CS 198:206; Introduction to Discrete Structures II

Final Exam **Name & section:**————————

- The approximate time required to complete this exam is 120 + 15 (to submit the exam) minutes.
- Please submit your exam answer as a **Single PDF file**. You can use any website or app to convert your pictures to a **single PDF file** as well as either of the following links:

  https://smallpdf.com/jpg-to-pdf

  https://tools.pdfforge.org/images-to-pdf
- You will get **3 points deduction** for any other submission aside from uploading file on Canvas. **Do Not** submit via email.
- You will get **3 points deduction** if you do not submit a SINGLE PDF file.
- You will get **2 points deduction** if you submit a paper without name.
- **For full grade, show and write all your work, a step by step. No work/ Just final answer, No credit.**
- You're not allowed to use second monitor or device during the exam.
- Do **NOT** use any electronic devices and calculator.
- To avoiding any missing or mistake, please read each question completely and carefully.
- To be able to protect the exam, your camera has to be on during the quiz/exam. It has to be face to you and your work place, with enough light around.

Theorem: A full m-ary tree with:
(i) n vertices has $i = (n-1)/m$ internal vertices and $l = [(m-1)n+1]/m$ leaves,
(ii) i internal vertices has $n = mi + 1$ vertices and $l = (m-1)i + 1$ leaves,
(iii) l leaves has $n = (ml-1)/(m-1)$ vertices and $i = (l-1)/(m-1)$ internal vertices.

* In each Distribution Problem, determine the type of the distribution and write the X's distribution.

X	*p(x)*	*Values of X*	*E(x)*	*V(x)*
	$\frac{1}{b-a+1}$	$a \le x \le b$	$\frac{b+a}{2}$	$\frac{(b-a+2)(b-a)}{12}$
	$\binom{n}{x}p^x(1-p)^{n-x}$	$x = 0,1,\ldots,n$	np	$np(1-p)$
	$\frac{e^{-\lambda}\lambda^x}{x!}$	$x = 0,1,2,\ldots$	λ	λ
	$(1-p)^{x-1}p$	$x = 1,2,3,\ldots$	$\frac{1}{p}$	$\frac{1-p}{p^2}$
	$\binom{x-1}{k-1}(1-p)^{x-k}p^k$	$x = k, k+1,\ldots$	$\frac{k}{p}$	$\frac{k(1-p)}{p^2}$
	$\frac{\binom{M}{x}\binom{N-M}{n-x}}{\binom{N}{n}}$	$\max(0, M+n-N) \le x \le \min(M,n)$	$n \cdot \frac{M}{N}$	$\frac{nM(N-M)(N-n)}{N^2(N-1)}$

1. ***(8 points; 2, 3, and 3 points respectively)*** A survey of used car salesmen revealed the following information:

W

24 wear white patent-leather shoes

28 wear plaid trousers P

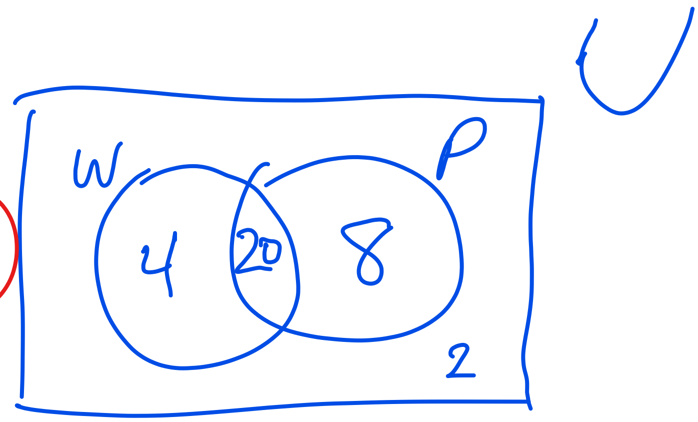
20 wear both of these things

2 wear neither of these things

i). draw the Venn diagram for the survey.

Each # worth 0.5 points + shape

2



Use the Venn diagram to respond the following questions. For each part use the "Set Operation" to mathematically describe which region(s) are satisfy in the question's answer.

1. How many were surveyed? 2

$n(U) = n(W \cup P) + n(W \cup P)' = 2 + 4 + 8 + 20 = \boxed{34}$ 1

2. How many wear plaid trousers but don't wear white patent-leather shoes? 2

$n(P - W) = 8$ 1

2. ***(4 points)*** In how many different ways can 12 different tax returns be assigned to 3 different employees, if each employee audits 4 returns.?

$_{12}C_4 \cdot {}_8C_4 \cdot {}_4C_4$ 4 or $\frac{12!}{4!\,4!\,4!}$

3. ***(16 points; 4 points each)*** Suppose a new cancer test has a 95% chance of correctly identifying that a sick patient has cancer and a 10% chance of incorrectly identifying that a healthy patient has cancer. Assume that 5% of the population has this form of cancer. Compute the following probabilities:

a) The probability that the test identifies a randomly chosen person as having cancer.

A: a randomly chosen person has Cancer 1

B: test result identifies a randomly chosen person has Cancer 1

$P(B) = P(A)P(B|A) + P(\bar{A})P(B|\bar{A}) = (0.95)(0.05) + (0.1)(0.95)$ 1 1

$\simeq 0.1425$

b) The probability that a person who tests positive for cancer actually has cancer.

$P(A|B) = \frac{P(A)\,P(B|A)}{P(B)} = \frac{(0.95)(0.05)}{0.1425}$ 2 2

c) The probability that a person who tests negative for cancer does not have cancer.

$$P(\bar{A}|\bar{B}) = \frac{P(\bar{A})P(\bar{B}|\bar{A})}{P(\bar{B})} = \frac{(0.9)(0.95)}{1-0.1425}$$

2 2

d) The probability that the test gives an incorrect result.

$$P(\bar{A} \cap B) + P(A \cap \bar{B}) = P(\bar{A})P(B|\bar{A}) + P(A)P(\bar{B}|A)$$
$$= (0.1)(0.95) + (0.05)(0.05)$$

2 2

4. ***(9 points; 5 and 4 points respectively)*** The following game is offered. There are 10 cards face-down numbered 1 through 10. You can pick one card. Your payoff is \$0.50 if the number on the card is less than 5 and is the dollar value on the card otherwise. What are the expected value and the variance of your payoff?

Let X = your payoff in \$\$ $\Rightarrow X = 0.5, 5, 6, 7, 8, 9, 10$

with respective prob. of $\frac{4}{10}, \frac{1}{10}, \frac{1}{10}, \frac{1}{10}, \frac{1}{10}, \frac{1}{10}, \frac{1}{10}$

$$\Rightarrow E(X) = \frac{4}{10}(0.5) + \sum_{j=5}^{10} j\left(\frac{1}{10}\right) = \frac{4}{20} + \frac{1}{10}(5+6+7+8+9+10) = 4.7$$

$$\sigma_X^2 = E(X^2) - E^2(X)$$
$$= (0.5)^2\left(\frac{4}{10}\right) + \frac{1}{10}(5^2+6^2+7^2+8^2+9^2+10^2) - (4.7)^2 = 35.6 - (4.7)^2$$

1 1 2 1 1 2

5. ***(11 points; 4, 3, and 4 respectively)*** The number of houses sold by an estate agent follows a Poisson distribution, with a mean of 2.5 houses per week.

a) Find the probability that in the next four weeks the estate agent sells ...

i. ... exactly 4 houses.

X = # houses sold per 4 weeks $\qquad X \sim \text{Poisson}(10)$ $\quad 4\cdot(2.5)$

$$P(X=4) = \frac{e^{-10}\cdot 10^4}{4!}$$

1 1 2

ii. ... more than 6 houses.

$$P(X>6) = P(X \geq 7) = 1 - P(X \leq 6)$$

$$= 1 - \left[\frac{e^{-10}\cdot 10^0}{0!} + \frac{e^{-10}\cdot 10^1}{1!} + \frac{e^{-10}\cdot 10^2}{2!} + \cdots + \frac{e^{-10}\cdot 10^6}{6!}\right]$$

≈ 0.8699

1 1 1

The estate agent monitors the house sales in periods of 4 weeks.
b) Find the probability that in the next twelve of those four week periods there are exactly 7 four week periods in which more than 6 houses are sold.

Y = # of 4 weeks period with at least 7 sales

$Y \sim B(12, 0.8699)$ 1 $P(Y=7) = \binom{12}{7}(0.8699)^7(0.1301)^5$

2

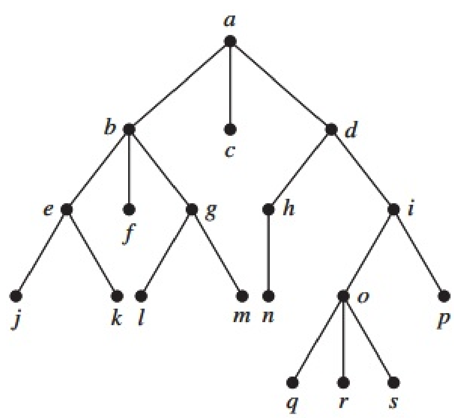
6. ***(2 points each)*** 1). For which values of "m" and "n" does the complete graph $K_{m,n}$ has:
i). an Euler circuit? why? m and n both has to be even numbers. 1

Because a graph has an Euler circuit if and only if each of the vertices has an even degree. 1

ii). a Hamiltonian Circuit? why? When m=n with minimum value 2 1

Since in Hamiltonian circuit, every vertex is visited only once. It must start and end at the same vertex. In complete bipartite graph Km, n, when m = n, then in that case, it has a Hamiltonian circuit. 1

7. ***(10 points; 2 points each part)*** Answer the following questions about the tree illustrated.



i) Which vertices are leaves? j, k, l, h, m, c, n, q, r, s, p 2

ii). Which vertices are children of "g"? l & m 2

iii). Which vertices are ancestors of "e"? b & a 2

iv). What's the height of the tree? 4 2

v). Is the tree a "full m-ary" tree for some positive integer m? Why? No, since Not all internal vertices have exactly same # of children. 2

8. ***(2 points each*** True or False? Explain your answer or give a counter example.

i). If every vertex in a tree has odd degree, the number of edges in the tree may be either odd or even.

1

FALSE. There must be an even number of vertices of odd degree in any graph, since the sum of the degrees is twice the number of edges. So the number of edges, which is one less than the number of vertices, must be odd.

1

ii) Let G be a connected graph. If G has no cut edge, then G has no cut vertices.

1

False. A counterexample is pretty easy to generate; two triangles with a common vertex will yield a graph with 5 vertices and 6 edges that contains 1 cut vertex and no cut edge.

1

9. ***(4 points*** Define:

i). rooted tree is a tree in which one vertex has been designated as the root and other vertices are away from the root or (every edge is away from the root).

2

ii). simple graph A graph without parallel edges and loops.

2

Total: 70 Points

Good Luck! ***:)***