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## 1 Hamiltonian in barycentric coordinates

Hmiltonian in barycentric coordinates $q_i, p_i \in \mathbb{R}^3,\ i = 0, \ldots, N$,

$$H(q,p) = \frac{1}{2} \sum_{i=0}^{N} \frac{\|p_i\|^2}{m_i} - G \sum_{0 \leq i < j \leq N}^{N} \frac{m_i m_j}{\|q_i - q_j\|}. \tag{1}$$

## 2 Canonical Heliocentric Coordinates

Planet indices

$$\begin{aligned}
&Q_0 : Sun \\
&Q_i (i = 1, \ldots, N-2) : \text{rest of the planets} \\
&Q_{N-1} : Earth \\
&Q_N : Moon
\end{aligned}$$

$$\begin{aligned}
\frac{1}{\mu_0} &= \frac{1}{M},\ M = \sum_{i=0}^{N} m_i \\
\frac{1}{\mu_i} &= \frac{1}{m_0} + \frac{1}{m_i}, \quad i < N-1 \\
\frac{1}{\mu_{N-1}} &= \frac{1}{m_0} + \frac{1}{(m_{N-1} + m_N)} \\
\frac{1}{\mu_N} &= \frac{1}{m_N} - \frac{1}{(m_{N-1} + m_N)}
\end{aligned}$$

$$\begin{aligned}
k_i &= (m_0 + m_i), \quad i < N-1 \\
k_{N-1} &= m_0 + m_{N-1} + m_N \\
k_N &= \frac{m_{N-1}^3}{(m_{N-1} + m_N)^2}
\end{aligned}$$

Hamiltonian

$$H(Q,V) = \frac{1}{M} V_0^2 + H_K(Q,V) + H_I(Q,V)$$

1. $H_k(Q,V)$

$$\sum_{i=1}^{N}\left(\frac{\|V_i\|^2\mu_i}{2}-\frac{\mu_i k_i}{\|Q_i\|}\right)$$

2. $H_I(Q,V)$

$$\begin{aligned}H_I(Q,V)=&\frac{1}{m_0}\sum_{1\leqslant i<j}^{N-1}\mu_i V_i\ \mu_j V_j-\sum_{1\leqslant i<j}^{N-2}\frac{m_i m_j}{\|Q_i-Q_j\|}\\&-\sum_{i=1}^{N-2}\left(\frac{m_i m_N}{\|Q_i-Q_{N-1}-Q_N\|}+\frac{m_i m_{N-1}}{\|Q_i-Q_{N-1}+\frac{m_N}{m_{N-1}}Q_N\|}\right)\\&+\frac{m_0(m_{N-1}+m_N)}{\|Q_{N-1}\|}-\frac{m_0 m_{N-1}}{\|Q_{N-1}-\frac{m_N}{m_{N-1}}Q_N\|}-\frac{m_0 m_N}{\|Q_{N-1}+Q_N\|}\end{aligned}$$

2.1 ODE(V,P)

1. Position

$$\begin{aligned}&\dot{Q}_i=V_i+\sum_{j\neq i,j=1}^{N-1}\frac{V_j\ \mu_j}{m_0},\quad i\leqslant N-1\\&\dot{Q}_{N-1}=V_{N-1}+\sum_{j\neq N-1,j=1}^{N-1}\frac{V_j\ \mu_j}{m_0}\\&\dot{Q}_N=V_N\end{aligned}$$

2. Velocity

$$\dot{V}_i = -\frac{k_i}{\|Q_i\|^3}Q_i - \frac{m_i}{\mu_i}\Bigg( \sum_{j\neq i, j=1}^{N-2} \frac{m_j}{\|Q_i - Qj\|^3}(Q_i - Q_j)$$
$$- \frac{m_N}{\|Q_i - Q_{N-1} - Q_N\|^3}(Q_i - Q_{N-1} - Q_N) - \frac{m_{N-1}}{\|Q_i - Q_{N-1} + \frac{m_N}{m_{N-1}}Q_N\|^3}(Q_i - Q_{N-1} + \frac{m_N}{m_{N-1}}Q_N)\Bigg)$$
$$i \leqslant N-2$$

$$\dot{V}_{N-1} = -\frac{k_{N-1}}{\|Q_{N-1}\|^3}Q_{N-1}$$
$$+ \frac{1}{\mu_{N-1}}\Bigg( \sum_{i=1}^{N-2}\Bigg( \frac{m_i m_N}{\|Q_j - Q_{N-1} - Q_N\|^3}(Q_j - Q_{N-1} - Q_N) + \frac{m_i m_{N-1}}{\|Q_j - Q_{N-1} + \frac{m_N}{m_{N-1}}Q_N\|^3}(Q_j - Q_{N-1} + \frac{m_N}{m_{N-1}}Q_N)\Bigg)$$
$$+ \frac{m_0(m_N + m_{N-1})}{\|Q_{N-1}\|^3}(Q_{N-1}) - \frac{m_0 m_{N-1}}{\|Q_{N-1} - \frac{m_N}{m_{N-1}}Q_N\|^3}(Q_{N-1} - \frac{m_N}{m_{N-1}}Q_N) - \frac{m_o m_N}{\|Q_{N-1} + Q_N\|^3}(Q_{N-1} + Q_N)\Bigg)$$

$$\dot{V}_N = -\frac{k_N}{\|Q_N\|^3}\ Q_N$$
$$+ \frac{1}{\mu_N}\Bigg( \sum_{i=1}^{N-2}\Bigg( \frac{m_i m_N}{\|Q_j - Q_{N-1} - Q_N\|^3}(Q_j - Q_{N-1} - Q_N) - \frac{m_i m_N}{\|Q_j - Q_{N-1} + \frac{m_N}{m_{N-1}}Q_N\|^3}(Q_j - Q_{N-1} + \frac{m_N}{m_{N-1}}Q_N)\Bigg)$$
$$+ \frac{m_0 m_N}{\|Q_{N-1} - \frac{m_N}{m_{N-1}}Q_N\|^3}(Q_{N-1} - \frac{m_N}{m_{N-1}}Q_N) - \frac{m_0 m_N}{\|Q_{N-1} + Q_N\|^3}(Q_{N-1} + Q_N)\Bigg)$$

## References