CS 198:206

Exam III **Name & Section:--------------------**

- The approximate time required to complete this exam is 60 minutes.
- **For full grade, show and write all your work, step by step. No work/Just final answer has no point.**
- To avoiding any missing or mistake, please read the question **carefully and completely**.
- You get 2 points deduction if you submit a paper without name.
- In case if you need more space, you might use the back side of your paper. I **DO NOT ACCEPT** any other sheet attached to the exam paper.
- **Do NOT USE** calculator or any electronic device.

* In each Distribution Problem, determine the type of the distribution and write the X's distribution.

X	*p(x)*	*Values of X*	*E(x)*	*V(x)*
	$\frac{1}{b-a+1}$	$a \le x \le b$	$\frac{b+a}{2}$	$\frac{(b-a+2)(b-a)}{12}$
	$\binom{n}{x} p^x (1-p)^{n-x}$	$x = 0,1,\ldots,n$	np	$np(1-p)$
	$\frac{e^{-\lambda}\lambda^x}{x!}$	$x = 0,1,2,\ldots$	λ	λ
	$(1-p)^{x-1}p$	$x = 1,2,3,\ldots$	$\frac{1}{p}$	$\frac{1-p}{p^2}$
	$\binom{x-1}{k-1}(1-p)^{x-k}p^k$	$x = k, k+1,\ldots$	$\frac{k}{p}$	$\frac{k(1-p)}{p^2}$
	$\frac{\binom{M}{x}\binom{N-M}{n-x}}{\binom{N}{n}}$	$\max(0, M+n-N) \le x \le \min(M,n)$	$n \cdot \frac{M}{N}$	$\frac{nM(N-M)(N-n)}{N^2(N-1)}$

Q 1. (12 points; 5, 5, and 2 points respectively) The occurrence of a tornado in a county can be modeled as a Poisson process. Twenty tornados have touched down in a county within the last twenty years. If there is at least one occurrences of tornadoes in a year, that year is classified as a "tornado year".

i). What is the probability that next year will be a tornado year?

$\lambda = 1$ X = # of torando in a year $X \sim \text{Poisson}(1)$

$P(X \geq 1) = 1 - p(X=0)$

$= 1 - \frac{e^{-1} 1^{0}}{0!} = \boxed{1 - e^{-1}} = 0.63$

ii). What's the probability that there will be two "tornado years" within the next 3 years?

y = # of torando year $y \sim \text{Binomial}(3, 0.63)$

$p(y=2) = \binom{3}{2}(0.63)^2(1-0.63)$

2

2

iii). On the average, over the 10 years, how many "tornado years" are expected to occur?

$E(X) = n \cdot p = 10(0.63) = 6.3$ years

2

r – success

is unknown

Q 2. (6 Points) The discrete random variable X represents the number of times a biased coin must be tossed until 3 "heads" have been obtained, with Var (X) =36. Find P (X = 9).

$\Rightarrow$ we know $X \sim NB(r, p)$ where p can

$$\mathrm{Var}(X) = \frac{r(1-p)}{p^2} \Rightarrow 36 = \frac{3(1-p)}{p^2}$$
$$\Rightarrow 36p^2 = 3(1-p)$$
$$\Rightarrow 12p^2 = 1-p$$
$$\Rightarrow 12p^2 + p - 1 = 0$$
$$\Rightarrow (4p-1)(3p+1) = 0$$
$$\Rightarrow p = \tfrac{1}{4} \quad \left(p \neq -\tfrac{1}{3}\right)$$

3

$\Rightarrow X \sim NB(3, 0.25)$ 1

$$P(X=9) = \binom{8}{2}(0.25)^3(0.75)^6$$

2

Q 3. (4 Points; 2 points each) True or false? Give a reasoning for each part.

- A graph with multiple components can contain a Eulerian cycle.

1

False. An Eulerian circuit requires crossing every edge exactly once, but if there are no edges between the components of the graph, there is no way to reach every edge.

1

- A graph with n ≥ 4 vertices that contains a triangle cannot be Hamiltonian.

1

False. Consider the complete graph K4. This graph contains many triangles but, as with all complete graphs, is Hamiltonian.

1

Q. 4. (**6 Points; 2 points each**) Define the bipartite graph.

A graph G= (V, E) is called a bipartite graph if its vertices V can be partitioned into two subsets V_1 and V_2 such that each edge of G connects a vertex of V_1 to a vertex V_2. In addition, neither of vertices in V_1 (or V_2) are adjacent.

2

i). For which integers m and n is $K_{m,n}$ Eulerian? Why?

$K_{m,n}$ is Eulerian if and only if m, n are both non-zero, even integers. This implies that degree of all vertices are even, and therefore graph has Euler circuit.

1

1

ii). For what integers m and n is $K_{m,n}$ Hamiltonian? Why?

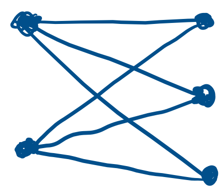
A graph $K_{m,n}$ is Hamiltonian if and only if m = n, with m, n ≥ 2. We note here that for m=n = 1, $K_{m,n}$ is a tree and is therefore not Hamiltonian.

0.5 0.5

The complete bipartite graph $K_{m,n}$ is not Hamiltonian when m ≠ n as it would not have Hamiltonian circuit (Hamiltonian circuit is a closed walk that visits each vertex exactly once).

Let's consider the case m<n (or similarly m>n),

so the graph doesn't have Hamiltonian circuit:

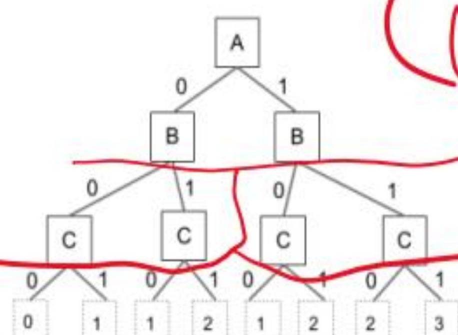


1

Q 5. (6 Points) Let's imagine that our data has 3 binary features A, B, C, which take values 0/1, and we want to learn a function which counts the number of features which have value 1. Draw the best decision tree which represents this function. How many leaf nodes does it have?

1

1



1

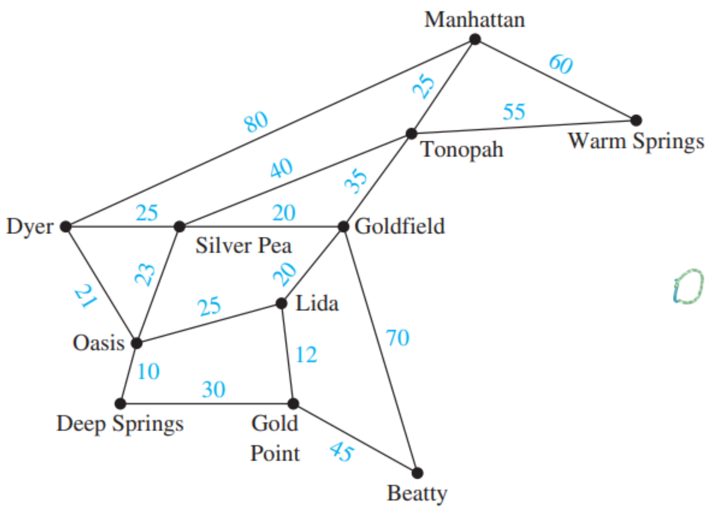
2

We can represent the function with a decision tree containing $\boxed{\text{8 nodes}}$.

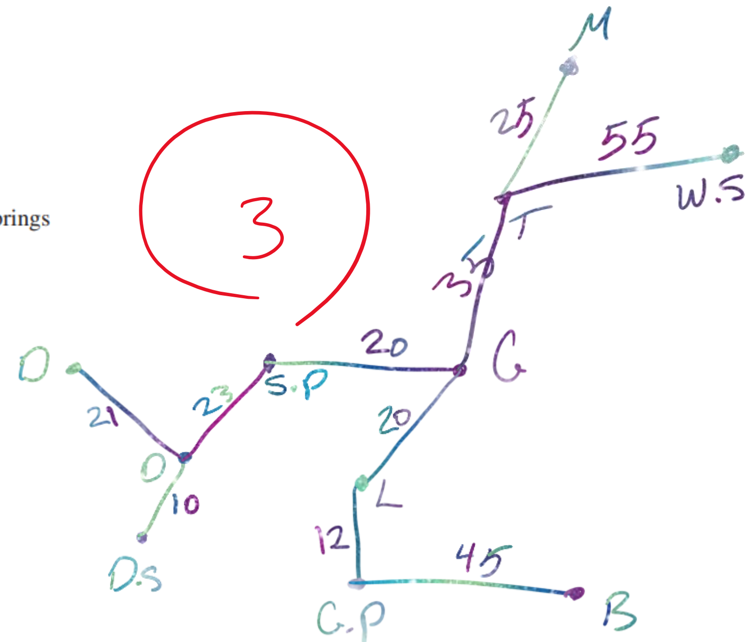
1

Q 6. (6 Points) The roads represented by this graph are all unpaved. The lengths of the roads between pairs of towns are represented by edge weights. Which roads should be paved so that there is a path of paved roads between each pair of towns so that a minimum road length is paved?

Draw the graph and explain your work step by step.



3



We can use minimum spanning tree using either Prime or Kruskal algorithm for this problem.

Using prime algorithm here: We

1. start from the edge with minimum value: Deep Springs–Oasis,
2. then we compare the adjacent edges to that edge and draw the one with minimum value (if that edge causes a circuit, we skip it and move on to other choice(s) as we cannot have circuit in tree0
3. we repeat step 2 till we have all the cities included in our graph, then we are done!

2.5

Edges that we get from above algorithms (in order) are:

Deep Springs–Oasis, Oasis–Dyer, Oasis–Silver Peak, Silver Peak–Goldfield, Goldfield -Lida, Lida- Gold Point, Gold Point–Beatty, Goldfield–Tonopah, Tonopah–Manhattan, Tonopah–Warm Springs

Therefore, total minimum road length is paved= 10+ 21+ 23+ 2 (20)+ 12+ 45+ 35+ 25+ 55

0.5

Total: 40 points

Good Luck! 😊