Explanation:

1. True

Reason: The dominant term in f(n) is log n and the dominant term in $g(n) = n^2$

Clearly, the dominant term in g(n) grows faster than that of f(n) for large values of n. So, f(n) is asymptotically upper bounded by g(n).

Thus, f(n) = O (g(n))

2. False

Reason: Assume base is 2

$f(n) = 2^{\log n} = n$

$g(n) = (\log n)^2$

No matter the value of n, $(\log n)^2$ will always be less than n.

So, g(n) asymptotically upper bounded by f(n)

Thus, g(n) = O (f(n))

Therefore, it is **false** to say f(n) = O(g(n))

3. False

Reason:

$f(n) = n^{\log n}$

$g(n) = (\log n)^n$

f(n) : When n approaches infinity, base grows faster than exponent

g(n) : When n approaches infinity, exponent grows faster than base

Exponent growing faster than base makes g(n) grow faster overall.

Thus, g(n) = O(f(n))

Therefore, it is **false** to say f(n) = O(g(n))

4. True

Reason:

$f(n) = 2^n$

$g(n) = \mathrm{sqrt}(2)^{(n\char`^2)}$

Both have positive constant base. But g(n) exponent is an order of magnitude higher than exponent of f(n).

This is because $n^2 > n$.

So, f(n) is upper bounded by g(n)

Thus, it is true that f(n) = O(g(n))

5. True

Reason:

$f(n) = 3^n$

$g(n) = \mathrm{sqrt}(2)^{2n}$

Both have positive constant base and both have exponent which are of same order of magnitude.

At infinitely large values of n, we can consider n to be asymptotically equal to 2n.

Thus, here both f(n) and g(n) are asympotically equal.

We can say f(n) upper bounds g(n) and also g(n) upper bounds f(n).

Thus, f(n) = O(g(n)) is also correct.

6. True
Reason:
f(n) = O(g(n)) means g(n) grows faster than f(n) when n tends to infinity
g(n) = O(h(n)) means h(n) grows faster than g(n) when n tends to infinity
Thus, transitively, h(n) grows faster than f(n)
Hence, f(n) = O(h(n)) is true.

7. False
Reason:
f(n) is upper bounded by g(n)
g(n) is lower bounded by h(n)
That doesn't tell whether h(n) is upper bounds f(n) or lower bounds f(n)
So, we can't tell the growth difference between f(n) and h(n) as either case may happen.
Example:
g(n) = n^5
f(n) = n^3
h(n) = n
In this case, statement is false
but if g(n) = n^5
f(n) = n^3
h(n) = n^3
Statement becomes true.
So, we can't say it is always true.
Thus, answer is False.

8. True
Reason:
f(n) + g(n) = O(g(n) + g(n)) because g(n) >= f(n)
So, f(n) + g(n) = O(g(n))
And g(n) = O(h(n))
Thus, transitively, f(n) + g(n) = O(h(n))

9. **(b) 2^ $^{n/2}$**
For extremely large numbers such as n tends to infinity, we can ignore the multiplication or division by 2.
Essentially for n tends to infinity, n/2 is equal to n.
So, we can say that $(n/2)^{(n/2)}$ = n^n = (n/2)^^{2n} for n tends to infinity.
Notice that for all these functions, we have approximately n * n * n * n ...... (n terms)
while n! = 1 * 2 * 3 * ..... n (n terms)
Clearly, having each term n makes the function grow faster.
So, option (a), (c), (d) grow faster than n! Thus, they can't lower bound n!
But for (b) 2 ^ (n/2), we can say:
2^$^{(n/2)}$ = 2 * 2 * 2 * .... (n times because when n tends to infinity, n = n/2)
n! = 1 * 2 * 3 * 4 * .... (n times)
So, n! upper bounds 2^$^{(n/2)}$
Thus, only correct option is (b)

10. **(a)** $(n/2)^{n/2}$, **(c)** n^n
Reason:
We have to find functions that grow faster than n!
(a)
$(n/2)^{(n/2)}$ is asymptotically equal to n^n when tends to infinity.
n^n = n * n * n * ..... (n times)
n! = 1 * 2 * 3 .... (n times)
Having each term as n, makes $(n/2)^{(n/2)}$ grow faster than n!
(b)
$2^{(n/2)}$ is asymptotically equal to 2^n when n tends to infinity.
2^n = 2 * 2 * 2 ... (n times)
n! = 1 * 2 * 3 * .... (n times)
So, $2^{(n/2)}$ doesn't grow faster than n!
(c) n^n grows faster than n! as seen in part (a)
(d) n^2 is much smaller than n! in growth.
So, only (a) and (c) upper bound on the value of n!

11. (a) log f(n) = O(log g(n))
Reason: log is increasing function.
So, if x <= y, then log x <= log y.
Thus, f(n) = O(g(n)) => log (f(n) = O(log g(n))

12. (a) $2^{f(n)} = O(2^{g(n)})$
Reason: exponential function is increasing function.
So, if x <= y, then $2^x <= 2^y$
Thus, if f(n) = O(g(n)) => $2^{f(n)} = O(2^{g(n)})$