CS 198:206

Final Exam **Name:————————**

- The approximate time required to complete this exam is 150 + 20 (to submit the exam) minutes.
- Please submit your exam answer as a **Single PDF file**. You can use any website or app to convert your pictures to a **single PDF file** as well as either of the following links:

  https://smallpdf.com/jpg-to-pdf

  https://tools.pdfforge.org/images-to-pdf
- You will get **3 points deduction** for any other submission aside from uploading file on Canvas. **Do Not** submit via email.
- You will get **3 points deduction** if you do not submit a SINGLE PDF file.
- You will get **2 points deduction** if you submit a paper without name.
- **For full grade, show and write all your work, a step by step. No work/ Just final answer, No credit.**
- You're not allowed to use second monitor or device during the exam.
- Do **NOT** use any electronic devices and calculator.
- To avoiding any missing or mistake, please read each question completely and carefully.
- To be able to protect the exam, your camera has to be on during the quiz/exam. It has to be face to you and your work place, with enough light around.

* In each Distribution Problem, determine the type of the distribution and write the X's distribution.

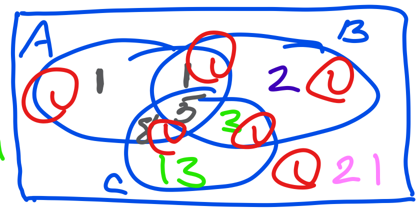
X	p(x)	Values of X	E(x)	V(x)
Discrete uniform	$\frac{1}{b-a+1}$	$a \le x \le b$	$\frac{b+a}{2}$	$\frac{(b-a+2)(b-a)}{12}$
Binomial	$\binom{n}{x} p^x (1-p)^{n-x}$	$x = 0,1,\ldots,n$	np	$np(1-p)$
Poisson	$\frac{e^{-\lambda}\lambda^x}{x!}$	$x = 0,1,2,\ldots$	λ	λ
Geometric	$(1-p)^{x-1}p$	$x = 1,2,3,\ldots$	$\frac{1}{p}$	$\frac{1-p}{p^2}$
Negative Binomial	$\binom{x-1}{k-1}(1-p)^{x-k}p^k$	$x = k, k+1, \ldots$	$\frac{k}{p}$	$\frac{k(1-p)}{p^2}$
Hyper - geometric	$\frac{\binom{M}{x}\binom{N-M}{n-x}}{\binom{N}{n}}$	$\max(0, M+n-N) \le x \le \min(M,n)$	$n * \frac{M}{N}$	$\frac{nM(N-M)(N-n)}{N^2(N-1)}$

1. ***(10 points)*** Draw a Venn diagram and use the given information to fill the number of elements in each region. Show all your calculations.

$n(A) = 15$, $n(A \cap B \cap C) = 5$, $n(A \cap C) = 13$, $n(A \cap B') = 9$ $= n(A-B)$, $n(B \cap C) = 8$, $n(A' \cap B' \cap C') = 21$, $n(B \cap C') = 3$ $= n(B-C)$, $n(B \cup C) = 32$

$$n(A \cap B) = n(A) - n(A-B) = 15 - 9 = 6$$

$$n(B \cup C) = n(B) + n(C) - n(B \cap C) \Rightarrow n(C) = 32 + 8 - 11 = 29$$



2. ***(5 points)*** How many even 4-digit numbers can be created using numbers: 0, 3, 4, 6, 7, 8, 9? (Repetition is allowed)

7 numbers

$$6 \times 7 \times 7 \times 4$$

3. ***(4 points)*** How many cards must be drawn (without replacement) from a standard deck of 52 to guarantee TWO of the cards will be of the same suit?

4 suits in 52 card deck $\Rightarrow 4(2-1)+1 = \boxed{5}$ cards

4. ***(8 points as follow)*** On a rainy days, Joe is late to work with probability 0.3; on non-rainy days, he is late with probability 0.1. With probability 0.7 it will rain tomorrow.

i). (4 Points) Find the probability Joe is early tomorrow.

ii). (4 Points)Given that Joe was early, what is the conditional probability that it rained?

$P(\text{rain}) = 0.7$ $\quad P(\text{late}|\text{rain}) = 0.3$ $\quad P(\text{Late}|\text{norain}) = 0.1$ $\quad P(\text{early}|\text{rain}) = 1 - 0.3 = 0.7$

i: $P(E) = P(E|R) \cdot P(R) + P(E|R^c)P(R^c) = 0.7(0.7) + 0.9(0.3)$

ii: $P(R|E) = \dfrac{P(E|R)P(R)}{P(E)} = \dfrac{0.7\,(0.7)}{0.9}$

5. ***(4 points)*** if one number is chosen randomly from the integers 1 through 10, find the probability of getting a number that is odd and prime.

$$P(\text{odd} \cap \text{prime}) = P(\text{odd}) \cdot P(\text{prime}|\text{odd}) = \frac{5}{10} \cdot \frac{3}{5} = \boxed{\frac{3}{10}}$$

6. ***(8 points)*** There are 3 coins in a box. One is two-headed coin, another is a fair coin, andthe third is biased coin that comes up heads 75 percent of the time. When one of the 3 coins isselected at random and flipped, it shows heads. What is the probability that it was the two-headedcoin?

$P(h|1^{st}) = 1$ $\quad P(h|2^{nd}) = 0.5$ $\quad P(h|3^{rd}) = 0.75$

$$P(1^{st}\text{ coin}|\text{heads}) = \frac{P(h|1^{st})\,P(1^{st})}{P(h)}$$

$$= \frac{1 \times \frac{1}{3}}{1 \times \frac{1}{3} + 0.5 \times \frac{1}{3} + 0.75\left(\frac{1}{3}\right)} = \frac{\frac{1}{3}}{\frac{1}{3} + \frac{1}{6} + \frac{1}{4}}$$

7. ***(12 points as follow)*** The occurrence of a tornado in a county can be modeled as a Poisson process. Twenty tornados have touched down in a county within the last twenty years. If there is at least one occurrences of tornadoes in a year, that year is classified as a "tornado year". $X \sim \text{Poisson}(1)$
i). (5 Points) What is the probability that next year will be a tornado year?
ii). (5 Points) What's the probability that there will be two "tornado years" within the next 3 years?
iii). (2 Points) On the average, over the 10 years, how many "tornado years" are expected to occur?

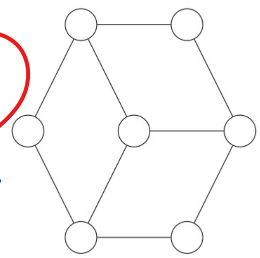
i. $\lambda = 1$ tornado/year $\quad P(X=0) = \frac{1^0 e^{-1}}{0!} = e^{-1} \quad P(\text{at least } 1) = 1 - e^{-1} = \boxed{0.63}$

ii. Binomial $\quad P(X=2) = \binom{3}{2}(0.63)^2(1-0.63)^1 = \boxed{0.44}$

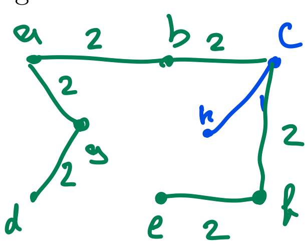
iii: $E(X) = n \cdot p = 10(0.63) = 6.3$ years

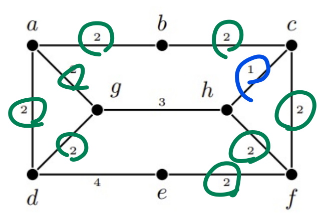
8. ***(3 points)*** Consider the graph G given below. Is G Eulerian? Give a valid reason.

G is NOT Eulerian as it has 4 vertices with odd degree. ⇒ No Euler circuit.



9. ***(7 points)*** Find a minimum spanning tree in the following weighted graph. What's the total minimum weight?





total weight $= 1 + 6(2) = 13$

10. ***(4 points)*** Define:
i): Leaf: In a rooted tree, a vertex with ~~no~~ any children is called a leaf.

ii). Path: is a walk that visit no vertex more than one.

Total: 65 Points

Good Luck! :)