CS 198:206; Introduction to Discrete Structures II

Exam III **Name & section:**———————————

- The approximate time required to complete this quiz is 80 minutes.
- **For full grade, show and write all of your work, step by step. No work/ Just final answer, No credit.**
- You will get **2 points deduction** if you submit a paper without name.
- In case if you need more space, you might use the back side of the your paper. **I DO NOT ACCEPT** any other sheet attached to the exam paper.
- **Do NOT USE** graphing calculator.
- To avoiding any missing or mistake, please read the question **carefully and completely**.

Theorem: A full m-ary tree with:
(i) n vertices has $i = (n-1)/m$ internal vertices and $l = [(m-1)n+1]/m$ leaves,
(ii) i internal vertices has $n = mi+1$ vertices and $l = (m-1)i+1$ leaves,
(iii) l leaves has $n = (ml-1)/(m-1)$ vertices and $i = (l-1)/(m-1)$ internal vertices.

1. ***(8 points)*** An insurance company offers a discount to homeowners who install smoke detectors in their homes. A company representative claims that 70% or more of policyholders have smoke detectors. You draw a random sample of eight policyholders. Let X be the number of policyholders in the sample who have smoke detectors. If exactly 70% of the policyholders have smoke detectors (so the representative's claim is true, but just barely), what is $P(X \geq 4)$?

$X \sim Bin(8, 0.7) \Rightarrow P(X=x) = \frac{8!}{x!(8-x)!}(0.7)^x(0.3)^{8-x} \quad x=0,\dots,8$

$P(X \geq 4) = 1 - P(X<4) = 1 - [P(X=0) + P(X=1) + P(X=2) + P(X=3)]$

$= 1 - \left[(0.3)^8 + \frac{8!}{7!}(0.7)(0.3)^7 + \frac{8!}{2!6!}(0.7)^2(0.3)^6 + \frac{8!}{3!5!}(0.7)^3(0.3)^5\right]$

2. ***(12 points; 5, 3, 2, 2 points respectively)*** A data center contains 1000 computer servers. Each server has probability 0.003 of failing on a given day.

a. What is the probability that exactly two servers fail?

X = # of servers fail $\quad X \sim Poisson(\lambda)$ where

$\lambda = np = 1000(0.003) = 3 \Rightarrow P(X=x) = \frac{e^{-\lambda}\lambda^x}{x!} \quad x=0,1,2,\dots$

$P(X=2) = \frac{e^{-3}3^2}{2!}$

b. What is the probability that fewer than 998 servers function? $\longrightarrow$ more than 2 servers fail

$P(X>2) = 1 - P(X \leq 2)$

$= 1 - \{P(X=0) + P(X=1) + P(X=2)\}$

$= 1 - \left\{e^{-3} + \frac{e^{-3}\cdot 3}{1!} + \frac{e^{-3}\cdot 3^2}{2!}\right\}$

c. What is the mean number of servers that fail?

$\mu_x = \lambda = 3$ 2

d. What is the standard deviation of the number of servers that fail?

$\sigma_x^2 = \lambda$

$\sigma_x = \sqrt{\sigma_x^2} = \sqrt{\lambda} = \sqrt{3}$

3. ***(1 point each)*** . For each situation, would you find an Euler circuit or a Hamilton Circuit?

a. The department of Public Works must inspect all streets in the city to remove dangerous debris.

Euler circuit

b. Relief food supplies must be delivered to eight emergency shelters located at different sites in a large city.

Hamilton Circuit?

c. The Department of Public Works must inspect traffic lights at intersections in the city to determine which are still working.

Hamilton Circuit?

d. An insurance claims adjuster must visit 11 homes in various neighborhoods to write reports.

Hamilton Circuit?

4. ***(4 points)*** Can a simple graph have 5 vertices and 12 edges? If so, draw it; if not, explain why it is not possible to have such a graph.

No!

In a simple graph, no pair of vertices can have more than one edge between them. In other words, there are no parallel edges.

For a simple graph, the "densest" graph we can get is one in which every vertex is connected to every other vertex. This is called a complete graph. The maximum number of edges in the complete graph containing 5 vertices is given by K5: which is C(5, 2) edges = "5 choose 2" edges = 10 edges. Since 12 > 10, it is not possible to have a simple graph with more than 10 edges.

5. ***(1 point each)*** True or False:

i). Every graph that has an Euler circuit also has an Euler path. False

ii). If every vertex in a tree has odd degree, the number of edges in the tree may be either odd or even. False

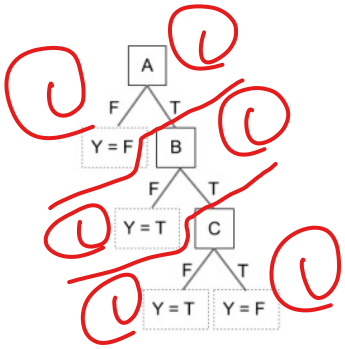
6. ***(4 points)*** How many leaves does a tree with 100 vertices have where each internal vertex has exactly 3 children.

n=100, m=3, therefore an application of (iii) of Theorem 4 implies:

n= (ml-1)/m-1 therefore 100= (3l-1)/2. Hence l= 201/3= 67.

7. ***(6 points)*** Using the dataset bellow, we want to build a decision tree (rotted at A) which classifies Y as T/F given the binary variables A, B, C. Draw the best decision tree that would go through the steps with zero training error. You do not need to show any computation.

A	B	C	Y
F	F	F	F
T	F	T	T
T	T	F	T
T	T	T	F



8. ***(2 points each)*** Define:

Forest

A forest is a graph with each connected component a tree.

Internal Vertex

Vertices that have children are called internal vertices.

Total: 44 Points

Good Luck! ***:)***