Renormalization of 2PIEA gap equations in the Hartree-Fock approximation

Author: Michael Brown
Supplement to thesis Chapter 3

Mathematica notebook to compute couter-terms for two loop truncations of the two particle irreducible effective action

```
ClearAll[veom, geom, neom, divergentpartrules, mg2soln, cteq, cts, δm, δλ];
```

Hartree-Fock gap equations with counterterms

Goldstone equation of motion. Quantities in reference to the thesis are:
p is the four-momentum flowing through the propagators $\Delta_G{}^{-1}$ and $\Delta_N{}^{-1}$,
mg2 is the Goldstone mass squared $m_G{}^2$,
mn2 is the Higgs mass squared $m_H{}^2$,
Z and ZΔ are the wavefunction a propagator renormalization constants,
m^2 is the (renormalized) Lagrangian mass parameter, $\delta m_0{}^2$, $\delta m_1{}^2$ are its counter-terms,
λ is the (renormalized) four point coupling,
$\delta\lambda_0$, $\delta\lambda_{1a}$, $\delta\lambda_{1b}$, $\delta\lambda_{2a}$, $\delta\lambda_{2b}$ are the independent coupling counter-terms,
v is the scalar field vacuum expectation value,
$\hbar$ is the reduced Planck constant,
n is the number of fields in the O(n) symmetry group,
t∞g, t∞n are the divergent tadpole integrals for the Goldstone, Higgs resp.,
tfing, tfinn are the finite parts of the tadpoles for the Goldstone, Higgs resp.

Equations of motion

Vev equation of motion

```
veom = ZΔ^-1 (m^2 + δm0^2) v + (λ + δλ0)/6 v^3 +
  ħ/6 ZΔ (n - 1) (λ + δλ1a) v (t∞g + tfing) + ħ/6 ZΔ (3 λ + δλ1a + 2 δλ1b) v (t∞n + tfinn)
finveom = m^2 v + λ/6 v^3 + ħ/6 (n - 1) λ v tfing + ħ/2 λ v tfinn
```

$$\frac{v\left(m^2+\delta m_0^2\right)}{Z\Delta}+\frac{1}{6}v^3\left(\lambda+\delta\lambda_0\right)+$$
$$\frac{1}{6}(-1+n)\left(\text{tfing}+\text{t}\infty\text{g}\right)v\,Z\Delta\,\hbar\left(\lambda+\delta\lambda_a\right)+\frac{1}{6}\left(\text{tfinn}+\text{t}\infty\text{n}\right)v\,Z\Delta\,\hbar\left(3\lambda+\delta\lambda_a+2\,\delta\lambda_b\right)$$

$$m^2 v+\frac{v^3\lambda}{6}+\frac{1}{6}(-1+n)\,\text{tfing}\,v\,\lambda\,\hbar+\frac{1}{2}\text{tfinn}\,v\,\lambda\,\hbar$$

Goldstone equation of motion

```
geom = p^2 - mg2 == Z ZΔ p^2 - m^2 - δm1^2 - ZΔ (λ + δλ1a)/6 v^2 -
  ħ/6 ((n + 1) λ + (n - 1) δλ2a + 2 δλ2b) ZΔ^2 (t∞g + tfing) - ħ/6 (λ + δλ2a) ZΔ^2 (t∞n + tfinn)
finmg2 = mg2 /. Solve[p^2 - mg2 == p^2 - m^2 - λ/6 v^2 - ħ/6 (n + 1) λ tfing - ħ/6 λ tfinn, mg2][[1]]
```

$$-\text{mg2}+p^2==-m^2+p^2\,Z\,Z\Delta-\delta m_1^2-\frac{1}{6}v^2\,Z\Delta\left(\lambda+\delta\lambda_a\right)-$$
$$\frac{1}{6}\left(\text{tfinn}+\text{t}\infty\text{n}\right)Z\Delta^2\,\hbar\left(\lambda+\delta\lambda_{2a}\right)-\frac{1}{6}\left(\text{tfing}+\text{t}\infty\text{g}\right)Z\Delta^2\,\hbar\left((1+n)\lambda+(-1+n)\,\delta\lambda_{2a}+2\,\delta\lambda_{2b}\right)$$

$$\frac{1}{6}\left(6\,m^2+v^2\lambda+\text{tfing}\,\lambda\,\hbar+n\,\text{tfing}\,\lambda\,\hbar+\text{tfinn}\,\lambda\,\hbar\right)$$

Higgs equation of motion

```
neom = p^2 - mn2 == Z ZΔ p^2 - m^2 - δm1^2 - ZΔ v^2 (3 λ + δλ1a + 2 δλ1b)/6 -
  ħ/6 (λ + δλ2a) (n - 1) ZΔ^2 (t∞g + tfing) - ħ/6 (3 λ + δλ2a + 2 δλ2b) ZΔ^2 (t∞n + tfinn)
finmn2 = mn2 /. Solve[p^2 - mn2 == p^2 - m^2 - v^2 λ/2 - ħ/6 λ (n - 1) tfing - ħ/2 λ tfinn, mn2][[1]]
```

$$-\text{mn2}+p^2==-m^2+p^2\,Z\,Z\Delta-\delta m_1^2-\frac{1}{6}(-1+n)\left(\text{tfing}+\text{t}\infty\text{g}\right)Z\Delta^2\,\hbar\left(\lambda+\delta\lambda_{2a}\right)-$$
$$\frac{1}{6}v^2\,Z\Delta\left(3\lambda+\delta\lambda_a+2\,\delta\lambda_b\right)-\frac{1}{6}\left(\text{tfinn}+\text{t}\infty\text{n}\right)Z\Delta^2\,\hbar\left(3\lambda+\delta\lambda_{2a}+2\,\delta\lambda_{2b}\right)$$

$$\frac{1}{6}\left(6\,m^2+3\,v^2\lambda-\text{tfing}\,\lambda\,\hbar+n\,\text{tfing}\,\lambda\,\hbar+3\,\text{tfinn}\,\lambda\,\hbar\right)$$

Infinite parts of tadpoles

c0, c1, Λ and μ are regularisation/renormalisation scheme dependent quantities

```
divergentpartrules = {t∞g → c0 Λ^2 + c1 mg2 Log[Λ^2 / μ^2], t∞n → c0 Λ^2 + c1 mn2 Log[Λ^2 / μ^2]}
```

$$\left\{t\infty g \to c0\,\Lambda^2 + c1\,mg2\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right],\ t\infty n \to c0\,\Lambda^2 + c1\,mn2\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\right\}$$

Sub in tadpole expressions, eliminate mn2 and solve for mg2

```
mn2fromneom = Solve[neom /. divergentpartrules, mn2][[1]]
```

$$\left\{mn2 \to \left(-m^2 - p^2 + p^2\,Z\,Z\Delta - \delta m_1^2 - \frac{1}{6}\,(-1+n)\,Z\Delta^2\,\hbar\left(tfing + c0\,\Lambda^2 + c1\,mg2\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\right)(\lambda + \delta\lambda_{2\,a}) - \frac{1}{6}\,v^2\,Z\Delta\,(3\,\lambda + \delta\lambda_a + 2\,\delta\lambda_b) - \frac{1}{6}\,tfinn\,Z\Delta^2\,\hbar\,(3\,\lambda + \delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b}) - \frac{1}{6}\,c0\,Z\Delta^2\,\Lambda^2\,\hbar\,(3\,\lambda + \delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b})\right) \Big/ \left(-1 + \frac{1}{6}\,c1\,Z\Delta^2\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right](3\,\lambda + \delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b})\right)\right\}$$

```
mg2soln = mg2 /. (geom /. divergentpartrules /. mn2fromneom // Solve[#, mg2][[1]] &)
```

$$\left(-m^2 - p^2 + p^2\, Z\, Z\Delta - \delta m_1^2 - \frac{1}{6} v^2\, Z\Delta\, (\lambda + \delta\lambda_a) - \frac{1}{6}\,\text{tfinn}\, Z\Delta^2\, \hbar\, (\lambda + \delta\lambda_{2\,a}) - \right.$$
$$\frac{1}{6}\,\text{c0}\, Z\Delta^2\, \Lambda^2\, \hbar\, (\lambda + \delta\lambda_{2\,a}) - \frac{1}{6}\,\text{tfing}\, Z\Delta^2\, \hbar\, ((1+n)\,\lambda + (-1+n)\,\delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b}) -$$
$$\frac{1}{6}\,\text{c0}\, Z\Delta^2\, \Lambda^2\, \hbar\, ((1+n)\,\lambda + (-1+n)\,\delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b}) +$$
$$\frac{\text{c1}\, m^2\, Z\Delta^2\, \hbar\, \text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] (\lambda + \delta\lambda_{2\,a})}{6\left(-1 + \frac{1}{6}\,\text{c1}\, Z\Delta^2\, \hbar\, \text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] (3\,\lambda + \delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b})\right)} +$$
$$\frac{\text{c1}\, p^2\, Z\Delta^2\, \hbar\, \text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] (\lambda + \delta\lambda_{2\,a})}{6\left(-1 + \frac{1}{6}\,\text{c1}\, Z\Delta^2\, \hbar\, \text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] (3\,\lambda + \delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b})\right)} -$$
$$\frac{\text{c1}\, p^2\, Z\, Z\Delta^3\, \hbar\, \text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] (\lambda + \delta\lambda_{2\,a})}{6\left(-1 + \frac{1}{6}\,\text{c1}\, Z\Delta^2\, \hbar\, \text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] (3\,\lambda + \delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b})\right)} +$$
$$\frac{\text{c1}\, Z\Delta^2\, \hbar\, \text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] \delta m_1^2\, (\lambda + \delta\lambda_{2\,a})}{6\left(-1 + \frac{1}{6}\,\text{c1}\, Z\Delta^2\, \hbar\, \text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] (3\,\lambda + \delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b})\right)} +$$
$$\frac{\text{c1}\,(-1+n)\,\text{tfing}\, Z\Delta^4\, \hbar^2\, \text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] (\lambda + \delta\lambda_{2\,a})^2}{36\left(-1 + \frac{1}{6}\,\text{c1}\, Z\Delta^2\, \hbar\, \text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] (3\,\lambda + \delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b})\right)} +$$
$$\frac{\text{c0}\,\text{c1}\,(-1+n)\, Z\Delta^4\, \Lambda^2\, \hbar^2\, \text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] (\lambda + \delta\lambda_{2\,a})^2}{36\left(-1 + \frac{1}{6}\,\text{c1}\, Z\Delta^2\, \hbar\, \text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] (3\,\lambda + \delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b})\right)} +$$
$$\frac{\text{c1}\, v^2\, Z\Delta^3\, \hbar\, \text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] (\lambda + \delta\lambda_{2\,a})\,(3\,\lambda + \delta\lambda_a + 2\,\delta\lambda_b)}{36\left(-1 + \frac{1}{6}\,\text{c1}\, Z\Delta^2\, \hbar\, \text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] (3\,\lambda + \delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b})\right)} +$$
$$\frac{\text{c1}\,\text{tfinn}\, Z\Delta^4\, \hbar^2\, \text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] (\lambda + \delta\lambda_{2\,a})\,(3\,\lambda + \delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b})}{36\left(-1 + \frac{1}{6}\,\text{c1}\, Z\Delta^2\, \hbar\, \text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] (3\,\lambda + \delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b})\right)} +$$
$$\left.\frac{\text{c0}\,\text{c1}\, Z\Delta^4\, \Lambda^2\, \hbar^2\, \text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] (\lambda + \delta\lambda_{2\,a})\,(3\,\lambda + \delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b})}{36\left(-1 + \frac{1}{6}\,\text{c1}\, Z\Delta^2\, \hbar\, \text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] (3\,\lambda + \delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b})\right)}\right) \Big/$$
$$\left(-1 + \frac{1}{6}\,\text{c1}\, Z\Delta^2\, \hbar\, \text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] ((1+n)\,\lambda + (-1+n)\,\delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b}) - \right.$$
$$\left.\frac{\text{c1}^2\,(-1+n)\, Z\Delta^4\, \hbar^2\, \text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]^2 (\lambda + \delta\lambda_{2\,a})^2}{36\left(-1 + \frac{1}{6}\,\text{c1}\, Z\Delta^2\, \hbar\, \text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] (3\,\lambda + \delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b})\right)}\right)$$

```
mn2soln = mn2 /. mn2fromneom /. mg2 → mg2soln // Simplify
```

$$-\frac{1}{6\left(-1+\frac{1}{6}\,\mathrm{c1}\,Z\Delta^2\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right](3\lambda+\delta\lambda_{2a}+2\,\delta\lambda_{2b})\right)}\Bigg(6m^2+6p^2-6p^2\,Z\,Z\Delta+6\,\delta m_1^2+v^2\,Z\Delta\,(3\lambda+\delta\lambda_a+2\,\delta\lambda_b)+$$
$$\mathrm{tfinn}\,Z\Delta^2\,\hbar\,(3\lambda+\delta\lambda_{2a}+2\,\delta\lambda_{2b})+\mathrm{c0}\,Z\Delta^2\,\Lambda^2\,\hbar\,(3\lambda+\delta\lambda_{2a}+2\,\delta\lambda_{2b})+$$
$$\Bigg((-1+n)\,Z\Delta^2\,\hbar\,(\lambda+\delta\lambda_{2a})\Bigg(18\,\mathrm{tfing}+18\,\mathrm{c0}\,\Lambda^2+18\,\mathrm{c1}\,m^2\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]+18\,\mathrm{c1}\,p^2\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]-$$
$$18\,\mathrm{c1}\,p^2\,Z\,Z\Delta\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]+3\,\mathrm{c1}\,v^2\,Z\Delta\,\lambda\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]-9\,\mathrm{c1}\,\mathrm{tfing}\,Z\Delta^2\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]+$$
$$3\,\mathrm{c1}\,\mathrm{tfinn}\,Z\Delta^2\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]-6\,\mathrm{c0}\,\mathrm{c1}\,Z\Delta^2\,\lambda\,\Lambda^2\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]-6\,\mathrm{c1}^2\,m^2\,Z\Delta^2\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]^2-$$
$$6\,\mathrm{c1}^2\,p^2\,Z\Delta^2\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]^2+6\,\mathrm{c1}^2\,p^2\,Z\,Z\Delta^3\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]^2-3\,\mathrm{c1}\,\mathrm{tfing}\,Z\Delta^2\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]$$
$$\delta\lambda_{2a}+3\,\mathrm{c1}\,\mathrm{tfinn}\,Z\Delta^2\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\delta\lambda_{2a}+\mathrm{c1}^2\,v^2\,Z\Delta^3\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]^2\delta\lambda_{2a}+\mathrm{c1}^2\,v^2\,Z\Delta^3\,\lambda$$
$$\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]^2\delta\lambda_b+\mathrm{c1}^2\,v^2\,Z\Delta^3\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]^2\delta\lambda_{2a}\,\delta\lambda_b-6\,\mathrm{c1}\,\mathrm{tfing}\,Z\Delta^2\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\delta\lambda_{2b}-$$
$$6\,\mathrm{c0}\,\mathrm{c1}\,Z\Delta^2\,\Lambda^2\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\delta\lambda_{2b}-6\,\mathrm{c1}^2\,m^2\,Z\Delta^2\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]^2\delta\lambda_{2b}-6\,\mathrm{c1}^2\,p^2\,Z\Delta^2\,\hbar$$
$$\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]^2\delta\lambda_{2b}+6\,\mathrm{c1}^2\,p^2\,Z\,Z\Delta^3\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]^2\delta\lambda_{2b}-\mathrm{c1}^2\,v^2\,Z\Delta^3\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]^2\delta\lambda_{2b}-$$
$$6\,\mathrm{c1}\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\delta m_1^2\left(-3+\mathrm{c1}\,Z\Delta^2\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]+\mathrm{c1}\,Z\Delta^2\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\delta\lambda_{2b}\right)-$$
$$\mathrm{c1}\,v^2\,Z\Delta\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\delta\lambda_a\left(-3+\mathrm{c1}\,Z\Delta^2\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]+\mathrm{c1}\,Z\Delta^2\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\delta\lambda_{2b}\right)\Bigg)\Bigg)\Bigg/$$
$$\Bigg(\left(-3+\mathrm{c1}\,Z\Delta^2\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]+\mathrm{c1}\,Z\Delta^2\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\delta\lambda_{2b}\right)\Bigg(-6+2\,\mathrm{c1}\,Z\Delta^2\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]+$$
$$\mathrm{c1}\,n\,Z\Delta^2\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]+\mathrm{c1}\,n\,Z\Delta^2\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\delta\lambda_{2a}+2\,\mathrm{c1}\,Z\Delta^2\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\delta\lambda_{2b}\Bigg)\Bigg)\Bigg)$$

Gather divergences proportional v, tfing and tfinn and set independently to zero

First we subtract the finite equation of motion, then gather coefficients of the remainder into a list and set each to zero (after some trimming and simplifying).

```
cteq = ((CoefficientList[mg2soln - finmg2, {p, v, tfing, tfinn}] // Flatten) //
    DeleteDuplicates // Simplify // FullSimplify) == 0 // Thread
```

$$\left\{-\left(\left(6\,\delta m_1^2 + Z\Delta^2\,\hbar\left(c0\,\Lambda^2 + c1\,m^2\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\right)\left((2+n)\,\lambda + n\,\delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b}\right)\right)\Big/\left(-6 + c1\,(2+n)\,Z\Delta^2\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right] + c1\,Z\Delta^2\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\left(n\,\delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b}\right)\right)\right) == 0,\right.$$

$$-\frac{\lambda\,\hbar}{6} + \left(3\,Z\Delta^2\,\hbar\,(\lambda + \delta\lambda_{2\,a})\right)\Big/\left(\left(-3 + c1\,Z\Delta^2\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right] + c1\,Z\Delta^2\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\delta\lambda_{2\,b}\right)\left(-6 + c1\,(2+n)\,Z\Delta^2\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right] + c1\,Z\Delta^2\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\left(n\,\delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b}\right)\right)\right) == 0,$$

$$-\frac{1}{6\,c1\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]}\left(6 + c1\,(1+n)\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right] + \frac{18}{n\left(-3 + c1\,Z\Delta^2\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right] + c1\,Z\Delta^2\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\delta\lambda_{2\,b}\right)} + (36\,(-1+n))\Big/\left(n\left(-6 + c1\,(2+n)\,Z\Delta^2\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right] + c1\,Z\Delta^2\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\left(n\,\delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b}\right)\right)\right)\right) == 0,\ \mathrm{True},$$

$$-\frac{\lambda}{6} + \frac{Z\Delta\,(\lambda + \delta\lambda_b)}{n\left(-3 + c1\,Z\Delta^2\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right] + c1\,Z\Delta^2\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\delta\lambda_{2\,b}\right)} - \left(Z\Delta\left((2+n)\,\lambda + n\,\delta\lambda_a + 2\,\delta\lambda_b\right)\right)\Big/\left(n\left(-6 + c1\,(2+n)\,Z\Delta^2\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right] + c1\,Z\Delta^2\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\left(n\,\delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b}\right)\right)\right) == 0,$$

$$\left.(-6 + 6\,Z\,Z\Delta)\Big/\left(-6 + c1\,(2+n)\,Z\Delta^2\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right] + c1\,Z\Delta^2\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\left(n\,\delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b}\right)\right) == 0\right\}$$

```
cteq2 = ((CoefficientList[mn2soln - finmn2, {p, v, tfing, tfinn}] // Flatten) //
      DeleteDuplicates // Simplify // FullSimplify) == 0 // Thread
```

$$\Big\{-\Big(\Big(6\,\delta m_1^2 + Z\Delta^2\,\hbar\,\Big(c0\,\Lambda^2 + c1\,m^2\,\mathrm{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big]\Big)\,((2+n)\,\lambda + n\,\delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b})\Big)\Big/\Big(-6 + c1\,(2+n)\,Z\Delta^2\,\lambda\,\hbar\,\mathrm{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big] + c1\,Z\Delta^2\,\hbar\,\mathrm{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big]\,(n\,\delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b})\Big)\Big) == 0,$$

$$-\frac{1}{2\,c1\,\mathrm{Log}\big[\frac{\Lambda^2}{\mu^2}\big]}\Bigg(2 + c1\,\lambda\,\hbar\,\mathrm{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big] + \frac{6\,(-1+n)}{n\,\Big(-3 + c1\,Z\Delta^2\,\lambda\,\hbar\,\mathrm{Log}\big[\frac{\Lambda^2}{\mu^2}\big] + c1\,Z\Delta^2\,\hbar\,\mathrm{Log}\big[\frac{\Lambda^2}{\mu^2}\big]\,\delta\lambda_{2\,b}\Big)} + 12\Big/\Big(n\,\Big(-6 + c1\,(2+n)\,Z\Delta^2\,\lambda\,\hbar\,\mathrm{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big] + c1\,Z\Delta^2\,\hbar\,\mathrm{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big]\,(n\,\delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b})\Big)\Big)\Bigg) == 0,$$

$$\Bigg((-1+n)\,\hbar\,\Bigg(-Z\Delta^2\,\delta\lambda_{2\,a}\,\Big(-18 + c1\,n\,\lambda\,\hbar\,\mathrm{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big]\,\Big(-3 + c1\,Z\Delta^2\,\lambda\,\hbar\,\mathrm{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big]\Big) + c1^2\,n\,Z\Delta^2\,\lambda\,\hbar^2\,\mathrm{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big]^2\,\delta\lambda_{2\,b}\Big) + \lambda\,\Big(18\,(-1 + Z\Delta^2) + c1\,Z\Delta^2\,\lambda\,\hbar\,\mathrm{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big]\,\Big(3\,(4+n) - c1\,(2+n)\,Z\Delta^2\,\lambda\,\hbar\,\mathrm{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big]\Big) - c1\,Z\Delta^2\,\hbar\,\mathrm{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big]\,\delta\lambda_{2\,b}\,\Big(-12 + c1\,(4+n)\,Z\Delta^2\,\lambda\,\hbar\,\mathrm{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big] + 2\,c1\,Z\Delta^2\,\hbar\,\mathrm{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big]\,\delta\lambda_{2\,b}\Big)\Big)\Bigg)\Bigg)\Big/\Bigg(6\,\Big(-3 + c1\,Z\Delta^2\,\lambda\,\hbar\,\mathrm{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big] + c1\,Z\Delta^2\,\hbar\,\mathrm{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big]\,\delta\lambda_{2\,b}\Big)\Big(-6 + c1\,(2+n)\,Z\Delta^2\,\lambda\,\hbar\,\mathrm{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big] + c1\,Z\Delta^2\,\hbar\,\mathrm{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big]\,(n\,\delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b})\Big)\Bigg) == 0,$$

$$\mathrm{True},\; -\frac{\lambda}{2} - \frac{(-1+n)\,Z\Delta\,(\lambda + \delta\lambda_b)}{n\,\Big(-3 + c1\,Z\Delta^2\,\lambda\,\hbar\,\mathrm{Log}\big[\frac{\Lambda^2}{\mu^2}\big] + c1\,Z\Delta^2\,\hbar\,\mathrm{Log}\big[\frac{\Lambda^2}{\mu^2}\big]\,\delta\lambda_{2\,b}\Big)} - (Z\Delta\,((2+n)\,\lambda + n\,\delta\lambda_a + 2\,\delta\lambda_b))\Big/\Big(n\,\Big(-6 + c1\,(2+n)\,Z\Delta^2\,\lambda\,\hbar\,\mathrm{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big] + c1\,Z\Delta^2\,\hbar\,\mathrm{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big]\,(n\,\delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b})\Big)\Big) == 0,$$

$$(-6 + 6\,Z\,Z\Delta)\Big/\Big(-6 + c1\,(2+n)\,Z\Delta^2\,\lambda\,\hbar\,\mathrm{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big] + c1\,Z\Delta^2\,\hbar\,\mathrm{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big]\,(n\,\delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b})\Big) == 0\Big\}$$

Solve for counterterms

Find counter-terms from the gap equations

```
cteqs = {cteq, cteq2} // Flatten // FullSimplify // DeleteDuplicates
```

$$\Big\{\left(6\,\delta m_1^2 + Z\Delta^2\,\hbar\left(c0\,\Lambda^2 + c1\,m^2\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\right)\left((2+n)\,\lambda + n\,\delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b}\right)\right)\Big/\left(-6 + c1\,(2+n)\,Z\Delta^2\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] + c1\,Z\Delta^2\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\left(n\,\delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b}\right)\right) == 0,$$

$$\hbar\left(\lambda - \left(18\,Z\Delta^2\,(\lambda + \delta\lambda_{2\,a})\right)\Big/\left(\left(-3 + c1\,Z\Delta^2\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] + c1\,Z\Delta^2\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\delta\lambda_{2\,b}\right)\left(-6 + c1\,(2+n)\,Z\Delta^2\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] + c1\,Z\Delta^2\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\left(n\,\delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b}\right)\right)\right)\right) == 0,$$

$$\frac{1}{c1\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]}\left(6 + c1\,(1+n)\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] + \frac{18}{n\left(-3 + c1\,Z\Delta^2\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] + c1\,Z\Delta^2\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\delta\lambda_{2\,b}\right)} + (36\,(-1+n))\Big/\left(n\left(-6 + c1\,(2+n)\,Z\Delta^2\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] + c1\,Z\Delta^2\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\left(n\,\delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b}\right)\right)\right)\right) == 0,$$

$$\text{True},\ \lambda + \left(6\,Z\Delta\left((2+n)\,\lambda + n\,\delta\lambda_a + 2\,\delta\lambda_b\right)\right)\Big/\left(n\left(-6 + c1\,(2+n)\,Z\Delta^2\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] + c1\,Z\Delta^2\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\left(n\,\delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b}\right)\right)\right) == \frac{6\,Z\Delta\,(\lambda + \delta\lambda_b)}{n\left(-3 + c1\,Z\Delta^2\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] + c1\,Z\Delta^2\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\delta\lambda_{2\,b}\right)},$$

$$(-1 + Z\,Z\Delta)\Big/\left(-6 + c1\,(2+n)\,Z\Delta^2\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] + c1\,Z\Delta^2\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\left(n\,\delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b}\right)\right) == 0,$$

$$\frac{1}{c1\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]}\left(2 + c1\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] + \frac{6\,(-1+n)}{n\left(-3 + c1\,Z\Delta^2\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] + c1\,Z\Delta^2\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\delta\lambda_{2\,b}\right)} + 12\Big/\left(n\left(-6 + c1\,(2+n)\,Z\Delta^2\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] + c1\,Z\Delta^2\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\left(n\,\delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b}\right)\right)\right)\right) == 0,$$

$$(-1+n)\,\hbar\left(\lambda - \left(18\,Z\Delta^2\,(\lambda + \delta\lambda_{2\,a})\right)\Big/\left(\left(-3 + c1\,Z\Delta^2\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] + c1\,Z\Delta^2\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\delta\lambda_{2\,b}\right)\left(-6 + c1\,(2+n)\,Z\Delta^2\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] + c1\,Z\Delta^2\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\left(n\,\delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b}\right)\right)\right)\right) == 0,$$

$$\lambda + \frac{2\,(-1+n)\,Z\Delta\,(\lambda + \delta\lambda_b)}{n\left(-3 + c1\,Z\Delta^2\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] + c1\,Z\Delta^2\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\delta\lambda_{2\,b}\right)} + \left(2\,Z\Delta\left((2+n)\,\lambda + n\,\delta\lambda_a + 2\,\delta\lambda_b\right)\right)\Big/\left(n\left(-6 + c1\,(2+n)\,Z\Delta^2\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] + c1\,Z\Delta^2\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\left(n\,\delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b}\right)\right)\right) == 0\Big\}$$

```
cts = {δm1^2, δλ1a, δλ2a, δλ1b, δλ2b, Z, ZΔ} /. Solve[cteqs,
    {δm1, δλ1a, δλ2a, δλ1b, δλ2b, Z, ZΔ}] // FullSimplify // DeleteDuplicates
```

Solve::svars : Equations may not give solutions for all "solve" variables. ≫

$$\left\{\left\{-\frac{(2+n)\,\lambda\,\hbar\left(c0\,\Lambda^2+c1\,m^2\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\right)}{6+c1\,(2+n)\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]},\right.\right.$$

$$\lambda\left(-1+\frac{6\,(2+n)}{n\,Z\Delta\left(6+c1\,(2+n)\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\right)}-\frac{6}{3\,n\,Z\Delta+c1\,n\,Z\Delta\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]}\right),$$

$$\lambda\left(-1+\frac{18}{Z\Delta^2\left(3+c1\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\right)\left(6+c1\,(2+n)\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\right)}\right),$$

$$\left.\left.\lambda\left(-1+\frac{3}{3\,Z\Delta+c1\,Z\Delta\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]}\right),\ \lambda\left(-1+\frac{3}{Z\Delta^2\left(3+c1\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\right)}\right),\ \frac{1}{Z\Delta},\ Z\Delta\right\}\right\}$$

ZΔ is redundant in this truncation, can remove it :

```
cts /. ZΔ → 1 // FullSimplify
```

$$\left\{\left\{-\frac{(2+n)\,\lambda\,\hbar\left(c0\,\Lambda^2+c1\,m^2\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\right)}{6+c1\,(2+n)\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]},\right.\right.$$

$$\lambda\left(-1-\frac{6}{3\,n+c1\,n\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]}+\frac{6\,(2+n)}{n\left(6+c1\,(2+n)\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\right)}\right),$$

$$\lambda\left(-1+\frac{18}{\left(3+c1\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\right)\left(6+c1\,(2+n)\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\right)}\right),$$

$$\left.\left.\lambda\left(-1+\frac{3}{3+c1\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]}\right),\ \lambda\left(-1+\frac{3}{3+c1\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]}\right),\ 1,\ 1\right\}\right\}$$

Verify that the finite gap equations come out right

```
finmg2 ==
  (mg2soln /. Solve[cteqs, {δm1, δλ1a, δλ2a, δλ2b, Z, ZΔ}] /. ZΔ → 1 // FullSimplify //
     DeleteDuplicates) [[2]] // Simplify
```

Solve::svars : Equations may not give solutions for all "solve" variables. >>

FullSimplify::infd : Expression $\left(-m^2 - \frac{-c0\,\Lambda^2 - c1\,m^2\,\text{Log[Power[≪2≫] Power[≪2≫]]}}{c1\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]} - \frac{v^2\,\lambda\,(\lambda + ≪1≫)}{2\,≪1≫\,(\lambda + ≪1≫)} - \frac{3\,\text{tfing}\,≪1≫\,\hbar\left(-(-1+n)\,\lambda + (1+n)\,\lambda + \frac{2\,(≪1≫)}{3\,≪3≫\,\text{Log}[≪1≫]}\right)}{2\,(3 + c1\,≪2≫\,\text{Log}[≪1≫])^2\,(\lambda + \delta\lambda_b)^2} - \left(3\,c0\,\lambda^2\,\Lambda^2\,\hbar\left(-(-1+n)\,\lambda + (1+n)\,\lambda + (2\,(9\,\text{Power}[≪2≫] + ≪7≫ + \text{Power}[≪2≫]\,\text{Power}[≪2≫]\,≪1≫\,\text{Power}[≪2≫]\,\text{Power}[≪2≫]))\Big/\left(3\,c1\,\lambda^2\,\hbar\,\text{Log[Times[≪2≫]]}\right)\right)\right)\Big/\left(2\,(3 + c1\,\lambda\,\hbar\,\text{Log[Times[≪2≫]]})^2\,(\lambda + \delta\lambda_b)^2\right)\right)\Big/\left(-1 + \left(3\,c1\,\lambda^2\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\left(-(-1+n)\,\lambda + (1+n)\,\lambda + (2\,(\text{Times}[≪2≫] + \text{Times}[≪5≫] + ≪5≫ + \text{Times}[≪6≫] + \text{Times}[≪5≫]))\Big/\left(3\,c1\,\lambda^2\,\hbar\,\text{Log}[≪1≫]\right)\right)\right)\Big/\left(2\,(3 + c1\,\lambda\,\hbar\,\text{Log}[≪1≫])^2\,(\lambda + \delta\lambda_b)^2\right)\right)$

simplified to ComplexInfinity. >>

```
True
```

```
finmn2 == mn2 /.
  ((neom /. divergentpartrules /. mg2 → mg2soln /. Solve[cteqs, {δm1, δλ1a,
           δλ2a, δλ1b, δλ2b, Z, ZΔ}] /. ZΔ → 1 // FullSimplify //
      DeleteDuplicates) // Solve[#, mn2] &) // FullSimplify
```

Solve::svars : Equations may not give solutions for all "solve" variables. >>

```
{True}
```

Find counter-terms for vev equation

```
rnveom = veom /. {mg2 → finmg2, mn2 → finmn2} // Simplify // DeleteDuplicates
```

$$\frac{1}{6}\,v\left(\frac{6\,(m^2 + \delta m_0^2)}{Z\Delta} + v^2\,(\lambda + \delta\lambda_0) + (-1+n)\,(\text{tfing} + \text{t}\infty\text{g})\,Z\Delta\,\hbar\,(\lambda + \delta\lambda_a) + (\text{tfinn} + \text{t}\infty\text{n})\,Z\Delta\,\hbar\,(3\,\lambda + \delta\lambda_a + 2\,\delta\lambda_b)\right)$$

```
ctegs3 =
 ((((CoefficientList[(1/v rnveom - 1/v finveom) /. divergentpartrules /. {mg2 → finmg2,
                    mn2 → finmn2} // Simplify // Expand // FullSimplify, {v,
                  tfing, tfinn}] // Simplify // Flatten) // DeleteDuplicates //
             Simplify // FullSimplify // DeleteDuplicates) == 0 // Thread) /.
          Solve[cteqs, {δm₁, δλ1a, δλ2a, δλ1b, δλ2b, Z}] /. ZΔ → 1 // Simplify //
        FullSimplify // DeleteDuplicates)[[1]] // Flatten // DeleteDuplicates
```

$$\left\{\frac{(2+n)\,\lambda\,\hbar\left(c0\,\Lambda^2+c1\,m^2\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\right)}{6+c1\,(2+n)\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]}+\delta m_0^2==0,\ \text{True},\right.$$
$$\left.3\,\lambda\left(1+\frac{2-2\,n}{3\,n+c1\,n\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]}-\frac{2\,(2+n)}{n\left(6+c1\,(2+n)\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\right)}\right)+\delta\lambda_0==0\right\}$$

Verify counter-term expressions in text

```
{δm₁² == δm₀², δλ1a == δλ2a, δλ1b == δλ2b} /.
    Solve[cteqs, {δm₁, δλ1a, δλ2a, δλ1b, δλ2b, Z, ZΔ}] /. Solve[ctegs3, {δm₀, δλ₀}] /.
  ZΔ → 1 // FullSimplify // Flatten // DeleteDuplicates
```

Solve::svars : Equations may not give solutions for all "solve" variables. >>

{True}

```
{δλ1a / δλ1b} /. Solve[cteqs, {δm₁, δλ1a, δλ2a, δλ1b, δλ2b, Z, ZΔ}] /.
   Solve[ctegs3, {δm₀, δλ₀}] /. ZΔ → 1 // FullSimplify // Flatten // DeleteDuplicates
```

Solve::svars : Equations may not give solutions for all "solve" variables. >>

$$\left\{1+\frac{3\,(2+n)}{6+c1\,(2+n)\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]}\right\}$$

```
δλ1b /. Solve[cteqs, {δm₁, δλ1a, δλ2a, δλ1b, δλ2b, Z, ZΔ}] /. ZΔ → 1 // FullSimplify //
 DeleteDuplicates
```

Solve::svars : Equations may not give solutions for all "solve" variables. >>

$$\left\{\lambda\left(-1+\frac{3}{3+c1\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]}\right)\right\}$$

```
{δλ₀ == 1 δλ1a + 2 δλ1b} /. Solve[cteqs, {δm₁, δλ1a, δλ2a, δλ1b, δλ2b, Z, ZΔ}] /.
   Solve[ctegs3, {δm₀, δλ₀}] /. ZΔ → 1 // FullSimplify // Flatten // DeleteDuplicates
```

Solve::svars : Equations may not give solutions for all "solve" variables. >>

{True}

```
{δm0^2 == -((n + 2) λ ℏ (c0 Λ^2 + c1 m^2 Log[Λ^2 / μ^2]))/6 (δλ1a + λ)/(δλ1b + λ)} /.
     Solve[cteqs, {δm1, δλ1a, δλ2a, δλ1b, δλ2b, Z, ZΔ}] /. Solve[ctegs3, {δm0, δλ0}] /.
   ZΔ → 1 // FullSimplify // Flatten // DeleteDuplicates
```

Solve::svars : Equations may not give solutions for all "solve" variables. >>

```
{True}
```

Total number of independent counter-term equations

```
Length[{cteqs, ctegs3} // Flatten // FullSimplify // DeleteDuplicates] -
 1 (* -1 because one of the "equations" is identically "True" *)
```

```
10
```