

Hartshorne Solutions

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1.1 Affine Varieties

1.8

Let Y be an affine variety of dimension r in \mathbb{A}^n . Let H be a hypersurface in \mathbb{A}^n , and assume that $Y \not\subseteq H$. Then every irreducible component of $Y \cap H$ has dimension $r - 1$.

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Suppose that $H = Z(f)$ with f irreducible in $k[x_1, \dots, x_n]$. Then the projection of f in $A(Y) := k[x_1, \dots, x_n]/I(Y)$ is not equal to $\bar{0}$ since by assumption $(f) \not\subseteq I(Y)$. Since $A(Y)$ is a domain (due to the irreducibility of Y), the element \bar{f} is not a zero divisor. Assuming that $Y \cap H \neq \emptyset$, we have that \bar{f} is not a unit in $A(Y)$. To see this, let $P \in Y \cap H$. Then $I(Y), (f) \subset \mathfrak{m}_P$, the maximal ideal of $k[x_1, \dots, x_n]$ corresponding to P . This implies $(\bar{f}) \subset \mathfrak{m}_P/I(Y)$, the latter being a maximal ideal in $A(Y)$. Thus, \bar{f} is not a unit.

We apply Theorem 1.11A to get that every minimal prime ideal \mathfrak{p} in $A(Y)$ containing \bar{f} has height 1. The irreducible components of $Y \cap H$ and the minimal prime ideals containing (\bar{f}) correspond, and since each of these prime ideals has height 1, the corresponding varieties have dimension $r - 1$ by Theorem 1.8A.