Lambda^2}{\mu^2}],\ \text{t}\infty\text{n} \to \text{c0}\,\Lambda^2 + \text{c1 mn2 Log}[\frac{\Lambda^2}{\mu^2}]\}$

Sub in tadpole expressions, eliminate mn2 and solve for mg2

In[6]:=
```
mg2soln = mg2 /.
  (geom /. msbarrules /. Solve[neom /. msbarrules, mn2][[1]] // Solve[#, mg2][[1]] &)
```

Out[6]=

$$
\begin{aligned}
&\Bigg(-m^2 - p^2 + p^2\, Z\, Z\Delta - \delta m_1^2 - \frac{1}{6}\, v^2\, Z\Delta\, (\lambda + \delta\lambda_{a}) - \frac{1}{6}\, \mathrm{tfinn}\, Z\Delta^2\, \hbar\, (\lambda + \delta\lambda_{2\,a}) - \\
&\quad \frac{1}{6}\, \mathrm{c0}\, Z\Delta^2\, \Lambda^2\, \hbar\, (\lambda + \delta\lambda_{2\,a}) - \frac{1}{6}\, \mathrm{ssi}\, Z\Delta^2\, \hbar\, ((1+n)\,\lambda + (-1+n)\,\delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b}) - \\
&\quad \frac{1}{6}\, \mathrm{tfing}\, Z\Delta^2\, \hbar\, ((1+n)\,\lambda + (-1+n)\,\delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b}) - \\
&\quad \frac{1}{6}\, \mathrm{c0}\, Z\Delta^2\, \Lambda^2\, \hbar\, ((1+n)\,\lambda + (-1+n)\,\delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b}) + \\
&\quad \frac{\mathrm{c1}\, m^2\, Z\Delta^2\, \hbar\, \mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\, (\lambda + \delta\lambda_{2\,a})}{6\left(-1 + \frac{1}{6}\, \mathrm{c1}\, Z\Delta^2\, \hbar\, \mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\, (3\,\lambda + \delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b})\right)} + \\
&\quad \frac{\mathrm{c1}\, p^2\, Z\Delta^2\, \hbar\, \mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\, (\lambda + \delta\lambda_{2\,a})}{6\left(-1 + \frac{1}{6}\, \mathrm{c1}\, Z\Delta^2\, \hbar\, \mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\, (3\,\lambda + \delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b})\right)} - \\
&\quad \frac{\mathrm{c1}\, p^2\, Z\, Z\Delta^3\, \hbar\, \mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\, (\lambda + \delta\lambda_{2\,a})}{6\left(-1 + \frac{1}{6}\, \mathrm{c1}\, Z\Delta^2\, \hbar\, \mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\, (3\,\lambda + \delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b})\right)} + \\
&\quad \frac{\mathrm{c1}\, Z\Delta^2\, \hbar\, \mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\, \delta m_1^2\, (\lambda + \delta\lambda_{2\,a})}{6\left(-1 + \frac{1}{6}\, \mathrm{c1}\, Z\Delta^2\, \hbar\, \mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\, (3\,\lambda + \delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b})\right)} + \\
&\quad \frac{\mathrm{c1}\, (-1+n)\, \mathrm{ssi}\, Z\Delta^4\, \hbar^2\, \mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\, (\lambda + \delta\lambda_{2\,a})^2}{36\left(-1 + \frac{1}{6}\, \mathrm{c1}\, Z\Delta^2\, \hbar\, \mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\, (3\,\lambda + \delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b})\right)} + \\
&\quad \frac{\mathrm{c1}\, (-1+n)\, \mathrm{tfing}\, Z\Delta^4\, \hbar^2\, \mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\, (\lambda + \delta\lambda_{2\,a})^2}{36\left(-1 + \frac{1}{6}\, \mathrm{c1}\, Z\Delta^2\, \hbar\, \mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\, (3\,\lambda + \delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b})\right)} + \\
&\quad \frac{\mathrm{c0}\, \mathrm{c1}\, (-1+n)\, Z\Delta^4\, \Lambda^2\, \hbar^2\, \mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\, (\lambda + \delta\lambda_{2\,a})^2}{36\left(-1 + \frac{1}{6}\, \mathrm{c1}\, Z\Delta^2\, \hbar\, \mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\, (3\,\lambda + \delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b})\right)} + \\
&\quad \frac{\mathrm{c1}\, v^2\, Z\Delta^3\, \hbar\, \mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\, (\lambda + \delta\lambda_{2\,a})\, (3\,\lambda + \delta\lambda_{a} + 2\,\delta\lambda_{b})}{36\left(-1 + \frac{1}{6}\, \mathrm{c1}\, Z\Delta^2\, \hbar\, \mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\, (3\,\lambda + \delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b})\right)} + \\
&\quad \frac{\mathrm{c1}\, \mathrm{tfinn}\, Z\Delta^4\, \hbar^2\, \mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\, (\lambda + \delta\lambda_{2\,a})\, (3\,\lambda + \delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b})}{36\left(-1 + \frac{1}{6}\, \mathrm{c1}\, Z\Delta^2\, \hbar\, \mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\, (3\,\lambda + \delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b})\right)} + \\
&\quad \frac{\mathrm{c0}\, \mathrm{c1}\, Z\Delta^4\, \Lambda^2\, \hbar^2\, \mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\, (\lambda + \delta\lambda_{2\,a})\, (3\,\lambda + \delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b})}{36\left(-1 + \frac{1}{6}\, \mathrm{c1}\, Z\Delta^2\, \hbar\, \mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\, (3\,\lambda + \delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b})\right)}\Bigg) \Bigg/ \\
&\Bigg(-1 + \frac{1}{6}\, \mathrm{c1}\, Z\Delta^2\, \hbar\, \mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\, ((1+n)\,\lambda + (-1+n)\,\delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b}) - \\
&\quad \frac{\mathrm{c1}^2\, (-1+n)\, Z\Delta^4\, \hbar^2\, \mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]^2\, (\lambda + \delta\lambda_{2\,a})^2}{36\left(-1 + \frac{1}{6}\, \mathrm{c1}\, Z\Delta^2\, \hbar\, \mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\, (3\,\lambda + \delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b})\right)}\Bigg)
\end{aligned}
$$

In[7]:= **mn2soln = mn2 /. (neom /. msbarrules /. mg2 → mg2soln // Solve[#, mn2][[1]] &)**

Out[7]=

$$\frac{1}{-1+\frac{1}{6}\,\text{c1}\,\text{Z}\Delta^2\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\,(3\,\lambda+\delta\lambda_{2\,a}+2\,\delta\lambda_{2\,b})}\left(-m^2-p^2+p^2\,\text{Z}\,\text{Z}\Delta-\delta m_1^2-\frac{1}{6}\,v^2\,\text{Z}\Delta\,(3\,\lambda+\delta\lambda_a+2\,\delta\lambda_b)-\frac{1}{6}\,\text{tfinn}\,\text{Z}\Delta^2\,\hbar\,(3\,\lambda+\delta\lambda_{2\,a}+2\,\delta\lambda_{2\,b})-\right.$$

$$\frac{1}{6}\,\text{c0}\,\text{Z}\Delta^2\,\Lambda^2\,\hbar\,(3\,\lambda+\delta\lambda_{2\,a}+2\,\delta\lambda_{2\,b})-\frac{1}{6}\,(-1+n)\,\text{Z}\Delta^2\,\hbar\,(\lambda+\delta\lambda_{2\,a})\left(\text{ssi}+\text{tfing}+\text{c0}\,\Lambda^2+\right.$$

$$\left(\text{c1}\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\left(-m^2-p^2+p^2\,\text{Z}\,\text{Z}\Delta-\delta m_1^2-\frac{1}{6}\,v^2\,\text{Z}\Delta\,(\lambda+\delta\lambda_a)-\frac{1}{6}\,\text{tfinn}\,\text{Z}\Delta^2\,\hbar\,(\lambda+\delta\lambda_{2\,a})-\right.\right.$$

$$\frac{1}{6}\,\text{c0}\,\text{Z}\Delta^2\,\Lambda^2\,\hbar\,(\lambda+\delta\lambda_{2\,a})-\frac{1}{6}\,\text{ssi}\,\text{Z}\Delta^2\,\hbar\,((1+n)\,\lambda+(-1+n)\,\delta\lambda_{2\,a}+2\,\delta\lambda_{2\,b})-$$

$$\frac{1}{6}\,\text{tfing}\,\text{Z}\Delta^2\,\hbar\,((1+n)\,\lambda+(-1+n)\,\delta\lambda_{2\,a}+2\,\delta\lambda_{2\,b})-\frac{1}{6}\,\text{c0}\,\text{Z}\Delta^2\,\Lambda^2\,\hbar\,((1+n)\,\lambda+$$

$$(-1+n)\,\delta\lambda_{2\,a}+2\,\delta\lambda_{2\,b})+\frac{\text{c1}\,m^2\,\text{Z}\Delta^2\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\,(\lambda+\delta\lambda_{2\,a})}{6\left(-1+\frac{1}{6}\,\text{c1}\,\text{Z}\Delta^2\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\,(3\,\lambda+\delta\lambda_{2\,a}+2\,\delta\lambda_{2\,b})\right)}+$$

$$\frac{\text{c1}\,p^2\,\text{Z}\Delta^2\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\,(\lambda+\delta\lambda_{2\,a})}{6\left(-1+\frac{1}{6}\,\text{c1}\,\text{Z}\Delta^2\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\,(3\,\lambda+\delta\lambda_{2\,a}+2\,\delta\lambda_{2\,b})\right)}-$$

$$\frac{\text{c1}\,p^2\,\text{Z}\,\text{Z}\Delta^3\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\,(\lambda+\delta\lambda_{2\,a})}{6\left(-1+\frac{1}{6}\,\text{c1}\,\text{Z}\Delta^2\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\,(3\,\lambda+\delta\lambda_{2\,a}+2\,\delta\lambda_{2\,b})\right)}+$$

$$\frac{\text{c1}\,\text{Z}\Delta^2\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\,\delta m_1^2\,(\lambda+\delta\lambda_{2\,a})}{6\left(-1+\frac{1}{6}\,\text{c1}\,\text{Z}\Delta^2\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\,(3\,\lambda+\delta\lambda_{2\,a}+2\,\delta\lambda_{2\,b})\right)}+$$

$$\frac{\text{c1}\,(-1+n)\,\text{ssi}\,\text{Z}\Delta^4\,\hbar^2\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\,(\lambda+\delta\lambda_{2\,a})^2}{36\left(-1+\frac{1}{6}\,\text{c1}\,\text{Z}\Delta^2\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\,(3\,\lambda+\delta\lambda_{2\,a}+2\,\delta\lambda_{2\,b})\right)}+$$

$$\frac{\text{c1}\,(-1+n)\,\text{tfing}\,\text{Z}\Delta^4\,\hbar^2\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\,(\lambda+\delta\lambda_{2\,a})^2}{36\left(-1+\frac{1}{6}\,\text{c1}\,\text{Z}\Delta^2\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\,(3\,\lambda+\delta\lambda_{2\,a}+2\,\delta\lambda_{2\,b})\right)}+$$

$$\frac{\text{c0}\,\text{c1}\,(-1+n)\,\text{Z}\Delta^4\,\Lambda^2\,\hbar^2\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\,(\lambda+\delta\lambda_{2\,a})^2}{36\left(-1+\frac{1}{6}\,\text{c1}\,\text{Z}\Delta^2\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\,(3\,\lambda+\delta\lambda_{2\,a}+2\,\delta\lambda_{2\,b})\right)}+$$

$$\frac{\text{c1}\,v^2\,\text{Z}\Delta^3\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\,(\lambda+\delta\lambda_{2\,a})\,(3\,\lambda+\delta\lambda_a+2\,\delta\lambda_b)}{36\left(-1+\frac{1}{6}\,\text{c1}\,\text{Z}\Delta^2\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\,(3\,\lambda+\delta\lambda_{2\,a}+2\,\delta\lambda_{2\,b})\right)}+$$

$$\left(\text{c1}\,\text{tfinn}\,\text{Z}\Delta^4\,\hbar^2\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\,(\lambda+\delta\lambda_{2\,a})\,(3\,\lambda+\delta\lambda_{2\,a}+2\,\delta\lambda_{2\,b})\right)\Big/$$

$$\left(36\left(-1+\frac{1}{6}\,\text{c1}\,\text{Z}\Delta^2\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\,(3\,\lambda+\delta\lambda_{2\,a}+2\,\delta\lambda_{2\,b})\right)\right)+$$

$$\left(\text{c0 c1 } Z\Delta^4\, \Lambda^2\, \hbar^2 \,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] (\lambda + \delta\lambda_{2\,a})\, (3\,\lambda + \delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b})\right) \Big/$$

$$\left(36 \left(-1 + \frac{1}{6}\,\text{c1 } Z\Delta^2\, \hbar\, \text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] (3\,\lambda + \delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b})\right)\right)\Bigg)\Bigg)\Bigg) \Big/$$

$$\Bigg(-1 + \frac{1}{6}\,\text{c1 } Z\Delta^2\, \hbar\, \text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] \left((1+n)\,\lambda + (-1+n)\,\delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b}\right) -$$

$$\frac{\text{c1}^2\,(-1+n)\, Z\Delta^4\, \hbar^2\, \text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]^2 (\lambda + \delta\lambda_{2\,a})^2}{36\left(-1 + \frac{1}{6}\,\text{c1 } Z\Delta^2\, \hbar\, \text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] (3\,\lambda + \delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b})\right)}\Bigg)\Bigg)\Bigg)$$

Gather divergences proportional v, tfing and tfinn and set independently to zero

First we subtract the finite equation of motion, then gather coefficients of the remainder into a list and set each to zero (after some trimming and simplifying).

In[8]:=
```
cteq =
 ((CoefficientList[mg2soln + (-m^2 - λ/6 v^2 - ħ/6 ((n + 1) λ) (tfing + ssi) - ħ/6 (λ) (tfinn)),
       {p, v, tfing, tfinn}] // Flatten) //
    DeleteDuplicates // Simplify // FullSimplify) == 0 // Thread
```

Out[8]=

$$\Bigg\{-\Bigg(\Bigg(\lambda\,\hbar\,\Bigg(-18\,(1+n)\,\text{ssi}\,\left(-1+Z\Delta^2\right) - 18\,\text{c0}\,(2+n)\,Z\Delta^2\,\Lambda^2 +$$

$$\text{c1 } Z\Delta^2\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] \Big(-18\,m^2\,(2+n) + 3\,\lambda\,\big(-(1+n)\,(4+n)\,\text{ssi} + 2\,(2+n)\,\text{ssi}\,Z\Delta^2 +$$

$$2\,\text{c0}\,(2+n)\,Z\Delta^2\,\Lambda^2\big)\,\hbar + \text{c1}\,(2+n)\,Z\Delta^2\,\lambda\,\hbar\,\left(6\,m^2 + (1+n)\,\text{ssi}\,\lambda\,\hbar\right)\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\Big)\Bigg)\Bigg) +$$

$$36\,\delta m_1^2 \left(-3 + \text{c1 } Z\Delta^2\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] + \text{c1 } Z\Delta^2\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\delta\lambda_{2\,b}\right) +$$

$$Z\Delta^2\,\hbar\,\Bigg(\delta\lambda_{2\,b}\,\Bigg(-36\,\left(\text{ssi} + \text{c0}\,\Lambda^2\right) +$$

$$\text{c1 Log}\left[\frac{\Lambda^2}{\mu^2}\right]\Big(-36\,m^2 + 6\,\lambda\,\left(-2\,(1+n)\,\text{ssi} + (4+n)\,\text{ssi}\,Z\Delta^2 + \text{c0}\,(4+n)\,Z\Delta^2\,\Lambda^2\right)\hbar +$$

$$\text{c1}\,(4+n)\,Z\Delta^2\,\lambda\,\hbar\,\left(6\,m^2 + (1+n)\,\text{ssi}\,\lambda\,\hbar\right)\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\Big) + 2\,\text{c1 } Z\Delta^2\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]$$

$$\left(6\,\left(\text{ssi} + \text{c0}\,\Lambda^2\right) + \text{c1}\,\left(6\,m^2 + (1+n)\,\text{ssi}\,\lambda\,\hbar\right)\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\right)\delta\lambda_{2\,b}\Bigg) + \delta\lambda_{2\,a}$$

$$\Bigg(-18\,\left((-1+n)\,\text{ssi} + \text{c0 n}\,\Lambda^2\right) + \text{c1 n Log}\left[\frac{\Lambda^2}{\mu^2}\right]\Big(-18\,m^2 - 3\,\lambda\,\big(\text{ssi} + \text{n ssi} - 2\,\text{ssi}\,Z\Delta^2 -$$

$$2\,\text{c0 } Z\Delta^2\,\Lambda^2\big)\,\hbar + \text{c1 } Z\Delta^2\,\lambda\,\hbar\,\left(6\,m^2 + (1+n)\,\text{ssi}\,\lambda\,\hbar\right)\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\Big) + \text{c1 n } Z\Delta^2\,\hbar$$

$$\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\left(6\left(\text{ssi}+\text{c0}\,\Lambda^2\right)+\text{c1}\left(6\,\text{m}^2+(1+\text{n})\,\text{ssi}\,\lambda\,\hbar\right)\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\right)\delta\lambda_{2\,\text{b}}\Big)\Big)\Big)\Big/$$
$$\left(6\left(-3+\text{c1}\,\text{Z}\Delta^2\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]+\text{c1}\,\text{Z}\Delta^2\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\delta\lambda_{2\,\text{b}}\right)\right.$$
$$\left.\left(-6+\text{c1}\,(2+\text{n})\,\text{Z}\Delta^2\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]+\text{c1}\,\text{Z}\Delta^2\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\left(\text{n}\,\delta\lambda_{2\,\text{a}}+2\,\delta\lambda_{2\,\text{b}}\right)\right)\right)\Big)=0,$$

$$-\frac{\lambda\,\hbar}{6}+\left(3\,\text{Z}\Delta^2\,\hbar\,(\lambda+\delta\lambda_{2\,\text{a}})\right)\Big/\left(\left(-3+\text{c1}\,\text{Z}\Delta^2\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]+\text{c1}\,\text{Z}\Delta^2\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\delta\lambda_{2\,\text{b}}\right)\right.$$
$$\left.\left(-6+\text{c1}\,(2+\text{n})\,\text{Z}\Delta^2\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]+\text{c1}\,\text{Z}\Delta^2\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\left(\text{n}\,\delta\lambda_{2\,\text{a}}+2\,\delta\lambda_{2\,\text{b}}\right)\right)\right)=0,$$

$$-\frac{1}{6\,\text{c1}\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]}\left(6+\text{c1}\,(1+\text{n})\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]+\right.$$
$$\frac{18}{\text{n}\left(-3+\text{c1}\,\text{Z}\Delta^2\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]+\text{c1}\,\text{Z}\Delta^2\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\delta\lambda_{2\,\text{b}}\right)}+$$
$$(36\,(-1+\text{n}))\Big/$$
$$\left.\left(\text{n}\left(-6+\text{c1}\,(2+\text{n})\,\text{Z}\Delta^2\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]+\text{c1}\,\text{Z}\Delta^2\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\left(\text{n}\,\delta\lambda_{2\,\text{a}}+2\,\delta\lambda_{2\,\text{b}}\right)\right)\right)\right)=0,$$

$$\text{True},\;-\frac{\lambda}{6}+\frac{\text{Z}\Delta\,(\lambda+\delta\lambda_{\text{b}})}{\text{n}\left(-3+\text{c1}\,\text{Z}\Delta^2\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]+\text{c1}\,\text{Z}\Delta^2\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\delta\lambda_{2\,\text{b}}\right)}-$$
$$\left(\text{Z}\Delta\,((2+\text{n})\,\lambda+\text{n}\,\delta\lambda_{\text{a}}+2\,\delta\lambda_{\text{b}})\right)\Big/$$
$$\left(\text{n}\left(-6+\text{c1}\,(2+\text{n})\,\text{Z}\Delta^2\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]+\text{c1}\,\text{Z}\Delta^2\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\left(\text{n}\,\delta\lambda_{2\,\text{a}}+2\,\delta\lambda_{2\,\text{b}}\right)\right)\right)=0,$$

$$(-6+6\,\text{Z}\,\text{Z}\Delta)\Big/\left(-6+\text{c1}\,(2+\text{n})\,\text{Z}\Delta^2\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]+\right.$$
$$\left.\text{c1}\,\text{Z}\Delta^2\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\left(\text{n}\,\delta\lambda_{2\,\text{a}}+2\,\delta\lambda_{2\,\text{b}}\right)\right)=0\Big\}$$

In[9]:=

```
cteq2 =
  ((CoefficientList[mn2soln + (-m^2 - λ/2 v^2 - ħ/6 ((n - 1) λ) (tfing + ssi) - ħ/2 (λ) (tfinn)),
        {p, v, tfing, tfinn}] // Flatten) //
      DeleteDuplicates // Simplify // FullSimplify) == 0 // Thread
```

Out[9]=

$$\Big\{-\Big(\Big(\Big(\lambda\,\hbar\,\Big(-18\,(-1+\text{n})\,\text{ssi}\left(-1+\text{Z}\Delta^2\right)-18\,\text{c0}\,(2+\text{n})\,\text{Z}\Delta^2\,\Lambda^2+$$
$$\text{c1}\,\text{Z}\Delta^2\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\Big(-18\,\text{m}^2\,(2+\text{n})+3\,\lambda\left(-(-1+\text{n})\,(4+\text{n})\,\text{ssi}+2\,\text{c0}\,(2+\text{n})\,\text{Z}\Delta^2\,\Lambda^2\right)\hbar+$$
$$\text{c1}\,(2+\text{n})\,\text{Z}\Delta^2\,\lambda\,\hbar\left(6\,\text{m}^2+(-1+\text{n})\,\text{ssi}\,\lambda\,\hbar\right)\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\Big)\Big)+$$
$$36\,\delta\text{m}_1^2\left(-3+\text{c1}\,\text{Z}\Delta^2\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]+\text{c1}\,\text{Z}\Delta^2\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\delta\lambda_{2\,\text{b}}\right)+\text{Z}\Delta^2\,\hbar$$

$$\left(\delta\lambda_{2\,b}\left(-36\,c0\,\Lambda^2+c1\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\left(6\left(-6\,m^2+\lambda\left(-2\,(-1+n)\,ssi+c0\,(4+n)\,Z\Delta^2\,\Lambda^2\right)\hbar\right)+c1\,(4+n)\,Z\Delta^2\,\lambda\,\hbar\left(6\,m^2+(-1+n)\,ssi\,\lambda\,\hbar\right)\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\right)+2\,c1\,Z\Delta^2\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\left(6\,c0\,\Lambda^2+c1\left(6\,m^2+(-1+n)\,ssi\,\lambda\,\hbar\right)\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\right)\delta\lambda_{2\,b}\right)+\delta\lambda_{2\,a}\left(-18\left((-1+n)\,ssi+c0\,n\,\Lambda^2\right)+c1\,n\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\left(3\left(-6\,m^2+\lambda\left(ssi-n\,ssi+2\,c0\,Z\Delta^2\,\Lambda^2\right)\hbar\right)+c1\,Z\Delta^2\,\lambda\,\hbar\left(6\,m^2+(-1+n)\,ssi\,\lambda\,\hbar\right)\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\right)+c1\,n\,Z\Delta^2\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\left(6\,c0\,\Lambda^2+c1\left(6\,m^2+(-1+n)\,ssi\,\lambda\,\hbar\right)\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\right)\delta\lambda_{2\,b}\right)\right)\Big/\left(6\left(-3+c1\,Z\Delta^2\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]+c1\,Z\Delta^2\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\delta\lambda_{2\,b}\right)\left(-6+c1\,(2+n)\,Z\Delta^2\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]+c1\,Z\Delta^2\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\left(n\,\delta\lambda_{2\,a}+2\,\delta\lambda_{2\,b}\right)\right)\right)=0,$$

$$-\frac{1}{2\,c1\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]}\left(2+c1\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]+\frac{6\,(-1+n)}{n\left(-3+c1\,Z\Delta^2\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]+c1\,Z\Delta^2\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\delta\lambda_{2\,b}\right)}+12\Big/\left(n\left(-6+c1\,(2+n)\,Z\Delta^2\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]+c1\,Z\Delta^2\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\left(n\,\delta\lambda_{2\,a}+2\,\delta\lambda_{2\,b}\right)\right)\right)\right)=0,$$

$$\left((-1+n)\,\hbar\left(-Z\Delta^2\,\delta\lambda_{2\,a}\left(-18+c1\,n\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\left(-3+c1\,Z\Delta^2\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\right)+c1^2\,n\,Z\Delta^2\,\lambda\,\hbar^2\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]^2\delta\lambda_{2\,b}\right)+\lambda\left(18\left(-1+Z\Delta^2\right)+c1\,Z\Delta^2\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\left(3\,(4+n)-c1\,(2+n)\,Z\Delta^2\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\right)-c1\,Z\Delta^2\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\delta\lambda_{2\,b}\left(-12+c1\,(4+n)\,Z\Delta^2\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]+2\,c1\,Z\Delta^2\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\delta\lambda_{2\,b}\right)\right)\right)\right)\Big/\left(6\left(-3+c1\,Z\Delta^2\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]+c1\,Z\Delta^2\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\delta\lambda_{2\,b}\right)\left(-6+c1\,(2+n)\,Z\Delta^2\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]+c1\,Z\Delta^2\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\left(n\,\delta\lambda_{2\,a}+2\,\delta\lambda_{2\,b}\right)\right)\right)=0,$$

$$\text{True},\;-\frac{\lambda}{2}-\frac{(-1+n)\,Z\Delta\,(\lambda+\delta\lambda_b)}{n\left(-3+c1\,Z\Delta^2\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]+c1\,Z\Delta^2\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\delta\lambda_{2\,b}\right)}-\left(Z\Delta\left((2+n)\,\lambda+n\,\delta\lambda_a+2\,\delta\lambda_b\right)\right)\Big/\left(n\left(-6+c1\,(2+n)\,Z\Delta^2\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]+c1\,Z\Delta^2\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\left(n\,\delta\lambda_{2\,a}+2\,\delta\lambda_{2\,b}\right)\right)\right)=0,$$

$$(-6+6\,Z\,Z\Delta)\Big/\left(-6+c1\,(2+n)\,Z\Delta^2\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]+\right.$$

$$\text{c1}\, Z\Delta^2\, \hbar\, \text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] \left(n\, \delta\lambda_{2\,a} + 2\, \delta\lambda_{2\,b}\right)\Bigg) == 0\Big\}$$

Solve for counterterms

In[10]:= `cteqs = {cteq, cteq2} // Flatten // FullSimplify // DeleteDuplicates`

Out[10]=

$$\Bigg\{\frac{1}{\text{c1}\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]}\Bigg(6\,\text{ssi} + 6\,\text{c0}\,\Lambda^2 + 6\,\text{c1}\,m^2\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] + \text{c1}\,\text{ssi}\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] + \text{c1}\,n\,\text{ssi}\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] + \frac{18\,\text{ssi}}{n\left(-3 + \text{c1}\,Z\Delta^2\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] + \text{c1}\,Z\Delta^2\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\delta\lambda_{2\,b}\right)} + \left(36\left((-1+n)\,\text{ssi} + \text{c0}\,n\,\Lambda^2 + \text{c1}\,m^2\,n\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] + \text{c1}\,n\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\delta m_1^2\right)\right) \Big/ \left(n\left(-6 + \text{c1}\,(2+n)\,Z\Delta^2\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] + \text{c1}\,Z\Delta^2\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\left(n\,\delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b}\right)\right)\right)\Bigg) == 0,$$

$$\hbar\left(\lambda - \left(18\,Z\Delta^2\,(\lambda + \delta\lambda_{2\,a})\right)\Big/\left(\left(-3 + \text{c1}\,Z\Delta^2\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] + \text{c1}\,Z\Delta^2\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\delta\lambda_{2\,b}\right)\left(-6 + \text{c1}\,(2+n)\,Z\Delta^2\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] + \text{c1}\,Z\Delta^2\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\left(n\,\delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b}\right)\right)\right)\right) == 0,$$

$$\frac{1}{\text{c1}\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]}\Bigg(6 + \text{c1}\,(1+n)\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] + \frac{18}{n\left(-3 + \text{c1}\,Z\Delta^2\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] + \text{c1}\,Z\Delta^2\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\delta\lambda_{2\,b}\right)} + (36\,(-1+n)) \Big/ \left(n\left(-6 + \text{c1}\,(2+n)\,Z\Delta^2\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] + \text{c1}\,Z\Delta^2\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\left(n\,\delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b}\right)\right)\right)\Bigg) == 0,$$

$$\text{True},\ \lambda + \left(6\,Z\Delta\,\left((2+n)\,\lambda + n\,\delta\lambda_a + 2\,\delta\lambda_b\right)\right) \Big/ \left(n\left(-6 + \text{c1}\,(2+n)\,Z\Delta^2\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] + \text{c1}\,Z\Delta^2\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\left(n\,\delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b}\right)\right)\right) == \frac{6\,Z\Delta\,(\lambda + \delta\lambda_b)}{n\left(-3 + \text{c1}\,Z\Delta^2\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] + \text{c1}\,Z\Delta^2\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\delta\lambda_{2\,b}\right)},$$

$$(-1 + Z\,Z\Delta) \Big/ \left(-6 + \text{c1}\,(2+n)\,Z\Delta^2\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] + \text{c1}\,Z\Delta^2\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\left(n\,\delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b}\right)\right) == 0,$$

$$\frac{1}{\text{c1}\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]}\Bigg(6\,\text{c0}\,\Lambda^2 + 6\,\text{c1}\,m^2\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] - \text{c1}\,\text{ssi}\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] + \text{c1}\,n\,\text{ssi}\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] - \frac{18\,(-1+n)\,\text{ssi}}{n\left(-3 + \text{c1}\,Z\Delta^2\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] + \text{c1}\,Z\Delta^2\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\delta\lambda_{2\,b}\right)} + \left(36\left((-1+n)\,\text{ssi} + \text{c0}\,n\,\Lambda^2 + \text{c1}\,m^2\,n\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] + \text{c1}\,n\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\delta m_1^2\right)\right) \Big/ \left(n\left(-6 + \text{c1}\,(2+n)\,Z\Delta^2\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] + \text{c1}\,Z\Delta^2\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\left(n\,\delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b}\right)\right)\right)\Bigg) == 0,$$

$$\frac{1}{\text{c1 Log}\left[\frac{\Lambda^2}{\mu^2}\right]}\left(2+\text{c1}\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]+\frac{6\,(-1+n)}{n\left(-3+\text{c1 Z}\Delta^2\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]+\text{c1 Z}\Delta^2\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\delta\lambda_{2\,b}\right)}+\right.$$
$$\left.12\Big/\left(n\left(-6+\text{c1}\,(2+n)\,\text{Z}\Delta^2\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]+\text{c1 Z}\Delta^2\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right](n\,\delta\lambda_{2\,a}+2\,\delta\lambda_{2\,b})\right)\right)\right)==0,$$

$$(-1+n)\,\hbar\left(\lambda-\left(18\,\text{Z}\Delta^2\,(\lambda+\delta\lambda_{2\,a})\right)\Big/\left(\left(-3+\text{c1 Z}\Delta^2\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]+\text{c1 Z}\Delta^2\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\delta\lambda_{2\,b}\right)\right.\right.$$
$$\left.\left.\left(-6+\text{c1}\,(2+n)\,\text{Z}\Delta^2\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]+\text{c1 Z}\Delta^2\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right](n\,\delta\lambda_{2\,a}+2\,\delta\lambda_{2\,b})\right)\right)\right)==0,$$

$$\lambda+\frac{2\,(-1+n)\,\text{Z}\Delta\,(\lambda+\delta\lambda_b)}{n\left(-3+\text{c1 Z}\Delta^2\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]+\text{c1 Z}\Delta^2\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\delta\lambda_{2\,b}\right)}+\left(2\,\text{Z}\Delta\,((2+n)\,\lambda+n\,\delta\lambda_a+2\,\delta\lambda_b)\right)\Big/$$
$$\left(n\left(-6+\text{c1}\,(2+n)\,\text{Z}\Delta^2\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]+\text{c1 Z}\Delta^2\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right](n\,\delta\lambda_{2\,a}+2\,\delta\lambda_{2\,b})\right)\right)==0\Big\}$$

In[11]:=
```
ctsolns =
  Solve[cteqs, {δm1, δλ1a, δλ2a, δλ1b, δλ2b, Z, ZΔ}] // FullSimplify // DeleteDuplicates
```

Solve::svars : Equations may not give solutions for all "solve" variables. ≫

Out[11]=
$$\Big\{\Big\{\delta m_1\to-\frac{i\sqrt{2+n}\,\sqrt{\lambda}\,\sqrt{\hbar}\,\sqrt{\text{c0}\,\Lambda^2+\text{c1}\,m^2\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]}}{\sqrt{6+\text{c1}\,(2+n)\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]}},$$
$$\delta\lambda_a\to\lambda\left(-1+\frac{6\,(2+n)}{n\,\text{Z}\Delta\left(6+\text{c1}\,(2+n)\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\right)}-\frac{6}{3\,n\,\text{Z}\Delta+\text{c1}\,n\,\text{Z}\Delta\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]}\right),$$
$$\delta\lambda_{2\,a}\to\lambda\left(-1+\frac{18}{\text{Z}\Delta^2\left(3+\text{c1}\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\right)\left(6+\text{c1}\,(2+n)\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\right)}\right),$$
$$\delta\lambda_b\to\lambda\left(-1+\frac{3}{3\,\text{Z}\Delta+\text{c1 Z}\Delta\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]}\right),\ \delta\lambda_{2\,b}\to\lambda\left(-1+\frac{3}{\text{Z}\Delta^2\left(3+\text{c1}\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\right)}\right),\ \text{Z}\to\frac{1}{\text{Z}\Delta}\Big\},$$

$$\Big\{\delta m_1\to\frac{i\sqrt{2+n}\,\sqrt{\lambda}\,\sqrt{\hbar}\,\sqrt{\text{c0}\,\Lambda^2+\text{c1}\,m^2\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]}}{\sqrt{6+\text{c1}\,(2+n)\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]}},$$
$$\delta\lambda_a\to\lambda\left(-1+\frac{6\,(2+n)}{n\,\text{Z}\Delta\left(6+\text{c1}\,(2+n)\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\right)}-\frac{6}{3\,n\,\text{Z}\Delta+\text{c1}\,n\,\text{Z}\Delta\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]}\right),$$
$$\delta\lambda_{2\,a}\to\lambda\left(-1+\frac{18}{\text{Z}\Delta^2\left(3+\text{c1}\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\right)\left(6+\text{c1}\,(2+n)\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\right)}\right),$$
$$\delta\lambda_b\to\lambda\left(-1+\frac{3}{3\,\text{Z}\Delta+\text{c1 Z}\Delta\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]}\right),\ \delta\lambda_{2\,b}\to\lambda\left(-1+\frac{3}{\text{Z}\Delta^2\left(3+\text{c1}\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\right)}\right),\ \text{Z}\to\frac{1}{\text{Z}\Delta}\Big\}\Big\}$$

In[12]:=
```
cts = {δm1^2, δλ1a, δλ2a, δλ1b, δλ2b, Z, ZΔ} /. ctsolns // FullSimplify // DeleteDuplicates
```

Out[12]=
$$\left\{\left\{-\frac{(2+n)\,\lambda\,\hbar\,\left(c0\,\Lambda^2+c1\,m^2\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\right)}{6+c1\,(2+n)\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]},\right.\right.$$
$$\lambda\left(-1+\frac{6\,(2+n)}{n\,Z\Delta\,\left(6+c1\,(2+n)\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\right)}-\frac{6}{3\,n\,Z\Delta+c1\,n\,Z\Delta\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]}\right),$$
$$\lambda\left(-1+\frac{18}{Z\Delta^2\,\left(3+c1\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\right)\,\left(6+c1\,(2+n)\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\right)}\right),$$
$$\left.\left.\lambda\left(-1+\frac{3}{3\,Z\Delta+c1\,Z\Delta\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]}\right),\ \lambda\left(-1+\frac{3}{Z\Delta^2\,\left(3+c1\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\right)}\right),\ \frac{1}{Z\Delta},\ Z\Delta\right\}\right\}$$

ZΔ is redundant in this truncation, can remove it :

In[13]:=
```
cts /. ZΔ → 1 // FullSimplify
```

Out[13]=
$$\left\{\left\{-\frac{(2+n)\,\lambda\,\hbar\,\left(c0\,\Lambda^2+c1\,m^2\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\right)}{6+c1\,(2+n)\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]},\right.\right.$$
$$\lambda\left(-1-\frac{6}{3\,n+c1\,n\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]}+\frac{6\,(2+n)}{n\,\left(6+c1\,(2+n)\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\right)}\right),$$
$$\lambda\left(-1+\frac{18}{\left(3+c1\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\right)\,\left(6+c1\,(2+n)\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\right)}\right),$$
$$\left.\left.\lambda\left(-1+\frac{3}{3+c1\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]}\right),\ \lambda\left(-1+\frac{3}{3+c1\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]}\right),\ 1,\ 1\right\}\right\}$$

In[14]:=
```
mg2soln /. ctsolns /. ZΔ → 1 // FullSimplify // DeleteDuplicates
```

Out[14]=
$$\left\{\frac{1}{6}\left(6\,m^2+\lambda\,\left(v^2+((1+n)\,(ssi+tfing)+tfinn)\,\hbar\right)\right)\right\}$$

In[15]:=
```
mn2 /. ((neom /. msbarrules /. mg2 → mg2soln /. ctsolns /. ZΔ → 1 // FullSimplify //
      DeleteDuplicates) // Solve[#, mn2] &) // FullSimplify
```

Out[15]=
$$\left\{\frac{1}{6}\left(6\,m^2+3\,v^2\,\lambda+((-1+n)\,(ssi+tfing)+3\,tfinn)\,\lambda\,\hbar\right)\right\}$$

In[16]:=
```
rnveom =
  veom /. {mg2 → m^2 + λ/6 v^2 + ħ/6 ((n + 1) λ) (tfing + ssi) + ħ/6 (λ) (tfinn), mn2 → m^2 + λ/2 v^2 +
       ħ/6 ((n - 1) λ) (tfing + ssi) + ħ/2 (λ) (tfinn)} // Simplify // DeleteDuplicates
```

Out[16]=
$$\frac{1}{6}\,v\left(6\,ssi2+\frac{6\,\left(m^2+\delta m_0^2\right)}{Z\Delta}+v^2\,(\lambda+\delta\lambda_0)+\right.$$
$$\left.(-1+n)\,(ssi+tfing+t\infty g)\,Z\Delta\,\hbar\,(\lambda+\delta\lambda_a)+(tfinn+t\infty n)\,Z\Delta\,\hbar\,(3\,\lambda+\delta\lambda_a+2\,\delta\lambda_b)\right)$$

In[17]:=
```
veomCtEqs =
 (((CoefficientList[((1/v) rnveom - (m^2 + λ/6 v^2 + ℏ/6 ((n - 1) λ) (tfing + ssi) + ℏ/2 (λ) (tfinn) +
          ssi2)) /. msbarrules /. {mg2 → m^2 + λ/6 v^2 + ℏ/6 ((n + 1) λ) (tfing +
             ssi) + ℏ/6 (λ) (tfinn), mn2 → m^2 + λ/2 v^2 + ℏ/6 ((n - 1) λ) (tfing +
             ssi) + ℏ/2 (λ) (tfinn)} // Simplify // Expand // FullSimplify,
       {v, tfing, tfinn}] // Simplify // Flatten) // DeleteDuplicates //
     Simplify // FullSimplify // DeleteDuplicates) == 0 // Thread)
```

Out[17]=

$$\Big\{\frac{1}{36\,Z\Delta}\Big(-36\,m^2\,(-1+Z\Delta)+6\,Z\Delta\,\lambda\,\big((-1+n)\,ssi\,(-1+Z\Delta)+c0\,(2+n)\,Z\Delta\,\Lambda^2\big)\,\hbar+ c1\,Z\Delta^2\,\lambda\,\hbar\,\big(6\,m^2\,(2+n)+(-1+n)\,(4+n)\,ssi\,\lambda\,\hbar\big)\,\mathrm{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big]+36\,\delta m_0^2+ Z\Delta^2\,\hbar\,\Big(\Big(6\,(-1+n)\,ssi+6\,c0\,n\,\Lambda^2+c1\,\big(6\,m^2\,n+(-2+n+n^2)\,ssi\,\lambda\,\hbar\big)\,\mathrm{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big]\Big)\,\delta\lambda_a+ 2\,\Big(6\,c0\,\Lambda^2+c1\,\big(6\,m^2+(-1+n)\,ssi\,\lambda\,\hbar\big)\,\mathrm{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big]\Big)\,\delta\lambda_b\Big)\Big)==0,$$

$$\frac{1}{36}\,\hbar\,\Big(\lambda\,\Big(18\,(-1+Z\Delta)+c1\,(8+n)\,Z\Delta\,\lambda\,\hbar\,\mathrm{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big]\Big)+Z\Delta\,\Big(6+c1\,(2+n)\,\lambda\,\hbar\,\mathrm{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big]\Big)\,\delta\lambda_a+ 6\,Z\Delta\,\Big(2+c1\,\lambda\,\hbar\,\mathrm{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big]\Big)\,\delta\lambda_b\Big)==0,$$

$$\frac{1}{36}\,(-1+n)\,\hbar\,\Big(Z\Delta\,\Big(6+c1\,(2+n)\,\lambda\,\hbar\,\mathrm{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big]\Big)\,\delta\lambda_a+ \lambda\,\Big(6\,(-1+Z\Delta)+c1\,(4+n)\,Z\Delta\,\lambda\,\hbar\,\mathrm{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big]+2\,c1\,Z\Delta\,\hbar\,\mathrm{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big]\,\delta\lambda_b\Big)\Big)==0,$$

$$\mathrm{True},\ \frac{1}{36}\Big(6\,\delta\lambda_0+c1\,Z\Delta\,\lambda\,\hbar\,\mathrm{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big]\,\big((8+n)\,\lambda+(2+n)\,\delta\lambda_a+6\,\delta\lambda_b\big)\Big)==0\Big\}$$

In[18]:=
```
ctegs3 = (veomCtEqs /. ctsolns // Simplify // DeleteDuplicates // FullSimplify)[[1]]
```

Out[18]=

$$\Big\{\Big(-6\,m^2\,(-1+Z\Delta)+c0\,(2+n)\,Z\Delta\,\lambda\,\Lambda^2\,\hbar+c1\,m^2\,(2+n)\,\lambda\,\hbar\,\mathrm{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big]+ \Big(6+c1\,(2+n)\,\lambda\,\hbar\,\mathrm{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big]\Big)\,\delta m_0^2\Big)\Big/\Big(Z\Delta\,\Big(6+c1\,(2+n)\,\lambda\,\hbar\,\mathrm{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big]\Big)\Big)==0,\ \mathrm{True},$$

$$\mathrm{True},\ \mathrm{True},\ 3\,\lambda\,\Bigg(1+\frac{2-2\,n}{3\,n+c1\,n\,\lambda\,\hbar\,\mathrm{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big]}-\frac{2\,(2+n)}{n\,\Big(6+c1\,(2+n)\,\lambda\,\hbar\,\mathrm{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big]\Big)}\Bigg)+\delta\lambda_0==0\Big\}$$

In[19]:=
```
{δm0^2, δλ0} /. Solve[ctegs3, {δm0, δλ0}] /. ZΔ → 1 // DeleteDuplicates // Simplify
```

Out[19]=

$$\Bigg\{\Bigg\{-\frac{(2+n)\,\lambda\,\hbar\,\Big(c0\,\Lambda^2+c1\,m^2\,\mathrm{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big]\Big)}{6+c1\,(2+n)\,\lambda\,\hbar\,\mathrm{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big]},\ -\frac{3\,c1\,\lambda^2\,\hbar\,\mathrm{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big]\,\Big(8+n+c1\,(2+n)\,\lambda\,\hbar\,\mathrm{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big]\Big)}{\Big(3+c1\,\lambda\,\hbar\,\mathrm{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big]\Big)\,\Big(6+c1\,(2+n)\,\lambda\,\hbar\,\mathrm{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big]\Big)}\Bigg\}\Bigg\}$$

In[39]:= $\{\delta m_1{}^2 == \delta m_0{}^2,\ \delta\lambda_{1\,a} == \delta\lambda_{2\,a},\ \delta\lambda_{1\,b} == \delta\lambda_{2\,b}\}$ /. ctsolns /. Solve[ctegs3, {δm_0, $\delta\lambda_0$}] /. Z$\Delta \to$ 1 // FullSimplify // Flatten // DeleteDuplicates

Out[39]= {True}

In[35]:= $\left\{\frac{\delta\lambda_{1\,a}}{\delta\lambda_{1\,b}}\right\}$ /. ctsolns /. Solve[ctegs3, {δm_0, $\delta\lambda_0$}] /. Z$\Delta \to$ 1 // FullSimplify // Flatten // DeleteDuplicates

Out[35]= $\left\{1+\frac{3\ (2+n)}{6+c1\ (2+n)\ \lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]}\right\}$

In[28]:= $\delta\lambda_{1\,b} == \delta\lambda_{2\,b}$ /. ctsolns /. Z$\Delta \to$ 1 // FullSimplify // DeleteDuplicates

Out[28]= {True}

In[31]:= $\delta\lambda_{1\,b}$ /. ctsolns /. Z$\Delta \to$ 1 // FullSimplify // DeleteDuplicates

Out[31]= $\left\{\lambda\left(-1+\frac{3}{3+c1\ \lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]}\right)\right\}$

In[36]:= $\{\delta\lambda_0 == 1\ \delta\lambda_{1\,a} + 2\ \delta\lambda_{1\,b}\}$ /. ctsolns /. Solve[ctegs3, {δm_0, $\delta\lambda_0$}] /. Z$\Delta \to$ 1 // FullSimplify // Flatten // DeleteDuplicates

Out[36]= {True}

In[38]:= $\left\{\delta m_0{}^2 == -\frac{\left(c0\ \Lambda^2 + c1\ m^2\ \text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\right)\lambda\,\hbar}{3}\left(\frac{\delta\lambda_{1\,a}}{\delta\lambda_{1\,b}}-1\right)\right\}$ /. ctsolns /. Solve[ctegs3, {δm_0, $\delta\lambda_0$}] /. Z$\Delta \to$ 1 // FullSimplify // Flatten // DeleteDuplicates

Out[38]= {True}