Renormalization of Soft Symmetry Improved 2PIEA gap equations in the Hartree-Fock approximation

Supplement to thesis Chapter 5 "Soft Symmetry Improvement"

Mathematica notebook to compute couter-terms for the Hartree-Fock truncation of the SSI-2PIEA

Author: Michael Brown

Hartree-Fock

```
ClearAll[geom, neom, intrules, msbarrules, mg2soln, cteq, cts, δm, δλ];
```

Hartree-Fock gap equations with counterterms

Goldstone equation of motion. Quantities in reference to the paper are:
p is the four-momentum flowing through the propagators $\Delta_G{}^{-1}$ and $\Delta_N{}^{-1}$,
mg2 is the Goldstone mass squared $m_G{}^2$,
mn2 is the Higgs mass squared $m_H{}^2$,
Z and ZΔ are the wavefunction a propagator renormalization constants,
m^2 is the (renormalized) Lagrangian mass parameter, $\delta \mathbf{m_0}^2$, $\delta \mathbf{m_1}^2$ are its counter-terms,
λ is the (renormalized) four point coupling,
$\delta\lambda_0$, $\delta\lambda_{1a}$, $\delta\lambda_{1b}$, $\delta\lambda_{2a}$, $\delta\lambda_{2b}$ are the independent coupling counter-terms,
v is the scalar field vacuum expectation value,
$\hbar$ is the reduced Planck constant,
n is the number of fields in the O(n) symmetry group,
ξ is the stiffness parameter,
ϵ is the solution of the Goldstone zero mode equation,
ssi = $\frac{1}{V\beta m_G{}^2}\left(\frac{1}{\epsilon}-1\right)$ is the soft symmetry improvement term in the propagator eoms,
ssi2 = $\frac{1}{\xi}(n-1)\,2\left(m_G{}^2\,\epsilon\right)^2$ is the other soft symmetry improvement term in the vev eom,
t∞g, t∞n are the divergent tadpole integrals for the Goldstone, Higgs resp.,
tfing, tfinn are the finite parts of the tadpoles for the Goldstone, Higgs resp.

Vev equation of motion

$$\mathrm{veom} = Z\Delta^{-1}\left(m^2 + \delta m_0{}^2\right) v + \frac{\lambda + \delta\lambda_0}{6} v^3 + \frac{\hbar}{6} Z\Delta \left(n - 1\right) (\lambda + \delta\lambda_{1\,a})\, v\, (t\infty g + \mathrm{tfing} + \mathrm{ssi}) + \frac{\hbar}{6} Z\Delta \left(3\,\lambda + \delta\lambda_{1\,a} + 2\,\delta\lambda_{1\,b}\right) v\, (t\infty n + \mathrm{tfinn}) + v\, \mathrm{ssi2}$$

Goldstone equation of motion

$$\mathrm{geom} = p^2 - \mathrm{mg2} == Z\, Z\Delta\, p^2 - m^2 - \delta m_1{}^2 - Z\Delta \frac{\lambda + \delta\lambda_{1\,a}}{6} v^2 - \frac{\hbar}{6}\left(\left(n+1\right)\lambda + \left(n-1\right)\delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b}\right) Z\Delta^2\, (t\infty g + \mathrm{tfing} + \mathrm{ssi}) - \frac{\hbar}{6}(\lambda + \delta\lambda_{2\,a})\, Z\Delta^2\, (t\infty n + \mathrm{tfinn})$$

Higgs equation of motion

$$\mathrm{neom} = p^2 - \mathrm{mn2} == Z\, Z\Delta\, p^2 - m^2 - \delta m_1{}^2 - Z\Delta\, v^2 \frac{\left(3\,\lambda + \delta\lambda_{1\,a} + 2\,\delta\lambda_{1\,b}\right)}{6} - \frac{\hbar}{6}(\lambda + \delta\lambda_{2\,a})\left(n - 1\right) Z\Delta^2\, (t\infty g + \mathrm{tfing} + \mathrm{ssi}) - \frac{\hbar}{6}\left(3\,\lambda + \delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b}\right) Z\Delta^2\, (t\infty n + \mathrm{tfinn})$$

Infinite parts of tadpoles in MSbar

MSbar rules for 4 - 2 ϵ dimensions

```
msbarrules = {t∞g → κ mg2, t∞n → κ mn2}
```

Sub in tadpole expressions, eliminate mn2 and solve for mg2

```
mg2soln = mg2 /.
   (geom /. msbarrules /. Solve[neom /. msbarrules, mn2][[1]] // Solve[#, mg2][[1]] &)

mn2soln = mn2 /. (neom /. msbarrules /. mg2 → mg2soln // Solve[#, mn2][[1]] &)
```

Gather divergences proportional v, tfing and tfinn and set independently to zero

First we subtract the finite equation of motion, then gather coefficients of the remainder into a list and set each to zero (after some trimming and simplifying).

$$\mathrm{cteq} = \left(\left(\mathrm{CoefficientList}\left[\mathrm{mg2soln} + \left(-m^2 - \frac{\lambda}{6} v^2 - \frac{\hbar}{6}\left(\left(n+1\right)\lambda\right)(\mathrm{tfing} + \mathrm{ssi}) - \frac{\hbar}{6}(\lambda)\,(\mathrm{tfinn})\right), \{p, v, \mathrm{tfing}, \mathrm{tfinn}\}\right] \,//\, \mathrm{Flatten}\right) \,//\, \mathrm{DeleteDuplicates} \,//\, \mathrm{Simplify} \,//\, \mathrm{FullSimplify}\right) == 0 \,//\, \mathrm{Thread}$$

```
cteq2 =
 ((CoefficientList[mn2soln + (-m^2 - λ/2 v^2 - ℏ/6 ((n - 1) λ) (tfing + ssi) - ℏ/2 (λ) (tfinn)),
        {p, v, tfing, tfinn}] // Flatten) //
      DeleteDuplicates // Simplify // FullSimplify) == 0 // Thread
```

Solve for counterterms

```
cteqs = {cteq, cteq2} // Flatten // FullSimplify // DeleteDuplicates
```

```
cts = {δm1^2, δλ1a, δλ2a, δλ1b, δλ2b, Z, ZΔ} /. Solve[cteqs,
     {δm1, δλ1a, δλ2a, δλ1b, δλ2b, Z, ZΔ}] // FullSimplify // DeleteDuplicates
```

ZΔ is redundant in this truncation, can remove it :

```
cts /. ZΔ → 1 // FullSimplify
```

```
mg2soln /. Solve[cteqs, {δm1, δλ1a, δλ2a, δλ2b, Z, ZΔ}] /. ZΔ → 1 // FullSimplify //
 DeleteDuplicates
```

```
mn2 /. ((neom /. msbarrules /. mg2 → mg2soln /.
         Solve[cteqs, {δm1, δλ1a, δλ2a, δλ1b, δλ2b, Z, ZΔ}] /. ZΔ → 1 // FullSimplify //
       DeleteDuplicates) // Solve[#, mn2] &) // FullSimplify
```

```
rnveom =
 veom /. {mg2 → m^2 + λ/6 v^2 + ℏ/6 ((n + 1) λ) (tfing + ssi) + ℏ/6 (λ) (tfinn), mn2 → m^2 + λ/2 v^2 +
       ℏ/6 ((n - 1) λ) (tfing + ssi) + ℏ/2 (λ) (tfinn)} // Simplify // DeleteDuplicates
```

```
ctegs3 =
 ((((CoefficientList[(1/v rnveom - (m^2 + λ/6 v^2 + ℏ/6 ((n - 1) λ) (tfing + ssi) + ℏ/2 (λ) (tfinn) +
                    ssi2)) /. msbarrules /. {mg2 → m^2 + λ/6 v^2 +
                 ℏ/6 ((n + 1) λ) (tfing + ssi) + ℏ/6 (λ) (tfinn), mn2 →
                m^2 + λ/2 v^2 + ℏ/6 ((n - 1) λ) (tfing + ssi) + ℏ/2 (λ) (tfinn)} //
              Simplify // Expand // FullSimplify, {v, tfing, tfinn}] //
            Simplify // Flatten) // DeleteDuplicates // Simplify //
          FullSimplify // DeleteDuplicates) == 0 // Thread) /.
      Solve[cteqs, {δm1, δλ1a, δλ2a, δλ1b, δλ2b, Z}] // Simplify //
    FullSimplify // DeleteDuplicates)[[1]]
```

```
{δm0^2, δλ0} /. Solve[ctegs3, {δm0, δλ0}] /. ZΔ → 1 // DeleteDuplicates // Simplify
```

```
{δm₁² == δm₀², δλ1a == δλ2a, δλ1b == δλ2b} /.
    Solve[cteqs, {δm₁, δλ1a, δλ2a, δλ1b, δλ2b, Z, ZΔ}] /.
   Solve[ctegs3, {δm₀, δλ₀}] /. ZΔ → 1 // FullSimplify
```

$$\left\{\delta\lambda_{1\,a} == \frac{\left(3\ (n+4) + \left(n+2\right)\kappa\,\lambda\,\hbar\right)}{\left(n+2\right)\kappa\,\lambda\,\hbar\ +6}\,\delta\lambda_{1\,b}\right\} /.$$

```
    Solve[cteqs, {δm₁, δλ1a, δλ2a, δλ1b, δλ2b, Z, ZΔ}] /.
   Solve[ctegs3, {δm₀, δλ₀}] /. ZΔ → 1 // FullSimplify
```

```
δλ1b /. Solve[cteqs, {δm₁, δλ1a, δλ2a, δλ1b, δλ2b, Z, ZΔ}] /. ZΔ → 1 // FullSimplify //
 DeleteDuplicates
```

```
{δλ₀ == 1 δλ1a + 2 δλ1b} /. Solve[cteqs, {δm₁, δλ1a, δλ2a, δλ1b, δλ2b, Z, ZΔ}] /.
   Solve[ctegs3, {δm₀, δλ₀}] /. ZΔ → 1 // FullSimplify
```

$$\left\{\delta m_0{}^2 == -\frac{m^2\,\kappa\,\lambda\,\hbar}{3}\left(\frac{\delta\lambda_{1\,a}}{\delta\lambda_{1\,b}}-1\right)\right\} /. \text{Solve[cteqs, \{}\delta m_1, \delta\lambda_{1\,a}, \delta\lambda_{2\,a}, \delta\lambda_{1\,b}, \delta\lambda_{2\,b}, Z, Z\Delta\}] /.$$

```
   Solve[ctegs3, {δm₀, δλ₀}] /. ZΔ → 1 // FullSimplify
```