CS 198:206

Exam II **Name & Section:**

- The approximate time required to complete this exam is 60 minutes.
- **For full grade, show and write all your work, step by step. No work/Just final answer has no point.**
- To avoiding any missing or mistake, please read the question **carefully and completely.**
- You get 2 points deduction if you submit a paper without name.
- In case if you need more space, you might use the back side of your paper. I **DO NOT ACCEPT** any other sheet attached to the exam paper.
- **Do NOT USE** calculator or any electronic device.

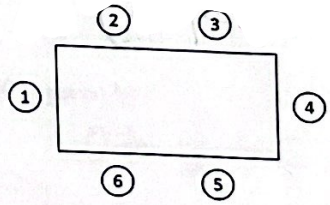
Q 1. (5 Points) Seven people are arranged in a row to take pictures, and three people A, B, and C cannot be adjacent to each other. How many ways are there?

Case A, B, C be next to each other: $5! \cdot 3!$ (2)

ways arranging 7 people: $7!$ (1)

$\Rightarrow$ # ways A, B, C NOT next to each other: $7! - 5! \cdot 3!$ (2)

Q 2. (5 Points) Given a table with 6 seats arranged as the figure below. We are trying to arrange Amy, Bobby, Cindy, Daniel, Eddy, and Fiona to this table. In a randomly arrangement, what's the probability of Amy can only sit on the left or right sides of the table and Bobby can not sit on the same side as Eddy?



$n = 6 \Rightarrow n(S) = 6!$

K is 4 tasks $\Rightarrow$

$n(K) = 2 \cdot 4 \cdot 3 \cdot 3!$

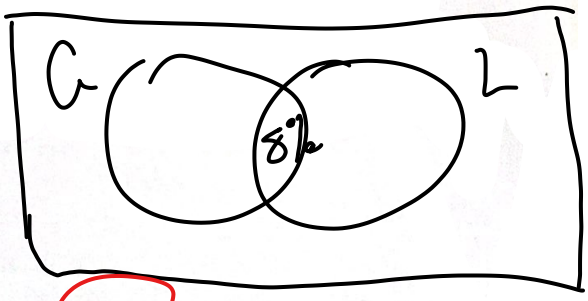
2: choices for A; 4: E; 3: B; $3!$: rest of people

$$\Rightarrow P(K) = \frac{n(K)}{n(S)} = \frac{2 \cdot 4 \cdot 3 \cdot 3!}{6!}$$

Q 3. (6 Points) Angela, who serves on the party planning committee, surveyed the whole office about people's dietary restrictions. She finds that a random employee has 30% chance to be gluten-free and 40% chance to be lactose-free (cannot drink milk). There are 20% chance a person is gluten-free given this person is lactose-free. What is the probability a random employee can eat gluten and drink milk?

Let G: gluten free L: lactose free

$P(G|L) = 20\% \Rightarrow P(G \cap L) = 20\%(40\%)$



$P(G \cup L) = 30\% + 40\% - 8\%$

$= 62\%$

$\Rightarrow P(G^C \cap L^C) = 1 - P(G \cup L) = 1 - 62\%$

4A/6B

3A/7B

7A/3B

Q 4. (7 Points) Suppose you have 3 10-side dice. Die 1 has 3 sides of A and 7 sides of B. Die 2 has 7 sides of A and 3 sides of B. Die 3 has 4 sides of A and 6 sides of B. For all three dice, it is equally likely to get any side on the top side after each roll. Given that a die is chosen and rolled without you seeing and the result is a B. What is the probability of die 2 was rolled given the chance of choosing die 1 is 30% and chance of choosing die 3 is 40%?

die	1	2	3
	0.3	0.3	0.4

$$P(\text{die}2|B) = \frac{P(B|2)\cdot P(2)}{P(B|1)\cdot P(1) + P(B|2)\cdot P(2) + P(B|3)\cdot P(3)}$$

$$= \frac{0.3 \times 0.3}{0.7 \times 0.3 + 0.3 \times 0.3 + 0.6 \times 0.4}$$

$$= \frac{0.09}{0.21 + 0.09 + 0.24}$$

$$= \frac{9}{54} = \frac{1}{6}$$

Q 5. (7 Points; 4 and 3 points respectively) Raffle tickets were sold at a gala dinner. One single ticket is sold for \$10, and the winner gets a ticket to the Super bowl final game (which worth \$5000). A person can buy multiple tickets and suppose Sam bought 8 tickets. There was total 800 ruffle tickets sold, find the following.

(a). The expected net winnings of Sam.

$5000-8(10)=4920$	-80
1%	99%

$E(X)=\sum xP(x)$

Expectation $= 1\%\,(4920)+(-80)(99\%)$

$=-30$

(b). Suppose this dinner and the ticket is sponsored by the NFL (who has free super bowl ticket), 10% of the earning going to the expanse for hosting the dinner and the rest is donated to charity. How much did they donate?

$800\,(\$10) = 8000$

$10\%\,(8000) = 800$

$\Rightarrow 8000-800$

$= \$7200$

Q 6. (10 Points) The probability distribution of a discrete random variable X is given by

$$P(X=x)=\begin{cases} k(2-x) & x=0,1,2 \\ \frac{1}{4} & x=3 \\ 0 & \text{Otherwise} \end{cases}$$

$\Rightarrow 0+\frac{1}{4}+0+k+2k=1$

$\Rightarrow k=\frac{1}{4}$

a). Find $E(X^2)$

$=\sum x^2 P(X=x)$

$=0^2(\frac{1}{2})+1(\frac{1}{4})+2^2(0)+3^2(\frac{1}{4})=\frac{5}{2}$

o) Determine Var $(3-X)$ $=\text{Var}\,3+(-1)^2\,\text{Var}\,x=\text{Var}\,x$

$=E(X^2)-(EX)^2$

$=\frac{5}{2}-1=\frac{3}{2}$

X	0	1	2	3
P	$\frac{1}{2}$	$\frac{1}{4}$	0	$\frac{1}{4}$
X^2	0	1	4	9

since $E(X)=\sum xP(X=x)$

$=0(\frac{1}{2})+1(\frac{1}{4})+2(0)+3(\frac{1}{4})=1$