CS 198:206; Introduction to Discrete Structures II

Final Exam   **Name** & section:——————————————

- The approximate time required to complete this quiz is 180 minutes.
- **For full grade, show and write all of your work, step by step. No work/ Just final answer, No credit.**
- You will get **2 points deduction** if you submit a paper without name.
- In case if you need more space, you might use the back side of the your paper. **I DO NOT ACCEPT** any other sheet attached to the exam paper.
- **Do NOT USE** graphing calculator.
- To avoiding any missing or mistake, please read the question **carefully and completely**.

1. *(14 points, 4 and then 2.5 each part)* A survey of faculty and graduate students at the University of Florida's film school revealed the following information: 51 admire Moe $\mathcal{M}$

49 admire Larry $\mathcal{L}$
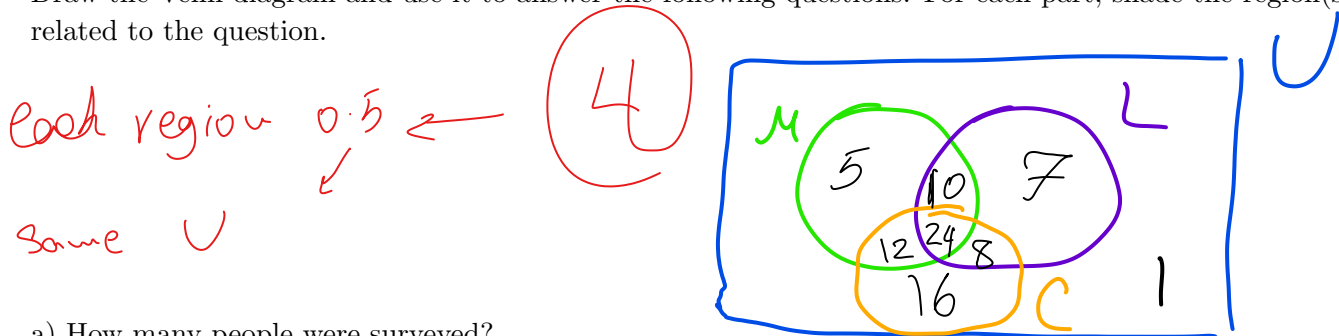60 admire Curly C
34 admire Moe and Larry
32 admire Larry and Curly
36 admire Moe and Curly
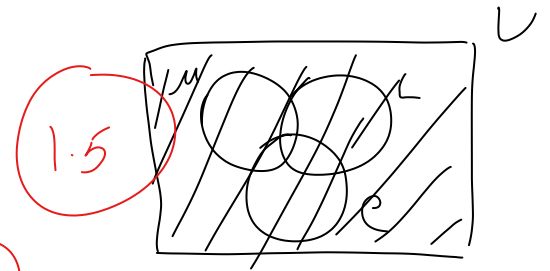24 admire all three of the Stooges
1 admires none of the Three Stooges

Draw the Venn diagram and use it to answer the following questions. For each part, shade the region(s) that are related to the question.
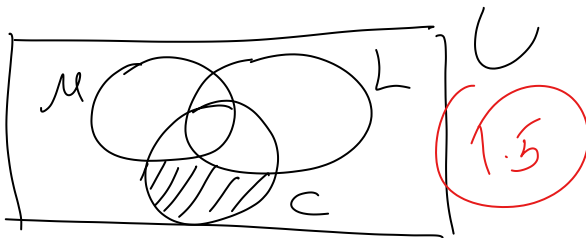
each region 0.5 ⟵ (4)

same ∪

a) How many people were surveyed?

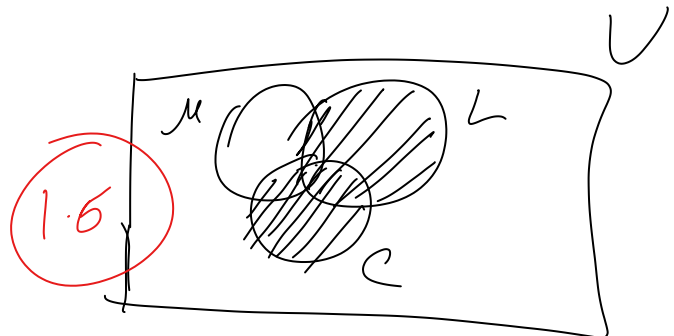$$n(U) = 5 + 10 + 7 + 12 + 24 + 8 + 16 + 1$$
$$= 83$$

(1.5)

b) How many admire Curly, but not Larry nor Moe? 16

(1.5)

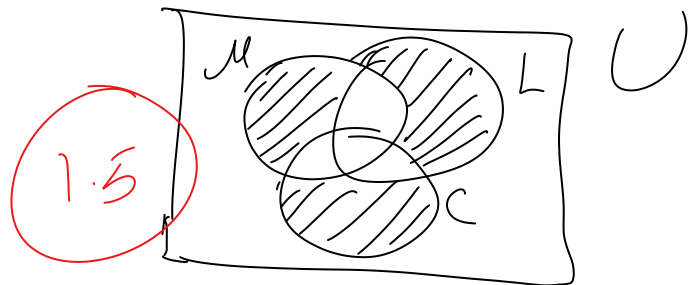c) How many admire Larry or Curly?

$$10 + 7 + 24 + 8 + 12 + 16$$
$$= 77$$

(1.5)

d) How many admire exactly one of the Stooges?

$$5 + 7 + 16 = 28$$

(1.5)

4

2. *(16 points;4 points each)* ABC computers routes its customers technical questions to three tech support offices in the US, Germany and China. The percent of calls handled by the US, Germany and China offices are 40%, 25% and 35% , respectively. The percent of costumers satisfied with the service by location are: 70% (US), 80% (Germany) and 75% (China). If a costumer who called with tech question is randomly selected, what is the probability his/her call

U

C

a) Was routed to Germany and resulted in a satisfied customer.

S: Satisfied Customer

$$P(G \cap S) = P(G) \cdot P(S|G) = 0.25(0.8)$$

b) resulted in a satisfied costumer.

$$P(S) = P(U) \cdot P(S|U) + P(G)P(S|G) + P(C)P(S|C)$$
$$= 0.4(0.7) + 0.25(0.8) + 0.35(0.75) = 0.7425$$

c) Given the customer was satisfied with the service, what is the probability her call was routed to Germany?

$$P(G|S) = \frac{P(G \cap S)}{P(S)} = \frac{0.2}{0.7425} = 0.27$$

d). Given the costumer is satisfied with the service, what is the probability her call was NOT routed to Germany?

$$P(G^c|S) = 1 - P(G|S) = 1 - 0.27$$

3. *(6 points)* An LG Dishwasher, which costs $800, has a 20% chance of needing to be replaced in the first 2 years of purchase. A two-year extended warrantee costs $112.10 on a dishwasher. What is the expected value of the extended warranty assuming it is replaced in the first 2 years?

$$800(0.2) - 112.1(0.8)$$
$$= 160 - 89.68 = 70.32$$

4. *(7 point)* It is known that in a certain town 30% of the people own an Apfone. A researcher asks people at random whether they own an Apfone. Calculate the probability of:

a) The 4th person who is been asked, be the first person who owns an Apfone.

X = # of people asked up to & including the first person who owns an Aphone

$X \sim Geo(0.3)$
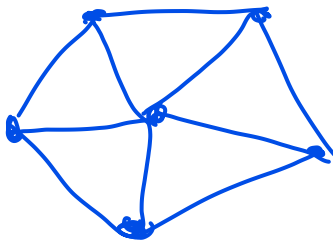
$P(X = 4) = (0.7)^3 (0.3) = 0.1029$

b) $P(X < 6)$?

$P(X < 6) = P(X = 1, 2, 3, 4, 5)$

$= 0.3 + (0.7)(0.3) + (0.7)^2(0.3) + (0.7)^3(0.3) + (0.7)^4(0.3)$

5. *(3 points)* A sequence is graphic if it is the degree sequence of a simple graph. Determine whether the sequence 5, 3, 3, 3, 3, 3 is graphic? If yes, draw a graph having the given degree sequence.

yes : $5 + 5(3) = 20$ which is even

6. *(1 point each)* True or False:

i). A graph is bipartite if and only if it has no odd cycles. T

ii). Every graph that has a Hamilton circuit also has a Hamilton path that is not a circuit. T

7. *(7 points; 4, 3 points respectively)* Suppose that there are five young women and five young men on an island. Each man is willing to marry some of the women on the island and each woman is willing to marry any man who is willing to marry her. Suppose that Sandeep is willing to marry Tina and Vandana; Barry is willing to marry Tina, Xia, and Uma; Teja is willing to marry Tina and Zelda; Anil is willing to marry Vandana and Zelda; and Emilio is willing to marry Tina and Zelda. a) Use a bipartite graph to model the problem.

4

S   B   Te   A   E

Ti   U   X   U   Z
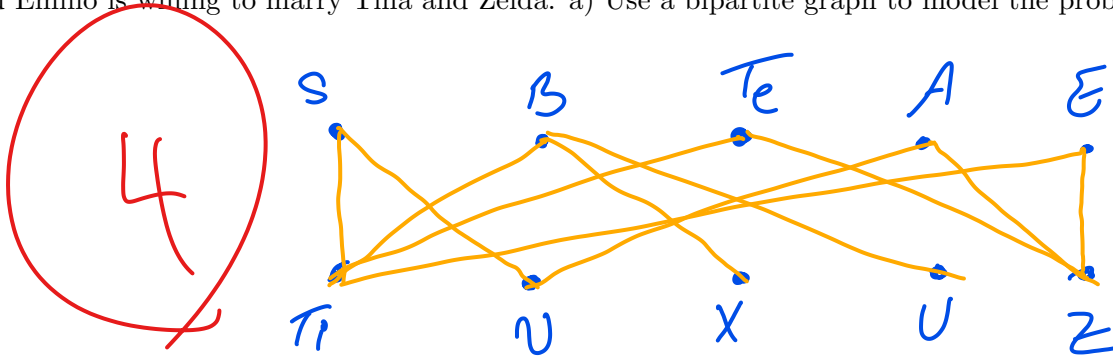
b) Use Hall's theorem to determine if there is any matching graph of the young men and young women on the island such that each young man is matched with a young woman he is willing to marry. If yes, draw it. If not, explain why?

0. 658, 659

Since $|V_1| = |V_2|$ it might be a matching **3**
However this is impossible because there are 2 women
X & U who are adjacent only to B. So we can't assign a woman
to each man. who is willing to marry with.

8. *(5 points)* Using Kruskal's algorithm, find the minimum spanning tree for the weighted graph shown below. What's the total weight of the minimum spanning tree?

$11 + 19 + 5 + 3 + 8 + 9 = 55$ **(2)**

each edge 0.5

3

11
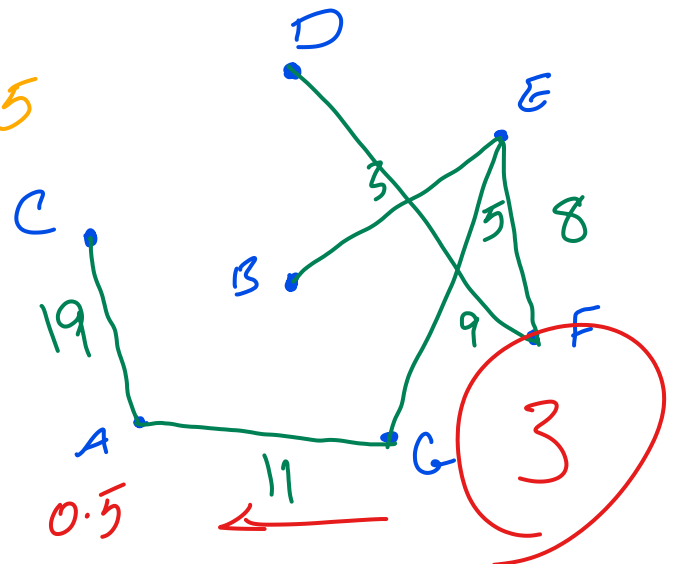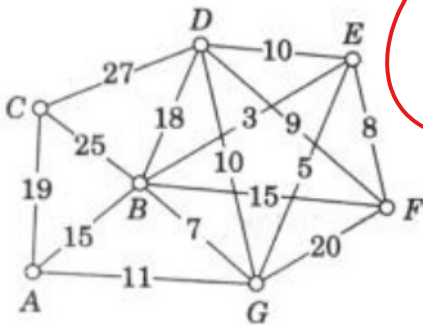
9. *(2 points each)* Define:

Cut edge : is an edge of graph whose deletion increases the # of connected components in graph. **(2)**

Internal Vertex : A vertex of tree which has children. 2

Total: 65 Points

Good Luck! *:)*