HW1 Numerical Methods Fall 2017

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1 1.2 Galerkin FE method

1.1 Part1

The analytical solution is found by integrating twice with respect to Φ and plugging in the provided boundary conditions.

$$\phi(x) = -\cos(x) + (1 + \cos(1))x + 1$$

We will follow the same procedure as Example 1.1, and choose the same top-hat linear base functions but with a different right hand side. Noticing that the same boundary conditions are used, we can infer that $a_1 = 0$ and $a_n = 1$ Using the same basis function gives us the same stiffness matrix:
if $i \neq j$, $K_{j,i} = 2/\Delta x$
if $i = j$, $K_{j,i} = -1/\Delta x$
The load vector is computed using integration by parts:

$$F_j = -\int_0^1 \cos x\psi(x)dx = \frac{-1}{\Delta x}\left(\int_{x_{i-1}}^{x_i} \cos x\,(x - x_{i-1})\,dx + \int_{x_i}^{x_{i+1}} \cos x\,(x_{i+1} - x)\,dx\right) =$$

$$\frac{-1}{\Delta x}\left(\sin x_i(2x_i - x_{i-1} - x_{i+1} - 2) + \sin x_{i-1} + \sin x_{i+1}\right)$$

which is valid for $2 \leq j \leq n-1$

Leading to the equation:

$$K_{j,i}a_i = F_j$$

solving for the remaining $n-2$ unknown nodes using the Scipy linear algebra solver: