. **(5 Points)** Consider three classes, each consisting of 10 students. From this group of 30 students, a group of 3 students is to be chosen.
i). How many choices are possible?
ii). How many choices are there in which all 3 students are in the same class?
iii). How many choices are there in which 2 of the 3 students are in the same class?

a) $\binom{30}{3}$

b) $3\binom{10}{3}$

c) $3\binom{10}{2}2\binom{10}{1}$

2. (9 Points; as follow) The chess clubs of two schools consist of, respectively, 8 and 9 players. Four members from each club are randomly chosen to participate in a contest between the two schools. The chosen players from one team are then randomly paired with those from the other team, and each pairing plays a game of chess. Suppose that Rebecca and her sister Elise are on the chess clubs at different schools. What is the probability that
i). (2 Points) Rebecca and Elise will be paired?
ii). (3 Points) Rebecca and Elise will be chosen to represent their schools but will not play each other?
iii). (4 Points) either Rebecca or Elise will be chosen to represent her school?

a) Total choices = $\binom{8}{4}\binom{9}{4}$

$P(R\&E) = \dfrac{3!\binom{7}{3}\binom{8}{3}}{4!\binom{8}{4}\binom{9}{4}} = \dfrac{1}{18}$

b) P(R&E but not paired) = P(R&E chosen) − P(R&E chosen & paired)

$= \dfrac{\binom{7}{3}\binom{8}{3}}{\binom{8}{4}\binom{9}{4}} - 1/18 = \dfrac{2}{9} - \dfrac{1}{18} = \dfrac{1}{6}$

c) P(either E or R are chosen) = 1 − P(neither chosen)

$= 1 - \dfrac{\binom{7}{4}\binom{8}{4}}{\binom{8}{4}\binom{9}{4}} = 1 - \dfrac{5}{18}$

$= 13/18$

3. **(7 Points; as follow)** On a rainy days, Joe is late to work with probability 0.3; on non rainy days, he is late with probability 0.1. With probability 0.7 it will rain tomorrow.

i). **(3 Points)** Find the probability Joe is early tomorrow.

ii). **(4 Points)** Given that Joe was early, what is the conditional probability that it rained?

$P(\text{rain}) = 0.7$

$P(\text{late}|\text{rain}) = 0.3$

$P(\text{late}|\text{non rain}) = 0.1$

i) To find $P(\text{early}) = 1 - P(\text{late})$

$= 1 - [0.7 \times 0.3 + 0.3 \times 0.1]$

$= 1 - [0.28 + 0.03]$

$= 1 - [0.31]$

$= 0.69$

$P(E) = P(E|R)P(R) + P(E|R^c)P(R^c)$

$= 0.7(0.7) + 0.9(0.3)$

$= 0.76$

ii) $P(\text{rain}|\text{early}) = \dfrac{P(\text{early}|\text{rain})P(\text{rain})}{P(\text{early})}$

$= \dfrac{0.7(0.7)}{0.69}$

$P(\text{early}|\text{rain}) = 1 - 0.3 = 0.7$

$= \dfrac{0.49}{0.76} = 0.71$

4. **(6 Points)** There are 3 coins in a box. One is two-headed coin, another is a fair coin, and the third is biased coin that comes up heads 75 percent of the time. When one of the 3 coins is selected at random and flipped, it shows heads. What is the probability that it was the two-headed coin?

$P(\text{head}|1^{st}) = 1$

$P(\text{head}|2^{nd}) = 0.5$

$P(\text{head}|3^{rd}) = 0.75$

$P(1^{st}\text{ coin}|\text{heads}) = \dfrac{P(\text{head}|1^{st})P(1^{st})}{P(\text{head})} = \dfrac{1 \times 1/3}{1 \times 1/3 + 0.5 \times 1/3 + \frac{3}{4} \times \frac{1}{3}}$

$= \dfrac{1/3}{1/3 + 1/6 + 1/4} = \dfrac{1/3}{8+4+6/24}$

$= \dfrac{1/3}{18/24} = 8/18 = 4/9$

Final

. **(4 Points)** A school class of 120 students is driven in 3 buses to a symphonic performance. There are 36 students in one of the buses, 40 in another, and 44 in the third bus. When the buses arrive, one of the 120 students is randomly chosen. Let X denote the number of students on the bus of that randomly chosen student. Write the cumulative distribution function of X.

$P(X=36) = 36/120$

$P(X=40) = 40/120$

$P(X=44) = 44/120$

$$F(X) = \begin{cases} 36/120 & 1 \le X \le 36 \\ \cancel{76}/120 & 37 \le X \le 76 \\ 120/120 & 77 \le X \le 120 \end{cases}$$

6. **(6 Points; 3 points each part)** A box contains 5 red and 5 blue marbles. Two marbles are withdrawn randomly. If they are the same color, then you win \$1.10; if they are different colors, then you lose \$1.00. Calculate:

i). the expected value of the amount you win.

ii). the variance of the amount you win.

X = amount you win.

$P(X=+1.1) = \dfrac{\binom{5}{2}+\binom{5}{2}}{\binom{10}{2}} = 4/9$

$P(X=-1) = \binom{5}{1}\binom{5}{1} / \binom{10}{2} = 5/9$

$E[X] =$

$E[X] = \sum x\,p(x)$

$= (+1.1)\left[\dfrac{\binom{5}{2}(2)}{\binom{10}{2}}\right] - 1\left(\dfrac{\binom{5}{1}\binom{5}{1}}{\binom{10}{2}}\right)$

$= (+1.1)(4/9) - 1(5/9) = \dfrac{-0.6}{9}$

$Var(X) = E[X^2] - (E[X])^2$

$= (+1.1)^2\dfrac{4}{9} + (-1)^2\dfrac{5}{9}$

$- \left(\dfrac{-0.6}{9}\right)^2$

$= \dfrac{4.84+5}{9} - \dfrac{0.36}{81}$

$= 1.089$

7. (4 Points) In response to an attack of 10 missiles, 500 antiballistic missiles are launched. The missile targets of the antiballistic missiles are independent, and each antiballstic missile is equally likely to go towards any of the target missiles. If each antiballistic missile independently hits its target with probability 0.1. Approximate the probability that all missiles are hit.

$X \sim \text{Poisson}(500 \times 0.01)$

X = # of antiballistic missile that have hit our fixed missile.

$P(X \geq 1) = 1 - P(X=0) = 1 - e^{-5}$.

As, all events are independent, prob that all missiles have been hit is $= (1-e^{-5})^{10} = 0.93462$.

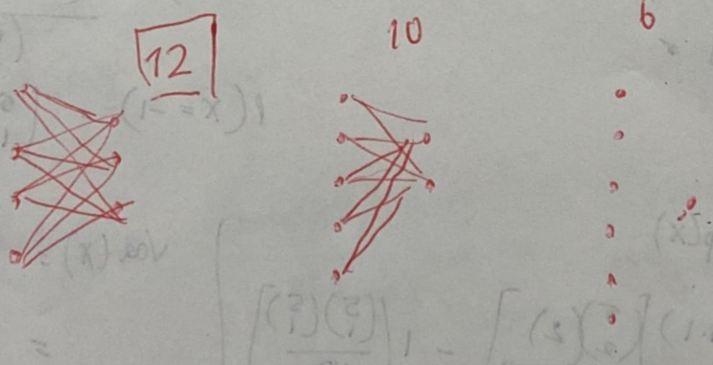
$X \sim \text{Poisson}(500 \times 0.01)$

Prob that same antibalistic missile choose fixed missile as target $= 0.1$ (since randomly)

Also, ~~prob~~ that it has been hit by that antiballistic missile is 0.1

$\therefore$ prob that some antiballistic missile will hit ours is 0.01

8. (4 Points) An evil host has 7 guests over for dinner, and has two rooms. He wants to put the guests into the two rooms such that no two people in any room know each other. What is the maximum possible number of relationships among the 7 guests if it is possible for the host to arrange them in this fashion.



. **(4 Points)** A sequence $d_1, d_2, \cdots, d_n$ is called graphic if it is the degree sequence of a simple graph. Determine whether each of the following sequences is graphic? (Give a reason for your answer). For those that are, draw a graph having the given degree sequence.

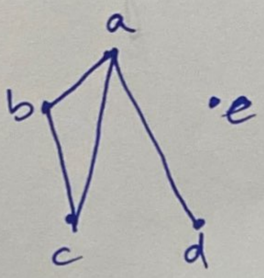
i). 4, 4, 3, 3, 3

$4+4+3+3+3 = 17$

$\sum deg(v) = 2e$

as $\sum deg(v)$ is not even, ∴ not graphic.

ii). 3, 2, 2, 1, 0



Sequence is graphic.

10. (3 Points) How many leaves does a full 3-ary tree with 100 vertices have?

$\ell = [(m-1)n+1]/m$

$= ((3-1)100+1)/3$

$= (200+1)/3 = 201/3 = 67$ leaves.

11. (3 Points) Define:

i). "Cut Vertices"

A cut vertex is a vertex v such that G−v has more components than G.

ii). "Hamiltonian Path"

A path where you can go from one vertex ~~to~~ and traverse to all vertices .

iii). "Full m-ary tree"

A tree where each internal node has exactly m children

Good Luck! :)