CS 198:206; Introduction to Discrete Structures II

Exam II **Name:**_____

- The approximate time required to complete this quiz is 75 minutes.
- **For full grade, show and write all of your work, step by step. No work/ Just final answer, No credit.**
- You will get **2 points deduction** if you submit a paper without name.
- In case if you need more space, you might use the back side of the your paper. **I DO NOT ACCEPT** any other sheet attached to the exam paper.
- **Do NOT USE** graphing calculator.
- To avoiding any missing or mistake, please read the question **carefully and completely**.

1. ***(3 points)*** How many four digit numbers have no repeat digits, do not contain zero, and have a sum of digits equal to 28?

Only the permutations of 9-8-7-4 & 9-8-6-5 have 4 digits with sum of 28. (0.5) (0.5)

$4! + 4! = 48$ (1)

such 4-digit numbers.

2. ***(6 points; 2 points each)*** If there are ten players on a basketball team, find the number of choices the coach has in selecting each of the following:

(a) two players for guard positions and two for forward positions?

$${}_{10}C_2 \cdot {}_8C_2$$ (2)

(b) two groups of four?

$$\frac{{}_{10}C_4 \cdot {}_6C_4}{2!}$$ (2)

(c) a group of 3 or more players?

$$2^{10} - ({}_{10}C_0 + {}_{10}C_1 + {}_{10}C_2) = 968$$ (2)

3. ***(3 points)*** Three men and three women are waiting to be interviewed for jobs. If they are all selected in random order, find the probability of no man will be interviewed until at least two women have been interviewed?

$P(WW) = \frac{3}{6} \times \frac{2}{5} = \frac{1}{5}$

2 women in a row are chosen first = No men will be interviewed 1st & 2nd

first picked be a W

second picked be a W

4. ***(10 points; 4 & 6 respectively)*** Items are inspected for flaws by two quality inspectors. If a flaw is present, it will be detected by the first inspector with a probability of 0.9, and by the second inspector with a probability of 0.7. Assume the inspectors function independently, and assume that if an item does not have a flaw, then neither inspector will report a flaw. Suppose 5 percent of the items are flawed.

(a) If an item has a flaw, what is the probability that it will be found by at least one of the inspectors?

$P(A|F) = 0.9 \quad P(B|F) = 0.7$

P(both will find the flaw) $= P(A \cap B|F)$

$= P(A|F) \cdot P(B|F)$

$= 0.9(0.7)$

$= 0.63$

at least by one: $P(A \cup B|F) = P(A|F) + P(B|F) - P(A \cap B|F)$

$= 0.9 + 0.7 - 0.63$

(b) If an item is passed by the first inspector, what is the probability that it actually has a flaw?

$P(F) = 0.05 \quad P(A^c|F^c) = 1$ Using the Bayes' Theorem:

$$P(F|A^c) = \frac{P(F \cap A^c)}{P(A^c)} = \frac{P(A^c|F) \cdot P(F)}{P(A^c|F) \cdot P(F) + P(A^c|F^c) \cdot P(F^c)}$$

$$= \frac{(0.1)(0.05)}{(0.1)(0.05) + 1(0.95)} \approx 0.0052$$

5. ***(7 points)*** Samsung, Panasonic, and LG are producing Single Board Computers (SBCs) for hobbyists. Samsung's SBCs take up 40% of the market, Panasonic's SBCs take up 25% of the market, and LG's SBCs take up the rest. 1% of all Samsung and Panasonic's SBCs are defective, whereas 2% of all LG SBCs are defective. If the SBC you bought was defective, what is the probability that it is a Panasonic SBC?

Let: S to represent Samsung

P " " Panasonic

L " " LG

D " " Defective computer

$P(S) = 0.4 \quad P(P) = 0.25$

$\Rightarrow P(L) = 0.35 \quad P(D|L) = 0.02$

$P(D|S) = P(D|P) = 0.01$

$\Rightarrow P(P|D) = ?$ Where $P(P|D) = \frac{P(D|P)P(P)}{P(D|L)P(L) + P(D|S)P(S) + P(D|P)P(P)}$

$$= \frac{0.01(0.25)}{0.02(0.35) + 0.01(0.4) + 0.01(0.25)}$$

6. ***(10 points)*** In a lottery a four-digit number is chosen at random from the range 0000 – 9999. A lottery ticket costs \$2. You win \$50 if your ticket matches the last two digits but not the last three, \$500 if your ticket matches the last three digits but not all four, and \$5,000 if your ticket matches all four digits. What is the expected payoff on a lottery ticket?

Let X = Payoff (\$) on a lottery ticket which takes on the values 0, 50, 500 and 5000

$$P(X=0) = \frac{1\times 9\times 10\times 10 + 9\times 10\times 10\times 10}{10000} = \frac{99}{100}$$

$$P(X=50) = \frac{1\times 1\times 9\times 10}{10000} = \frac{9}{1000} \qquad P(X=500) = \frac{1\times 1\times 1\times 9}{10000} = \frac{9}{10000}$$

$$P(X=5000) = \frac{1}{10000}$$

$$\Rightarrow E(X) = \sum x_i P(X=x_i) = 0\times\left(\frac{99}{100}\right) + 50\left(\frac{9}{1000}\right) + 500\left(\frac{9}{10000}\right) + 5000\left(\frac{1}{10000}\right)$$

$$= 1.4$$

Extra Points (5 points) Fifteen dots are evenly spaced on the circumference of a circle. How many combinations of three dots can we pick from these 15 that do not form an equilateral triangle?

$\frac{15\times 14\times 13}{3!} = 455$ total triangle

order

equilateral triangle → 3 points equally spaced across the whole circle:

$\{1,6,11\}, \{2,7,12\}, \{3,8,13\}, \{4,9,14\}, \{5,10,15\}$

$\Rightarrow 455 - 5 = 450$ are Not equilateral

Total: 40 Points

Good Luck! *:)*