Renormalization of Soft Symmetry Improved 2PIEA gap equations in the Hartree-Fock approximation

Supplement to thesis Chapter 5 "Soft Symmetry Improvement"

Mathematica notebook to compute couter-terms for the Hartree-Fock truncation of the SSI-2PIEA

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Hartree-Fock

In[41]:=
```
ClearAll[veom, geom, neom, regularisedtadpoles, mg2soln, mn2soln,
  cteq, cteq2, cteqs, ctsolns, cts, ctegs3, rnveom, veomCtEqs, δm, δλ];
```

Hartree-Fock gap equations with counterterms

Goldstone equation of motion. Quantities in reference to the paper are:
p is the four-momentum flowing through the propagators $\Delta_G{}^{-1}$ and $\Delta_N{}^{-1}$,
mg2 is the Goldstone mass squared $m_G{}^2$,
mn2 is the Higgs mass squared $m_H{}^2$,
Z and ZΔ are the wavefunction a propagator renormalization constants,
m^2 is the (renormalized) Lagrangian mass parameter, $\delta\mathbf{m_0}^2$, $\delta\mathbf{m_1}^2$ are its counter-terms,
λ is the (renormalized) four point coupling,
$\delta\lambda_0$, $\delta\lambda_{1\,a}$, $\delta\lambda_{1\,b}$, $\delta\lambda_{2\,a}$, $\delta\lambda_{2\,b}$ are the independent coupling counter-terms,
v is the scalar field vacuum expectation value,
$\hbar$ is the reduced Planck constant,
n is the number of fields in the O(n) symmetry group,
ξ is the stiffness parameter,
ϵ is the solution of the Goldstone zero mode equation,
ssi = $\frac{1}{V\beta m_G{}^2}\left(\frac{1}{\epsilon}-1\right)$ is the soft symmetry improvement term in the propagator eoms,
ssi2 = $\frac{1}{\xi}(n-1)\,2\left(m_G{}^2\,\epsilon\right)^2$ is the other soft symmetry improvement term in the vev eom,
t∞g, t∞n are the divergent tadpole integrals for the Goldstone, Higgs resp.,
tfing, tfinn are the finite parts of the tadpoles for the Goldstone, Higgs resp.

Vev equation of motion

In[42]:= $\mathbf{veom} = Z\Delta^{-1}\left(m^2+\delta m_0{}^2\right) v + \frac{\lambda+\delta\lambda_0}{6} v^3 + \frac{\hbar}{6} Z\Delta \left(n-1\right) (\lambda+\delta\lambda_{1\,a})\, v\, (\text{t}\infty\text{g}+\text{tfing}+\text{ssi}) + \frac{\hbar}{6} Z\Delta \left(3\lambda+\delta\lambda_{1\,a}+2\,\delta\lambda_{1\,b}\right) v\, (\text{t}\infty\text{n}+\text{tfinn}) + v\,\text{ssi2}$

Out[42]= $\text{ssi2}\, v + \frac{v\left(m^2+\delta m_0^2\right)}{Z\Delta} + \frac{1}{6} v^3 (\lambda+\delta\lambda_0) + \frac{1}{6}(-1+n)(\text{ssi}+\text{tfing}+\text{t}\infty\text{g})\, v\, Z\Delta\, \hbar\, (\lambda+\delta\lambda_a) + \frac{1}{6}(\text{tfinn}+\text{t}\infty\text{n})\, v\, Z\Delta\, \hbar\, (3\lambda+\delta\lambda_a+2\,\delta\lambda_b)$

Goldstone equation of motion

In[43]:= $\mathbf{geom} = p^2 - \text{mg2} == Z\, Z\Delta\, p^2 - m^2 - \delta m_1{}^2 - Z\Delta \frac{\lambda+\delta\lambda_{1\,a}}{6} v^2 - \frac{\hbar}{6}\left(\left(n+1\right)\lambda + \left(n-1\right)\delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b}\right) Z\Delta^2\, (\text{t}\infty\text{g}+\text{tfing}+\text{ssi}) - \frac{\hbar}{6}(\lambda+\delta\lambda_{2\,a})\, Z\Delta^2\, (\text{t}\infty\text{n}+\text{tfinn})$

Out[43]= $-\text{mg2}+p^2 == -m^2 + p^2\, Z\, Z\Delta - \delta m_1^2 - \frac{1}{6} v^2\, Z\Delta\, (\lambda+\delta\lambda_a) - \frac{1}{6}(\text{tfinn}+\text{t}\infty\text{n})\, Z\Delta^2\, \hbar\, (\lambda+\delta\lambda_{2\,a}) - \frac{1}{6}(\text{ssi}+\text{tfing}+\text{t}\infty\text{g})\, Z\Delta^2\, \hbar\, ((1+n)\lambda + (-1+n)\,\delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b})$

Higgs equation of motion

In[44]:= $\mathbf{neom} = p^2 - \text{mn2} == Z\, Z\Delta\, p^2 - m^2 - \delta m_1{}^2 - Z\Delta\, v^2 \frac{\left(3\lambda+\delta\lambda_{1\,a}+2\,\delta\lambda_{1\,b}\right)}{6} - \frac{\hbar}{6}(\lambda+\delta\lambda_{2\,a})\left(n-1\right) Z\Delta^2\, (\text{t}\infty\text{g}+\text{tfing}+\text{ssi}) - \frac{\hbar}{6}\left(3\lambda+\delta\lambda_{2\,a}+2\,\delta\lambda_{2\,b}\right) Z\Delta^2\, (\text{t}\infty\text{n}+\text{tfinn})$

Out[44]= $-\text{mn2}+p^2 == -m^2 + p^2\, Z\, Z\Delta - \delta m_1^2 - \frac{1}{6}(-1+n)(\text{ssi}+\text{tfing}+\text{t}\infty\text{g})\, Z\Delta^2\, \hbar\, (\lambda+\delta\lambda_{2\,a}) - \frac{1}{6} v^2\, Z\Delta\, (3\lambda+\delta\lambda_a+2\,\delta\lambda_b) - \frac{1}{6}(\text{tfinn}+\text{t}\infty\text{n})\, Z\Delta^2\, \hbar\, (3\lambda+\delta\lambda_{2\,a}+2\,\delta\lambda_{2\,b})$

Infinite parts of tadpoles in MSbar

MSbar rules for 4 - 2 ϵ dimensions

In[45]:= $\mathbf{regularisedtadpoles} = \left\{\text{t}\infty\text{g} \to \text{c0}\,\Lambda^2 + \text{c1}\,\text{mg2}\,\text{Log}\left[\Lambda^2/\mu^2\right],\ \text{t}\infty\text{n} \to \text{c0}\,\Lambda^2 + \text{c1}\,\text{mn2}\,\text{Log}\left[\Lambda^2/\mu^2\right]\right\}$

Out[45]= $\left\{\text{t}\infty\text{g} \to \text{c0}\,\Lambda^2 + \text{c1}\,\text{mg2}\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right],\ \text{t}\infty\text{n} \to \text{c0}\,\Lambda^2 + \text{c1}\,\text{mn2}\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\right\}$

Sub in tadpole expressions, eliminate mn2 and solve for mg2

In[46]:=
```
mg2soln = mg2 /. (geom /. regularisedtadpoles /.
      Solve[neom /. regularisedtadpoles, mn2][[1]] // Solve[#, mg2][[1]] &)
```

Out[46]=

$$\Bigg(-m^2 - p^2 + p^2\, Z\, Z\Delta - \delta m_1^2 - \frac{1}{6} v^2\, Z\Delta\, (\lambda + \delta\lambda_a) - \frac{1}{6}\, \text{tfinn}\, Z\Delta^2\, \hbar\, (\lambda + \delta\lambda_{2a}) -$$
$$\frac{1}{6}\, \text{c0}\, Z\Delta^2\, \Lambda^2\, \hbar\, (\lambda + \delta\lambda_{2a}) - \frac{1}{6}\, \text{ssi}\, Z\Delta^2\, \hbar\, ((1+n)\,\lambda + (-1+n)\,\delta\lambda_{2a} + 2\,\delta\lambda_{2b}) -$$
$$\frac{1}{6}\, \text{tfing}\, Z\Delta^2\, \hbar\, ((1+n)\,\lambda + (-1+n)\,\delta\lambda_{2a} + 2\,\delta\lambda_{2b}) -$$
$$\frac{1}{6}\, \text{c0}\, Z\Delta^2\, \Lambda^2\, \hbar\, ((1+n)\,\lambda + (-1+n)\,\delta\lambda_{2a} + 2\,\delta\lambda_{2b}) +$$
$$\frac{\text{c1}\, m^2\, Z\Delta^2\, \hbar\, \text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\, (\lambda + \delta\lambda_{2a})}{6\left(-1 + \frac{1}{6}\, \text{c1}\, Z\Delta^2\, \hbar\, \text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\, (3\,\lambda + \delta\lambda_{2a} + 2\,\delta\lambda_{2b})\right)} +$$
$$\frac{\text{c1}\, p^2\, Z\Delta^2\, \hbar\, \text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\, (\lambda + \delta\lambda_{2a})}{6\left(-1 + \frac{1}{6}\, \text{c1}\, Z\Delta^2\, \hbar\, \text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\, (3\,\lambda + \delta\lambda_{2a} + 2\,\delta\lambda_{2b})\right)} -$$
$$\frac{\text{c1}\, p^2\, Z\, Z\Delta^3\, \hbar\, \text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\, (\lambda + \delta\lambda_{2a})}{6\left(-1 + \frac{1}{6}\, \text{c1}\, Z\Delta^2\, \hbar\, \text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\, (3\,\lambda + \delta\lambda_{2a} + 2\,\delta\lambda_{2b})\right)} +$$
$$\frac{\text{c1}\, Z\Delta^2\, \hbar\, \text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\, \delta m_1^2\, (\lambda + \delta\lambda_{2a})}{6\left(-1 + \frac{1}{6}\, \text{c1}\, Z\Delta^2\, \hbar\, \text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\, (3\,\lambda + \delta\lambda_{2a} + 2\,\delta\lambda_{2b})\right)} +$$
$$\frac{\text{c1}\, (-1+n)\, \text{ssi}\, Z\Delta^4\, \hbar^2\, \text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\, (\lambda + \delta\lambda_{2a})^2}{36\left(-1 + \frac{1}{6}\, \text{c1}\, Z\Delta^2\, \hbar\, \text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\, (3\,\lambda + \delta\lambda_{2a} + 2\,\delta\lambda_{2b})\right)} +$$
$$\frac{\text{c1}\, (-1+n)\, \text{tfing}\, Z\Delta^4\, \hbar^2\, \text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\, (\lambda + \delta\lambda_{2a})^2}{36\left(-1 + \frac{1}{6}\, \text{c1}\, Z\Delta^2\, \hbar\, \text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\, (3\,\lambda + \delta\lambda_{2a} + 2\,\delta\lambda_{2b})\right)} +$$
$$\frac{\text{c0}\, \text{c1}\, (-1+n)\, Z\Delta^4\, \Lambda^2\, \hbar^2\, \text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\, (\lambda + \delta\lambda_{2a})^2}{36\left(-1 + \frac{1}{6}\, \text{c1}\, Z\Delta^2\, \hbar\, \text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\, (3\,\lambda + \delta\lambda_{2a} + 2\,\delta\lambda_{2b})\right)} +$$
$$\frac{\text{c1}\, v^2\, Z\Delta^3\, \hbar\, \text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\, (\lambda + \delta\lambda_{2a})\, (3\,\lambda + \delta\lambda_a + 2\,\delta\lambda_b)}{36\left(-1 + \frac{1}{6}\, \text{c1}\, Z\Delta^2\, \hbar\, \text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\, (3\,\lambda + \delta\lambda_{2a} + 2\,\delta\lambda_{2b})\right)} +$$
$$\frac{\text{c1}\, \text{tfinn}\, Z\Delta^4\, \hbar^2\, \text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\, (\lambda + \delta\lambda_{2a})\, (3\,\lambda + \delta\lambda_{2a} + 2\,\delta\lambda_{2b})}{36\left(-1 + \frac{1}{6}\, \text{c1}\, Z\Delta^2\, \hbar\, \text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\, (3\,\lambda + \delta\lambda_{2a} + 2\,\delta\lambda_{2b})\right)} +$$
$$\frac{\text{c0}\, \text{c1}\, Z\Delta^4\, \Lambda^2\, \hbar^2\, \text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\, (\lambda + \delta\lambda_{2a})\, (3\,\lambda + \delta\lambda_{2a} + 2\,\delta\lambda_{2b})}{36\left(-1 + \frac{1}{6}\, \text{c1}\, Z\Delta^2\, \hbar\, \text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\, (3\,\lambda + \delta\lambda_{2a} + 2\,\delta\lambda_{2b})\right)}\Bigg) \Bigg/$$
$$\Bigg(-1 + \frac{1}{6}\, \text{c1}\, Z\Delta^2\, \hbar\, \text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\, ((1+n)\,\lambda + (-1+n)\,\delta\lambda_{2a} + 2\,\delta\lambda_{2b}) -$$
$$\frac{\text{c1}^2\, (-1+n)\, Z\Delta^4\, \hbar^2\, \text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]^2\, (\lambda + \delta\lambda_{2a})^2}{36\left(-1 + \frac{1}{6}\, \text{c1}\, Z\Delta^2\, \hbar\, \text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\, (3\,\lambda + \delta\lambda_{2a} + 2\,\delta\lambda_{2b})\right)}\Bigg)$$

In[47]:=
```
mn2soln = mn2 /. (neom /. regularisedtadpoles /. mg2 → mg2soln // Solve[#, mn2][[1]] &)
```

Out[47]=

$$\frac{1}{-1+\frac{1}{6}\,\mathrm{c1}\,\mathrm{Z\Delta}^2\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\,(3\,\lambda+\delta\lambda_{2\,\mathrm{a}}+2\,\delta\lambda_{2\,\mathrm{b}})}$$

$$\Bigg(-\mathrm{m}^2-\mathrm{p}^2+\mathrm{p}^2\,\mathrm{Z}\,\mathrm{Z\Delta}-\delta\mathrm{m}_1^2-\frac{1}{6}\,\mathrm{v}^2\,\mathrm{Z\Delta}\,(3\,\lambda+\delta\lambda_{\mathrm{a}}+2\,\delta\lambda_{\mathrm{b}})-\frac{1}{6}\,\mathrm{tfinn}\,\mathrm{Z\Delta}^2\,\hbar\,(3\,\lambda+\delta\lambda_{2\,\mathrm{a}}+2\,\delta\lambda_{2\,\mathrm{b}})-$$

$$\frac{1}{6}\,\mathrm{c0}\,\mathrm{Z\Delta}^2\,\Lambda^2\,\hbar\,(3\,\lambda+\delta\lambda_{2\,\mathrm{a}}+2\,\delta\lambda_{2\,\mathrm{b}})-\frac{1}{6}\,(-1+\mathrm{n})\,\mathrm{Z\Delta}^2\,\hbar\,(\lambda+\delta\lambda_{2\,\mathrm{a}})\,\Bigg(\mathrm{ssi}+\mathrm{tfing}+\mathrm{c0}\,\Lambda^2+$$

$$\Bigg(\mathrm{c1}\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\,\Bigg(-\mathrm{m}^2-\mathrm{p}^2+\mathrm{p}^2\,\mathrm{Z}\,\mathrm{Z\Delta}-\delta\mathrm{m}_1^2-\frac{1}{6}\,\mathrm{v}^2\,\mathrm{Z\Delta}\,(\lambda+\delta\lambda_{\mathrm{a}})-\frac{1}{6}\,\mathrm{tfinn}\,\mathrm{Z\Delta}^2\,\hbar\,(\lambda+\delta\lambda_{2\,\mathrm{a}})-$$

$$\frac{1}{6}\,\mathrm{c0}\,\mathrm{Z\Delta}^2\,\Lambda^2\,\hbar\,(\lambda+\delta\lambda_{2\,\mathrm{a}})-\frac{1}{6}\,\mathrm{ssi}\,\mathrm{Z\Delta}^2\,\hbar\,((1+\mathrm{n})\,\lambda+(-1+\mathrm{n})\,\delta\lambda_{2\,\mathrm{a}}+2\,\delta\lambda_{2\,\mathrm{b}})-$$

$$\frac{1}{6}\,\mathrm{tfing}\,\mathrm{Z\Delta}^2\,\hbar\,((1+\mathrm{n})\,\lambda+(-1+\mathrm{n})\,\delta\lambda_{2\,\mathrm{a}}+2\,\delta\lambda_{2\,\mathrm{b}})-\frac{1}{6}\,\mathrm{c0}\,\mathrm{Z\Delta}^2\,\Lambda^2\,\hbar\,((1+\mathrm{n})\,\lambda+$$

$$(-1+\mathrm{n})\,\delta\lambda_{2\,\mathrm{a}}+2\,\delta\lambda_{2\,\mathrm{b}})+\frac{\mathrm{c1}\,\mathrm{m}^2\,\mathrm{Z\Delta}^2\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\,(\lambda+\delta\lambda_{2\,\mathrm{a}})}{6\,\left(-1+\frac{1}{6}\,\mathrm{c1}\,\mathrm{Z\Delta}^2\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\,(3\,\lambda+\delta\lambda_{2\,\mathrm{a}}+2\,\delta\lambda_{2\,\mathrm{b}})\right)}+$$

$$\frac{\mathrm{c1}\,\mathrm{p}^2\,\mathrm{Z\Delta}^2\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\,(\lambda+\delta\lambda_{2\,\mathrm{a}})}{6\,\left(-1+\frac{1}{6}\,\mathrm{c1}\,\mathrm{Z\Delta}^2\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\,(3\,\lambda+\delta\lambda_{2\,\mathrm{a}}+2\,\delta\lambda_{2\,\mathrm{b}})\right)}-$$

$$\frac{\mathrm{c1}\,\mathrm{p}^2\,\mathrm{Z}\,\mathrm{Z\Delta}^3\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\,(\lambda+\delta\lambda_{2\,\mathrm{a}})}{6\,\left(-1+\frac{1}{6}\,\mathrm{c1}\,\mathrm{Z\Delta}^2\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\,(3\,\lambda+\delta\lambda_{2\,\mathrm{a}}+2\,\delta\lambda_{2\,\mathrm{b}})\right)}+$$

$$\frac{\mathrm{c1}\,\mathrm{Z\Delta}^2\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\,\delta\mathrm{m}_1^2\,(\lambda+\delta\lambda_{2\,\mathrm{a}})}{6\,\left(-1+\frac{1}{6}\,\mathrm{c1}\,\mathrm{Z\Delta}^2\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\,(3\,\lambda+\delta\lambda_{2\,\mathrm{a}}+2\,\delta\lambda_{2\,\mathrm{b}})\right)}+$$

$$\frac{\mathrm{c1}\,(-1+\mathrm{n})\,\mathrm{ssi}\,\mathrm{Z\Delta}^4\,\hbar^2\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\,(\lambda+\delta\lambda_{2\,\mathrm{a}})^2}{36\,\left(-1+\frac{1}{6}\,\mathrm{c1}\,\mathrm{Z\Delta}^2\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\,(3\,\lambda+\delta\lambda_{2\,\mathrm{a}}+2\,\delta\lambda_{2\,\mathrm{b}})\right)}+$$

$$\frac{\mathrm{c1}\,(-1+\mathrm{n})\,\mathrm{tfing}\,\mathrm{Z\Delta}^4\,\hbar^2\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\,(\lambda+\delta\lambda_{2\,\mathrm{a}})^2}{36\,\left(-1+\frac{1}{6}\,\mathrm{c1}\,\mathrm{Z\Delta}^2\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\,(3\,\lambda+\delta\lambda_{2\,\mathrm{a}}+2\,\delta\lambda_{2\,\mathrm{b}})\right)}+$$

$$\frac{\mathrm{c0}\,\mathrm{c1}\,(-1+\mathrm{n})\,\mathrm{Z\Delta}^4\,\Lambda^2\,\hbar^2\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\,(\lambda+\delta\lambda_{2\,\mathrm{a}})^2}{36\,\left(-1+\frac{1}{6}\,\mathrm{c1}\,\mathrm{Z\Delta}^2\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\,(3\,\lambda+\delta\lambda_{2\,\mathrm{a}}+2\,\delta\lambda_{2\,\mathrm{b}})\right)}+$$

$$\frac{\mathrm{c1}\,\mathrm{v}^2\,\mathrm{Z\Delta}^3\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\,(\lambda+\delta\lambda_{2\,\mathrm{a}})\,(3\,\lambda+\delta\lambda_{\mathrm{a}}+2\,\delta\lambda_{\mathrm{b}})}{36\,\left(-1+\frac{1}{6}\,\mathrm{c1}\,\mathrm{Z\Delta}^2\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\,(3\,\lambda+\delta\lambda_{2\,\mathrm{a}}+2\,\delta\lambda_{2\,\mathrm{b}})\right)}+$$

$$\left(\mathrm{c1}\,\mathrm{tfinn}\,\mathrm{Z\Delta}^4\,\hbar^2\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\,(\lambda+\delta\lambda_{2\,\mathrm{a}})\,(3\,\lambda+\delta\lambda_{2\,\mathrm{a}}+2\,\delta\lambda_{2\,\mathrm{b}})\right)\Big/$$

$$\left(36\,\left(-1+\frac{1}{6}\,\mathrm{c1}\,\mathrm{Z\Delta}^2\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\,(3\,\lambda+\delta\lambda_{2\,\mathrm{a}}+2\,\delta\lambda_{2\,\mathrm{b}})\right)\right)+$$

$$\left(c0\ c1\ Z\Delta^4\ \Lambda^2\ \hbar^2\ \mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\ (\lambda + \delta\lambda_{2\,a})\ (3\,\lambda + \delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b})\right) \Big/$$

$$\left(36\left(-1 + \frac{1}{6}\ c1\ Z\Delta^2\ \hbar\ \mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\ (3\,\lambda + \delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b})\right)\right)\Bigg)\Bigg) \Big/$$

$$\left(-1 + \frac{1}{6}\ c1\ Z\Delta^2\ \hbar\ \mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\ ((1+n)\ \lambda + (-1+n)\ \delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b}) - \frac{c1^2\ (-1+n)\ Z\Delta^4\ \hbar^2\ \mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]^2\ (\lambda + \delta\lambda_{2\,a})^2}{36\left(-1 + \frac{1}{6}\ c1\ Z\Delta^2\ \hbar\ \mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\ (3\,\lambda + \delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b})\right)}\right)\Bigg)\Bigg)$$

Gather divergences proportional v, tfing and tfinn and set independently to zero

First we subtract the finite equation of motion, then gather coefficients of the remainder into a list and set each to zero (after some trimming and simplifying).

In[48]:=
```
cteq =
 ((CoefficientList[mg2soln + (-m^2 - λ/6 v^2 - ℏ/6 ((n + 1) λ) (tfing + ssi) - ℏ/6 (λ) (tfinn)),
        {p, v, tfing, tfinn}] // Flatten) //
     DeleteDuplicates // Simplify // FullSimplify) == 0 // Thread
```

Out[48]=

$$\Bigg\{-\Bigg(\Bigg(\lambda\,\hbar\,\Bigg(-18\,(1+n)\ ssi\ (-1+Z\Delta^2) - 18\ c0\ (2+n)\ Z\Delta^2\ \Lambda^2 +$$
$$c1\ Z\Delta^2\ \mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\Bigg(-18\,m^2\,(2+n) + 3\,\lambda\,\big(-(1+n)\ (4+n)\ ssi + 2\,(2+n)\ ssi\ Z\Delta^2 +$$
$$2\ c0\ (2+n)\ Z\Delta^2\ \Lambda^2\big)\ \hbar + c1\ (2+n)\ Z\Delta^2\ \lambda\,\hbar\,\left(6\,m^2 + (1+n)\ ssi\ \lambda\,\hbar\right)\ \mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\Bigg)\Bigg) +$$
$$36\ \delta m_1^2\ \left(-3 + c1\ Z\Delta^2\ \lambda\,\hbar\ \mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right] + c1\ Z\Delta^2\ \hbar\ \mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\ \delta\lambda_{2\,b}\right) +$$
$$Z\Delta^2\ \hbar\ \Bigg(\delta\lambda_{2\,b}\ \Bigg(-36\,\left(ssi + c0\ \Lambda^2\right) +$$
$$c1\ \mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\Bigg(-36\,m^2 + 6\,\lambda\,\left(-2\,(1+n)\ ssi + (4+n)\ ssi\ Z\Delta^2 + c0\ (4+n)\ Z\Delta^2\ \Lambda^2\right)\ \hbar +$$
$$c1\ (4+n)\ Z\Delta^2\ \lambda\,\hbar\,\left(6\,m^2 + (1+n)\ ssi\ \lambda\,\hbar\right)\ \mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\Bigg) + 2\ c1\ Z\Delta^2\ \hbar\ \mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]$$
$$\left(6\,\left(ssi + c0\ \Lambda^2\right) + c1\ \left(6\,m^2 + (1+n)\ ssi\ \lambda\,\hbar\right)\ \mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\right)\ \delta\lambda_{2\,b}\Bigg) + \delta\lambda_{2\,a}$$
$$\Bigg(-18\,\left((-1+n)\ ssi + c0\ n\ \Lambda^2\right) + c1\ n\ \mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\ \Bigg(-18\,m^2 - 3\,\lambda\,\big(ssi + n\ ssi - 2\ ssi\ Z\Delta^2 -$$
$$2\ c0\ Z\Delta^2\ \Lambda^2\big)\ \hbar + c1\ Z\Delta^2\ \lambda\,\hbar\,\left(6\,m^2 + (1+n)\ ssi\ \lambda\,\hbar\right)\ \mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\Bigg) + c1\ n\ Z\Delta^2\ \hbar$$

$$\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\left(6\left(\mathrm{ssi}+\mathrm{c0}\,\Lambda^2\right)+\mathrm{c1}\left(6\,\mathrm{m}^2+(1+\mathrm{n})\,\mathrm{ssi}\,\lambda\,\hbar\right)\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\right)\delta\lambda_{2\,\mathrm{b}}\Big)\Big)\Big)\Big/$$
$$\Big(6\left(-3+\mathrm{c1}\,\mathrm{Z}\Delta^2\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]+\mathrm{c1}\,\mathrm{Z}\Delta^2\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\delta\lambda_{2\,\mathrm{b}}\right)$$
$$\left(-6+\mathrm{c1}\,(2+\mathrm{n})\,\mathrm{Z}\Delta^2\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]+\mathrm{c1}\,\mathrm{Z}\Delta^2\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\left(\mathrm{n}\,\delta\lambda_{2\,\mathrm{a}}+2\,\delta\lambda_{2\,\mathrm{b}}\right)\right)\Big)\Big)==0,$$

$$-\frac{\lambda\,\hbar}{6}+\left(3\,\mathrm{Z}\Delta^2\,\hbar\,(\lambda+\delta\lambda_{2\,\mathrm{a}})\right)\Big/\Big(\left(-3+\mathrm{c1}\,\mathrm{Z}\Delta^2\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]+\mathrm{c1}\,\mathrm{Z}\Delta^2\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\delta\lambda_{2\,\mathrm{b}}\right)$$
$$\left(-6+\mathrm{c1}\,(2+\mathrm{n})\,\mathrm{Z}\Delta^2\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]+\mathrm{c1}\,\mathrm{Z}\Delta^2\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\left(\mathrm{n}\,\delta\lambda_{2\,\mathrm{a}}+2\,\delta\lambda_{2\,\mathrm{b}}\right)\right)\Big)==0,$$

$$-\frac{1}{6\,\mathrm{c1}\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]}\Bigg(6+\mathrm{c1}\,(1+\mathrm{n})\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]+$$
$$\frac{18}{\mathrm{n}\left(-3+\mathrm{c1}\,\mathrm{Z}\Delta^2\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]+\mathrm{c1}\,\mathrm{Z}\Delta^2\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\delta\lambda_{2\,\mathrm{b}}\right)}+$$
$$(36\,(-1+\mathrm{n}))\Big/$$
$$\left(\mathrm{n}\left(-6+\mathrm{c1}\,(2+\mathrm{n})\,\mathrm{Z}\Delta^2\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]+\mathrm{c1}\,\mathrm{Z}\Delta^2\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\left(\mathrm{n}\,\delta\lambda_{2\,\mathrm{a}}+2\,\delta\lambda_{2\,\mathrm{b}}\right)\right)\right)\Bigg)==0,$$

$$\mathrm{True},\ -\frac{\lambda}{6}+\frac{\mathrm{Z}\Delta\,(\lambda+\delta\lambda_{\mathrm{b}})}{\mathrm{n}\left(-3+\mathrm{c1}\,\mathrm{Z}\Delta^2\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]+\mathrm{c1}\,\mathrm{Z}\Delta^2\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\delta\lambda_{2\,\mathrm{b}}\right)}-$$
$$(\mathrm{Z}\Delta\,((2+\mathrm{n})\,\lambda+\mathrm{n}\,\delta\lambda_{\mathrm{a}}+2\,\delta\lambda_{\mathrm{b}}))\Big/$$
$$\left(\mathrm{n}\left(-6+\mathrm{c1}\,(2+\mathrm{n})\,\mathrm{Z}\Delta^2\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]+\mathrm{c1}\,\mathrm{Z}\Delta^2\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\left(\mathrm{n}\,\delta\lambda_{2\,\mathrm{a}}+2\,\delta\lambda_{2\,\mathrm{b}}\right)\right)\right)==0,$$

$$(-6+6\,\mathrm{Z}\,\mathrm{Z}\Delta)\Big/\Big(-6+\mathrm{c1}\,(2+\mathrm{n})\,\mathrm{Z}\Delta^2\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]+$$
$$\mathrm{c1}\,\mathrm{Z}\Delta^2\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\left(\mathrm{n}\,\delta\lambda_{2\,\mathrm{a}}+2\,\delta\lambda_{2\,\mathrm{b}}\right)\Big)==0\Big\}$$

In[49]:= **cteq2 =**

$$\mathbf{\left(\left(\mathrm{CoefficientList}\left[\mathrm{mn2soln}+\left(-\mathrm{m}^2-\frac{\lambda}{2}\,\mathrm{v}^2-\frac{\hbar}{6}\left((\mathrm{n}-1)\,\lambda\right)(\mathrm{tfing}+\mathrm{ssi})-\frac{\hbar}{2}\,(\lambda)\,(\mathrm{tfinn})\right),\right.\right.\right.}$$
$$\mathbf{\left.\{\mathrm{p},\mathrm{v},\mathrm{tfing},\mathrm{tfinn}\}\right]//\mathrm{Flatten}\Big)//}$$
$$\mathbf{\mathrm{DeleteDuplicates}//\mathrm{Simplify}//\mathrm{FullSimplify}\Big)==0//\mathrm{Thread}}$$

Out[49]=

$$\Big\{-\Big(\Big(\lambda\,\hbar\,\Big(-18\,(-1+\mathrm{n})\,\mathrm{ssi}\left(-1+\mathrm{Z}\Delta^2\right)-18\,\mathrm{c0}\,(2+\mathrm{n})\,\mathrm{Z}\Delta^2\,\Lambda^2+$$
$$\mathrm{c1}\,\mathrm{Z}\Delta^2\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\Big(-18\,\mathrm{m}^2\,(2+\mathrm{n})+3\,\lambda\left(-(-1+\mathrm{n})\,(4+\mathrm{n})\,\mathrm{ssi}+2\,\mathrm{c0}\,(2+\mathrm{n})\,\mathrm{Z}\Delta^2\,\Lambda^2\right)\hbar+$$
$$\mathrm{c1}\,(2+\mathrm{n})\,\mathrm{Z}\Delta^2\,\lambda\,\hbar\left(6\,\mathrm{m}^2+(-1+\mathrm{n})\,\mathrm{ssi}\,\lambda\,\hbar\right)\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\Big)\Big)+$$
$$36\,\delta\mathrm{m}_1^2\left(-3+\mathrm{c1}\,\mathrm{Z}\Delta^2\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]+\mathrm{c1}\,\mathrm{Z}\Delta^2\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\delta\lambda_{2\,\mathrm{b}}\right)+\mathrm{Z}\Delta^2\,\hbar$$

$$\left(\delta\lambda_{2\,b}\left(-36\,c0\,\Lambda^2+c1\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\left(6\left(-6\,m^2+\lambda\left(-2\,(-1+n)\,ssi+c0\,(4+n)\,Z\Delta^2\,\Lambda^2\right)\hbar\right)+c1\,(4+n)\,Z\Delta^2\,\lambda\,\hbar\left(6\,m^2+(-1+n)\,ssi\,\lambda\,\hbar\right)\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\right)+2\,c1\,Z\Delta^2\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\left(6\,c0\,\Lambda^2+c1\left(6\,m^2+(-1+n)\,ssi\,\lambda\,\hbar\right)\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\right)\delta\lambda_{2\,b}\right)+\delta\lambda_{2\,a}\left(-18\left((-1+n)\,ssi+c0\,n\,\Lambda^2\right)+c1\,n\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\left(3\left(-6\,m^2+\lambda\left(ssi-n\,ssi+2\,c0\,Z\Delta^2\,\Lambda^2\right)\hbar\right)+c1\,Z\Delta^2\,\lambda\,\hbar\left(6\,m^2+(-1+n)\,ssi\,\lambda\,\hbar\right)\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\right)+c1\,n\,Z\Delta^2\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\left(6\,c0\,\Lambda^2+c1\left(6\,m^2+(-1+n)\,ssi\,\lambda\,\hbar\right)\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\right)\delta\lambda_{2\,b}\right)\right)\Big/\left(6\left(-3+c1\,Z\Delta^2\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]+c1\,Z\Delta^2\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\delta\lambda_{2\,b}\right)\left(-6+c1\,(2+n)\,Z\Delta^2\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]+c1\,Z\Delta^2\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\left(n\,\delta\lambda_{2\,a}+2\,\delta\lambda_{2\,b}\right)\right)\right)=0,$$

$$-\frac{1}{2\,c1\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]}\left(2+c1\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]+\frac{6\,(-1+n)}{n\left(-3+c1\,Z\Delta^2\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]+c1\,Z\Delta^2\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\delta\lambda_{2\,b}\right)}+12\Big/\left(n\left(-6+c1\,(2+n)\,Z\Delta^2\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]+c1\,Z\Delta^2\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\left(n\,\delta\lambda_{2\,a}+2\,\delta\lambda_{2\,b}\right)\right)\right)\right)=0,$$

$$\left((-1+n)\,\hbar\left(-Z\Delta^2\,\delta\lambda_{2\,a}\left(-18+c1\,n\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\left(-3+c1\,Z\Delta^2\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\right)+c1^2\,n\,Z\Delta^2\,\lambda\,\hbar^2\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]^2\delta\lambda_{2\,b}\right)+\lambda\left(18\left(-1+Z\Delta^2\right)+c1\,Z\Delta^2\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\left(3\,(4+n)-c1\,(2+n)\,Z\Delta^2\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\right)-c1\,Z\Delta^2\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\delta\lambda_{2\,b}\left(-12+c1\,(4+n)\,Z\Delta^2\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]+2\,c1\,Z\Delta^2\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\delta\lambda_{2\,b}\right)\right)\right)\right)\Big/\left(6\left(-3+c1\,Z\Delta^2\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]+c1\,Z\Delta^2\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\delta\lambda_{2\,b}\right)\left(-6+c1\,(2+n)\,Z\Delta^2\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]+c1\,Z\Delta^2\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\left(n\,\delta\lambda_{2\,a}+2\,\delta\lambda_{2\,b}\right)\right)\right)=0,$$

$$\mathrm{True},\ -\frac{\lambda}{2}-\frac{(-1+n)\,Z\Delta\,(\lambda+\delta\lambda_b)}{n\left(-3+c1\,Z\Delta^2\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]+c1\,Z\Delta^2\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\delta\lambda_{2\,b}\right)}-\left(Z\Delta\left((2+n)\,\lambda+n\,\delta\lambda_a+2\,\delta\lambda_b\right)\right)\Big/\left(n\left(-6+c1\,(2+n)\,Z\Delta^2\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]+c1\,Z\Delta^2\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\left(n\,\delta\lambda_{2\,a}+2\,\delta\lambda_{2\,b}\right)\right)\right)=0,$$

$$(-6+6\,Z\,Z\Delta)\Big/\left(-6+c1\,(2+n)\,Z\Delta^2\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]+\right.$$

$$\text{c1}\, Z\Delta^2\, \hbar\, \text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] (n\, \delta\lambda_{2\,a} + 2\, \delta\lambda_{2\,b})\Big) == 0\Big\}$$

Solve for counterterms

In[50]:= **cteqs = {cteq, cteq2} // Flatten // FullSimplify // DeleteDuplicates**

Out[50]=

$$\Bigg\{\frac{1}{\text{c1}\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]}\Bigg(6\,\text{ssi} + 6\,\text{c0}\,\Lambda^2 + 6\,\text{c1}\,m^2\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] + \text{c1}\,\text{ssi}\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] + \text{c1}\,n\,\text{ssi}\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] + \frac{18\,\text{ssi}}{n\left(-3 + \text{c1}\,Z\Delta^2\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] + \text{c1}\,Z\Delta^2\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\delta\lambda_{2\,b}\right)} + \Big(36\Big((-1+n)\,\text{ssi} + \text{c0}\,n\,\Lambda^2 + \text{c1}\,m^2\,n\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] + \text{c1}\,n\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\delta m_1^2\Big)\Big) \Big/ \Big(n\Big(-6 + \text{c1}\,(2+n)\,Z\Delta^2\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] + \text{c1}\,Z\Delta^2\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right](n\,\delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b})\Big)\Big)\Bigg) == 0,$$

$$\hbar\Bigg(\lambda - \left(18\,Z\Delta^2\,(\lambda + \delta\lambda_{2\,a})\right) \Big/ \Bigg(\Big(-3 + \text{c1}\,Z\Delta^2\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] + \text{c1}\,Z\Delta^2\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\delta\lambda_{2\,b}\Big)\Big(-6 + \text{c1}\,(2+n)\,Z\Delta^2\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] + \text{c1}\,Z\Delta^2\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right](n\,\delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b})\Big)\Bigg)\Bigg) == 0,$$

$$\frac{1}{\text{c1}\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]}\Bigg(6 + \text{c1}\,(1+n)\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] + \frac{18}{n\left(-3 + \text{c1}\,Z\Delta^2\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] + \text{c1}\,Z\Delta^2\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\delta\lambda_{2\,b}\right)} + (36\,(-1+n)) \Big/ \Big(n\Big(-6 + \text{c1}\,(2+n)\,Z\Delta^2\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] + \text{c1}\,Z\Delta^2\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right](n\,\delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b})\Big)\Big)\Bigg) == 0,$$

$$\text{True},\ \lambda + (6\,Z\Delta\,((2+n)\,\lambda + n\,\delta\lambda_a + 2\,\delta\lambda_b)) \Big/ \Big(n\Big(-6 + \text{c1}\,(2+n)\,Z\Delta^2\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] + \text{c1}\,Z\Delta^2\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right](n\,\delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b})\Big)\Big) == \frac{6\,Z\Delta\,(\lambda + \delta\lambda_b)}{n\left(-3 + \text{c1}\,Z\Delta^2\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] + \text{c1}\,Z\Delta^2\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\delta\lambda_{2\,b}\right)},$$

$$(-1 + Z\,Z\Delta) \Big/ \Big(-6 + \text{c1}\,(2+n)\,Z\Delta^2\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] + \text{c1}\,Z\Delta^2\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right](n\,\delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b})\Big) == 0,$$

$$\frac{1}{\text{c1}\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]}\Bigg(6\,\text{c0}\,\Lambda^2 + 6\,\text{c1}\,m^2\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] - \text{c1}\,\text{ssi}\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] + \text{c1}\,n\,\text{ssi}\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] - \frac{18\,(-1+n)\,\text{ssi}}{n\left(-3 + \text{c1}\,Z\Delta^2\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] + \text{c1}\,Z\Delta^2\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\delta\lambda_{2\,b}\right)} + \Big(36\Big((-1+n)\,\text{ssi} + \text{c0}\,n\,\Lambda^2 + \text{c1}\,m^2\,n\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] + \text{c1}\,n\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\delta m_1^2\Big)\Big) \Big/ \Big(n\Big(-6 + \text{c1}\,(2+n)\,Z\Delta^2\,\lambda\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right] + \text{c1}\,Z\Delta^2\,\hbar\,\text{Log}\left[\frac{\Lambda^2}{\mu^2}\right](n\,\delta\lambda_{2\,a} + 2\,\delta\lambda_{2\,b})\Big)\Big)\Bigg) == 0,$$

$$\frac{1}{c1\ \mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]}\left(2 + c1\ \lambda\ \hbar\ \mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right] + \frac{6\ (-1+n)}{n\left(-3 + c1\ Z\Delta^2\ \lambda\ \hbar\ \mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right] + c1\ Z\Delta^2\ \hbar\ \mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\ \delta\lambda_{2\,b}\right)} + 12\Big/\left(n\left(-6 + c1\ (2+n)\ Z\Delta^2\ \lambda\ \hbar\ \mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right] + c1\ Z\Delta^2\ \hbar\ \mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\ (n\ \delta\lambda_{2\,a} + 2\ \delta\lambda_{2\,b})\right)\right)\right) == 0,$$

$$(-1+n)\ \hbar\left(\lambda - \left(18\ Z\Delta^2\ (\lambda + \delta\lambda_{2\,a})\right)\Big/\left(\left(-3 + c1\ Z\Delta^2\ \lambda\ \hbar\ \mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right] + c1\ Z\Delta^2\ \hbar\ \mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\ \delta\lambda_{2\,b}\right)\left(-6 + c1\ (2+n)\ Z\Delta^2\ \lambda\ \hbar\ \mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right] + c1\ Z\Delta^2\ \hbar\ \mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\ (n\ \delta\lambda_{2\,a} + 2\ \delta\lambda_{2\,b})\right)\right)\right) == 0,$$

$$\lambda + \frac{2\ (-1+n)\ Z\Delta\ (\lambda + \delta\lambda_b)}{n\left(-3 + c1\ Z\Delta^2\ \lambda\ \hbar\ \mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right] + c1\ Z\Delta^2\ \hbar\ \mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\ \delta\lambda_{2\,b}\right)} + (2\ Z\Delta\ ((2+n)\ \lambda + n\ \delta\lambda_a + 2\ \delta\lambda_b))\Big/\left(n\left(-6 + c1\ (2+n)\ Z\Delta^2\ \lambda\ \hbar\ \mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right] + c1\ Z\Delta^2\ \hbar\ \mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\ (n\ \delta\lambda_{2\,a} + 2\ \delta\lambda_{2\,b})\right)\right) == 0\}$$

In[51]:=
```
ctsolns =
  Solve[cteqs, {δm₁, δλ1a, δλ2a, δλ1b, δλ2b, Z, ZΔ}] // FullSimplify // DeleteDuplicates
```

Solve::svars : Equations may not give solutions for all "solve" variables. ≫

Out[51]=

$$\left\{\left\{\delta m_1 \to -\frac{\mathrm{i}\ \sqrt{2+n}\ \sqrt{\lambda}\ \sqrt{\hbar}\ \sqrt{c0\ \Lambda^2 + c1\ m^2\ \mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]}}{\sqrt{6 + c1\ (2+n)\ \lambda\ \hbar\ \mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]}},\right.\right.$$

$$\delta\lambda_a \to \lambda\left(-1 + \frac{6\ (2+n)}{n\ Z\Delta\left(6 + c1\ (2+n)\ \lambda\ \hbar\ \mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\right)} - \frac{6}{3\ n\ Z\Delta + c1\ n\ Z\Delta\ \lambda\ \hbar\ \mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]}\right),$$

$$\delta\lambda_{2\,a} \to \lambda\left(-1 + \frac{18}{Z\Delta^2\left(3 + c1\ \lambda\ \hbar\ \mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\right)\left(6 + c1\ (2+n)\ \lambda\ \hbar\ \mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\right)}\right),$$

$$\left.\delta\lambda_b \to \lambda\left(-1 + \frac{3}{3\ Z\Delta + c1\ Z\Delta\ \lambda\ \hbar\ \mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]}\right),\ \delta\lambda_{2\,b} \to \lambda\left(-1 + \frac{3}{Z\Delta^2\left(3 + c1\ \lambda\ \hbar\ \mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\right)}\right),\ Z \to \frac{1}{Z\Delta}\right\},$$

$$\left\{\delta m_1 \to \frac{\mathrm{i}\ \sqrt{2+n}\ \sqrt{\lambda}\ \sqrt{\hbar}\ \sqrt{c0\ \Lambda^2 + c1\ m^2\ \mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]}}{\sqrt{6 + c1\ (2+n)\ \lambda\ \hbar\ \mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]}},\right.$$

$$\delta\lambda_a \to \lambda\left(-1 + \frac{6\ (2+n)}{n\ Z\Delta\left(6 + c1\ (2+n)\ \lambda\ \hbar\ \mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\right)} - \frac{6}{3\ n\ Z\Delta + c1\ n\ Z\Delta\ \lambda\ \hbar\ \mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]}\right),$$

$$\delta\lambda_{2\,a} \to \lambda\left(-1 + \frac{18}{Z\Delta^2\left(3 + c1\ \lambda\ \hbar\ \mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\right)\left(6 + c1\ (2+n)\ \lambda\ \hbar\ \mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\right)}\right),$$

$$\left.\left.\delta\lambda_b \to \lambda\left(-1 + \frac{3}{3\ Z\Delta + c1\ Z\Delta\ \lambda\ \hbar\ \mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]}\right),\ \delta\lambda_{2\,b} \to \lambda\left(-1 + \frac{3}{Z\Delta^2\left(3 + c1\ \lambda\ \hbar\ \mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\right)}\right),\ Z \to \frac{1}{Z\Delta}\right\}\right\}$$

In[52]:= `cts = {δm1^2, δλ1a, δλ2a, δλ1b, δλ2b, Z, ZΔ} /. ctsolns // FullSimplify // DeleteDuplicates`

Out[52]=
$$\left\{\left\{-\frac{(2+n)\,\lambda\,\hbar\,\left(c0\,\Lambda^2+c1\,m^2\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\right)}{6+c1\,(2+n)\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]},\right.\right.$$
$$\lambda\left(-1+\frac{6\,(2+n)}{n\,Z\Delta\,\left(6+c1\,(2+n)\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\right)}-\frac{6}{3\,n\,Z\Delta+c1\,n\,Z\Delta\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]}\right),$$
$$\lambda\left(-1+\frac{18}{Z\Delta^2\,\left(3+c1\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\right)\,\left(6+c1\,(2+n)\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\right)}\right),$$
$$\left.\left.\lambda\left(-1+\frac{3}{3\,Z\Delta+c1\,Z\Delta\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]}\right),\ \lambda\left(-1+\frac{3}{Z\Delta^2\,\left(3+c1\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\right)}\right),\ \frac{1}{Z\Delta},\ Z\Delta\right\}\right\}$$

ZΔ is redundant in this truncation, can remove it :

In[53]:= `cts /. ZΔ → 1 // FullSimplify`

Out[53]=
$$\left\{\left\{-\frac{(2+n)\,\lambda\,\hbar\,\left(c0\,\Lambda^2+c1\,m^2\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\right)}{6+c1\,(2+n)\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]},\right.\right.$$
$$\lambda\left(-1-\frac{6}{3\,n+c1\,n\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]}+\frac{6\,(2+n)}{n\,\left(6+c1\,(2+n)\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\right)}\right),$$
$$\lambda\left(-1+\frac{18}{\left(3+c1\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\right)\,\left(6+c1\,(2+n)\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\right)}\right),$$
$$\left.\left.\lambda\left(-1+\frac{3}{3+c1\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]}\right),\ \lambda\left(-1+\frac{3}{3+c1\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]}\right),\ 1,\ 1\right\}\right\}$$

In[54]:= `mg2soln /. ctsolns /. ZΔ → 1 // FullSimplify // DeleteDuplicates`

Out[54]=
$$\left\{\frac{1}{6}\,\left(6\,m^2+\lambda\,\left(v^2+((1+n)\,(ssi+tfing)+tfinn)\,\hbar\right)\right)\right\}$$

In[55]:=
```
mn2 /.
  ((neom /. regularisedtadpoles /. mg2 → mg2soln /. ctsolns /. ZΔ → 1 // FullSimplify //
      DeleteDuplicates) // Solve[#, mn2] &) // FullSimplify
```

Out[55]=
$$\left\{\frac{1}{6}\,\left(6\,m^2+3\,v^2\,\lambda+((-1+n)\,(ssi+tfing)+3\,tfinn)\,\lambda\,\hbar\right)\right\}$$

In[56]:= **rnveom =**

$$\texttt{veom /. }\left\{mg2 \to m^2 + \frac{\lambda}{6} v^2 + \frac{\hbar}{6}\left(\left(n+1\right)\lambda\right)(tfing + ssi) + \frac{\hbar}{6}(\lambda)(tfinn),\ mn2 \to m^2 + \frac{\lambda}{2} v^2 + \frac{\hbar}{6}\left(\left(n-1\right)\lambda\right)(tfing + ssi) + \frac{\hbar}{2}(\lambda)(tfinn)\right\} \texttt{ // Simplify // DeleteDuplicates}$$

Out[56]=
$$\frac{1}{6} v \left(6\, ssi2 + \frac{6\left(m^2 + \delta m_0^2\right)}{Z\Delta} + v^2 (\lambda + \delta\lambda_0) + (-1+n)(ssi + tfing + t\infty g) Z\Delta \hbar (\lambda + \delta\lambda_a) + (tfinn + t\infty n) Z\Delta \hbar (3\lambda + \delta\lambda_a + 2\delta\lambda_b)\right)$$

In[57]:= **veomCtEqs =**

$$\left(\left(\left(\texttt{CoefficientList}\left[\left(\frac{1}{v} rnveom - \left(m^2 + \frac{\lambda}{6} v^2 + \frac{\hbar}{6}\left(\left(n-1\right)\lambda\right)(tfing + ssi) + \frac{\hbar}{2}(\lambda)(tfinn) + ssi2\right)\right) \texttt{ /. regularisedtadpoles /. } \left\{mg2 \to m^2 + \frac{\lambda}{6} v^2 + \frac{\hbar}{6}\left(\left(n+1\right)\lambda\right)(tfing + ssi) + \frac{\hbar}{6}(\lambda)(tfinn),\ mn2 \to m^2 + \frac{\lambda}{2} v^2 + \frac{\hbar}{6}\left(\left(n-1\right)\lambda\right)(tfing + ssi) + \frac{\hbar}{2}(\lambda)(tfinn)\right\} \texttt{ // Simplify // Expand // FullSimplify},\ \{v, tfing, tfinn\}\right] \texttt{ // Simplify // Flatten}\right) \texttt{ // DeleteDuplicates // Simplify // FullSimplify // DeleteDuplicates}\right) == 0 \texttt{ // Thread}\right)$$

Out[57]=
$$\left\{\frac{1}{36 Z\Delta}\left(-36 m^2 (-1 + Z\Delta) + 6 Z\Delta \lambda \left((-1+n)\, ssi\, (-1 + Z\Delta) + c0 (2+n) Z\Delta \Lambda^2\right)\hbar + c1\, Z\Delta^2 \lambda \hbar \left(6 m^2 (2+n) + (-1+n)(4+n)\, ssi\, \lambda \hbar\right) \mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right] + 36\,\delta m_0^2 + Z\Delta^2 \hbar \left(\left(6(-1+n)\, ssi + 6\, c0\, n \Lambda^2 + c1 \left(6 m^2 n + \left(-2 + n + n^2\right) ssi\, \lambda \hbar\right) \mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\right)\delta\lambda_a + 2\left(6\, c0\, \Lambda^2 + c1 \left(6 m^2 + (-1+n)\, ssi\, \lambda \hbar\right) \mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\right)\delta\lambda_b\right)\right) == 0,\right.$$

$$\frac{1}{36}\hbar\left(\lambda\left(18(-1 + Z\Delta) + c1 (8+n) Z\Delta \lambda \hbar \mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\right) + Z\Delta\left(6 + c1 (2+n) \lambda \hbar \mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\right)\delta\lambda_a + 6 Z\Delta \left(2 + c1 \lambda \hbar \mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\right)\delta\lambda_b\right) == 0,$$

$$\frac{1}{36}(-1+n)\hbar\left(Z\Delta\left(6 + c1 (2+n)\lambda\hbar \mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\right)\delta\lambda_a + \lambda\left(6(-1+Z\Delta) + c1 (4+n) Z\Delta \lambda \hbar \mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right] + 2\, c1\, Z\Delta \hbar \mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\delta\lambda_b\right)\right) == 0,$$

$$\left.\text{True},\ \frac{1}{36}\left(6\,\delta\lambda_0 + c1\, Z\Delta \lambda \hbar \mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\left((8+n)\lambda + (2+n)\delta\lambda_a + 6\,\delta\lambda_b\right)\right) == 0\right\}$$

In[58]:= **ctegs3 = (veomCtEqs /. ctsolns // Simplify // DeleteDuplicates // FullSimplify)[[1]]**

Out[58]= $\left\{\left(-6\,m^2\,(-1+Z\Delta)+c0\,(2+n)\,Z\Delta\,\lambda\,\Lambda^2\,\hbar+c1\,m^2\,(2+n)\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]+\left(6+c1\,(2+n)\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\right)\delta m_0^2\right)\Big/\left(Z\Delta\left(6+c1\,(2+n)\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\right)\right)=0,\ \mathrm{True},\ \mathrm{True},\ \mathrm{True},\ 3\,\lambda\left(1+\frac{2-2\,n}{3\,n+c1\,n\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]}-\frac{2\,(2+n)}{n\left(6+c1\,(2+n)\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\right)}\right)+\delta\lambda_0=0\right\}$

In[59]:= **$\{\delta m_0{}^2,\ \delta\lambda_0\}$ /. Solve[ctegs3, {δm₀, δλ₀}] /. ZΔ → 1 // DeleteDuplicates // Simplify**

Out[59]= $\left\{\left\{-\frac{(2+n)\,\lambda\,\hbar\left(c0\,\Lambda^2+c1\,m^2\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\right)}{6+c1\,(2+n)\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]},\ -\frac{3\,c1\,\lambda^2\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\left(8+n+c1\,(2+n)\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\right)}{\left(3+c1\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\right)\left(6+c1\,(2+n)\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\right)}\right\}\right\}$

In[60]:= **$\{\delta m_1{}^2 == \delta m_0{}^2,\ \delta\lambda_{1\,a} == \delta\lambda_{2\,a},\ \delta\lambda_{1\,b} == \delta\lambda_{2\,b}\}$ /. ctsolns /. Solve[ctegs3, {δm₀, δλ₀}] /. ZΔ → 1 // FullSimplify // Flatten // DeleteDuplicates**

Out[60]= {True}

In[61]:= **$\left\{\frac{\delta\lambda_{1\,a}}{\delta\lambda_{1\,b}}\right\}$ /. ctsolns /. Solve[ctegs3, {δm₀, δλ₀}] /. ZΔ → 1 // FullSimplify // Flatten // DeleteDuplicates**

Out[61]= $\left\{1+\frac{3\,(2+n)}{6+c1\,(2+n)\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]}\right\}$

In[62]:= **$\delta\lambda_{1\,b} == \delta\lambda_{2\,b}$ /. ctsolns /. ZΔ → 1 // FullSimplify // DeleteDuplicates**

Out[62]= {True}

In[63]:= **$\delta\lambda_{1\,b}$ /. ctsolns /. ZΔ → 1 // FullSimplify // DeleteDuplicates**

Out[63]= $\left\{\lambda\left(-1+\frac{3}{3+c1\,\lambda\,\hbar\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]}\right)\right\}$

In[64]:= **$\{\delta\lambda_0 == 1\,\delta\lambda_{1\,a}+2\,\delta\lambda_{1\,b}\}$ /. ctsolns /. Solve[ctegs3, {δm₀, δλ₀}] /. ZΔ → 1 // FullSimplify // Flatten // DeleteDuplicates**

Out[64]= {True}

In[65]:= **$\left\{\delta m_0{}^2 == -\frac{\left(c0\,\Lambda^2+c1\,m^2\,\mathrm{Log}\left[\frac{\Lambda^2}{\mu^2}\right]\right)\lambda\,\hbar}{3}\left(\frac{\delta\lambda_{1\,a}}{\delta\lambda_{1\,b}}-1\right)\right\}$ /. ctsolns /. Solve[ctegs3, {δm₀, δλ₀}] /. ZΔ → 1 // FullSimplify // Flatten // DeleteDuplicates**

Out[65]= {True}