Few-body integrator with time-reversible adaptivity in Julia

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Purpose

- **Simulation**: long-time numerical integration of few-body gravitational problem involving close encounters:

$$\frac{dq_i}{dt} = v_i$$
$$\frac{dv_i}{dt} = \sum_{j \neq i} \frac{Gm_j}{\|q_j - q_i\|^3}(q_j - q_i)$$

- **Integrator**: based on IRKGL16 symmetric and symplectic Implicit Runge-Kutta method of order 16
- **Automatic Step Size Control**:
  - Standard: good long-time behaviour is lost
  - Time-reversible adaptivity better long-time behaviour

Time-reversible adaptivity

- Consider the initial value problem

$$\frac{du}{dt} = f(u), \quad u_0 = u(t_0)$$

where approximations are denoted as $u_n \approx u(t_n)$.

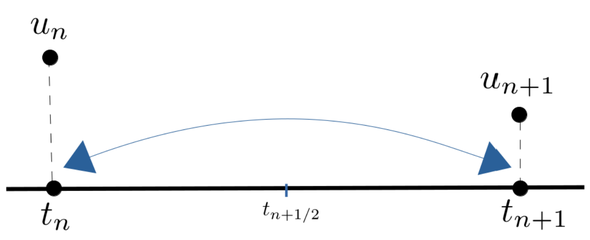


Figure 1. **Forward** and **backward integration**: same step size

Approach

- **Time-reversible adaptive fbirkgl16 algorithm**: for $n = 0, 1, 2, \ldots$

$$u_{n+1} = u_n + h_n \sum_{i=1}^{s} b_i f(U_{n,i}),$$

where $U_{n,i}$ and h_n are implicitly defined at each step by

$$U_{n,i} = u_n + h_n \sum_{j=1}^{s} a_{ij} f(U_{n,j}),$$
$$h_n = \frac{\Delta\tau}{\sum_{i=1}^{s} b_i K(U_{n,i})}.$$

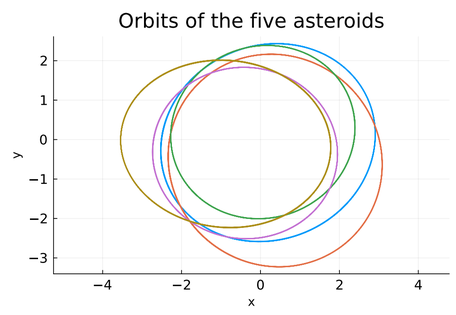
- $\Delta\tau$=user supplied constant to specify the tolerance
- $\boldsymbol{\sigma(u)}$**= step-size function for the** n**-body problem**:

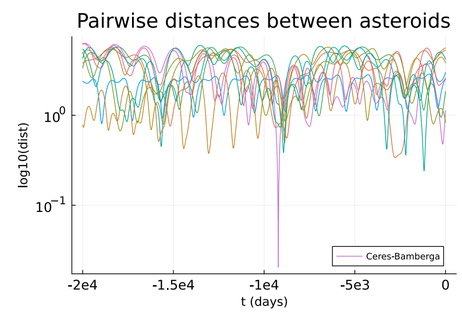
$$\sigma^{-1}(u_n) = \sum_{i=1}^{s} b_i K(U_{n,i}) \approx K(u(t_{n+1/2}))$$

where $K(u)$ is a time renormalization function for the n-body problem derived from the lower bound of the radius of convergence of the solution (see [3, p.179])

Test problem

- 15-body model of the Solar System
  - The Sun, all eight planets of the Solar System, Pluto and the five main bodies of the asteroid belt
  - Close encounters between some of the asteroids

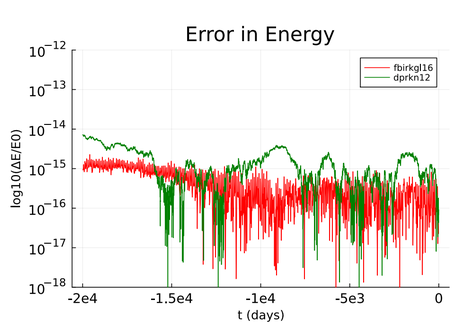


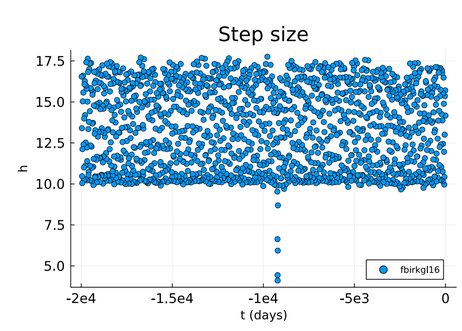


Results

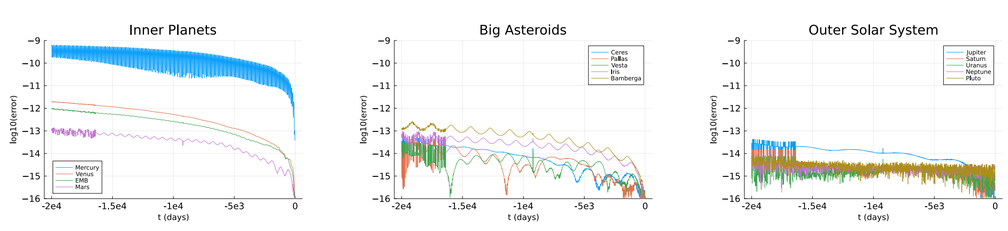
- Numerical integrations

package	method	implementation	tolerance	@btime	steps
	fbirkgl16	generic	$\Delta\tau = 2.1$	215ms	$1,487$
	fbirkgl16	simd	$\Delta\tau = 2.1$	46ms	$1,487$
OrdinaryDiffEq.jl	dprkn12		atol=rtol=$1e-14$	46ms	$5,250$

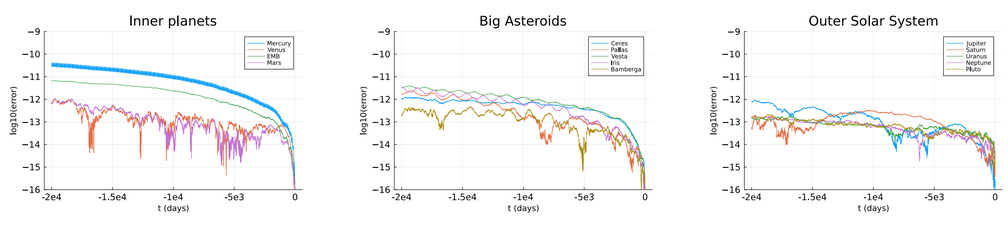




Error in Position: fbirkgl16



Error in Position: dprkn12



Our contribution

- Integrator for few-body problems that incorporate a time-reversible adaptivity mechanism
- Outperform state-of-the-art high order explicit RK schemes thanks to SIMD-vectorization.
- **but, is there still room for improvement?**

References

[1] E.Hairer and D. Stoffer.
Reversible long-term integration with variable stepsizes.
SIAM Journal on Scientific Computing, 1997.

[2] J. Makazaga M. Antoñana, P. Chartier and A. Murua.
Global time-renormalization of the gravitational n-body problem.
SIAM Journal on Applied Dynamical System, 2020.

[3] J. Makazaga M. Antoñana, P. Chartier and A. Murua.
Majorant series for the n-body problem.
International Journal of Computer Mathematics, 2022.

Julia package

- https://github.com/mikelehu/NbodyIRKGL16.jl