CS 198:206; Introduction to Discrete Structures II

Exam II **Name & section:**———————————

- The approximate time required to complete this exam is 60 + 15 (to submit the exam) minutes.
- Please submit your exam answer as a **Single PDF file**. You can use any website or app to convert your pictures to a **single PDF file** as well as either of the following links:

$$https://smallpdf.com/jpg-to-pdf$$

$$https://tools.pdfforge.org/images-to-pdf$$

- You will get **3 points deduction** for any other submission aside from uploading file on Canvas. **Do Not** submit via email.
- You will get **3 points deduction** if you do not submit a SINGLE PDF file.
- You will get **2 points deduction** if you submit a paper without name.
- **For full grade, show and write all your work, a step by step. No work/ Just final answer, No credit.**
- You're not allowed to use second monitor or device during the exam.
- Do **NOT** use any electronic devices and calculator.
- To avoiding any missing or mistake, please read each question completely and carefully.
- To be able to protect the exam, your camera has to be on during the quiz/exam. It has to be face to you and your work place, with enough light around.

1. ***(4 points)*** One number chosen randomly from the integers 1 to 20. Find the probability of getting a number that is odd and prime.

$n(S) = 20$

A: set of odd # in $S = \{1, 3, 5, 7, 9, 11, 13, 15, 17, 19\}$ $n(A) = 10$

B: " " prime " " $= \{2, 3, 5, 7, 11, 13, 17, 19\}$ $n(B) = 8$

$$P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{4}{20}$$

2. ***(7 points)*** You draw at random five cards from a standard deck of 52 cards. What is the probability that there is "an ace" among the five cards and "a king or queen"?

Let A be the event that there is no ace among the five cards and B be the event that there is neither a king nor a queen among the five cards.
The desired probability is given by

$1 - P(A \cup B) = 1 - [P(A) + P(B) - P(A \cap B)]$

$$= 1 - \left[\frac{\binom{48}{5}}{\binom{52}{5}} + \frac{\binom{44}{5}}{\binom{52}{5}} - \frac{\binom{40}{5}}{\binom{52}{5}}\right] = 0.1765$$

3. ***(7 points)*** A professor gives only two types of exams, "easy" and "hard". You will get a hard exam with probability 0.80. The probability that the first question on the exam will be marked as difficult is 0.90 if the exam is hard and is 0.15 otherwise. What is the probability that the first question on your exam is marked as difficult. What is the probability that your exam is hard given that the first question on the exam is marked as difficult?

Let A be the event that the first question on the exam is marked as difficult. Let B_1 be the event that the exam is hard and B_2 be the event that the exam is easy. Applying the formula

$$P(A) = \sum_{i=1}^{2} P(B_i)\, P(A|B_i) = (0.9)(0.8) + (0.15)(0.1) = 0.735$$

The probability that the exam is hard given that the first question on the exam is marked as difficult is equal to

$$P(B_1|A) = \frac{P(B_1 \cap A)}{P(A)} = \frac{P(B_1)\,P(A|B_1)}{P(A)} = \frac{(0.9)(0.8)}{(0.735)}$$

4. ***(16 points, 5, 3, 4, and 4 respectively*** A survey of cars on a certain stretch of highway during morning commute hours showed that 70% had only one occupant, 15% had 2, 10% had 3, 3% had 4, and 2% had 5. Let X represent the number of occupants in a randomly chosen car.

Find the Probability Mass Function of X.

x	1	2	3	4	5
$P(X=x)$	0.7	0.15	0.1	0.03	0.02

b) Find $P(X \leq 2)$?

$$P(X \leq 2) = P(X=1) + P(X=2)$$
$$= 0.7 + 0.15$$

c) Find μ_X and σ_X^2?

$$\mu_X = \sum_{i=1}^{5} x_i P(X=x_i)$$
$$= 1(0.7) + 2(0.15) + 3(0.1) + 4(0.03) + 5(0.02)$$
$$= 1.52$$

$$\sigma_X^2 = E(X^2) - E^2(X) = \sum_{i=1}^{5} x_i^2 P(X=x_i) - (1.52)^2$$
$$= (0.7) + 4(0.5) + 9(0.1) + 16(0.03) + 25(0.02) - (1.52)^2$$

Extra point problem: (3 points) In how many different ways you can pick hands of 5-Cards Poker such that to have "3 cards of one denomination,plus 2 cards of a second denomination"?

3 cards of one denomination $\simeq$ 3 of a kind

52 cards — 13 different denominations — 4 suits

Diff. ways to have 3 of a kind: $13 \cdot {}_4C_3$

Diff. ways to have a pair: $12 \cdot {}_4C_2$

$\Rightarrow$ both together: $(13 \cdot {}_4C_3)(12 \cdot {}_4C_2) = 3744$

Total: ~~44~~ 34 Points

Good Luck! *:)*