Few-body integrator with time-reversible adaptivity in Julia

Mikel Antoñana and Ander Murua

Department of Computer Science and Artificial Intelligence, University of of the Basque Country

Purpose

- **Simulation**: long-time numerical integration of few-body gravitational problem involving close encounters:

$$\frac{dq_i}{dt} = v_i$$
$$\frac{dv_i}{dt} = \sum_{j \neq i} \frac{G m_j}{\|q_j - q_i\|^3} (q_j - q_i)$$

- **Integrator**: based on IRKGL16 symmetric and symplectic Implicit Runge-Kutta method of order 16
- **Automatic Step Size Control**:
  - Standard: good long-time behaviour is lost
  - Time-reversible adaptivity better long-time behaviour

Time-reversible adaptivity

- Consider the initial value problem

$$\frac{du}{dt} = f(u), \quad u_0 = u(t_0)$$

where approximations are denoted as $u_n \approx u(t_n)$.

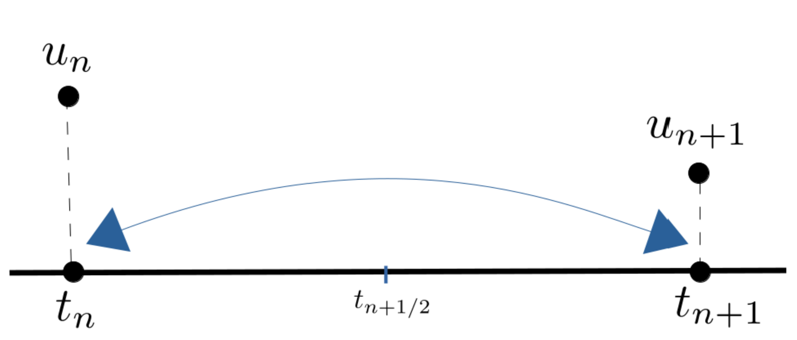


Figure 1. **Forward** and **backward integration**: same step size

Approach

- **Time-reversible adaptive fbirkgl16 algorithm**:
  - for $n = 0, 1, 2, \ldots$

$$u_{n+1} = u_n + h_n \sum_{i=1}^{s} b_i f(U_{n,i})$$

where $U_{n,i}$ and h_n are implicitly defined at each step by

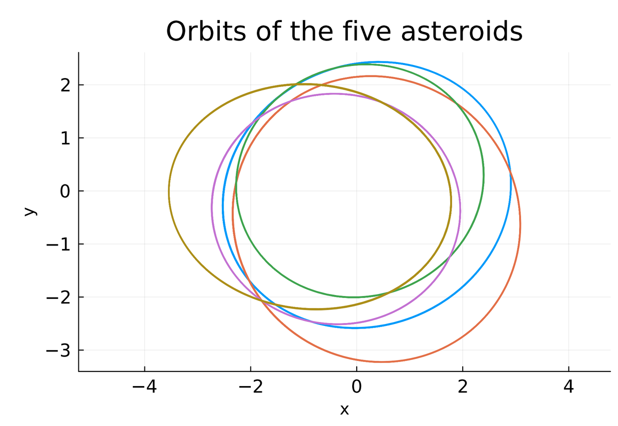
$$U_{n,i} = u_n + h_n \sum_{j=1}^{s} a_{ij} f(U_{n,j}),$$
$$h_n = \frac{\Delta\tau}{\sum_{i=1}^{s} b_i K(U_{n,i})}.$$

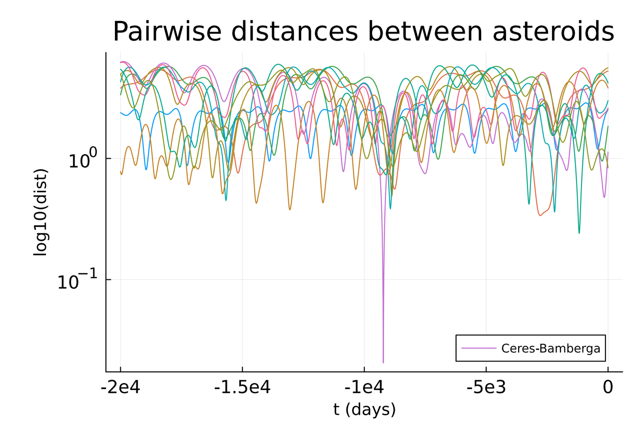
  - Step-size function is based on $K(u)$ time renormalized function (see [3, p.179]),

$$\frac{d\tau}{dt} = K(u(t)), \quad \tau(t_0) = 0$$
$$t_{n+1} = t_n + h_n \approx t_n + \int_0^{\Delta\tau} K^{-1}(u(\tau))\, d\tau.$$

Test problem

- 15-body model of the Solar System
  - The Sun, all eight planets of the Solar System, Pluto and the five main bodies of the asteroid belt
  - Close encounters between some of the asteroids during backward integration in time for $2e4$ days

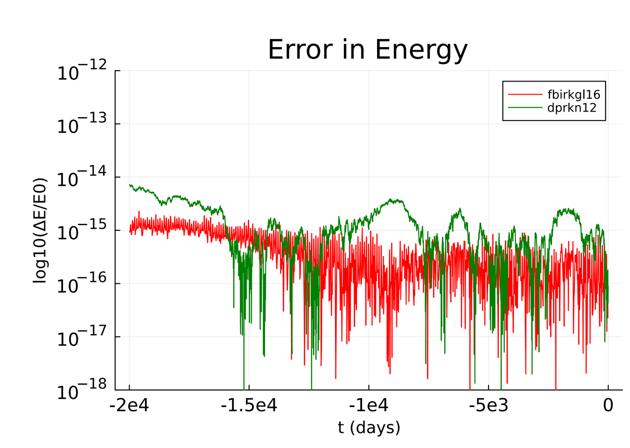


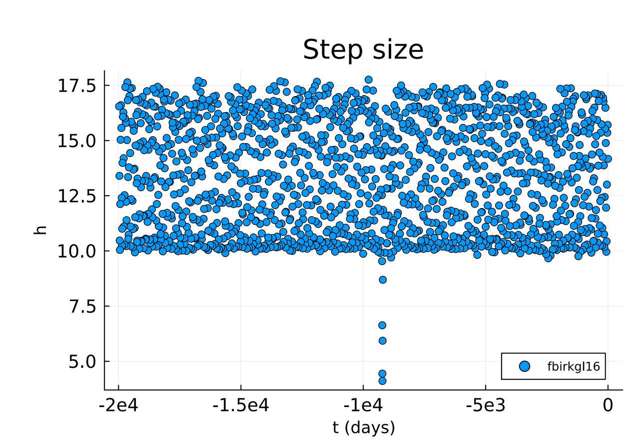


Results

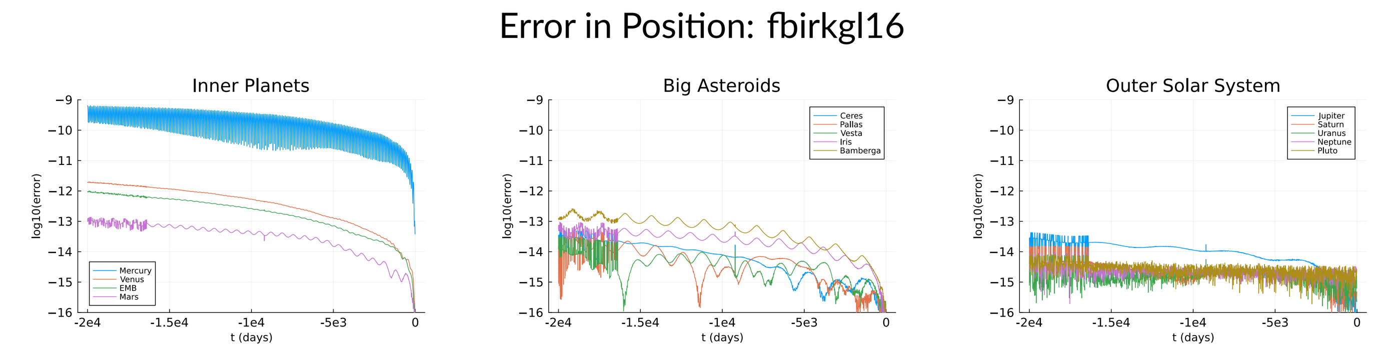
- Numerical experiment

package	method	implementation	tolerance	@btime	steps
NbodyIRKGL16.jl	fbirkgl16	generic	$\Delta\tau = 2.1$	215ms	$1,487$
NbodyIRKGL16.jl	fbirkgl16	simd	$\Delta\tau = 2.1$	46ms	$1,487$
OrdinaryDiffEq.jl	dprkn12		atol=rtol=$1e$-14	46ms	$5,250$

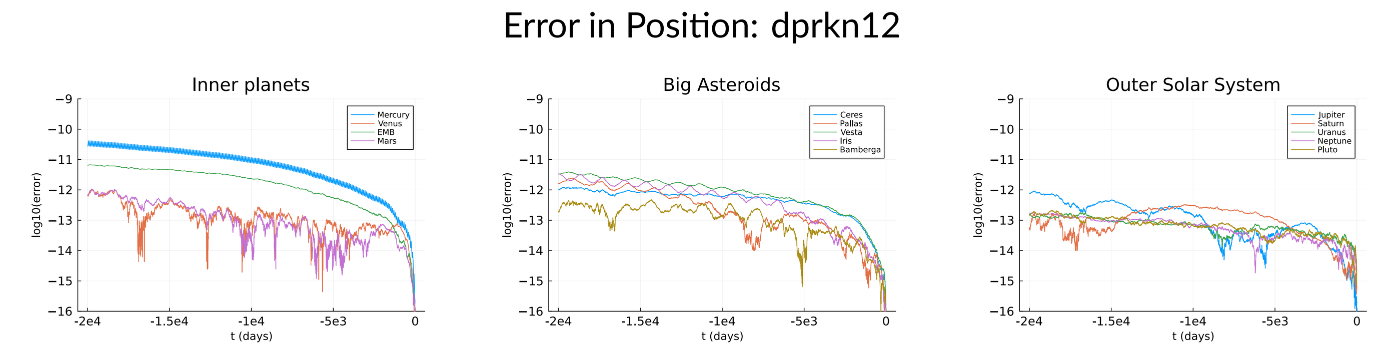




Error in Position: fbirkgl16



Error in Position: dprkn12



Our contribution

- Few-body integrator that incorporates a time-reversible adaptivity mechanism
- Outperform state-of-the-art high order explicit RK schemes thanks to SIMD-vectorization
- **Is there still room for improvement?**

References

[1] E.Hairer and D. Stoffer.
Reversible long-term integration with variable stepsizes.
SIAM Journal on Scientific Computing, 1997.

[2] J. Makazaga M. Antoñana, P. Chartier and A. Murua.
Global time-renormalization of the gravitational n-body problem.
SIAM Journal on Applied Dynamical System, 2020.

[3] J. Makazaga M. Antoñana, P. Chartier and A. Murua.
Majorant series for the n-body problem.
International Journal of Computer Mathematics, 2022.

Github repository

- https://github.com/mikelehu/NbodyIRKGL16.jl
- Jupyter notebooks are shared for reproducibility of the experiments