5body_IRK16_luzea

December 15, 2022

0.0.1 Loading packages and functions

[1]:
```julia
using LinearAlgebra
using Plots
using SIMD
using IRKGaussLegendre

PATH_SRC="../../src_simd/"
include(string(PATH_SRC,"IRKGL_SIMD.jl"))
using .IRKGL_SIMD
```

[2]:
```julia
PATH_ODES="../../ODEProblems/"

include(string(PATH_ODES,"Initial5Body.jl"))
include(string(PATH_ODES,"Nbody.jl"))
```

[2]:
```
NbodyODE! (generic function with 2 methods)
```

[3]:
```julia
PATH_SRC="../../src_seq/"

include(string(PATH_SRC,"IRKGL_SEQ.jl"))

using .IRKGL_SEQ
```

[4]:
```julia
#export JULIA_NUM_THREADS=4
Threads.nthreads()
```

[4]:
```
4
```

0.1 Definition of the N-body problem

In Nbody.jl below, the following functions are defined: NbodyEnergy(u,Gm), NbodyODE!(du,u,Gm,t), and NbodyODE1!(du,u,Gm,t), where

$$u = \begin{pmatrix} q_1 & v_1 \\ \vdots & \vdots \\ q_N & v_N \end{pmatrix} \in \mathbb{R}^{2\times 3\times N}, \quad Gm = (G\, m_1, \dots, G\, m_N) \in \mathbb{R}^N.$$

The energy, as a function of the positions $q_1, \dots, q_N \in \mathbb{R}^3$ and the velocities $v_1, \dots, v_N \in \mathbb{R}^3$ of the N bodies is:

$$\text{Energy}(q_1, \dots, q_N, v_1, \dots, v_N) = \frac{1}{2}\sum_{i=1}^{N} m_i \, \|v_i\|^2 - G \sum_{1 \le i < j \le N} \frac{m_i m_j}{\|q_i - q_j\|}.$$

The ODE system of the N-body problem, as a function of the positions $q_1, \dots, q_N \in \mathbb{R}^3$ and the velocities $v_1, \dots, v_N \in \mathbb{R}^3$ of the N bodies is:

$$\begin{aligned} \frac{d}{dt} q_i &= v_i, \\ \frac{d}{dt} v_i &= G \sum_{j \neq i} \frac{m_j}{\|q_j - q_i\|^3} \, (q_j - q_i). \end{aligned}$$

This system of ODEs can be writen in compact form as

$$\frac{du}{dt} = f(t, u, Gm)$$

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0.2 Initial value problem: 5-body problem (outer solar system)

We consider $N = 5$ bodies of the outer solar system: the Sun, Jupiter, Saturn, Uranus, and Neptune. The initial values u_{00} are taken from DE430, Julian day (TDB) 2440400.5 (June 28, 1969).

[5]:
```
u0, Gm, bodylist = Initial5Body(Float64)
q0=u0[:,:,1]
v0=u0[:,:,2]
dim=length(size(u0))

N = length(Gm)

show(bodylist)
E0=NbodyEnergy(u0,Gm)
```

```
["Sun" "Jupiter" "Saturn" "Uranus" "Neptune"]
```

[5]:
```
-9.522696242724855e-12
```

0.2.1 Integrazio luzea

[6]:
```
t0 = 0.
dt = 200.
tF = 1e11
tF = 1e10
tF = 1e8    # 17 sec
tF = 1e9    # 195 sec
```

```
#tF = 1e10

prob = ODEProblem(NbodyODE!, u0, (t0,tF), Gm)

m = Int64(ceil((tF-t0)/(1000*dt)))
```

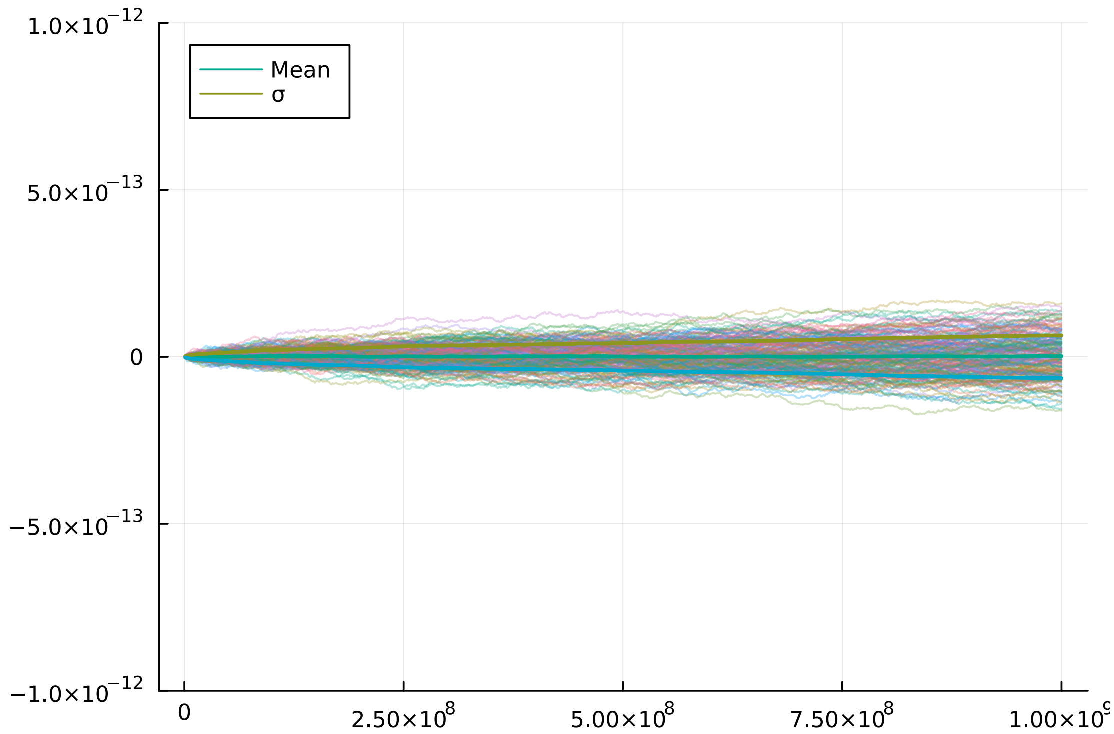
[6]: 5000

[7]:
```
sols = []
iters = []
kmax=24
kmax=200

@time Threads.@threads for k in 0:kmax-1
   u0_ = copy(u0)
   prob_ = ODEProblem(NbodyODE!, u0_, (t0,tF), Gm);

   u0_[1] = u0[1] + k*1e-14
#   u0_[1] = u0[1] + kmax*1e-14*(rand()-0.5)

alg=IRKGL_simd(s=8,initial_interp=1,dim=3,floatType=Float64, m=m,␣
 ↪myoutputs=true)
solk,iterk=solve(prob_,alg,dt=dt, adaptive=false);

    push!(sols,solk)
#    push!(iters,iterk)

end
```

1455.086079 seconds (12.55 M allocations: 803.517 MiB, 0.01% gc time, 0.01% compilation time)

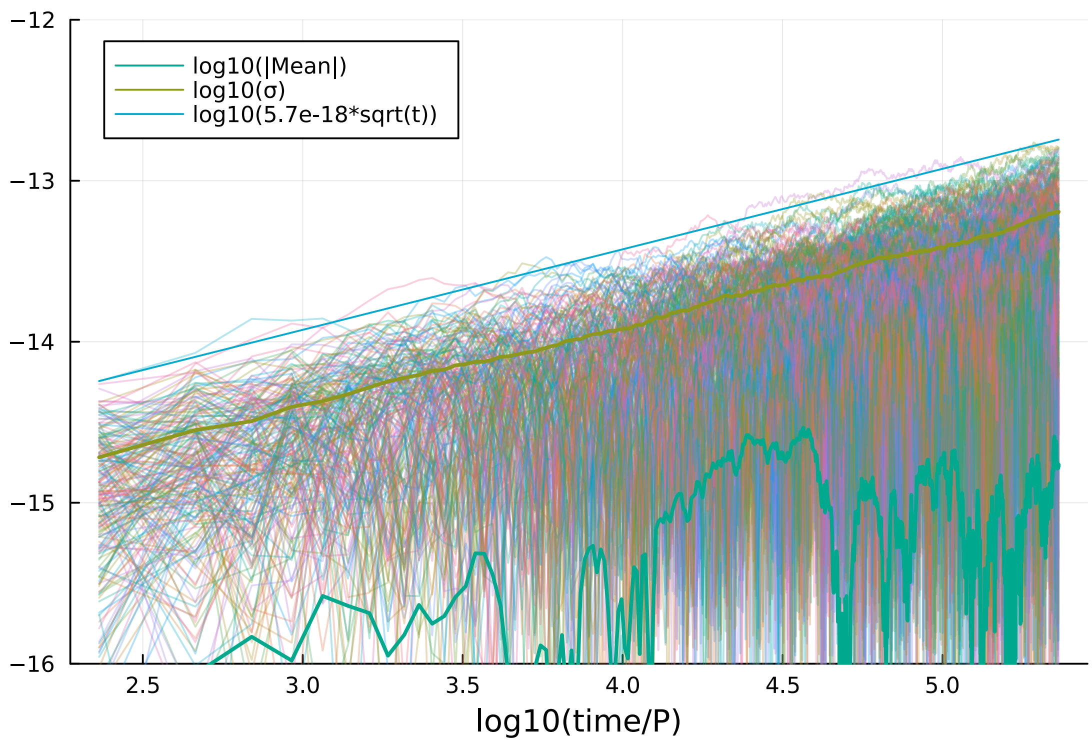
[8]:
```
function energy_plot(sols; title="")
    tt = sols[1].t[2:end]
    yrange = (-1e-12,1e-12)
    P = 4331.89
    times = tt/P
    pl = plot(title=title, legend=:topleft, ylims=yrange)
    n = length(tt)
    errors = zeros(n)
    mean = zeros(n)
    sdev = zeros(n)
    energies = zeros(n)
    U = BigFloat.(u0)


    mean .= 0
```

```
    sdev .= 0
    for sl in sols
        U .= sl.u[1]
        E0 = NbodyEnergy(U,Gm)
        for i in 1:n
            U .= sl.u[i+1]
            errors[i] = NbodyEnergy(U,Gm)/E0 - 1
            mean[i] += errors[i]
            sdev[i] += errors[i]^2
        end
        plot!(tt,  errors, linewidth=1, linealpha=0.3, label=:none)
    end
    kinv = 1/length(sols)
    @. mean = kinv * mean
    @. sdev = sqrt(kinv*sdev-mean^2)
    plot!(tt, mean, linewidth=2,label="Mean")
    plot!(tt, sdev, linewidth=2,label=" ")
    plot!(tt, -sdev, linewidth=2, label=:none)
#    plot!(tt, 5.7e-18sqrt.(tt), label="5.7e-18*sqrt(t)")
#    plot!(tt, -5.7e-18sqrt.(tt), label=:none)
    return pl, tt, mean, sdev
end



pl0, tt, mean, sdev = energy_plot(sols);

display(pl0)
```




```
[9]: function energy_plot(sols; title="")
         tt = sols[1].t[2:end]
         yrange = (-16,-12)
         P = 4331.89
         logtimes = log10.(tt/P)
         pl = plot(title=title, ylims=yrange, legend=:topleft, xlabel="log10(time/
     ↪P)")
         n = length(tt)
         errors = zeros(n)
         mean = zeros(n)
         sdev = zeros(n)
         energies = zeros(n)
         U = BigFloat.(u0)


         mean .= 0
         sdev .= 0
         for sl in sols
             U .= sl.u[1]
             E0 = NbodyEnergy(U,Gm)
             for i in 1:n
                 U .= sl.u[i+1]
                 errorsi = NbodyEnergy(U,Gm)/E0 - 1
```

```
            errors[i] = log10(abs(errorsi))
            mean[i] += errorsi
            sdev[i] += errorsi^2
        end
        plot!(logtimes,  errors, linewidth=1, linealpha=0.3, label=:none)
    end
    kinv = 1/length(sols)
    @. mean = abs(kinv * mean)
    @. sdev = log10(sqrt(kinv*sdev-mean^2))
    @. mean = log10(mean)
    plot!(logtimes, mean, linewidth=2, label="log10(|Mean|)")
    plot!(logtimes, sdev, linewidth=2, label="log10( )")
    plot!(logtimes, log10.(5.7e-18sqrt.(tt)), label="log10(5.7e-18*sqrt(t))")
    return pl, tt, mean, sdev
end



pl0, tt, mean, sdev = energy_plot(sols);

display(pl0)
```



[ ]: