CS 198:206

Exam II **Name:**——————

- The approximate time required to complete this exam is 60 + 15 (to submit the exam) minutes.
- Please submit your exam answer as a **Single PDF file**. You can use any website or app to convert your pictures to a **single PDF file** as well as either of the following links:

  https : //smallpdf.com/jpg – to – pdf

  https : //tools.pdfforge.org/images – to – pdf

- You will get **3 points deduction** for any other submission aside from uploading file on Canvas. **Do Not** submit via email.
- You will get **3 points deduction** if you do not submit a SINGLE PDF file.
- You will get **2 points deduction** if you submit a paper without name.
- **For full grade, show and write all your work, a step by step. No work/ Just final answer, No credit.**
- You're not allowed to use second monitor or device during the exam.
- Do **NOT** use any electronic devices and calculator.
- To avoiding any missing or mistake, please read each question completely and carefully.
- To be able to protect the exam, your camera has to be on during the quiz/exam. It has to be face to you and your work place, with enough light around.

* In each Distribution Problem, determine the type of the distribution and write the X's distribution.

1. ***(5 points)*** There is a 95% chance of passing any exam. What is variance in number of attempts until third exam is passed?

$X \sim NB(3, 0.95)$

$$\sigma_x^2 = \frac{r(1-p)}{p} = \frac{3(0.05)}{(0.95)^2} \simeq 0.166$$

2. ***(6 points)*** Average of 7 particles hit a magnetic detection field per millisecond. What is probability at most 5 particles hit in one second?

$X \sim Poisson(7)$ $\lambda = \lambda t = 7(1000) = 7000$

$$P(X \le 5) = P(X=0) + P(X=1) + P(X=2) + P(X=3) + P(X=4) + P(X=5) = e^{-7000}\left[1 + \frac{7000}{1!} + \frac{7000^2}{2!} + \frac{7000^3}{3!} + \frac{7000^4}{4!} + \frac{7000^5}{5!}\right]$$

3. ***(8 points)*** An insurance company offers a discount to homeowners who install smoke detectors in their homes. A company representative claims that 70% or more of policyholders have smoke detectors. You draw a random sample of eight policyholders. Let X be the number of policyholders in the sample who have smoke detectors.

a). If exactly 70% of the policyholders have smoke detectors (so the representative's claim is true, but just barely), what is $P(X \geq 4)$?

$$X \sim Bin(8, 0.7) \Rightarrow P(X) = \frac{8!}{x!(8-x)!}(0.7)^x(0.3)^{8-x} \quad x = 0, 1, \dots, 8$$

$$P(X \ge 4) = 1 - P(X < 4) = 1 - [P(X=0) + P(X=1) + P(X=2) + P(X=3)]$$

$$= 1 - \left[(0.3)^8 + \frac{8!}{7!}(0.7)(0.3)^7 + \frac{8!}{2!6!}(0.7)^2(0.3)^6 + \frac{8!}{3!5!}(0.7)^3(0.3)^5\right]$$

4. ***(8 points)*** A system is tested for faults once per hour. If there is no fault, none will be detected. If there is a fault, the probability is 0.8 that it will be detected. The tests are independent of one another. Given that a fault has gone undetected for 2 hours, what is the probability that it will be detected in the next hour?

$P(X=3 \mid X>2) = \frac{P(X=3) \cap P(X>2)}{P(X>2)} = \frac{P(X=3)}{P(X>2)} = \frac{0.032}{0.04} = \boxed{0.8}$ 2 1

$P(X=3) = pq^{x-1} = 0.8(0.2)^2 = 0.032$ 2

$P(X>2) = 1 - P(X\leq 2) = 1 - [P(X=1) + P(X=2)] = 1 - [0.8(0.2)^0 + (0.8)(0.2)] = 0.04$ 2 1

5. ***(3 points)*** How many vertices and edges does the complete graph K_n have? degree of each $v = n-1$

\# of vertices: n 1 $\quad \sum_{i=1}^{n} d(v) = 2e$ 1 $\Rightarrow 2e = n(n-1) \Rightarrow e = \frac{n(n-1)}{2}$ edges 1

6. ***(8 points)*** A sequence $d_1, d_2, ..., d_n$ is called graphic if it is the degree sequence of a simple graph. Determine whether each of these sequences is graphic (Give a reason to your answer). For those that are, draw a graph having the given degree sequence.

a) 5, 4, 3, 2, 1, 0

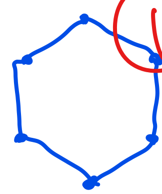
1 No; 6 vertices Max deg = 5 1 ⇒ min deg must be 1

b) 6, 5, 4, 3, 2, 1

1 No: Sum of deg is odd. Also a simple graph with 6 vertices cannot have any deg greater than 5. 1

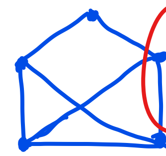
c) 2, 2, 2, 2, 2, 2

1 Yes, C_6 1



d) 3, 3, 3, 3, 2

Yes 1 1



7. ***(7 points)*** Find a route with the least total airfare that visits each of the cities in this graph, where the weight on an edge is the least price available for a flight between the two cities.

The following table shows the twelve different Hamilton circuits and their weights, where we abbreviate the cities with the beginning letter of their name, except that Detroit is M (for Motor City, of course!):

Circuit	Weight
S-M-N-D-L-S	$329 + 189 + 279 + 209 + 69 = 1075$
S-M-N-L-D-S	$329 + 189 + 379 + 209 + 179 = 1285$
S-M-D-N-L-S	$329 + 229 + 279 + 379 + 69 = 1285$
S-M-D-L-N-S	$329 + 229 + 209 + 379 + 359 = 1505$
S-M-L-N-D-S	$329 + 349 + 379 + 279 + 179 = 1515$
S-M-L-D-N-S	$329 + 349 + 209 + 279 + 359 = 1525$
S-N-M-D-L-S	$359 + 189 + 229 + 209 + 69 = 1055$
S-N-M-L-D-S	$359 + 189 + 349 + 209 + 179 = 1285$
S-N-D-M-L-S	$359 + 279 + 229 + 349 + 69 = 1285$
S-N-L-M-D-S	$359 + 379 + 349 + 229 + 179 = 1495$
S-D-M-N-L-S	$179 + 229 + 189 + 379 + 69 = 1045$
S-D-N-M-L-S	$179 + 279 + 189 + 349 + 69 = 1065$

Goo

3 even some (min 3) is fine

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As a check of our arithmetic, we can compute the total weight (price) of all the trips (it comes to 15420) and check that it is equal to 6 times the sum of the weights (which here is 2570), since each edge appears in six paths (and sure enough, $15420 = 6 \cdot 2570$). We see that the circuit S-D-M-N-L-S (or the same circuit starting at some other point but traversing the vertices in the same or exactly opposite order) is the one with minimum total weight, 1045. Note that we might have guessed this route by looking at the drawing (which is more or less to scale in terms of the actual locations of these cities).

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