CS 198:206; Introduction to Discrete Structures II

Exam III **Name & section:**——————————

- The approximate time required to complete this exam is 70 + 15 (to submit the exam) minutes.
- Please submit your exam answer as a **Single PDF file**. You can use any website or app to convert your pictures to a **single PDF file** as well as either of the following links:

  https://smallpdf.com/jpg-to-pdf

  https://tools.pdfforge.org/images-to-pdf
- You will get **3 points deduction** for any other submission aside from uploading file on Canvas. **Do Not** submit via email.
- You will get **3 points deduction** if you do not submit a SINGLE PDF file.
- You will get **2 points deduction** if you submit a paper without name.
- **For full grade, show and write all your work, a step by step. No work/ Just final answer, No credit.**
- You're not allowed to use second monitor or device during the exam.
- Do **NOT** use any electronic devices and calculator.
- To avoiding any missing or mistake, please read each question completely and carefully.
- To be able to protect the exam, your camera has to be on during the quiz/exam. It has to be face to you and your work place, with enough light around.

Theorem: A full m-ary tree with:
(i) n vertices has $i = (n-1)/m$ internal vertices and $l = [(m-1)n+1]/m$ leaves,
(ii) i internal vertices has $n = mi+1$ vertices and $l = (m-1)i+1$ leaves,
(iii) l leaves has $n = (ml-1)/(m-1)$ vertices and $i = (l-1)/(m-1)$ internal vertices.

* In each Distribution Problem, determine the type of the distribution and write the X's distribution.

X	*p(x)*	*Values of X*	*E(x)*	*V(x)*
	$\frac{1}{b-a+1}$	$a \le x \le b$	$\frac{b+a}{2}$	$\frac{(b-a+2)(b-a)}{12}$
	$\binom{n}{x} p^x (1-p)^{n-x}$	$x = 0,1,\ldots,n$	np	$np(1-p)$
	$\frac{e^{-\lambda}\lambda^x}{x!}$	$x = 0,1,2\ldots$	λ	λ
	$(1-p)^{x-1}p$	$x = 1,2,3,\ldots$	$\frac{1}{p}$	$\frac{1-p}{p^2}$
	$\binom{x-1}{k-1}(1-p)^{x-k}p^k$	$x = k, k+1, \ldots$	$\frac{k}{p}$	$\frac{k(1-p)}{p^2}$
	$\frac{\binom{M}{x}\binom{N-M}{n-x}}{\binom{N}{n}}$	$\max(0, M+n-N) \le x \le \min(M,n)$	$n \cdot \frac{M}{N}$	$\frac{nM(N-M)(N-n)}{N^2(N-1)}$

1. ***(17 points; 4, 4, 4, and 5 points respectively)*** It has been established over a long period of time that the probability that an electricity company operative will be able to take a reading from a house meter, due to the resident being at home, is 0.3 .

a). Determine the probability that the first reading to be taken will be on, or before, the 8^{th} house visit.

Let X= # of houses to be visited up to and including the first reading meter 1

X~ Geom (0.3) 1

P(X<= 8) = 1-P(X>=9)= 1-(0.7)^8 2

b). Find the probability that the operative will be able to take …

i. … exactly 4 readings in his first 8 visits. 1

X= # of success in reading the meter   X~ Binomial (8, 0.3) 1

$P(X=3)=\binom{8}{4}(0.3)^4\ (0.7)^4$ 2

ii. … his 4^{th} reading on his 8^{th} visit. 1

Y= # of success

Y~NB(4, 0.3) 1 $P(Y=8)=\binom{7}{3}(0.3)^4\ (0.7)^4$ 2

iii. … his 4^{th} reading on, or before his 8^{th} visit. 2

P(Y<=8)= P(Y= 4, 5, 6, 7, 8)

$= (0.3)^4+\binom{4}{3}(0.3)^4(0.7)+\binom{5}{3}(0.3)^4(0.7)^2+\binom{6}{3}(0.3)^4(0.7)^3+\binom{7}{3}(0.3)^4(0.7)^4$

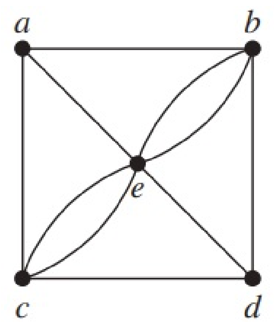
1   1

2. ***(6 points)*** Determine whether the given graph has an Euler circuit? How about Hamiltonian circuit? If it does, find such a circuit(s). If it doesn't, give a reason to show why no such a circuit exists?

2

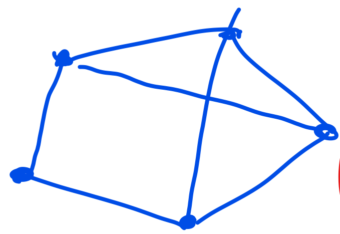
This graph doesn't have an Euler circuit as not all vertices has even degree. 1

But has Hamiltonian circuit like 2 e,b,d,c,a,e or some other more circuits. 1



3. ***(4 points)*** A sequence $d_1, d_2, ..., d_n$ is graphic if it is the degree sequence of a simple graph. Determine whether the sequence 3, 3, 3, 3, 2 is graphic? If yes, draw a graph having the given degree sequence.

Since SUM of d(v_i)= 3+ 3+ 3+ 3+ 2= 14 is even so there is a possibility of existence of such a graph. 2



2

4. ***(2 points each*** True or False? Explain your answer or give a counter example.

i). If every vertex in a tree has odd degree, the number of edges in the tree may be either odd or even. 2

FALSE. There must be an even number of vertices of odd degree in any graph, since the sum of the degrees is twice the number of edges. So the number of edges, which is one less than the number of vertices, must be odd.

ii) Let G be a connected graph. If G has no cut edge, then G has no cut vertices. 2

False. A counterexample is pretty easy to generate; two triangles with a common vertex will yield a graph with 5 vertices and 6 edges that contains 1 cut vertex and no cut edge.

5. ***(4 points*** A chain letter starts when a person sends a letter to 5 people. Each person who sends the letter to 5 other people who have never received it or does not send it to anyone. Suppose that 10,000 people send out the letter before the chain ends and that no one receives more than one letter. How many people receive the letter? How many people do not send it out?

Notice that since every person who sends out the letter sends it to exactly 5 other people, and no two people receive the letter twice, this situation can be modeled using a full rooted 5-ary tree. The root represents the person who first sends out the letter, and the children of any vertex represent the 5 people that the related person sent letters. From this, we see that i = 10, 000.

The total number of people who received the letter can be found by computing the number total number of vertices in the rooted tree. Using Theorem 4, this is n = mi + 1 = 5(10, 000) + 1 = 50, 001. This counts the root, who started the letter but did not receive it, so 50,000 people received the letter. 2

Notice that leaves represent people who did not mail the letter to 5 other people. Thus, again using Theorem 4, l = (m 1)i + 1 = 4(10, 000) + 1 = 40, 001, so 40,001 people did not send it out after they received it.

2

6. ***(4 points*** Define the graph $K_{m,n}$. How many edges does that graph has? 2

Graph Let Km,n is a complete bipartite graph with two partite sets X and Y having m and n vertices, respectively.

Graph Km,n has exactly one edge for each pair (x,y) of vertices such that xX and yY.

So, if you can count those pairs, you can count the edges, which is mn edges. 2

Total: 39 Points

Good Luck! ***:)***