

Adversarial Search - Bayesian Networks

Assignment 3

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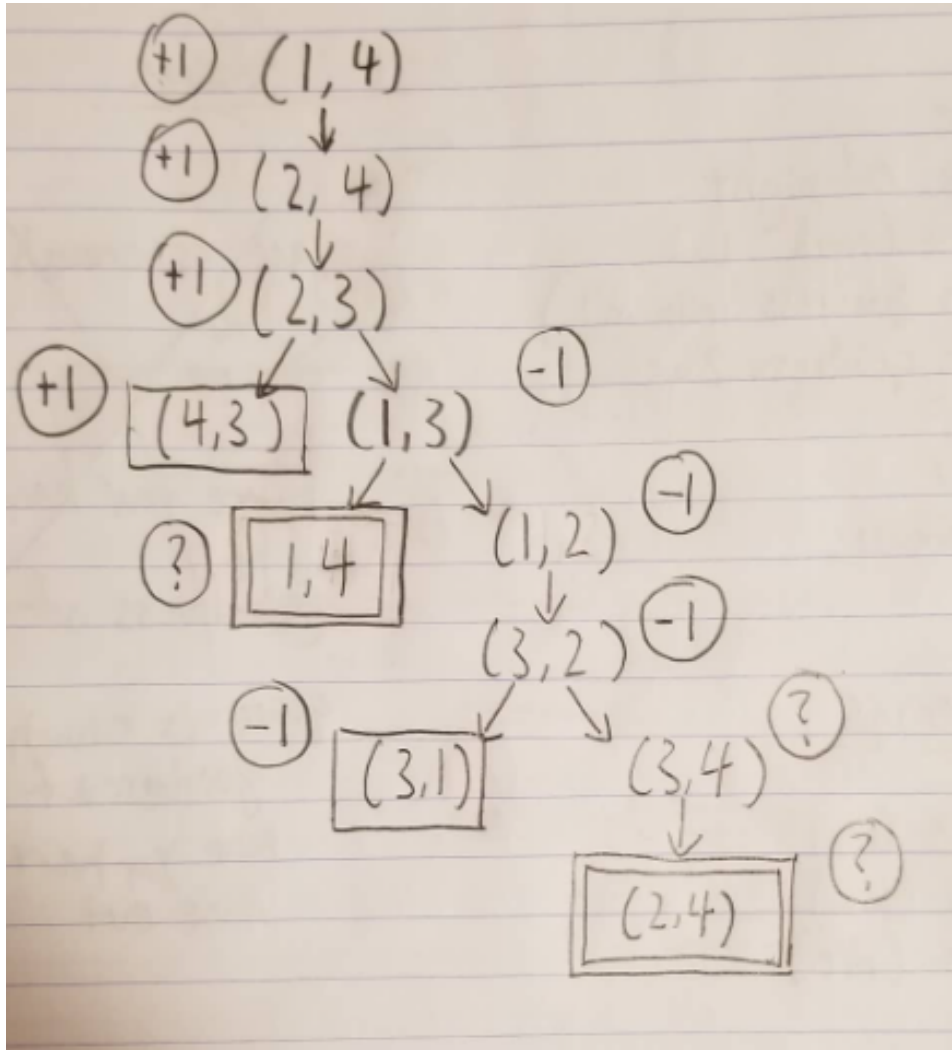
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Problem 1



"?" values are used to annotate loop states. When an agent has to choose between a +1 value and a "?" value (+1,?) the max player will always choose +1 while the min player will choose ?, and vice versa for (-1,?), with the min player choosing -1 and the max player choosing "?". Since the game value of a loop state is unknown, it is best for a max/min player to choose the corresponding max/min value if it is available, however if that value is not available the "?" state is a viable option to explore as it cannot be worse than an immediate loss. If all the successors of a state have a "?" value, the backed-up value is also "?".

Problem 2

(a) Computing the joint probability distribution:

$$P(a, b, c, d, e) = P(a) * P(b) * P(c) * P(d|a, b) * P(e|b, c)$$

$$P(a, b, c, d, e) = 0.2 * 0.5 * 0.8 * 0.1 * 0.3$$

$$P(a, b, c, d, e) = 0.0024$$

(b) Computing the joint probability distribution:

$$P(\neg a, \neg b, \neg c, \neg d, \neg e) = P(\neg a) * P(\neg b) * P(\neg c) * P(\neg d|\neg a, \neg b) * P(\neg e|\neg b, \neg c)$$

$$P(\neg a, \neg b, \neg c, \neg d, \neg e) = 0.8 * 0.5 * 0.2 * 0.1 * 0.8$$

$$P(\neg a, \neg b, \neg c, \neg d, \neg e) = 0.0064$$

(c) Using the conditional probability rule $P(A|B) = \frac{P(A \cap B)}{P(B)}$

$$P(\neg a|b, c, d, e) = \frac{P(\neg a, b, c, d, e)}{P(b, c, d, e)}$$

$$P(\neg a|b, c, d, e) = \frac{P(\neg a, b, c, d, e)}{P(a, b, c, d, e) + P(\neg a, b, c, d, e)}$$

$$P(\neg a|b, c, d, e) = \frac{P(\neg a) * P(b) * P(c) * P(d|\neg a, b) * P(e|b, c)}{0.0024 + P(\neg a) * P(b) * P(c) * P(d|\neg a, b) * P(e|b, c)}$$

$$P(\neg a|b, c, d, e) = \frac{0.8 * 0.5 * 0.8 * 0.6 * 0.3}{0.0024 + 0.8 * 0.5 * 0.8 * 0.6 * 0.3}$$

$$P(\neg a|b, c, d, e) = \frac{0.0576}{0.0024 + 0.0576}$$

$$P(\neg a|b, c, d, e) = 0.96$$

Problem 3

(a)

$$P(b|j, m) = \alpha * P(b, j, m)$$

$$\alpha = \frac{1}{P(\neg b, j, m) + P(b, j, m)}$$

$$\begin{aligned}
P(b, j, m) &= \sum_e \sum_a P(b, j, m, e, a) \\
P(b, j, m) &= P(b) \sum_e P(e) \sum_a P(a|b, e) P(j|a) P(m|a) \\
P(b, j, m) &= 0.001 * 0.002 * 0.95 * 0.9 * 0.7 + 0.001 * 0.002 * 0.05 * 0.05 * 0.01 + \\
&\quad 0.001 * 0.998 * 0.94 * 0.9 * 0.7 + 0.001 * 0.998 * 0.06 * 0.05 * 0.01 \\
P(b, j, m) &= 0.00059224259
\end{aligned}$$

$$\begin{aligned}
P(\neg b, j, m) &= \sum_e \sum_a P(\neg b, j, m, e, a) \\
P(\neg b, j, m) &= P(\neg b) \sum_e P(e) \sum_a P(a|\neg b, e) P(j|a) P(m|a) \\
P(\neg b, j, m) &= 0.999 * 0.002 * 0.29 * 0.9 * 0.7 + 0.999 * 0.002 * 0.71 * 0.05 * 0.01 + \\
&\quad 0.999 * 0.998 * 0.001 * 0.9 * 0.7 + 0.999 * 0.998 * 0.999 * 0.05 * 0.01 \\
P(\neg b, j, m) &= 0.00149185764
\end{aligned}$$

$$\begin{aligned}
\alpha &= \frac{1}{P(\neg b, j, m) + P(b, j, m)} \\
\alpha &= \frac{1}{0.00149185764 + 0.00059224259} \\
\alpha &= \frac{1}{0.00208410023}
\end{aligned}$$

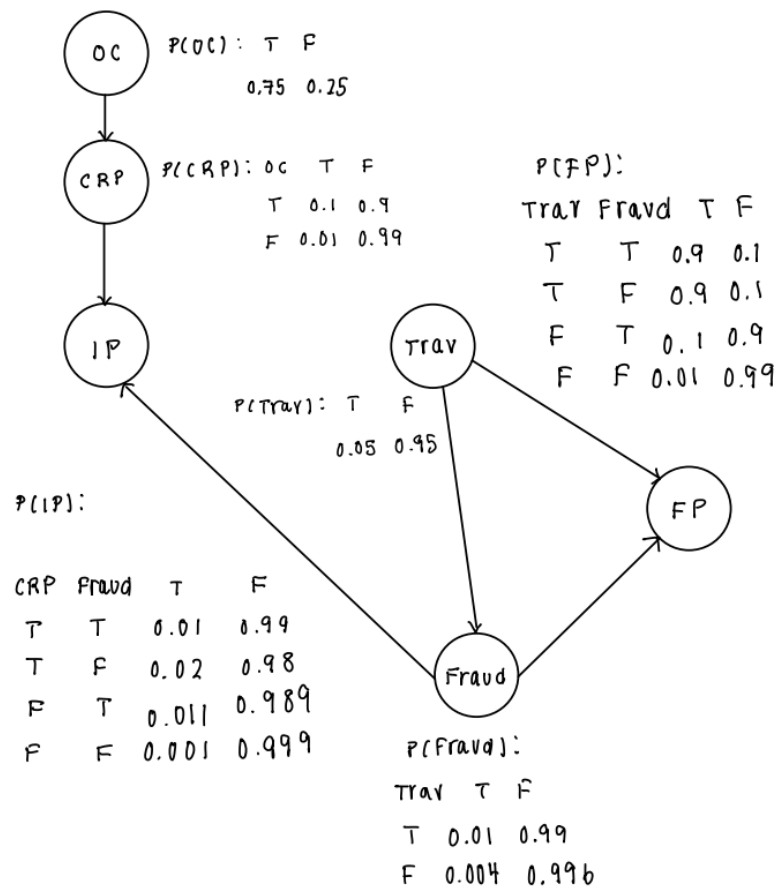
$$\begin{aligned}
P(b|j, m) &= \alpha * P(b, j, m) \\
P(b|j, m) &= \frac{0.00059224259}{0.00208410023} \\
P(b|j, m) &= 0.28417183659
\end{aligned}$$

Therefore, the probability of a burglary given that John and Mary both called, is 0.28417183659.

- (b) In a Bayesian network in the form of a chain, using the enumeration method has a worst case runtime of $\mathcal{O}(2^n)$, as moving down each level in the chain generates another level in a binary tree.

Using the variable elimination method, moving down each level in the Bayesian network chain only adds one additional problem to compute (point-wise product of factors), as each node in the network only has one parent. Thus, the variable elimination method has a worst case runtime of $\mathcal{O}(n)$.

Problem 4



(a)

- (b) Probability the current transaction is a fraud prior to searching for previous computer related purchases and before verifying if it is a foreign and/or an internet purchase:

$$\begin{aligned}
P(Fraud) = & \\
& P(Fraud = True|Trav = True) * P(Trav = True) + \\
& P(Fraud = True|Trav = False) * P(Trav = False) = \\
& 0.01 * 0.05 + 0.004 * 0.95 = \\
& 0.0043
\end{aligned}$$

Probability the current transaction is a fraud given that it is a foreign transaction, not an internet purchase, and card holder purchased computer related purchases in the past week:

$$\begin{aligned}
& P(Fraud = True|IP = False, CRP = True, OC = True) = \\
& P(IP = False|CRP = True, Fraud = True) * P(CRP = True) + \\
& P(IP = False|CRP = False, Fraud = True) * P(CRP = False) = \\
& P(IP = False|CRP = True, Fraud = True) * (P(CRP = True|OC = True) * \\
& \quad P(OC = True) + \\
& \quad P(CRP = True|OC = False) * P(OC = False)) + \\
& P(IP = False|CRP = False, Fraud = True) * (P(CRP = False|OC = True) * \\
& \quad P(OC = True) + \\
& \quad P(CRP = False|OC = False) * P(OC = False)) = \\
& 0.99 * (0.1 * 0.75 + 0.01 * 0.25) + 0.89 * (0.9 * 0.75 + 0.99 * 0.25) = \\
& \quad 0.89775
\end{aligned}$$

$$\begin{aligned}
& P(Fraud = True|FP = True) = \\
& P(Fraud = True|Trav = True) * P(FP = True|Trav = True, Fraud = True) * \\
& \quad P(Trav = True) + \\
& P(Fraud = True|Trav = False) * P(FP = True|Trav = False, Fraud = True) * \\
& \quad P(Trav = False) = \\
& 0.01 * 0.90 * 0.05 + 0.004 * 0.10 * 0.95 = \\
& \quad 0.00083
\end{aligned}$$

$$\begin{aligned}
& P(Fraud = True|IP = False, CRP = True, OC = True, FP = True) = \\
& 0.89775 * 0.00083 = \\
& \quad 0.000745
\end{aligned}$$