6 Body Solar System_IRKNGL LUZEA

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1 6-Body Solar System IRKNGL

Loading packages and functions

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1.1 Loading packages and functions

[1]:
```
using LinearAlgebra
using Plots
using OrdinaryDiffEq
using JLD2, FileIO
using Parameters,NBInclude
#using IRKGaussLegendre
using LaTeXStrings
using BenchmarkTools
#using RecursiveArrayTools
```

[2]:
```
PATH_DATA="./Data/"

PATH_SRC="../../src_simd/"
include(string(PATH_SRC,"IRKGL_SIMD.jl"))
using .IRKGL_SIMD


PATH_SRC="../../src_seq/"
include(string(PATH_SRC,"IRKGL_SEQ.jl"))
using .IRKGL_SEQ


PATH_SRC="../../src/"
include(string(PATH_SRC,"MyBenchmarksTools.jl"))
```

[2]:
```
launch_method_tests (generic function with 1 method)
```

[3]:
```
PATH_SRC2="../../src_seq2/"
include(string(PATH_SRC2,"IRKGL_SEQ.jl"))
#include(string(PATH_SRC2,"IRKGL_Seq_Solver.jl"))
#include(string(PATH_SRC2,"IRKGL_Step_Functions.jl"))

using .IRKGL_SEQ2
```

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1.2 Definition of the N-body problem

In Nbody.jl below, the following functions are defined: NbodyEnergy(u,Gm), NbodyODE!(du,u,Gm,t), and NbodyODE1!(du,u,Gm,t), where

$$u = \begin{pmatrix} q_1 & v_1 \\ \vdots & \vdots \\ q_N & v_N \end{pmatrix} \in \mathbb{R}^{2\times 3\times N}, \quad Gm = (G\,m_1, \dots, G\,m_N) \in \mathbb{R}^N.$$

The energy, as a function of the positions $q_1, \dots, q_N \in \mathbb{R}^3$ and the velocities $v_1, \dots, v_N \in \mathbb{R}^3$ of the N bodies is:

$$\text{Energy}(q_1, \dots, q_N, v_1, \dots, v_N) = \frac{1}{2}\sum_{i=1}^{N} m_i \|v_i\|^2 - G \sum_{1\le i<j\le N} \frac{m_i m_j}{\|q_i - q_j\|}.$$

The ODE system of the N-body problem, as a function of the positions $q_1, \dots, q_N \in \mathbb{R}^3$ and the velocities $v_1, \dots, v_N \in \mathbb{R}^3$ of the N bodies is:

$$\begin{aligned} \frac{d}{dt} q_i &= v_i, \\ \frac{d}{dt} v_i &= G \sum_{j\neq i} \frac{m_j}{\|q_j - q_i\|^3} (q_j - q_i). \end{aligned}$$

This system of ODEs can be writen in compact form as

$$\frac{du}{dt} = f(t, u, Gm)$$

[4]:
```
PATH_ODES="../../ODEProblems/"

include(string(PATH_ODES,"Initial6Body.jl"))
include(string(PATH_ODES,"Nbody.jl"))
include(string(PATH_ODES,"Nbody2nd.jl"))
include(string(PATH_ODES,"NbodyDyn.jl"));
```

1.2.1 Initial value problem: 6-body problem (outer solar system)

We consider $N = 6$ bodies of the outer solar system: the Sun, Jupiter, Saturn, Uranus, Neptune and Pluto. The initial values u_{00} are taken from DE430, Julian day (TDB) 2440400.5 (June 28, 1969).

[5]:
```
nruns=1

u0, Gm, bodylist = Initial6Body(Float64)
u0_B, Gm_B, bodylist = Initial6Body(BigFloat)

q0=u0[:,:,1]
v0=u0[:,:,2]
dim=length(size(u0))

N = length(Gm)

show(bodylist)
E0=NbodyEnergy(u0,Gm)
```

```
["Sun" "Jupiter" "Saturn" "Uranus" "Neptune" "Pluto"]
```

[5]:
```
-9.522620605966966e-12
```

[6]:
```
# Comprobación: centro de masas del sistema solar exterior en el origen de
 ↪coordenadas con velocidad nula

q0_ = zeros(3)
v0_ = zeros(3)
Gm0_ = 0.
for i in 1:6
    Gmi = Gm[i]
    qi = u0[:,i,1]
    vi = u0[:,i,2]
    q0_ += Gmi*qi
    v0_ += Gmi*vi
    Gm0_ += Gmi
end
(q0_/Gm0_, v0_/Gm0_)
```

[6]:
```
([1.417048569677335e-18, 9.119357686204585e-20, 1.5082339679877506e-19],
[1.0942040830907497e-22, -2.709864045560887e-22, 1.6237832716049163e-22])
```

[7]:
```
t0=0.
tF=1e9  # days

tspan= (t0,tF)
tspan_B=(BigFloat(t0),BigFloat(tF))

dim=length(size(u0))
prob = ODEProblem(NbodyODE!, u0,tspan , Gm)
probDyn= DynamicalODEProblem(NbodyODEq!,NbodyODEv!,q0,v0,tspan,Gm)
prob2nd = SecondOrderODEProblem(NbodyODE2nd!,v0,q0,(t0,tF),Gm)
```

```
prob_B = ODEProblem(NbodyODE!, u0_B, tspan_B, Gm_B);
```

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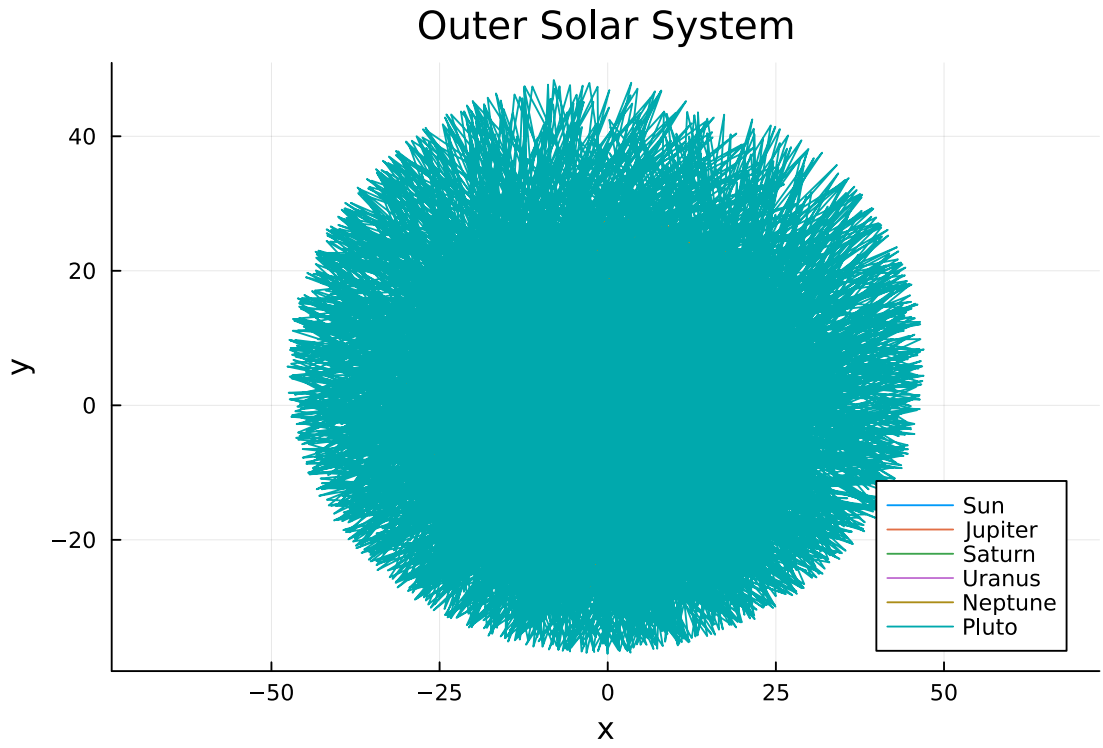
1.3 Integrations

1.3.1 IRKNGL integrazioa-0 (exact solution)

[8]:
```
m=5000
dt0=100.
alg=IRKNGL_simd(s=8, initial_interp=1, m=m,myoutputs=true)
saveat=m*dt0
sol0,iters0,steps0=solve(prob,alg,dt=dt0, saveat=saveat,adaptive=false);
```

[9]:
```
pl = plot(title="Outer Solar System",
            xlabel="x", ylabel="y",  aspect_ratio=1)

for j = 1:6
    x  = [u[1,j,1] for u in sol0.u]
    y  = [u[2,j,1] for u in sol0.u]
    pl = plot!(x,y, label=bodylist[j])
end
display(pl)
```



1.3.2 IRKNGL

[10]:
```
m=1000
dt=500.
alg=IRKNGL_Seq(s=8, initial_interp=1, m=m,myoutputs=true)
alg2=IRKNGL_simd(s=8, initial_interp=1, m=m, myoutputs=true)

sol1,iters1,steps1=solve(prob,alg,dt=dt, adaptive=false);
sol2,iters2,steps2=solve(prob,alg2,dt=dt, adaptive=false);
```

[11]:
```
solve(prob,alg,dt=dt, save_everystep=false, adaptive=false);
cpu_IRKGLseq=0.
for i in 1:nruns
    cpu_IRKGLseq+=@elapsed solve(prob,alg,dt=dt, save_everystep=false,␣
 ↪adaptive=false);
end
cpu_IRKGLseq=cpu_IRKGLseq/nruns

solve(prob,alg2,dt=dt,save_everystep=false, adaptive=false);
cpu_IRKGLsimd=0.
for i in 1:nruns
    cpu_IRKGLsimd+=@elapsed solve(prob,alg2,dt=dt, save_everystep=false,␣
 ↪adaptive=false);
end
cpu_IRKGLsimd=cpu_IRKGLsimd/nruns;
```

1.3.3 Sofronio

[12]:
```
m=2000
dt3=250.0
method = SofSpa10

m=4000
dt3=250.0/2
method = KahanLi8

saveat=m*dt3
sol3=solve(prob2nd, method(), dt=dt3, saveat=saveat, adaptive=false);
T=typeof(u0)
solu3=Array{T}(undef,length(sol3.u));
for k in 1:length(sol3.u)
    solu3[k]=zero(u0)
    solu3[k][:,:,1].=sol3.u[k].x[2]
    solu3[k][:,:,2].=sol3.u[k].x[1]
end
```

[13]:
```
solve(prob2nd, method(), dt=dt3, save_everystep=false, adaptive=false);
cpu_SofSpa10=0.
for i in 1:nruns
    cpu_SofSpa10+=@elapsed solve(prob2nd, method(), dt=dt3,␣
 ↪save_everystep=false, adaptive=false)
end
cpu_SofSpa10=cpu_SofSpa10/nruns;
```

1.3.4 DPRKN12

[14]:
```
m=2500
dt4=200.
saveat=m*dt4
sol4=solve(prob2nd, DPRKN12(), dt=dt4, saveat=saveat,adaptive=false);
T=typeof(u0)
solu4=Array{T}(undef,length(sol4.u));
for k in 1:length(sol4.u)
    solu4[k]=zero(u0)
    solu4[k][:,:,1].=sol4.u[k].x[2]
    solu4[k][:,:,2].=sol4.u[k].x[1]
end
```

[15]:
```
solve(prob2nd, DPRKN12(), dt=dt4, save_everystep=false, adaptive=false);
cpu_DPRKN12=0.
for i in 1:nruns
    cpu_DPRKN12+=@elapsed solve(prob2nd, DPRKN12(), dt=dt4,␣
 ↪save_everystep=false, adaptive=false)
end
cpu_DPRKN12=cpu_DPRKN12/nruns;
```

1.3.5 Some checks

[16]:
```
sols=[sol0, sol1, sol2, sol3, sol4]
for sol in sols
    println("length=",length(sol.t))
end
```

```
length=2001
length=2001
length=2001
length=2001
length=2001
```

[17]:
```
k=20
for sol in sols[1:3]
    println("norm=", norm(sol0.u[1]-sol.u[1]), ", sol.t[k]=",sol.t[k], ", sol.
 ↪t[end]=", sol.t[end])
```

```
end
```

```
norm=0.0, sol.t[k]=9.5e6, sol.t[end]=1.0e9
norm=0.0, sol.t[k]=9.5e6, sol.t[end]=1.0e9
norm=0.0, sol.t[k]=9.5e6, sol.t[end]=1.0e9
```

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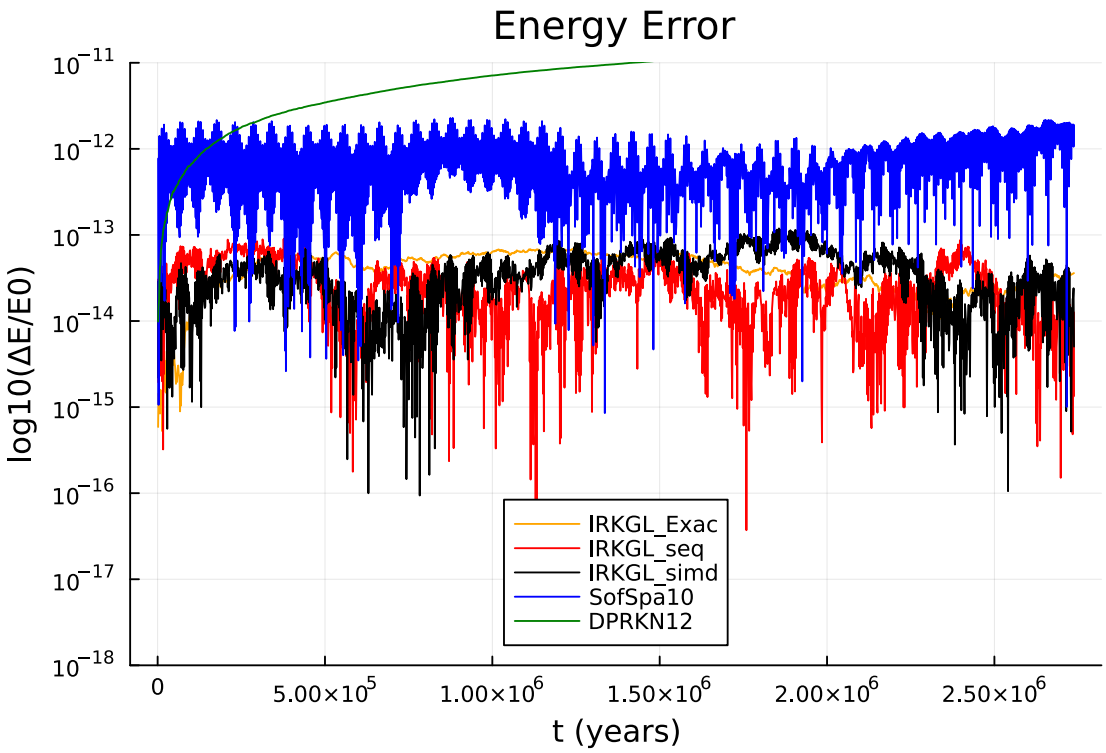
1.4 Errors in energy

[18]:
```
println("CPU IRKNGL_seq=$cpu_IRKGLseq, CPU IRKNGL_simd=$cpu_IRKGLsimd, CPU␣
 ↪IRKNGL_seq/CPU IRKNGL_simd=$(cpu_IRKGLseq/cpu_IRKGLsimd)")
println("CPU SofSpa10=$cpu_SofSpa10, CPU DPRKN12=$cpu_DPRKN12" )
```

```
CPU IRKNGL_seq=52.813707364, CPU IRKNGL_simd=9.954895036, CPU IRKNGL_seq/CPU
IRKNGL_simd=5.305300274187642
CPU SofSpa10=14.770874216, CPU DPRKN12=10.259620822
```

[19]:
```
E0=NbodyEnergy(u0_B, Gm_B)
year=365.25
yrange=(1e-18,1e-11)
ΔE0 = map(x->NbodyEnergy(BigFloat.(x),Gm_B), sol0.u)./E0.-1
ΔE1 = map(x->NbodyEnergy(BigFloat.(x),Gm_B), sol1.u)./E0.-1
ΔE2 = map(x->NbodyEnergy(BigFloat.(x),Gm_B), sol2.u)./E0.-1
ΔE3 = map(x->NbodyEnergy(BigFloat.(x),Gm_B), solu3)./E0.-1
ΔE4 = map(x->NbodyEnergy(BigFloat.(x),Gm_B), solu4)./E0.-1
plot(title="Energy Error",xlabel="t (years)", ylabel="log10(ΔE/E0)",
     yscale=:log10, ylims=yrange, legend=:bottom)
plot!(sol0.t./year,abs.(ΔE0),label="IRKGL_Exac", color="orange")
plot!(sol1.t./year,abs.(ΔE1),label="IRKGL_seq", color="red")
plot!(sol2.t/year,abs.(ΔE2),label="IRKGL_simd", color="black")
plot!(sol3.t/year,abs.(ΔE3),label="SofSpa10", color="blue")
plot!(sol4.t/year,abs.(ΔE4),label="DPRKN12", color="green")
```

[19]:



[20]:
```
Jupiter_period = 11.9*365
tF/Jupiter_period
```

[20]:
```
230229.0779325429
```

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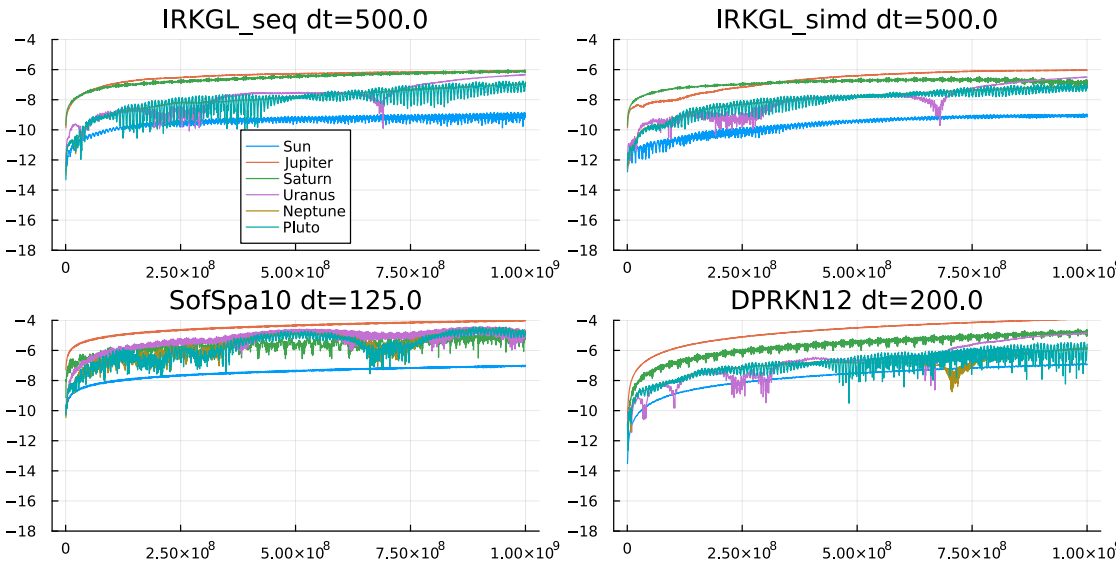
1.5 Errors in position

[21]:
```
qe1=Array{Array{Float64,1}}(undef,N)
qe2=Array{Array{Float64,1}}(undef,N)
qe3=Array{Array{Float64,1}}(undef,N)
qe4=Array{Array{Float64,1}}(undef,N)
for i in 1:N
    qe1[i]=map((u0,u1)-> log10(norm(u0[:,i,1]-u1[:,i,1])), sol0.u[2:end],sol1.
 ↪u[2:end])
    qe2[i]=map((u0,u1)-> log10(norm(u0[:,i,1]-u1[:,i,1])), sol0.u[2:end],sol2.
 ↪u[2:end])
    qe3[i]=map((u0,u1)-> log10(norm(u0[:,i,1]-u1[:,i,1])), sol0.u[2:
 ↪end],solu3[2:end])
    qe4[i]=map((u0,u1)-> log10(norm(u0[:,i,1]-u1[:,i,1])), sol0.u[2:
 ↪end],solu4[2:end])
end
```

[22]:
```
MinE=-18
MaxE=-4

pe1=plot(sol0.t[2:end], qe1,
         title="IRKGL_seq dt=$dt",
         ylims=(MinE,MaxE),
         label=bodylist,legend=:bottom)

pe2=plot(sol0.t[2:end], qe2,
     title="IRKGL_simd dt=$dt",
     ylims=(MinE,MaxE),
     legend=false)

pe3=plot(sol0.t[2:end], qe3,
         title="SofSpa10 dt=$dt3",
         ylims=(MinE,MaxE),
         legend=false)

pe4=plot(sol0.t[2:end], qe4,
         title="DPRKN12 dt=$dt4",
         ylims=(MinE,MaxE),
         legend=false)


plot(pe1, pe2, pe3, pe4,
layout=(2,2), size=(900,450),plot_title="")
```

[22]:



[ ]:

[ ]: