Renormalization of Soft Symmetry Improved 2PIEA gap equations in the Hartree-Fock approximation

Supplement to thesis Chapter 5 "Soft Symmetry Improvement"

Mathematica notebook to compute couter-terms for the Hartree-Fock truncation of the SSI-2PIEA

Author: Michael Brown

Hartree-Fock

In[224]:=
```
ClearAll[geom, neom, intrules, msbarrules, mg2soln, cteq, cts, δm, δλ];
```

Hartree-Fock gap equations with counterterms

Goldstone equation of motion. Quantities in reference to the paper are:
p is the four-momentum flowing through the propagators $\Delta_G{}^{-1}$ and $\Delta_N{}^{-1}$,
mg2 is the Goldstone mass squared $m_G{}^2$,
mn2 is the Higgs mass squared $m_H{}^2$,
Z and ZΔ are the wavefunction a propagator renormalization constants,
m^2 is the (renormalized) Lagrangian mass parameter, $\delta\mathbf{m_0}^2$, $\delta\mathbf{m_1}^2$ are its counter-terms,
λ is the (renormalized) four point coupling,
$\delta\lambda_0$, $\delta\lambda_{1\,a}$, $\delta\lambda_{1\,b}$, $\delta\lambda_{2\,a}$, $\delta\lambda_{2\,b}$ are the independent coupling counter-terms,
v is the scalar field vacuum expectation value,
$\hbar$ is the reduced Planck constant,
n is the number of fields in the O(n) symmetry group,
ξ is the stiffness parameter,
ϵ is the solution of the Goldstone zero mode equation,
ssi = $\frac{1}{V\beta m_G{}^2}\left(\frac{1}{\epsilon}-1\right)$ is the soft symmetry improvement term in the propagator eoms,
ssi2 = $\frac{1}{\xi}(n-1)\,2\left(m_G{}^2\,\epsilon\right)^2$ is the other soft symmetry improvement term in the vev eom,
t∞g, t∞n are the divergent tadpole integrals for the Goldstone, Higgs resp.,
tfing, tfinn are the finite parts of the tadpoles for the Goldstone, Higgs resp.

Vev equation of motion

In[225]:= $$\mathbf{veom} = Z\Delta^{-1}\left(m^2+\delta m_0{}^2\right) v + \frac{\lambda+\delta\lambda_0}{6} v^3 + \frac{\hbar}{6} Z\Delta \left(n-1\right) \left(\lambda+\delta\lambda_{1\,a}\right) v\,(t\infty g + tfing + ssi) + \frac{\hbar}{6} Z\Delta \left(3\,\lambda+\delta\lambda_{1\,a}+2\,\delta\lambda_{1\,b}\right) v\,(t\infty n + tfinn) + v\, ssi2$$

Out[225]= $$ssi2\, v + \frac{v\left(m^2+\delta m_0^2\right)}{Z\Delta} + \frac{1}{6} v^3 \left(\lambda+\delta\lambda_0\right) + \frac{1}{6}(-1+n)\,(ssi + tfing + t\infty g)\, v\, Z\Delta\, \hbar\, (\lambda+\delta\lambda_a) + \frac{1}{6}(tfinn + t\infty n)\, v\, Z\Delta\, \hbar\, (3\,\lambda + \delta\lambda_a + 2\,\delta\lambda_b)$$

Goldstone equation of motion

In[226]:= $$\mathbf{geom} = p^2 - mg2 == Z\, Z\Delta\, p^2 - m^2 - \delta m_1{}^2 - Z\Delta \frac{\lambda+\delta\lambda_{1\,a}}{6} v^2 - \frac{\hbar}{6}\left(\left(n+1\right)\lambda + \left(n-1\right)\delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b}\right) Z\Delta^2\,(t\infty g + tfing + ssi) - \frac{\hbar}{6}\left(\lambda+\delta\lambda_{2\,a}\right) Z\Delta^2\,(t\infty n + tfinn)$$

Out[226]= $$-mg2 + p^2 == -m^2 + p^2\, Z\, Z\Delta - \delta m_1^2 - \frac{1}{6} v^2\, Z\Delta\,(\lambda+\delta\lambda_a) - \frac{1}{6}(tfinn + t\infty n)\, Z\Delta^2\, \hbar\, (\lambda + \delta\lambda_{2\,a}) - \frac{1}{6}(ssi + tfing + t\infty g)\, Z\Delta^2\, \hbar\, ((1+n)\,\lambda + (-1+n)\,\delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b})$$

Higgs equation of motion

In[227]:= $$\mathbf{neom} = p^2 - mn2 == Z\, Z\Delta\, p^2 - m^2 - \delta m_1{}^2 - Z\Delta\, v^2 \frac{\left(3\,\lambda+\delta\lambda_{1\,a}+2\,\delta\lambda_{1\,b}\right)}{6} - \frac{\hbar}{6}\left(\lambda+\delta\lambda_{2\,a}\right)\left(n-1\right) Z\Delta^2\,(t\infty g + tfing + ssi) - \frac{\hbar}{6}\left(3\,\lambda+\delta\lambda_{2\,a}+2\,\delta\lambda_{2\,b}\right) Z\Delta^2\,(t\infty n + tfinn)$$

Out[227]= $$-mn2 + p^2 == -m^2 + p^2\, Z\, Z\Delta - \delta m_1^2 - \frac{1}{6}(-1+n)\,(ssi + tfing + t\infty g)\, Z\Delta^2\, \hbar\, (\lambda+\delta\lambda_{2\,a}) - \frac{1}{6} v^2\, Z\Delta\,(3\,\lambda + \delta\lambda_a + 2\,\delta\lambda_b) - \frac{1}{6}(tfinn + t\infty n)\, Z\Delta^2\, \hbar\, (3\,\lambda + \delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b})$$

Infinite parts of tadpoles in MSbar

MSbar rules for 4 - 2 ϵ dimensions

In[228]:= $\mathbf{msbarrules} = \{t\infty g \to \kappa\, mg2,\ t\infty n \to \kappa\, mn2\}$

Out[228]= $\{t\infty g \to mg2\,\kappa,\ t\infty n \to mn2\,\kappa\}$

Sub in tadpole expressions, eliminate mn2 and solve for mg2

In[229]:=
```
mg2soln = mg2 /.
  (geom /. msbarrules /. Solve[neom /. msbarrules, mn2][[1]] // Solve[#, mg2][[1]] &)
```

Out[229]=

$$\left(-m^2 - p^2 + p^2\, Z\, Z\Delta - \delta m_1^2 - \frac{1}{6} v^2\, Z\Delta\, (\lambda + \delta\lambda_a) - \frac{1}{6}\,\mathrm{tfinn}\, Z\Delta^2\, \hbar\, (\lambda + \delta\lambda_{2\,a}) - \frac{1}{6}\,\mathrm{ssi}\, Z\Delta^2\, \hbar\, ((1+n)\,\lambda + (-1+n)\,\delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b}) - \frac{1}{6}\,\mathrm{tfing}\, Z\Delta^2\, \hbar\, ((1+n)\,\lambda + (-1+n)\,\delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b}) + \frac{m^2\, Z\Delta^2\, \kappa\, \hbar\, (\lambda + \delta\lambda_{2\,a})}{6\left(-1 + \frac{1}{6} Z\Delta^2\, \kappa\, \hbar\, (3\,\lambda + \delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b})\right)} + \frac{p^2\, Z\Delta^2\, \kappa\, \hbar\, (\lambda + \delta\lambda_{2\,a})}{6\left(-1 + \frac{1}{6} Z\Delta^2\, \kappa\, \hbar\, (3\,\lambda + \delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b})\right)} - \frac{p^2\, Z\, Z\Delta^3\, \kappa\, \hbar\, (\lambda + \delta\lambda_{2\,a})}{6\left(-1 + \frac{1}{6} Z\Delta^2\, \kappa\, \hbar\, (3\,\lambda + \delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b})\right)} + \frac{Z\Delta^2\, \kappa\, \hbar\, \delta m_1^2\, (\lambda + \delta\lambda_{2\,a})}{6\left(-1 + \frac{1}{6} Z\Delta^2\, \kappa\, \hbar\, (3\,\lambda + \delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b})\right)} + \frac{(-1+n)\,\mathrm{ssi}\, Z\Delta^4\, \kappa\, \hbar^2\, (\lambda + \delta\lambda_{2\,a})^2}{36\left(-1 + \frac{1}{6} Z\Delta^2\, \kappa\, \hbar\, (3\,\lambda + \delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b})\right)} + \frac{(-1+n)\,\mathrm{tfing}\, Z\Delta^4\, \kappa\, \hbar^2\, (\lambda + \delta\lambda_{2\,a})^2}{36\left(-1 + \frac{1}{6} Z\Delta^2\, \kappa\, \hbar\, (3\,\lambda + \delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b})\right)} + \frac{v^2\, Z\Delta^3\, \kappa\, \hbar\, (\lambda + \delta\lambda_{2\,a})\, (3\,\lambda + \delta\lambda_a + 2\,\delta\lambda_b)}{36\left(-1 + \frac{1}{6} Z\Delta^2\, \kappa\, \hbar\, (3\,\lambda + \delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b})\right)} + \frac{\mathrm{tfinn}\, Z\Delta^4\, \kappa\, \hbar^2\, (\lambda + \delta\lambda_{2\,a})\, (3\,\lambda + \delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b})}{36\left(-1 + \frac{1}{6} Z\Delta^2\, \kappa\, \hbar\, (3\,\lambda + \delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b})\right)}\right) \Bigg/ \left(-1 + \frac{1}{6} Z\Delta^2\, \kappa\, \hbar\, ((1+n)\,\lambda + (-1+n)\,\delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b}) - \frac{(-1+n)\, Z\Delta^4\, \kappa^2\, \hbar^2\, (\lambda + \delta\lambda_{2\,a})^2}{36\left(-1 + \frac{1}{6} Z\Delta^2\, \kappa\, \hbar\, (3\,\lambda + \delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b})\right)}\right)$$

In[230]:= **mn2soln = mn2 /. (neom /. msbarrules /. mg2 → mg2soln // Solve[#, mn2][[1]] &)**

Out[230]=

$$\frac{1}{-1+\frac{1}{6}Z\Delta^2\kappa\hbar\,(3\lambda+\delta\lambda_{2a}+2\,\delta\lambda_{2b})}\Bigg(-m^2-p^2+p^2\,Z\,Z\Delta-\delta m_1^2-\frac{1}{6}v^2\,Z\Delta\,(3\lambda+\delta\lambda_a+2\,\delta\lambda_b)-\frac{1}{6}\,\text{tfinn}\,Z\Delta^2\hbar\,(3\lambda+\delta\lambda_{2a}+2\,\delta\lambda_{2b})-$$

$$\frac{1}{6}(-1+n)\,Z\Delta^2\hbar\,(\lambda+\delta\lambda_{2a})\Bigg(\text{ssi}+\text{tfing}+\Bigg(\kappa\Bigg(-m^2-p^2+p^2\,Z\,Z\Delta-\delta m_1^2-\frac{1}{6}v^2\,Z\Delta\,(\lambda+\delta\lambda_a)-$$

$$\frac{1}{6}\,\text{tfinn}\,Z\Delta^2\hbar\,(\lambda+\delta\lambda_{2a})-\frac{1}{6}\,\text{ssi}\,Z\Delta^2\hbar\,((1+n)\lambda+(-1+n)\,\delta\lambda_{2a}+2\,\delta\lambda_{2b})-\frac{1}{6}\,\text{tfing}$$

$$Z\Delta^2\hbar\,((1+n)\lambda+(-1+n)\,\delta\lambda_{2a}+2\,\delta\lambda_{2b})+\frac{m^2\,Z\Delta^2\kappa\hbar\,(\lambda+\delta\lambda_{2a})}{6\left(-1+\frac{1}{6}Z\Delta^2\kappa\hbar\,(3\lambda+\delta\lambda_{2a}+2\,\delta\lambda_{2b})\right)}+$$

$$\frac{p^2\,Z\Delta^2\kappa\hbar\,(\lambda+\delta\lambda_{2a})}{6\left(-1+\frac{1}{6}Z\Delta^2\kappa\hbar\,(3\lambda+\delta\lambda_{2a}+2\,\delta\lambda_{2b})\right)}-\frac{p^2\,Z\,Z\Delta^3\kappa\hbar\,(\lambda+\delta\lambda_{2a})}{6\left(-1+\frac{1}{6}Z\Delta^2\kappa\hbar\,(3\lambda+\delta\lambda_{2a}+2\,\delta\lambda_{2b})\right)}+$$

$$\frac{Z\Delta^2\kappa\hbar\,\delta m_1^2\,(\lambda+\delta\lambda_{2a})}{6\left(-1+\frac{1}{6}Z\Delta^2\kappa\hbar\,(3\lambda+\delta\lambda_{2a}+2\,\delta\lambda_{2b})\right)}+\frac{(-1+n)\,\text{ssi}\,Z\Delta^4\kappa\hbar^2\,(\lambda+\delta\lambda_{2a})^2}{36\left(-1+\frac{1}{6}Z\Delta^2\kappa\hbar\,(3\lambda+\delta\lambda_{2a}+2\,\delta\lambda_{2b})\right)}+$$

$$\frac{(-1+n)\,\text{tfing}\,Z\Delta^4\kappa\hbar^2\,(\lambda+\delta\lambda_{2a})^2}{36\left(-1+\frac{1}{6}Z\Delta^2\kappa\hbar\,(3\lambda+\delta\lambda_{2a}+2\,\delta\lambda_{2b})\right)}+\frac{v^2\,Z\Delta^3\kappa\hbar\,(\lambda+\delta\lambda_{2a})\,(3\lambda+\delta\lambda_a+2\,\delta\lambda_b)}{36\left(-1+\frac{1}{6}Z\Delta^2\kappa\hbar\,(3\lambda+\delta\lambda_{2a}+2\,\delta\lambda_{2b})\right)}+$$

$$\frac{\text{tfinn}\,Z\Delta^4\kappa\hbar^2\,(\lambda+\delta\lambda_{2a})\,(3\lambda+\delta\lambda_{2a}+2\,\delta\lambda_{2b})}{36\left(-1+\frac{1}{6}Z\Delta^2\kappa\hbar\,(3\lambda+\delta\lambda_{2a}+2\,\delta\lambda_{2b})\right)}\Bigg)\Bigg)\Bigg/$$

$$\Bigg(-1+\frac{1}{6}Z\Delta^2\kappa\hbar\,((1+n)\lambda+(-1+n)\,\delta\lambda_{2a}+2\,\delta\lambda_{2b})-$$

$$\frac{(-1+n)\,Z\Delta^4\kappa^2\hbar^2\,(\lambda+\delta\lambda_{2a})^2}{36\left(-1+\frac{1}{6}Z\Delta^2\kappa\hbar\,(3\lambda+\delta\lambda_{2a}+2\,\delta\lambda_{2b})\right)}\Bigg)\Bigg)\Bigg)$$

Gather divergences proportional v, tfing and tfinn and set independently to zero

First we subtract the finite equation of motion, then gather coefficients of the remainder into a list and set each to zero (after some trimming and simplifying).

In[231]:=
```
cteq =
 ((CoefficientList[mg2soln + (-m^2 - λ/6 v^2 - ℏ/6 ((n + 1) λ) (tfing + ssi) - ℏ/6 (λ) (tfinn)),
       {p, v, tfing, tfinn}] // Flatten) //
    DeleteDuplicates // Simplify // FullSimplify) == 0 // Thread
```

Out[231]=

$$\Big\{-\Big(\Big(\lambda\hbar\Big(6 m^2 (2+n) Z\Delta^2 \kappa \big(-3 + Z\Delta^2 \kappa\lambda\hbar\big) + ssi\Big(-18 (1+n)\big(-1+Z\Delta^2\big) - 3 Z\Delta^2\big(4 + 5 n + n^2 - 2 (2+n) Z\Delta^2\big)\kappa\lambda\hbar + (1+n)(2+n) Z\Delta^4 \kappa^2 \lambda^2 \hbar^2\Big)\Big) + 36\,\delta m_1^2\big(-3 + Z\Delta^2\kappa\lambda\hbar + Z\Delta^2\kappa\hbar\,\delta\lambda_{2b}\big) + Z\Delta^2\hbar\Big(\delta\lambda_{2b}\Big(6 m^2\kappa\big(-6 + (4+n) Z\Delta^2\kappa\lambda\hbar\big) + ssi\Big(-36 + \kappa\lambda\hbar\big(-12 (1+n) + 6 (4+n) Z\Delta^2 + (1+n)(4+n) Z\Delta^2\kappa\lambda\hbar\big)\Big) + 2 Z\Delta^2\kappa\hbar\big(6 m^2\kappa + ssi\,(6 + (1+n)\kappa\lambda\hbar)\big)\delta\lambda_{2b}\Big) + \delta\lambda_{2a}\Big(6 m^2 n\kappa\big(-3 + Z\Delta^2\kappa\lambda\hbar\big) + ssi\big(18 + n (6 + (1+n)\kappa\lambda\hbar)\big)\big(-3 + Z\Delta^2\kappa\lambda\hbar\big) + n Z\Delta^2\kappa\hbar\big(6 m^2\kappa + ssi\,(6 + (1+n)\kappa\lambda\hbar)\big)\delta\lambda_{2b}\Big)\Big)\Big) \Big/ \Big(6\big(-3 + Z\Delta^2\kappa\lambda\hbar + Z\Delta^2\kappa\hbar\,\delta\lambda_{2b}\big)\big(-6 + (2+n) Z\Delta^2\kappa\lambda\hbar + Z\Delta^2\kappa\hbar\,(n\,\delta\lambda_{2a} + 2\,\delta\lambda_{2b})\big)\Big)\Big) == 0,$$

$$-\frac{\lambda\hbar}{6} + \big(3 Z\Delta^2\hbar\,(\lambda + \delta\lambda_{2a})\big) \Big/ \Big(\big(-3 + Z\Delta^2\kappa\lambda\hbar + Z\Delta^2\kappa\hbar\,\delta\lambda_{2b}\big)\big(-6 + (2+n) Z\Delta^2\kappa\lambda\hbar + Z\Delta^2\kappa\hbar\,(n\,\delta\lambda_{2a} + 2\,\delta\lambda_{2b})\big)\Big) == 0,$$

$$-\frac{1}{6\kappa}\left(6 + (1+n)\kappa\lambda\hbar + \frac{18}{n\big(-3 + Z\Delta^2\kappa\lambda\hbar + Z\Delta^2\kappa\hbar\,\delta\lambda_{2b}\big)} + \frac{36(-1+n)}{n\big(-6 + (2+n) Z\Delta^2\kappa\lambda\hbar + Z\Delta^2\kappa\hbar\,(n\,\delta\lambda_{2a} + 2\,\delta\lambda_{2b})\big)}\right) == 0,$$

$$\text{True},\; -\frac{\lambda}{6} + \frac{Z\Delta\,(\lambda + \delta\lambda_b)}{n\big(-3 + Z\Delta^2\kappa\lambda\hbar + Z\Delta^2\kappa\hbar\,\delta\lambda_{2b}\big)} - \frac{Z\Delta\,((2+n)\lambda + n\,\delta\lambda_a + 2\,\delta\lambda_b)}{n\big(-6 + (2+n) Z\Delta^2\kappa\lambda\hbar + Z\Delta^2\kappa\hbar\,(n\,\delta\lambda_{2a} + 2\,\delta\lambda_{2b})\big)} == 0,$$

$$\frac{-6 + 6\,Z\,Z\Delta}{-6 + (2+n) Z\Delta^2\kappa\lambda\hbar + Z\Delta^2\kappa\hbar\,(n\,\delta\lambda_{2a} + 2\,\delta\lambda_{2b})} == 0\Big\}$$

In[232]:=
```
cteq2 =
  ((CoefficientList[mn2soln + (-m^2 - λ/2 v^2 - ℏ/6 ((n - 1) λ) (tfing + ssi) - ℏ/2 (λ) (tfinn)),
        {p, v, tfing, tfinn}] // Flatten) //
      DeleteDuplicates // Simplify // FullSimplify) == 0 // Thread
```

Out[232]=

$$\Big\{\Big(\lambda\hbar\big(-6\,m^2\,(2+n)\,Z\Delta^2\,\kappa\,(-3+Z\Delta^2\,\kappa\,\lambda\,\hbar)+(-1+n)\,ssi\,(-18+Z\Delta^2\,(18+\kappa\,\lambda\,\hbar\,(3\,(4+n)-(2+n)\,Z\Delta^2\,\kappa\,\lambda\,\hbar)))\big)-36\,\delta m_1^2\,(-3+Z\Delta^2\,\kappa\,\lambda\,\hbar+Z\Delta^2\,\kappa\,\hbar\,\delta\lambda_{2\,b})+Z\Delta^2\,\hbar\,\big((-6\,m^2\,n\,\kappa\,(-3+Z\Delta^2\,\kappa\,\lambda\,\hbar)+(-1+n)\,ssi\,(18+n\,\kappa\,\lambda\,\hbar\,(3-Z\Delta^2\,\kappa\,\lambda\,\hbar)))\,\delta\lambda_{2\,a}-\kappa\,\big((-1+n)\,ssi\,\lambda\,\hbar\,(-12+(4+n)\,Z\Delta^2\,\kappa\,\lambda\,\hbar)+6\,m^2\,(-6+(4+n)\,Z\Delta^2\,\kappa\,\lambda\,\hbar)+n\,Z\Delta^2\,\kappa\,\hbar\,(6\,m^2+(-1+n)\,ssi\,\lambda\,\hbar)\,\delta\lambda_{2\,a}\big)\,\delta\lambda_{2\,b}-2\,Z\Delta^2\,\kappa^2\,\hbar\,(6\,m^2+(-1+n)\,ssi\,\lambda\,\hbar)\,\delta\lambda_{2\,b}^2\big)\Big)\Big/\Big(6\,(-3+Z\Delta^2\,\kappa\,\lambda\,\hbar+Z\Delta^2\,\kappa\,\hbar\,\delta\lambda_{2\,b})\,(-6+(2+n)\,Z\Delta^2\,\kappa\,\lambda\,\hbar+Z\Delta^2\,\kappa\,\hbar\,(n\,\delta\lambda_{2\,a}+2\,\delta\lambda_{2\,b}))\Big)==0,$$

$$-\frac{1}{2\kappa}\left(2+\kappa\,\lambda\,\hbar+\frac{6\,(-1+n)}{n\,(-3+Z\Delta^2\,\kappa\,\lambda\,\hbar+Z\Delta^2\,\kappa\,\hbar\,\delta\lambda_{2\,b})}+\frac{12}{n\,(-6+(2+n)\,Z\Delta^2\,\kappa\,\lambda\,\hbar+Z\Delta^2\,\kappa\,\hbar\,(n\,\delta\lambda_{2\,a}+2\,\delta\lambda_{2\,b}))}\right)==0,$$

$$\frac{1}{6}\,(-1+n)\,\Big(-\lambda\,\hbar+\big(18\,Z\Delta^2\,\hbar\,(\lambda+\delta\lambda_{2\,a})\big)\Big/\big((-3+Z\Delta^2\,\kappa\,\lambda\,\hbar+Z\Delta^2\,\kappa\,\hbar\,\delta\lambda_{2\,b})\,(-6+(2+n)\,Z\Delta^2\,\kappa\,\lambda\,\hbar+Z\Delta^2\,\kappa\,\hbar\,(n\,\delta\lambda_{2\,a}+2\,\delta\lambda_{2\,b}))\big)\Big)==0,$$

$$\text{True},\ -\frac{\lambda}{2}-\frac{(-1+n)\,Z\Delta\,(\lambda+\delta\lambda_b)}{n\,(-3+Z\Delta^2\,\kappa\,\lambda\,\hbar+Z\Delta^2\,\kappa\,\hbar\,\delta\lambda_{2\,b})}-\frac{Z\Delta\,((2+n)\,\lambda+n\,\delta\lambda_a+2\,\delta\lambda_b)}{n\,(-6+(2+n)\,Z\Delta^2\,\kappa\,\lambda\,\hbar+Z\Delta^2\,\kappa\,\hbar\,(n\,\delta\lambda_{2\,a}+2\,\delta\lambda_{2\,b}))}==0,$$

$$\frac{-6+6\,Z\,Z\Delta}{-6+(2+n)\,Z\Delta^2\,\kappa\,\lambda\,\hbar+Z\Delta^2\,\kappa\,\hbar\,(n\,\delta\lambda_{2\,a}+2\,\delta\lambda_{2\,b})}==0\Big\}$$

Solve for counterterms

In[233]:=
```
cteqs = {cteq, cteq2} // Flatten // FullSimplify // DeleteDuplicates
```

Out[233]=

$$\Big\{\frac{1}{\kappa}\left(6\,\mathrm{ssi}+6\,m^2\kappa+\mathrm{ssi}\,\kappa\lambda\hbar+n\,\mathrm{ssi}\,\kappa\lambda\hbar+\frac{18\,\mathrm{ssi}}{n\left(-3+Z\Delta^2\kappa\lambda\hbar+Z\Delta^2\kappa\hbar\,\delta\lambda_{2\,b}\right)}+\frac{36\left((-1+n)\,\mathrm{ssi}+m^2 n\kappa+n\kappa\,\delta m_1^2\right)}{n\left(-6+(2+n)\,Z\Delta^2\kappa\lambda\hbar+Z\Delta^2\kappa\hbar\,(n\,\delta\lambda_{2\,a}+2\,\delta\lambda_{2\,b})\right)}\right)==0,\ \hbar\left(\lambda-\left(18\,Z\Delta^2(\lambda+\delta\lambda_{2\,a})\right)\Big/\left(\left(-3+Z\Delta^2\kappa\lambda\hbar+Z\Delta^2\kappa\hbar\,\delta\lambda_{2\,b}\right)\left(-6+(2+n)\,Z\Delta^2\kappa\lambda\hbar+Z\Delta^2\kappa\hbar\,(n\,\delta\lambda_{2\,a}+2\,\delta\lambda_{2\,b})\right)\right)\right)==0,$$

$$\frac{1}{\kappa}\left(6+(1+n)\kappa\lambda\hbar+\frac{18}{n\left(-3+Z\Delta^2\kappa\lambda\hbar+Z\Delta^2\kappa\hbar\,\delta\lambda_{2\,b}\right)}+\frac{36\,(-1+n)}{n\left(-6+(2+n)\,Z\Delta^2\kappa\lambda\hbar+Z\Delta^2\kappa\hbar\,(n\,\delta\lambda_{2\,a}+2\,\delta\lambda_{2\,b})\right)}\right)==0,$$

$$\mathrm{True},\ \lambda+\frac{6\,Z\Delta\,((2+n)\lambda+n\,\delta\lambda_a+2\,\delta\lambda_b)}{n\left(-6+(2+n)\,Z\Delta^2\kappa\lambda\hbar+Z\Delta^2\kappa\hbar\,(n\,\delta\lambda_{2\,a}+2\,\delta\lambda_{2\,b})\right)}==\frac{6\,Z\Delta\,(\lambda+\delta\lambda_b)}{n\left(-3+Z\Delta^2\kappa\lambda\hbar+Z\Delta^2\kappa\hbar\,\delta\lambda_{2\,b}\right)},$$

$$\frac{-1+Z\,Z\Delta}{-6+(2+n)\,Z\Delta^2\kappa\lambda\hbar+Z\Delta^2\kappa\hbar\,(n\,\delta\lambda_{2\,a}+2\,\delta\lambda_{2\,b})}==0,$$

$$6\,m^2+n\,\mathrm{ssi}\,\lambda\hbar+\frac{36\left((-1+n)\,\mathrm{ssi}+m^2 n\kappa+n\kappa\,\delta m_1^2\right)}{n\kappa\left(-6+(2+n)\,Z\Delta^2\kappa\lambda\hbar+Z\Delta^2\kappa\hbar\,(n\,\delta\lambda_{2\,a}+2\,\delta\lambda_{2\,b})\right)}==\mathrm{ssi}\,\lambda\hbar+\frac{18\,(-1+n)\,\mathrm{ssi}}{n\kappa\left(-3+Z\Delta^2\kappa\lambda\hbar+Z\Delta^2\kappa\hbar\,\delta\lambda_{2\,b}\right)},$$

$$\frac{1}{\kappa}\left(2+\kappa\lambda\hbar+\frac{6\,(-1+n)}{n\left(-3+Z\Delta^2\kappa\lambda\hbar+Z\Delta^2\kappa\hbar\,\delta\lambda_{2\,b}\right)}+\frac{12}{n\left(-6+(2+n)\,Z\Delta^2\kappa\lambda\hbar+Z\Delta^2\kappa\hbar\,(n\,\delta\lambda_{2\,a}+2\,\delta\lambda_{2\,b})\right)}\right)==0,$$

$$(-1+n)\,\hbar\left(\lambda-\left(18\,Z\Delta^2(\lambda+\delta\lambda_{2\,a})\right)\Big/\left(\left(-3+Z\Delta^2\kappa\lambda\hbar+Z\Delta^2\kappa\hbar\,\delta\lambda_{2\,b}\right)\left(-6+(2+n)\,Z\Delta^2\kappa\lambda\hbar+Z\Delta^2\kappa\hbar\,(n\,\delta\lambda_{2\,a}+2\,\delta\lambda_{2\,b})\right)\right)\right)==0,$$

$$\lambda+\frac{2\,(-1+n)\,Z\Delta\,(\lambda+\delta\lambda_b)}{n\left(-3+Z\Delta^2\kappa\lambda\hbar+Z\Delta^2\kappa\hbar\,\delta\lambda_{2\,b}\right)}+\frac{2\,Z\Delta\,((2+n)\lambda+n\,\delta\lambda_a+2\,\delta\lambda_b)}{n\left(-6+(2+n)\,Z\Delta^2\kappa\lambda\hbar+Z\Delta^2\kappa\hbar\,(n\,\delta\lambda_{2\,a}+2\,\delta\lambda_{2\,b})\right)}==0\Big\}$$

In[234]:=
```
cts = {δm₁², δλ1a, δλ2a, δλ1b, δλ2b, Z, ZΔ} /. Solve[cteqs,
    {δm₁, δλ1a, δλ2a, δλ1b, δλ2b, Z, ZΔ}] // FullSimplify // DeleteDuplicates
```

Solve::svars : Equations may not give solutions for all "solve" variables. >>

Out[234]=

$$\Big\{\Big\{-\frac{m^2(2+n)\kappa\lambda\hbar}{6+(2+n)\kappa\lambda\hbar},\ \frac{\lambda\,(18-Z\Delta\,(3+\kappa\lambda\hbar)\,(6+(2+n)\kappa\lambda\hbar))}{Z\Delta\,(3+\kappa\lambda\hbar)\,(6+(2+n)\kappa\lambda\hbar)},\ \frac{\lambda\left(18-Z\Delta^2(3+\kappa\lambda\hbar)\,(6+(2+n)\kappa\lambda\hbar)\right)}{Z\Delta^2(3+\kappa\lambda\hbar)\,(6+(2+n)\kappa\lambda\hbar)},\ \lambda\left(-1+\frac{3}{3\,Z\Delta+Z\Delta\,\kappa\lambda\hbar}\right),\ \lambda\left(-1+\frac{3}{Z\Delta^2(3+\kappa\lambda\hbar)}\right),\ \frac{1}{Z\Delta},\ Z\Delta\Big\}\Big\}$$

ZΔ is redundant in this truncation, can remove it :

In[235]:=
```
cts /. ZΔ → 1 // FullSimplify
```

Out[235]=

$$\left\{\left\{-\frac{m^2\,(2+n)\,\kappa\,\lambda\,\hbar}{6+(2+n)\,\kappa\,\lambda\,\hbar},\,-\frac{\kappa\,\lambda^2\,\hbar\,(3\,(4+n)+(2+n)\,\kappa\,\lambda\,\hbar)}{(3+\kappa\,\lambda\,\hbar)\,(6+(2+n)\,\kappa\,\lambda\,\hbar)},\,-\frac{\kappa\,\lambda^2\,\hbar\,(3\,(4+n)+(2+n)\,\kappa\,\lambda\,\hbar)}{(3+\kappa\,\lambda\,\hbar)\,(6+(2+n)\,\kappa\,\lambda\,\hbar)},\,\lambda\left(-1+\frac{3}{3+\kappa\,\lambda\,\hbar}\right),\,\lambda\left(-1+\frac{3}{3+\kappa\,\lambda\,\hbar}\right),\,1,\,1\right\}\right\}$$

In[236]:=
```
mg2soln /. Solve[cteqs, {δm1, δλ1a, δλ2a, δλ2b, Z, ZΔ}] /. ZΔ → 1 // FullSimplify //
  DeleteDuplicates
```

Out[236]=

$$\left\{\frac{1}{6}\left(6\,m^2+\lambda\left(v^2+((1+n)\,(\text{ssi}+\text{tfing})+\text{tfinn})\,\hbar\right)\right)\right\}$$

In[237]:=
```
mn2 /. ((neom /. msbarrules /. mg2 → mg2soln /.
        Solve[cteqs, {δm1, δλ1a, δλ2a, δλ1b, δλ2b, Z, ZΔ}] /. ZΔ → 1 // FullSimplify //
     DeleteDuplicates) // Solve[#, mn2] &) // FullSimplify
```

Solve::svars : Equations may not give solutions for all "solve" variables. ≫

Out[237]=

$$\left\{\frac{1}{6}\left(6\,m^2+3\,v^2\,\lambda+((-1+n)\,(\text{ssi}+\text{tfing})+3\,\text{tfinn})\,\lambda\,\hbar\right)\right\}$$

In[238]:=

$$\text{rnveom} = \text{veom} \;/.\; \left\{\text{mg2}\to m^2+\frac{\lambda}{6}v^2+\frac{\hbar}{6}\left((n+1)\,\lambda\right)(\text{tfing}+\text{ssi})+\frac{\hbar}{6}(\lambda)\,(\text{tfinn}),\;\text{mn2}\to m^2+\frac{\lambda}{2}v^2+\frac{\hbar}{6}\left((n-1)\,\lambda\right)(\text{tfing}+\text{ssi})+\frac{\hbar}{2}(\lambda)\,(\text{tfinn})\right\}\;//\;\text{Simplify}\;//\;\text{DeleteDuplicates}$$

Out[238]=

$$\frac{1}{6}\,v\left(6\,\text{ssi2}+\frac{6\left(m^2+\delta m_0^2\right)}{Z\Delta}+v^2\,(\lambda+\delta\lambda_0)+(-1+n)\,(\text{ssi}+\text{tfing}+t\infty g)\,Z\Delta\,\hbar\,(\lambda+\delta\lambda_a)+(\text{tfinn}+t\infty n)\,Z\Delta\,\hbar\,(3\,\lambda+\delta\lambda_a+2\,\delta\lambda_b)\right)$$

In[239]:=
```
ctegs3 =
 ((((CoefficientList[(1/v rnveom - (m^2 + λ/6 v^2 + ℏ/6 ((n - 1) λ) (tfing + ssi) + ℏ/2 (λ) (tfinn) +
          ssi2)) /. msbarrules /. {mg2 → m^2 + λ/6 v^2 +
          ℏ/6 ((n + 1) λ) (tfing + ssi) + ℏ/6 (λ) (tfinn), mn2 →
          m^2 + λ/2 v^2 + ℏ/6 ((n - 1) λ) (tfing + ssi) + ℏ/2 (λ) (tfinn)} //
        Simplify // Expand // FullSimplify, {v, tfing, tfinn}] //
       Simplify // Flatten) // DeleteDuplicates // Simplify //
     FullSimplify // DeleteDuplicates) == 0 // Thread) /.
   Solve[cteqs, {δm1, δλ1a, δλ2a, δλ1b, δλ2b, Z}] // Simplify //
  FullSimplify // DeleteDuplicates)[[1]]
```

Out[239]=
$$\left\{\frac{m^2\ (6-6\,Z\Delta+(2+n)\,\kappa\,\lambda\,\hbar)+(6+(2+n)\,\kappa\,\lambda\,\hbar)\,\delta m_0^2}{Z\Delta\ (6+(2+n)\,\kappa\,\lambda\,\hbar)}==0,\ \text{True},\ \text{True},\ \text{True},\ \frac{3\,\kappa\,\lambda^2\,\hbar\ (8+n+(2+n)\,\kappa\,\lambda\,\hbar)}{(3+\kappa\,\lambda\,\hbar)\ (6+(2+n)\,\kappa\,\lambda\,\hbar)}+\delta\lambda_0==0\right\}$$

In[240]:=
```
{δm0^2, δλ0} /. Solve[ctegs3, {δm0, δλ0}] /. ZΔ → 1 // DeleteDuplicates // Simplify
```

Out[240]=
$$\left\{\left\{-\frac{m^2\ (2+n)\,\kappa\,\lambda\,\hbar}{6+(2+n)\,\kappa\,\lambda\,\hbar},\ -\frac{3\,\kappa\,\lambda^2\,\hbar\ (8+n+2\,\kappa\,\lambda\,\hbar+n\,\kappa\,\lambda\,\hbar)}{(3+\kappa\,\lambda\,\hbar)\ (6+(2+n)\,\kappa\,\lambda\,\hbar)}\right\}\right\}$$

In[241]:=
```
{δm1^2 == δm0^2, δλ1a == δλ2a, δλ1b == δλ2b} /.
   Solve[cteqs, {δm1, δλ1a, δλ2a, δλ1b, δλ2b, Z, ZΔ}] /.
  Solve[ctegs3, {δm0, δλ0}] /. ZΔ → 1 // FullSimplify
```

Solve::svars : Equations may not give solutions for all "solve" variables. ≫

Out[241]= {{{True, True, True}, {True, True, True}}, {{True, True, True}, {True, True, True}}}

In[242]:=
```
{δλ1a == ((3 (n + 4) + (n + 2) κ λ ℏ)/((n + 2) κ λ ℏ + 6)) δλ1b} /.
   Solve[cteqs, {δm1, δλ1a, δλ2a, δλ1b, δλ2b, Z, ZΔ}] /.
  Solve[ctegs3, {δm0, δλ0}] /. ZΔ → 1 // FullSimplify
```

Solve::svars : Equations may not give solutions for all "solve" variables. ≫

Out[242]= {{{True}, {True}}, {{True}, {True}}}

In[243]:=
```
δλ1b /. Solve[cteqs, {δm1, δλ1a, δλ2a, δλ1b, δλ2b, Z, ZΔ}] /. ZΔ → 1 // FullSimplify //
  DeleteDuplicates
```

Solve::svars : Equations may not give solutions for all "solve" variables. ≫

Out[243]=
$$\left\{-\frac{\kappa\,\lambda^2\,\hbar}{3+\kappa\,\lambda\,\hbar}\right\}$$

In[244]:=
```
{δλ₀ == 1 δλ₁ₐ + 2 δλ₁b} /. Solve[cteqs, {δm₁, δλ₁ₐ, δλ₂ₐ, δλ₁b, δλ₂b, Z, ZΔ}] /.
    Solve[ctegs3, {δm₀, δλ₀}] /. ZΔ → 1 // FullSimplify
```

Solve::svars : Equations may not give solutions for all "solve" variables. ≫

Out[244]= {{{True}, {True}}, {{True}, {True}}}

In[245]:=

$$\left\{\delta m_0{}^2 == -\frac{m^2\,\kappa\,\lambda\,\hbar}{3}\left(\frac{\delta\lambda_{1\,a}}{\delta\lambda_{1\,b}}-1\right)\right\}$$

```
 /. Solve[cteqs, {δm₁, δλ₁ₐ, δλ₂ₐ, δλ₁b, δλ₂b, Z, ZΔ}] /.
    Solve[ctegs3, {δm₀, δλ₀}] /. ZΔ → 1 // FullSimplify
```

Solve::svars : Equations may not give solutions for all "solve" variables. ≫

Out[245]= {{{True}, {True}}, {{True}, {True}}}