CS 198:206

Exam II **Name & Section:--------------------**

- The approximate time required to complete this exam is 60 minutes.
- **For full grade, show and write all your work, step by step. No work/Just final answer has no point.**
- To avoiding any missing or mistake, please read the question **carefully and completely**.
- You get 2 points deduction if you submit a paper without name.
- In case if you need more space, you might use the back side of your paper. I **DO NOT ACCEPT** any other sheet attached to the exam paper.
- **Do NOT USE** calculator or any electronic device.

Q 1. (5 Points) Arne, Bobbette, Chuck, Deirdre, Ed, and Fran have reserved six seats in a row at the theater, starting at an aisle seat. In how many ways can they arrange themselves if the men and women are to alternate seats and a man must sit on the aisle? Arne, Chuck, and Ed are men, and the others are women.

aisel $\frac{3}{M} \times \frac{3}{W} \times \frac{2}{M} \times \frac{2}{W} \times \frac{1}{M} \times \frac{1}{W}$

5

Q 2. (8 Points; 4 points each) In a certain four engine vintage aircraft, now quite unreliable, each engine has a 10% chance of failure on any flight, as long as it is carrying its one-fourth share of the load. But if one engine fails, then the chance of failure increases to 20% for each of the other three engines. And if a second engine fails, each of the remaining two has a 30% chance of failure. Assuming that no two engines ever fail simultaneously, and that the aircraft can continue flying with as few as two operating engines, find each probability for a given flight of this aircraft.

a) no engine failures.

P(an engine wont fail | $\frac{1}{4}$ share load) = 0.9

P(no engine failures) = 0.9(0.9)(0.9)(0.9)

4

b) exactly two engine failures (any two of four engines)

P(an engine wont fail | ½ share load) = 1 − 0.3

P(exactly 2 engine failures) = $\binom{4}{2}(0.1)(0.2)(0.8)(0.8)$

4

Q 3. (5 Points) The probability that a visit to a particular car dealer results in neither point second-hand car nor a Japanese car is 55%. Of those coming to the dealer, 25% buy second-hand car and 30% buy a Japanese car what's the probability that a visit leads to buying a second-hand Japanese car?

Let A be the event of buying a second hand car and B of buying a japanese car. In the problem it is given that $P(\bar{A} \cap \bar{B}) = 0.55$, $P(A) = 0.25$ and $P(B) = 0.3$. We are asked to calculate $P(A \cap B)$. We know that $P(A \cup B) = P(A) + P(B) - P(A \cap B)$. Moreover, $P(\bar{A} \cap \bar{B}) = P(\overline{A \cup B}) = 1 - P(A \cup B)$. 2
Hence, $P(A \cup B) = 0.45$. Thus, $P(A \cap B) = 0.25 + 0.3 - 0.45 = 0.10$.

1 1 1

Q 4. (6 Points) Three urns are there containing white and black balls; first urn has 3 white and 2 black balls; second urn has 2 white and 3 black balls and third urn has 4 white and 1 black balls. Without any biasing one urn is chosen from that one ball is chosen randomly which was white. What is probability that it came from the third urn?

	white	Black
U_1	3	2
U_2	2	3
U_3	4	1

A = white ball is drawn

$$P(U_3|A) = \frac{P(U_3 \cap A)}{P(A)} = \frac{P(U_3)\,P(A|U_3)}{P(U_1)P(A|U_1) + P(U_2)P(A|U_2) + P(U_3)P(A|U_3)}$$

2

$$= \frac{\frac{1}{3}\left(\frac{4}{5}\right)}{\frac{1}{3}\left(\frac{3}{5}\right) + \frac{1}{3}\left(\frac{2}{5}\right) + \frac{1}{3}\left(\frac{4}{5}\right)}$$

1 1 1 1

Q 5. (6 Points; 4 and 2 points respectively) college foundation raises funds by selling raffle tickets for a new car worth $36,000. If 600 tickets are sold for $120 each, determine each of the following.

(a) The expected net winnings of a person buying one of the tickets

$$E(x) = (36000 - 120)\frac{1}{600} + (-120)\left(\frac{599}{600}\right) = -\$60$$

2 2

2

(b) The total profit for the foundation, assuming that the car was donated.

Since car is free → total profit: 600(120) = $72000

Q 6. (10 Points) The probability distribution of a discrete random variable X is given by

$$P(X=x)=\begin{cases} k(2-x) & x=0,1,2 \\ \frac{1}{4} & x=3 \\ 0 & Otherwise \end{cases}$$

a). Find E(X^2) b) Determine Var (3 − X)

GENERATING A TABLE FOR PROBABILITIES

x	0	1	2	3
$P(X=x)$	$2k$	k	0	$\frac{1}{4}$

1

$2k+k+\frac{1}{4}=1$

$3k=\frac{3}{4}$

$k=\frac{1}{4}$

1

$\Longrightarrow$

$E(X^2)=\sum x^2 P(X=x)$

$=(0^2\times 2k)+(1^2\times k)+(2^2\times 0)+(3^2\times\frac{1}{4})$

$=0+k+0+\frac{9}{4}$

$=\frac{5}{2}$

1

1

it's better to substitue k from beginning

2

$Var(3-X)=Var(-X+3)=(-1)^2\,Var(X)$

$=Var(X)$

$=E(X^2)-(E(X))^2$

$=2.5-1^2=1.5$

1

1

where

$E(X)=\sum x P(X=x)$

$=(0\times 2k)+(1\times k)+(2\times 0)+(3\times\frac{1}{4})$

$=0+k+0+\frac{3}{4}$

$=1$

2

Total: 40 points

Good Luck! 😊