Few-body integrator with time-reversible adaptivity in Julia

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Purpose

- **Simulation**: long-time numerical integration of few-body gravitational problem involving close encounters:

$$\frac{dq_i}{dt} = v_i, \qquad \frac{dv_i}{dt} = \sum_{j\neq i} \frac{Gm_j}{\|q_j - q_i\|^3}(q_j - q_i).$$

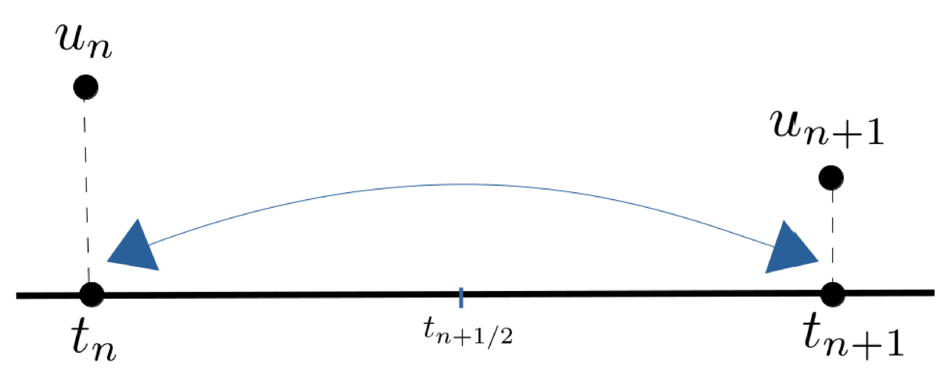
- **Integrator**: based on IRKGL16 symmetric and symplectic Implicit Runge-Kutta method of order 16
- **Automatic Step Size Control**:
  - Standard: good long-time behaviour is lost
  - Time-reversible adaptivity better long-time behaviour

Time-reversible adaptivity

- Consider the initial value problem

$$\frac{du}{dt} = f(u), \qquad u_0 = u(t_0)$$

where approximations are denoted as $u_n \approx u(t_n)$.



Forward and **backward integration**: same step size

Approach

- **Motivation**: constant step size in fictitious time τ

$$\frac{d\tau}{dt} = K(u(t))$$

with $K(u)$ proposed for n-body problems in [1, 2]

$$\Delta\tau = \tau_{n+1} - \tau_n = \int_{t_n}^{t_{n+1}} K(u(t))\,dt, \quad \approx (t_{n+1} - t_n)\sum_{i=1}^{s} b_i K(U_{n,i}).$$

- **Time-reversible adaptive NBIRKGL16 algorithm**:
  - for $n = 0, 1, 2, \dots$

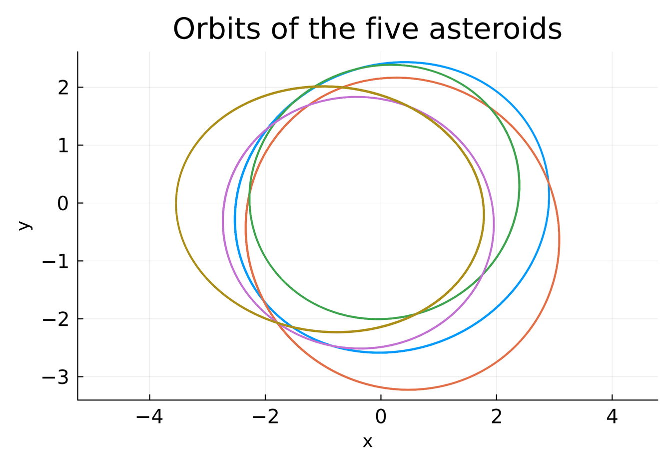
$$u_{n+1} = u_n + h_n \sum_{i=1}^{s} b_i f(U_{n,i})$$

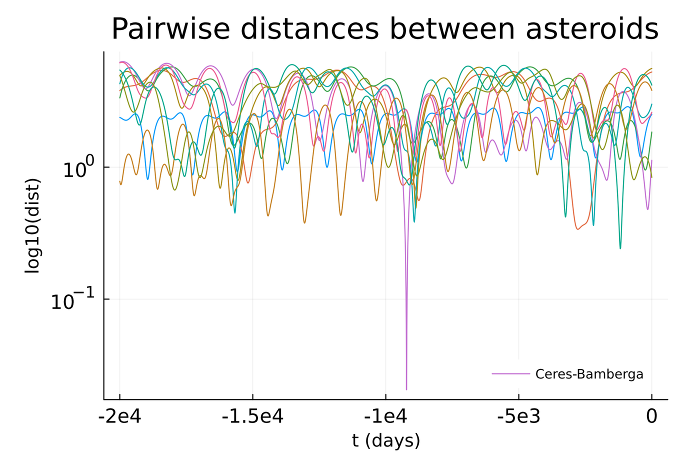
where $U_{n,i}$ and h_n are implicitly defined by

$$U_{n,i} = u_n + h_n \sum_{j=1}^{s} a_{ij} f(U_{n,j}), \qquad h_n = \frac{\Delta\tau}{\sum_{i=1}^{s} b_i K(U_{n,i})}.$$

Test problem

- 15-body model of the Solar System
  - The Sun, all eight planets of the Solar System, Pluto and the five main bodies of the asteroid belt
  - Close encounters between some of the asteroids during backward integration in time for $2e4$ days

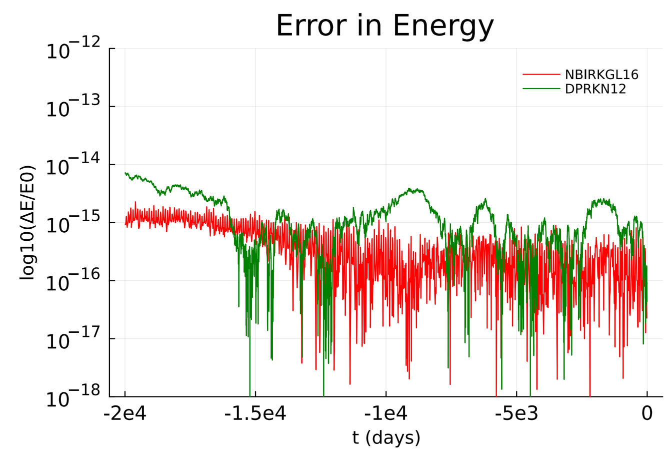


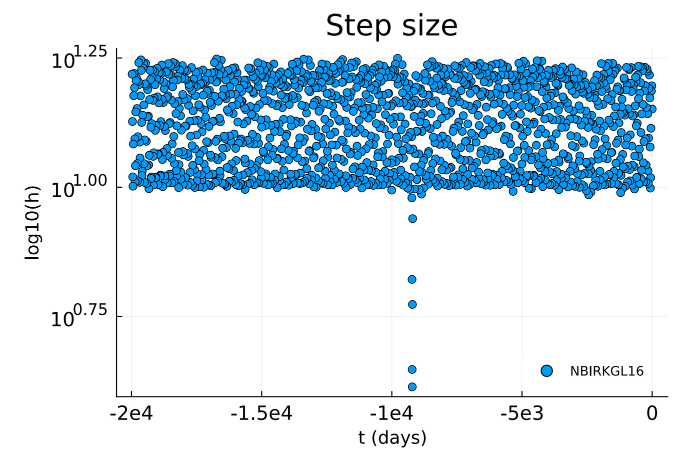


Results

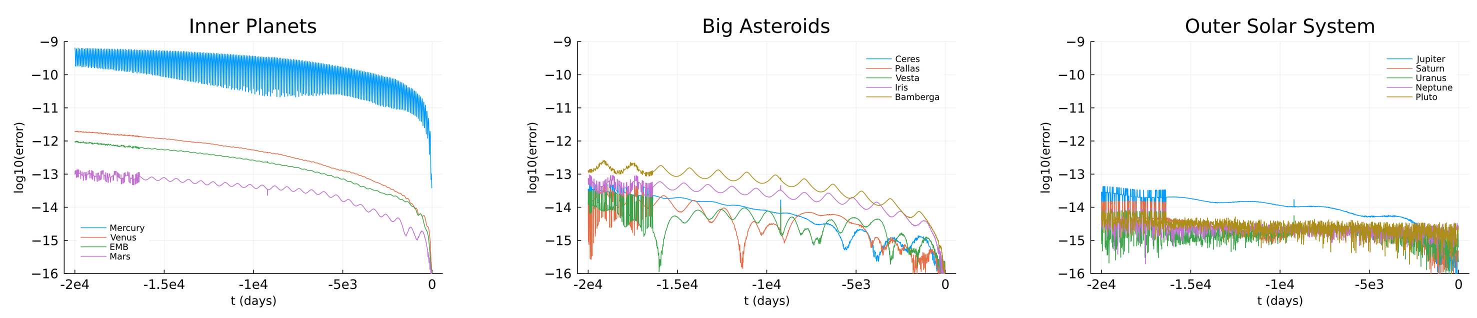
- Numerical experiment

package	method	implementation	tolerance	@btime	steps
NbodyIRKGL16.jl	NBIRKGL16	generic	$\Delta\tau = 2.1$	215ms	$1,487$
NbodyIRKGL16.jl	NBIRKGL16	simd	$\Delta\tau = 2.1$	46ms	$1,487$
OrdinaryDiffEq.jl	DPRKN12		atol=rtol=$1e$-14	46ms	$5,250$

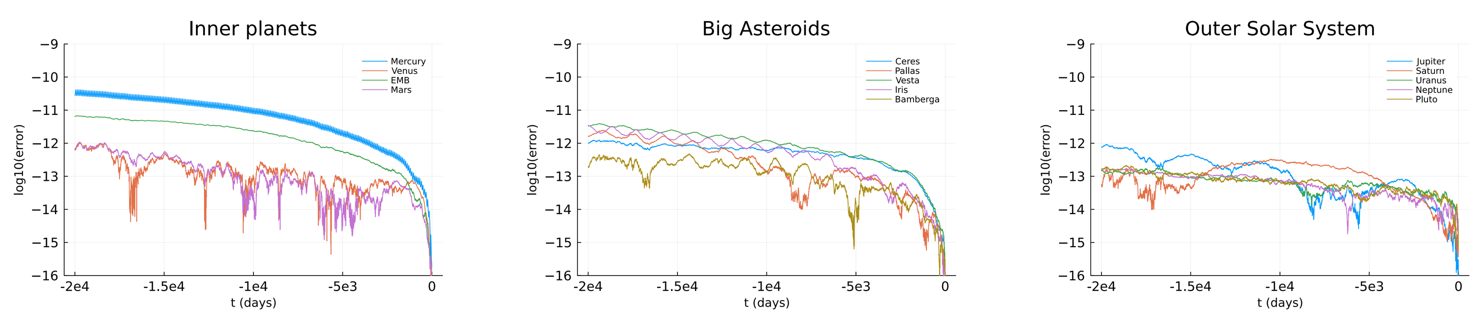




Error in Position: NBIRKGL16



Error in Position: DPRKN12



Our contribution

- Few-body integrator with time-reversible adaptivity that outperforms state-of-the-art high order explicit RKN schemes thanks to SIMD vectorization
- Is there still room for improvement?

References

[1] M. Antoñana, P. Chartier, J. Makazaga, and A. Murua. Global time-renormalization of the gravitational n-body problem (2020).

[2] M. Antoñana, P. Chartier, and A. Murua. Majorant series for the n-body problem (2022).

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Github repository

- https://github.com/mikelehu/NbodyIRKGL16.jl
- Jupyter notebooks for reproducibility of the experiments