Renormalization of SI-2PIEA gap equations in the Hartree-Fock approximation

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Supplement to thesis Chapter 3

Mathematica notebook to compute couter-terms for two loop truncations of the two particle irreducible effective action

In[90]:=
```
ClearAll[veom, geom, neom, regularisedtadpoles, mg2soln, cteq, cts, δm, δλ];
```

Hartree-Fock gap equations with counterterms

Goldstone equation of motion. Quantities in reference to the thesis are:
p is the four-momentum flowing through the propagators $\Delta_G{}^{-1}$ and $\Delta_N{}^{-1}$,
mg2 is the Goldstone mass squared $m_G{}^2$,
mn2 is the Higgs mass squared $m_H{}^2$,
Z and ZΔ are the wavefunction a propagator renormalization constants,
m^2 is the (renormalized) Lagrangian mass parameter, $\delta m_0{}^2$, $\delta m_1{}^2$ are its counter-terms,
λ is the (renormalized) four point coupling,
$\delta\lambda_0$, $\delta\lambda_{1a}$, $\delta\lambda_{1b}$, $\delta\lambda_{2a}$, $\delta\lambda_{2b}$ are the independent coupling counter-terms,
v is the scalar field vacuum expectation value,
$\hbar$ is the reduced Planck constant,
n is the number of fields in the O(n) symmetry group,
t∞g, t∞n are the divergent tadpole integrals for the Goldstone, Higgs resp.,
tfing, tfinn are the finite parts of the tadpoles for the Goldstone, Higgs resp.

Equations of motion

Vev equation of motion

In[91]:=
(*veom=

$$Z\Delta^{-1}\left(m^2+\delta m_0{}^2\right)v+\frac{\lambda+\delta\lambda_0}{6}v^3+\frac{\hbar}{6}Z\Delta\left(n-1\right)(\lambda+\delta\lambda_{1a})\,v\,(t\infty g+tfing)+\frac{\hbar}{6}Z\Delta\ \left(3\lambda+\delta\lambda_{1a}+2\delta\lambda_{1b}\right)v\,(t\infty n+tfinn)$$

$$finveom=m^2v+\frac{\lambda}{6}v^3+\frac{\hbar}{6}\left(n-1\right)\lambda\ v\ tfing+\frac{\hbar}{2}\lambda\ v\ tfinn$$ *)

```
veom = v mg2
```

Out[91]= mg2 v

Goldstone equation of motion

In[92]:=
$$\text{geom} = p^2 - \text{mg2} == Z\, Z\Delta\, p^2 - m^2 - \delta m_1{}^2 - Z\Delta \frac{\lambda + \delta\lambda_{1\,a}}{6} v^2 - \frac{\hbar}{6}\left(\left(n+1\right)\lambda + \left(n-1\right)\delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b}\right) Z\Delta^2\, (\text{t}\infty\text{g} + \text{tfing}) - \frac{\hbar}{6}(\lambda + \delta\lambda_{2\,a})\, Z\Delta^2\, (\text{t}\infty\text{n} + \text{tfinn})$$

$$\text{finmg2} = \text{mg2} \;/.\; \text{Solve}\left[p^2 - \text{mg2} == p^2 - m^2 - \frac{\lambda}{6} v^2 - \frac{\hbar}{6}\left(n+1\right)\lambda\,\text{tfing} - \frac{\hbar}{6}\lambda\,\text{tfinn},\ \text{mg2}\right][[1]]$$

Out[92]=
$$-\text{mg2} + p^2 == -m^2 + p^2\, Z\, Z\Delta - \delta m_1^2 - \frac{1}{6} v^2\, Z\Delta\, (\lambda + \delta\lambda_a) - \frac{1}{6}(\text{tfinn} + \text{t}\infty\text{n})\, Z\Delta^2\, \hbar\, (\lambda + \delta\lambda_{2\,a}) - \frac{1}{6}(\text{tfing} + \text{t}\infty\text{g})\, Z\Delta^2\, \hbar\, ((1+n)\,\lambda + (-1+n)\,\delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b})$$

Out[93]=
$$\frac{1}{6}\left(6\, m^2 + v^2\,\lambda + \text{tfing}\,\lambda\,\hbar + n\,\text{tfing}\,\lambda\,\hbar + \text{tfinn}\,\lambda\,\hbar\right)$$

Higgs equation of motion

In[94]:=
$$\text{neom} = p^2 - \text{mn2} == Z\, Z\Delta\, p^2 - m^2 - \delta m_1{}^2 - Z\Delta\, v^2 \frac{\left(3\,\lambda + \delta\lambda_{1\,a} + 2\,\delta\lambda_{1\,b}\right)}{6} - \frac{\hbar}{6}(\lambda + \delta\lambda_{2\,a})\left(n-1\right) Z\Delta^2\, (\text{t}\infty\text{g} + \text{tfing}) - \frac{\hbar}{6}\left(3\,\lambda + \delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b}\right) Z\Delta^2\, (\text{t}\infty\text{n} + \text{tfinn})$$

$$\text{finmn2} = \text{mn2} \;/.\; \text{Solve}\left[p^2 - \text{mn2} == p^2 - m^2 - v^2 \frac{\lambda}{2} - \frac{\hbar}{6}\lambda\left(n-1\right)\text{tfing} - \frac{\hbar}{2}\lambda\,\text{tfinn},\ \text{mn2}\right][[1]]$$

Out[94]=
$$-\text{mn2} + p^2 == -m^2 + p^2\, Z\, Z\Delta - \delta m_1^2 - \frac{1}{6}(-1+n)\,(\text{tfing} + \text{t}\infty\text{g})\, Z\Delta^2\, \hbar\, (\lambda + \delta\lambda_{2\,a}) - \frac{1}{6} v^2\, Z\Delta\, (3\,\lambda + \delta\lambda_a + 2\,\delta\lambda_b) - \frac{1}{6}(\text{tfinn} + \text{t}\infty\text{n})\, Z\Delta^2\, \hbar\, (3\,\lambda + \delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b})$$

Out[95]=
$$\frac{1}{6}\left(6\, m^2 + 3\, v^2\,\lambda - \text{tfing}\,\lambda\,\hbar + n\,\text{tfing}\,\lambda\,\hbar + 3\,\text{tfinn}\,\lambda\,\hbar\right)$$

Infinite parts of tadpoles

c0, c1, Λ and μ are regularisation/renormalisation scheme dependent quantities

In[96]:=
$$\text{regularisedtadpoles} = \left\{\text{t}\infty\text{g} \to \text{c0}\,\Lambda^2 + \text{c1}\,\text{mg2}\,\text{Log}\left[\Lambda^2 / \mu^2\right],\ \text{t}\infty\text{n} \to \text{c0}\,\Lambda^2 + \text{c1}\,\text{mn2}\,\text{Log}\left[\Lambda^2 / \mu^2\right]\right\}$$

Out[96]=
$$\left\{\text{t}\infty\text{g} \to \text{c0}\,\Lambda^2 + \text{c1}\,\text{mg2}\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right],\ \text{t}\infty\text{n} \to \text{c0}\,\Lambda^2 + \text{c1}\,\text{mn2}\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\right\}$$

Sub in tadpole expressions, eliminate mn2 and solve for mg2

In[97]:=
```
mn2fromneom = Solve[neom /. regularisedtadpoles, mn2][[1]]
```

Out[97]=
$$\left\{mn2 \to \left(-m^2 - p^2 + p^2\, Z\, Z\Delta - \delta m_1^2 - \frac{1}{6}(-1+n)\, Z\Delta^2\, \hbar \left(tfing + c0\, \Lambda^2 + c1\, mg2\, Log\left[\frac{\Lambda^2}{\mu^2}\right]\right)(\lambda + \delta\lambda_{2\,a}) - \frac{1}{6} v^2\, Z\Delta\, (3\,\lambda + \delta\lambda_a + 2\,\delta\lambda_b) - \frac{1}{6} tfinn\, Z\Delta^2\, \hbar\, (3\,\lambda + \delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b}) - \frac{1}{6} c0\, Z\Delta^2\, \Lambda^2\, \hbar\, (3\,\lambda + \delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b})\right) \Big/ \left(-1 + \frac{1}{6} c1\, Z\Delta^2\, \hbar\, Log\left[\frac{\Lambda^2}{\mu^2}\right] (3\,\lambda + \delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b})\right)\right\}$$

In[98]:= **mg2soln = mg2 /. (geom /. regularisedtadpoles /. mn2fromneom // Solve[#, mg2][[1]] &)**

Out[98]=

$$
\begin{aligned}
&\Bigg(-m^2 - p^2 + p^2\, Z\, Z\Delta - \delta m_1^2 - \frac{1}{6}\, v^2\, Z\Delta\, (\lambda + \delta\lambda_a) - \frac{1}{6}\, \text{tfinn}\, Z\Delta^2\, \hbar\, (\lambda + \delta\lambda_{2\,a}) - \\
&\quad \frac{1}{6}\, \text{c0}\, Z\Delta^2\, \Lambda^2\, \hbar\, (\lambda + \delta\lambda_{2\,a}) - \frac{1}{6}\, \text{tfing}\, Z\Delta^2\, \hbar\, ((1+n)\,\lambda + (-1+n)\,\delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b}) - \\
&\quad \frac{1}{6}\, \text{c0}\, Z\Delta^2\, \Lambda^2\, \hbar\, ((1+n)\,\lambda + (-1+n)\,\delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b}) + \\
&\quad \frac{\text{c1}\, m^2\, Z\Delta^2\, \hbar\, \text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] (\lambda + \delta\lambda_{2\,a})}{6\left(-1 + \frac{1}{6}\, \text{c1}\, Z\Delta^2\, \hbar\, \text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] (3\,\lambda + \delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b})\right)} + \\
&\quad \frac{\text{c1}\, p^2\, Z\Delta^2\, \hbar\, \text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] (\lambda + \delta\lambda_{2\,a})}{6\left(-1 + \frac{1}{6}\, \text{c1}\, Z\Delta^2\, \hbar\, \text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] (3\,\lambda + \delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b})\right)} - \\
&\quad \frac{\text{c1}\, p^2\, Z\, Z\Delta^3\, \hbar\, \text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] (\lambda + \delta\lambda_{2\,a})}{6\left(-1 + \frac{1}{6}\, \text{c1}\, Z\Delta^2\, \hbar\, \text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] (3\,\lambda + \delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b})\right)} + \\
&\quad \frac{\text{c1}\, Z\Delta^2\, \hbar\, \text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] \delta m_1^2\, (\lambda + \delta\lambda_{2\,a})}{6\left(-1 + \frac{1}{6}\, \text{c1}\, Z\Delta^2\, \hbar\, \text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] (3\,\lambda + \delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b})\right)} + \\
&\quad \frac{\text{c1}\, (-1+n)\, \text{tfing}\, Z\Delta^4\, \hbar^2\, \text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] (\lambda + \delta\lambda_{2\,a})^2}{36\left(-1 + \frac{1}{6}\, \text{c1}\, Z\Delta^2\, \hbar\, \text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] (3\,\lambda + \delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b})\right)} + \\
&\quad \frac{\text{c0}\, \text{c1}\, (-1+n)\, Z\Delta^4\, \Lambda^2\, \hbar^2\, \text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] (\lambda + \delta\lambda_{2\,a})^2}{36\left(-1 + \frac{1}{6}\, \text{c1}\, Z\Delta^2\, \hbar\, \text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] (3\,\lambda + \delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b})\right)} + \\
&\quad \frac{\text{c1}\, v^2\, Z\Delta^3\, \hbar\, \text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] (\lambda + \delta\lambda_{2\,a})\, (3\,\lambda + \delta\lambda_a + 2\,\delta\lambda_b)}{36\left(-1 + \frac{1}{6}\, \text{c1}\, Z\Delta^2\, \hbar\, \text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] (3\,\lambda + \delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b})\right)} + \\
&\quad \frac{\text{c1}\, \text{tfinn}\, Z\Delta^4\, \hbar^2\, \text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] (\lambda + \delta\lambda_{2\,a})\, (3\,\lambda + \delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b})}{36\left(-1 + \frac{1}{6}\, \text{c1}\, Z\Delta^2\, \hbar\, \text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] (3\,\lambda + \delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b})\right)} + \\
&\quad \frac{\text{c0}\, \text{c1}\, Z\Delta^4\, \Lambda^2\, \hbar^2\, \text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] (\lambda + \delta\lambda_{2\,a})\, (3\,\lambda + \delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b})}{36\left(-1 + \frac{1}{6}\, \text{c1}\, Z\Delta^2\, \hbar\, \text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] (3\,\lambda + \delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b})\right)}\Bigg) \Bigg/ \\
&\Bigg(-1 + \frac{1}{6}\, \text{c1}\, Z\Delta^2\, \hbar\, \text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] ((1+n)\,\lambda + (-1+n)\,\delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b}) - \\
&\quad \frac{\text{c1}^2\, (-1+n)\, Z\Delta^4\, \hbar^2\, \text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]^2 (\lambda + \delta\lambda_{2\,a})^2}{36\left(-1 + \frac{1}{6}\, \text{c1}\, Z\Delta^2\, \hbar\, \text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] (3\,\lambda + \delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b})\right)}\Bigg)
\end{aligned}
$$

In[99]:=
```
mn2soln = mn2 /. mn2fromneom /. mg2 → mg2soln // Simplify
```

Out[99]=

$$-\Bigg(\Bigg(6\,m^2+6\,p^2-6\,p^2\,Z\,Z\Delta+6\,\delta m_1^2+v^2\,Z\Delta\,(3\,\lambda+\delta\lambda_a+2\,\delta\lambda_b)+\text{tfinn}\,Z\Delta^2\,\hbar\,(3\,\lambda+\delta\lambda_{2\,a}+2\,\delta\lambda_{2\,b})+c0\,Z\Delta^2\,\Lambda^2\,\hbar\,(3\,\lambda+\delta\lambda_{2\,a}+2\,\delta\lambda_{2\,b})+\Bigg((-1+n)\,Z\Delta^2\,\hbar\,(\lambda+\delta\lambda_{2\,a})\Bigg(18\,\text{tfing}+18\,c0\,\Lambda^2+18\,c1\,m^2\,\text{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big]+18\,c1\,p^2\,\text{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big]-18\,c1\,p^2\,Z\,Z\Delta\,\text{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big]+3\,c1\,v^2\,Z\Delta\,\lambda\,\text{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big]-9\,c1\,\text{tfing}\,Z\Delta^2\,\lambda\,\hbar\,\text{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big]+3\,c1\,\text{tfinn}\,Z\Delta^2\,\lambda\,\hbar\,\text{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big]-6\,c0\,c1\,Z\Delta^2\,\lambda\,\Lambda^2\,\hbar\,\text{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big]-6\,c1^2\,m^2\,Z\Delta^2\,\lambda\,\hbar\,\text{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big]^2-6\,c1^2\,p^2\,Z\Delta^2\,\lambda\,\hbar\,\text{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big]^2+6\,c1^2\,p^2\,Z\,Z\Delta^3\,\lambda\,\hbar\,\text{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big]^2-3\,c1\,\text{tfing}\,Z\Delta^2\,\hbar\,\text{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big]\,\delta\lambda_{2\,a}+3\,c1\,\text{tfinn}\,Z\Delta^2\,\hbar\,\text{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big]\,\delta\lambda_{2\,a}+c1^2\,v^2\,Z\Delta^3\,\lambda\,\hbar\,\text{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big]^2\,\delta\lambda_{2\,a}+c1^2\,v^2\,Z\Delta^3\,\lambda\,\hbar\,\text{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big]^2\,\delta\lambda_b+c1^2\,v^2\,Z\Delta^3\,\hbar\,\text{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big]^2\,\delta\lambda_{2\,a}\,\delta\lambda_b-6\,c1\,\text{tfing}\,Z\Delta^2\,\hbar\,\text{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big]\,\delta\lambda_{2\,b}-6\,c0\,c1\,Z\Delta^2\,\Lambda^2\,\hbar\,\text{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big]\,\delta\lambda_{2\,b}-6\,c1^2\,m^2\,Z\Delta^2\,\hbar\,\text{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big]^2\,\delta\lambda_{2\,b}-6\,c1^2\,p^2\,Z\Delta^2\,\hbar\,\text{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big]^2\,\delta\lambda_{2\,b}+6\,c1^2\,p^2\,Z\,Z\Delta^3\,\hbar\,\text{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big]^2\,\delta\lambda_{2\,b}-c1^2\,v^2\,Z\Delta^3\,\lambda\,\hbar\,\text{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big]^2\,\delta\lambda_{2\,b}-6\,c1\,\text{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big]\,\delta m_1^2\Big(-3+c1\,Z\Delta^2\,\lambda\,\hbar\,\text{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big]+c1\,Z\Delta^2\,\hbar\,\text{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big]\,\delta\lambda_{2\,b}\Big)-c1\,v^2\,Z\Delta\,\text{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big]\,\delta\lambda_a\Big(-3+c1\,Z\Delta^2\,\lambda\,\hbar\,\text{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big]+c1\,Z\Delta^2\,\hbar\,\text{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big]\,\delta\lambda_{2\,b}\Big)\Bigg)\Bigg)\Bigg/\Bigg(\Big(-3+c1\,Z\Delta^2\,\lambda\,\hbar\,\text{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big]+c1\,Z\Delta^2\,\hbar\,\text{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big]\,\delta\lambda_{2\,b}\Big)\Big(-6+2\,c1\,Z\Delta^2\,\lambda\,\hbar\,\text{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big]+c1\,n\,Z\Delta^2\,\lambda\,\hbar\,\text{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big]+c1\,n\,Z\Delta^2\,\hbar\,\text{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big]\,\delta\lambda_{2\,a}+2\,c1\,Z\Delta^2\,\hbar\,\text{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big]\,\delta\lambda_{2\,b}\Big)\Bigg)\Bigg)\Bigg/\Bigg(6\,\Big(-1+\frac{1}{6}\,c1\,Z\Delta^2\,\hbar\,\text{Log}\Big[\frac{\Lambda^2}{\mu^2}\Big]\,(3\,\lambda+\delta\lambda_{2\,a}+2\,\delta\lambda_{2\,b})\Big)\Bigg)\Bigg)$$

Gather divergences proportional v, tfing and tfinn and set independently to zero

First we subtract the finite equation of motion, then gather coefficients of the remainder into a list and set each to zero (after some trimming and simplifying).

In[100]:=
```
cteq = ((CoefficientList[mg2soln - finmg2, {p, v, tfing, tfinn}] // Flatten) //
    DeleteDuplicates // Simplify // FullSimplify) == 0 // Thread
```

Out[100]=

$$\Bigg\{-\Bigg(\Bigg(6\,\delta m_1^2 + Z\Delta^2\,\hbar\left(c0\,\Lambda^2 + c1\,m^2\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\right)\left((2+n)\,\lambda + n\,\delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b}\right)\Bigg)\Big/\left(-6 + c1\,(2+n)\,Z\Delta^2\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right] + c1\,Z\Delta^2\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\left(n\,\delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b}\right)\right)\Bigg) == 0,$$

$$-\frac{\lambda\,\hbar}{6} + \left(3\,Z\Delta^2\,\hbar\,(\lambda + \delta\lambda_{2\,a})\right)\Big/\Bigg(\left(-3 + c1\,Z\Delta^2\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right] + c1\,Z\Delta^2\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\delta\lambda_{2\,b}\right)\left(-6 + c1\,(2+n)\,Z\Delta^2\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right] + c1\,Z\Delta^2\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\left(n\,\delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b}\right)\right)\Bigg) == 0,$$

$$-\frac{1}{6\,c1\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]}\Bigg(6 + c1\,(1+n)\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right] + \frac{18}{n\left(-3 + c1\,Z\Delta^2\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right] + c1\,Z\Delta^2\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\delta\lambda_{2\,b}\right)} + (36\,(-1+n))\Big/\left(n\left(-6 + c1\,(2+n)\,Z\Delta^2\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right] + c1\,Z\Delta^2\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\left(n\,\delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b}\right)\right)\right)\Bigg) == 0,\ \mathrm{True},$$

$$-\frac{\lambda}{6} + \frac{Z\Delta\,(\lambda + \delta\lambda_b)}{n\left(-3 + c1\,Z\Delta^2\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right] + c1\,Z\Delta^2\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\delta\lambda_{2\,b}\right)} - \left(Z\Delta\left((2+n)\,\lambda + n\,\delta\lambda_a + 2\,\delta\lambda_b\right)\right)\Big/\left(n\left(-6 + c1\,(2+n)\,Z\Delta^2\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right] + c1\,Z\Delta^2\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\left(n\,\delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b}\right)\right)\right) == 0,$$

$$(-6 + 6\,Z\,Z\Delta)\Big/\left(-6 + c1\,(2+n)\,Z\Delta^2\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right] + c1\,Z\Delta^2\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\left(n\,\delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b}\right)\right) == 0\Bigg\}$$

In[101]:=
```
cteq2 = ((CoefficientList[mn2soln - finmn2, {p, v, tfing, tfinn}] // Flatten) //
    DeleteDuplicates // Simplify // FullSimplify) == 0 // Thread
```

Out[101]=
$$\left\{-\left(\left(6\,\delta m_1^2 + Z\Delta^2\,\hbar\left(c0\,\Lambda^2 + c1\,m^2\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\right)\left((2+n)\,\lambda + n\,\delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b}\right)\right)\Big/\left(-6 + c1\,(2+n)\,Z\Delta^2\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right] + c1\,Z\Delta^2\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\left(n\,\delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b}\right)\right)\right) == 0,\right.$$

$$-\frac{1}{2\,c1\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]}\left(2 + c1\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right] + \frac{6\,(-1+n)}{n\left(-3 + c1\,Z\Delta^2\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right] + c1\,Z\Delta^2\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\delta\lambda_{2\,b}\right)} + 12\Big/\left(n\left(-6 + c1\,(2+n)\,Z\Delta^2\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right] + c1\,Z\Delta^2\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\left(n\,\delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b}\right)\right)\right)\right) == 0,$$

$$\left((-1+n)\,\hbar\left(-Z\Delta^2\,\delta\lambda_{2\,a}\left(-18 + c1\,n\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\right)\left(-3 + c1\,Z\Delta^2\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\right) + c1^2\,n\,Z\Delta^2\,\lambda\,\hbar^2\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]^2\delta\lambda_{2\,b}\right) + \lambda\left(18\left(-1 + Z\Delta^2\right) + c1\,Z\Delta^2\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\left(3\,(4+n) - c1\,(2+n)\,Z\Delta^2\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\right) - c1\,Z\Delta^2\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\delta\lambda_{2\,b}\left(-12 + c1\,(4+n)\,Z\Delta^2\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right] + 2\,c1\,Z\Delta^2\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\delta\lambda_{2\,b}\right)\right)\right)\Big/\left(6\left(-3 + c1\,Z\Delta^2\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right] + c1\,Z\Delta^2\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\delta\lambda_{2\,b}\right)\left(-6 + c1\,(2+n)\,Z\Delta^2\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right] + c1\,Z\Delta^2\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\left(n\,\delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b}\right)\right)\right) == 0,$$

$$\mathrm{True},\ -\frac{\lambda}{2} - \frac{(-1+n)\,Z\Delta\,(\lambda + \delta\lambda_b)}{n\left(-3 + c1\,Z\Delta^2\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right] + c1\,Z\Delta^2\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\delta\lambda_{2\,b}\right)} - \left(Z\Delta\left((2+n)\,\lambda + n\,\delta\lambda_a + 2\,\delta\lambda_b\right)\right)\Big/\left(n\left(-6 + c1\,(2+n)\,Z\Delta^2\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right] + c1\,Z\Delta^2\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\left(n\,\delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b}\right)\right)\right) == 0,$$

$$\left.(-6 + 6\,Z\,Z\Delta)\Big/\left(-6 + c1\,(2+n)\,Z\Delta^2\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right] + c1\,Z\Delta^2\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\left(n\,\delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b}\right)\right) == 0\right\}$$

Solve for counterterms

Find counter-terms from the gap equations

In[102]:=
```
cteqs = {cteq, cteq2} // Flatten // FullSimplify // DeleteDuplicates
```

Out[102]=

$$\Bigg\{\left(6\,\delta m_1^2 + Z\Delta^2\,\hbar\left(c0\,\Lambda^2 + c1\,m^2\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\right)\left((2+n)\,\lambda + n\,\delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b}\right)\right)\Big/ \left(-6 + c1\,(2+n)\,Z\Delta^2\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] + c1\,Z\Delta^2\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\left(n\,\delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b}\right)\right) == 0,$$

$$\hbar\left(\lambda - \left(18\,Z\Delta^2\,(\lambda + \delta\lambda_{2\,a})\right)\Big/\left(\left(-3 + c1\,Z\Delta^2\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] + c1\,Z\Delta^2\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\delta\lambda_{2\,b}\right)\left(-6 + c1\,(2+n)\,Z\Delta^2\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] + c1\,Z\Delta^2\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\left(n\,\delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b}\right)\right)\right)\right) == 0,$$

$$\frac{1}{c1\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]}\left(6 + c1\,(1+n)\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] + \frac{18}{n\left(-3 + c1\,Z\Delta^2\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] + c1\,Z\Delta^2\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\delta\lambda_{2\,b}\right)} + (36\,(-1+n))\Big/\left(n\left(-6 + c1\,(2+n)\,Z\Delta^2\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] + c1\,Z\Delta^2\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\left(n\,\delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b}\right)\right)\right)\right) == 0,$$

$$\text{True},\ \lambda + \left(6\,Z\Delta\left((2+n)\,\lambda + n\,\delta\lambda_a + 2\,\delta\lambda_b\right)\right)\Big/\left(n\left(-6 + c1\,(2+n)\,Z\Delta^2\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] + c1\,Z\Delta^2\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\left(n\,\delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b}\right)\right)\right) == \frac{6\,Z\Delta\,(\lambda + \delta\lambda_b)}{n\left(-3 + c1\,Z\Delta^2\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] + c1\,Z\Delta^2\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\delta\lambda_{2\,b}\right)},$$

$$(-1 + Z\,Z\Delta)\Big/\left(-6 + c1\,(2+n)\,Z\Delta^2\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] + c1\,Z\Delta^2\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\left(n\,\delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b}\right)\right) == 0,$$

$$\frac{1}{c1\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]}\left(2 + c1\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] + \frac{6\,(-1+n)}{n\left(-3 + c1\,Z\Delta^2\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] + c1\,Z\Delta^2\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\delta\lambda_{2\,b}\right)} + 12\Big/\left(n\left(-6 + c1\,(2+n)\,Z\Delta^2\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] + c1\,Z\Delta^2\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\left(n\,\delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b}\right)\right)\right)\right) == 0,$$

$$(-1+n)\,\hbar\left(\lambda - \left(18\,Z\Delta^2\,(\lambda + \delta\lambda_{2\,a})\right)\Big/\left(\left(-3 + c1\,Z\Delta^2\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] + c1\,Z\Delta^2\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\delta\lambda_{2\,b}\right)\left(-6 + c1\,(2+n)\,Z\Delta^2\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] + c1\,Z\Delta^2\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\left(n\,\delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b}\right)\right)\right)\right) == 0,$$

$$\lambda + \frac{2\,(-1+n)\,Z\Delta\,(\lambda + \delta\lambda_b)}{n\left(-3 + c1\,Z\Delta^2\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] + c1\,Z\Delta^2\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\delta\lambda_{2\,b}\right)} + \left(2\,Z\Delta\left((2+n)\,\lambda + n\,\delta\lambda_a + 2\,\delta\lambda_b\right)\right)\Big/\left(n\left(-6 + c1\,(2+n)\,Z\Delta^2\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] + c1\,Z\Delta^2\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\left(n\,\delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b}\right)\right)\right) == 0\Bigg\}$$

In[103]:=
```
cts = {δm1^2, δλ1a, δλ2a, δλ1b, δλ2b, Z, ZΔ} /. Solve[cteqs,
    {δm1, δλ1a, δλ2a, δλ1b, δλ2b, Z, ZΔ}] // FullSimplify // DeleteDuplicates
```

Solve::svars : Equations may not give solutions for all "solve" variables. ≫

Out[103]=
$$\Big\{\Big\{-\frac{(2+n)\,\lambda\,\hbar\,\left(c0\,\Lambda^2+c1\,m^2\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\right)}{6+c1\,(2+n)\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]},$$
$$\lambda\left(-1+\frac{6\,(2+n)}{n\,Z\Delta\left(6+c1\,(2+n)\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\right)}-\frac{6}{3\,n\,Z\Delta+c1\,n\,Z\Delta\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]}\right),$$
$$\lambda\left(-1+\frac{18}{Z\Delta^2\left(3+c1\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\right)\left(6+c1\,(2+n)\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\right)}\right),$$
$$\lambda\left(-1+\frac{3}{3\,Z\Delta+c1\,Z\Delta\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]}\right),\ \lambda\left(-1+\frac{3}{Z\Delta^2\left(3+c1\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\right)}\right),\ \frac{1}{Z\Delta},\ Z\Delta\Big\}\Big\}$$

ZΔ is redundant in this truncation, can remove it :

In[104]:=
```
cts /. ZΔ → 1 // FullSimplify
```

Out[104]=
$$\Big\{\Big\{-\frac{(2+n)\,\lambda\,\hbar\,\left(c0\,\Lambda^2+c1\,m^2\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\right)}{6+c1\,(2+n)\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]},$$
$$\lambda\left(-1-\frac{6}{3\,n+c1\,n\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]}+\frac{6\,(2+n)}{n\left(6+c1\,(2+n)\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\right)}\right),$$
$$\lambda\left(-1+\frac{18}{\left(3+c1\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\right)\left(6+c1\,(2+n)\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\right)}\right),$$
$$\lambda\left(-1+\frac{3}{3+c1\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]}\right),\ \lambda\left(-1+\frac{3}{3+c1\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]}\right),\ 1,\ 1\Big\}\Big\}$$

Verify that the finite gap equations come out right

In[105]:=
```
finmg2 ==
  (mg2soln /. Solve[cteqs, {δm₁, δλ₁ a, δλ₂ a, δλ₂ b, Z, ZΔ}] /. ZΔ → 1 // FullSimplify //
     DeleteDuplicates)[[2]] // Simplify
```

Solve::svars : Equations may not give solutions for all "solve" variables. ≫

FullSimplify::infd : Expression $\left(-m^2 - \frac{-c0\,\Lambda^2 - c1\,m^2\,\text{Log[Power[≪2≫] Power[≪2≫]]}}{c1\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]} - \frac{≪1≫}{≪1≫} - \frac{3\,\text{tfing}\,≪1≫\,\hbar\,(≪1≫)}{2\,≪1≫^2\,(≪1≫)^2} - \frac{3\,c0\,\lambda^2\,\Lambda^2\,\hbar\left(-(-1+n)\,\lambda + (1+n)\,\lambda + \frac{2\,(9\,\text{Power[≪2≫]+≪7≫+≪1≫})}{3\,c1\,\lambda^2\,\hbar\,\text{Log[Times[≪2≫]]}}\right)}{2\,(3 + c1\,\lambda\,\hbar\,\text{Log[Times[≪2≫]]})^2\,(\lambda + \delta\lambda_b)^2}\right) \Big/ \left(-1 + \left(3\,c1\,\lambda^2\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\left(-(-1+n)\,\lambda + (1+n)\,\lambda + \frac{2\,(\text{Times[≪2≫]} + ≪7≫ + \text{Times[≪5≫]})}{3\,c1\,\lambda^2\,\hbar\,\text{Log[≪1≫]}}\right)\right) \Big/ \left(2\,(3 + c1\,\lambda\,\hbar\,\text{Log[≪1≫]})^2\,(\lambda + \delta\lambda_b)^2\right)\right)$ simplified to ComplexInfinity. ≫

Out[105]= True

In[106]:=
```
finmn2 == mn2 /.
  ((neom /. regularisedtadpoles /. mg2 → mg2soln /. Solve[cteqs, {δm₁, δλ₁ a,
          δλ₂ a, δλ₁ b, δλ₂ b, Z, ZΔ}] /. ZΔ → 1 // FullSimplify //
     DeleteDuplicates) // Solve[#, mn2] &) // FullSimplify
```

Solve::svars : Equations may not give solutions for all "solve" variables. ≫

Out[106]= {True}

Verify counter-term expressions in text

In[107]:=
$$\left\{\delta m_1{}^2 == \frac{-\hbar\,\lambda\,(n+2)}{6}\left(c0\,\Lambda^2 + c1\,m^2\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\right)\frac{\delta\lambda_{1\,a}+\lambda}{\delta\lambda_{1\,b}+\lambda}\right\} /.$$
```
    Solve[cteqs, {δm₁, δλ₁ a, δλ₂ a, δλ₁ b, δλ₂ b, Z, ZΔ}] /. ZΔ → 1 //
  FullSimplify // Flatten // DeleteDuplicates
```

Solve::svars : Equations may not give solutions for all "solve" variables. ≫

Out[107]= {True}

In[108]:=
```
{δλ₁ a == δλ₂ a, δλ₁ b == δλ₂ b} /. Solve[cteqs, {δm₁, δλ₁ a, δλ₂ a, δλ₁ b, δλ₂ b, Z, ZΔ}] /.
    ZΔ → 1 // FullSimplify // Flatten // DeleteDuplicates
```

Solve::svars : Equations may not give solutions for all "solve" variables. ≫

Out[108]= {True}

In[109]:=
```
{δλ1 a / δλ1 b} /. Solve[cteqs, {δm1, δλ1 a, δλ2 a, δλ1 b, δλ2 b, Z, ZΔ}] /. ZΔ → 1 //
    FullSimplify // Flatten // DeleteDuplicates
```

Solve::svars : Equations may not give solutions for all "solve" variables. ≫

Out[109]= $\left\{1+\frac{3\,(2+n)}{6+c1\,(2+n)\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]}\right\}$

In[110]:=
```
δλ1 b /. Solve[cteqs, {δm1, δλ1 a, δλ2 a, δλ1 b, δλ2 b, Z, ZΔ}] /. ZΔ → 1 // FullSimplify //
  DeleteDuplicates
```

Solve::svars : Equations may not give solutions for all "solve" variables. ≫

Out[110]= $\left\{\lambda\left(-1+\frac{3}{3+c1\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]}\right)\right\}$

Total number of independent counter-term equations

In[111]:=
```
Length[{cteqs} // Flatten // FullSimplify // DeleteDuplicates] -
  1 (* -1 because one of the "equations" is identically "True" *)
```

Out[111]= 8