SYS 4581/6581 Project

Dynamic Delta Trading Strategy

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Contents

1 Introduction

Delta hedging is an options strategy that aims to reduce or hedge, the risk associated with price movements in the underlying asset. Here, the delta represents the change in the value of an option in relation to the movement in the market price of the underlying asset. This approach uses options to offset the risk to either a single other option holding or an entire portfolio of holdings. Since delta hedging attempts to neutralize or reduce the extent of the move in an option's price relative to the asset's price, it requires a constant rebalancing of the hedge. It works best when successive price movements are small, which in other words, slow accumulations of low intensity information. However, delta hedging does not work well when prices move due to large information shocks, because traders may over hedge if the delta is offset by too much or the markets change unexpectedly after the hedge is in place. Also, numerous transactions needed to constantly adjust the delta hedge will lead to costly fees.

2 Theoretical Base

2.1 European Option

A European call option is a financial security that gives its owner the right, but not the obligation, to purchase a predetermined asset at a pre-determined price and future time. The buyer of a European call option on stock A has the right, but not the obligation, to purchase A at a fixed strike price K on the expiration date T. Let S_T be the value of the stock on the expiration date, C_T be the value of the option at maturity. Then $C_T = max\{0, S_T - K\}$. Notice that the value of an option is always non-negative, since the buyer will not exercise if the expected return is below 0.

2.2 Black Scholes Model

Let $C(S,t)$ be the value of the option as a function of S, the price of the underlying stock, and the time t. Then for each share of underlying stock an investor owns, he must sell short $1/C_1(S,t)$ options to hedge his position, where $C_1(S,t) = \frac{\partial C}{\partial S}(S,t)$. The resulting portfolio value E is given by $E = S\frac{C}{C_1}$. Note that for such a hedged position, for small ΔS and a small time interval Δt, $\Delta E = \Delta S \frac{\Delta C}{C_1} \approx 0$.

Independent of whether the position is hedged continuously or not, the return on the above hedged position is certain, and by the arbitrage principle, the risk-free rate. If the position is hedged continuously, the return on the hedged position becomes certain. If the position is not hedged continuously, the portfolio's risk is small even for large changes in the underlying stock's price, and consists entirely of unsystematic risk. Hence, again by the arbitrage principle, the return on the hedged position is certain. Thus, the rate of return is given by the short term risk-free rate $r\Delta t$, and the total return is given by the equity position multiplied by this rate: $\Delta E = (S - \frac{C}{C_1})r\Delta t$, where r is the risk-free rate of return.
By stochastic calculus,

$$\Delta C = C_1 \Delta S + 1/2 C_{11} \sigma^2 S^2 \Delta t + C_2 \Delta t,$$

where σ^2 is the variance of the stock return. Then

$$\Delta E = -(\frac{1}{2}C_{11}\sigma^2 S^2 + C_2)\frac{\Delta t}{C_1} = (S-\frac{C}{C_1})r\Delta t$$

The equivalent partial differential equation is

$$C_2 = rC - rSC_1 - \frac{1}{2}\sigma^2 S^2 C_{11}.$$

Now consider the boundary conditions imposed by the structure of the call option. Let T be the maturity date of the option. Then

$$C(S,T) = max\{S-K, 0\}.$$

By considering the substitution

$$C(S,t) = e^{-r(T-t)}y[(2/\sigma^2)(r-\frac{\sigma^2}{2})[ln(S/K)-(r-\sigma^2/2)(T-t)]-(2/\sigma^2)(r-\frac{\sigma^2}{2})^2(T-t)]-(2/\sigma^2)(r-\frac{\sigma^2}{2})^2(T-t)]$$

the differential equation becomes

$$y_2 = y_{11}$$

with boundary conditions

$$y(u,0) = \begin{cases} 0, u < 0 \\ K[e^{u(\sigma^2/2)/(r-\sigma^2/2)}-1], u \geq 0 \end{cases}$$

Observe that the differential equation has reduced to the heat equation. We obtain

$$C(S,t) = S\mathcal{N}(d_1) - Ke^{-r(T-t)}\mathcal{N}(d_2)$$
$$d_1 = \frac{ln(S/K) + (r+\sigma^2/2)(T-t)}{\sigma\sqrt{T-t}}$$
$$d_2 = \frac{ln(S/K) + (r-\sigma^2/2)(T-t)}{\sigma\sqrt{T-t}}$$

where $\mathcal{N}(\cdot)$ is the standard normal cumulative density function.

2.3 Delta

First we note that by using the chain rule we find

$$\frac{\partial d_1}{\partial S} = \frac{\partial d_2}{\partial S} = \frac{1}{S} \cdot \frac{1}{\sigma\sqrt{T-t}}$$

We can then differentiate $C(S,t)$ equation with respect to S yields

$$\frac{\partial C}{\partial S} = \mathcal{N}(d_1) + \frac{1}{S} \cdot \frac{1}{\sigma\sqrt{T-t}}[S \cdot \phi(d_1) - Ke^{-r(T-t)} \cdot \phi(d_2)]$$

where $S \cdot \phi(d_1) - Ke^{-r(T-t)} \cdot \phi(d_2) = 0$, thus left

$$\Delta = \frac{\partial C}{\partial S} = \mathcal{N}(d_1).$$

2.4 Assumptions

All options are European options, in other words, they can only be exercised at maturity T.

3 Data and initial testing

3.1 Data Summary

We will use the the past five-year data of AAPL stock to test our dynamic delta hedging strategy. However, we had a hard time finding the the historical option data for the stock, since the data is hardly posted in any public database and most U.S stocks do not offer the trade for European options. Therefore, we will use the Black-Scholes model to price the options. As for the risk-free rate, we will always assume it is 2.00%. As for the volatility of the stock, which is one parameter cannot be directly observed in the market, we use the standard deviation formula for the volatility:

$$volatility = \frac{\sum(daily\ return - average\ return)^2}{trading\ period}$$

We get the AAPL stock data from Yahoo Finance. The data set contains the monthly data of AAPL from December 1, 2014 to November 1, 2019 (5-year time period) . Here are some simple statistics about the data:

Table 1: Numerical Summary of AAPL Stock in the Past Five Years

Stock	monthly return	monthly volatility	start	end
AAPL	1.67%	7.60%	110.83	267.25

3.2 Strategies

Here is our strategy implementation. Suppose at the beginning day of each monthly, the stock price is S_0. We short a European call option for 100 shares of AAPL stock with strike price $K = S_0(1+1.67\%)^3$. The maturity date is the beginning day after two months (so the option has a maturity of a quarter period). Convert the monthly volatility in the above table into quarter volatility and apply it into the Black-Scholes model to price the option. In the following days before the option mature, to "net" the value of the option, we long AAPL stock according to the

Delta on the daily base. We repeat this strategy for each quarters during this 5-year time period and compare the return with the two conventional strategies. In sum, these strategies are labeled as following,

- Dynamics 1: the delta hedging strategy as we just described above.
- Convention 1: only short the option at the beginning.
- Convention 2: Short the option and long 100 shares of AAPL stock at the beginning.

3.3 Test with Historical Data

Following figures and table show the test results using the historical data of AAPL.

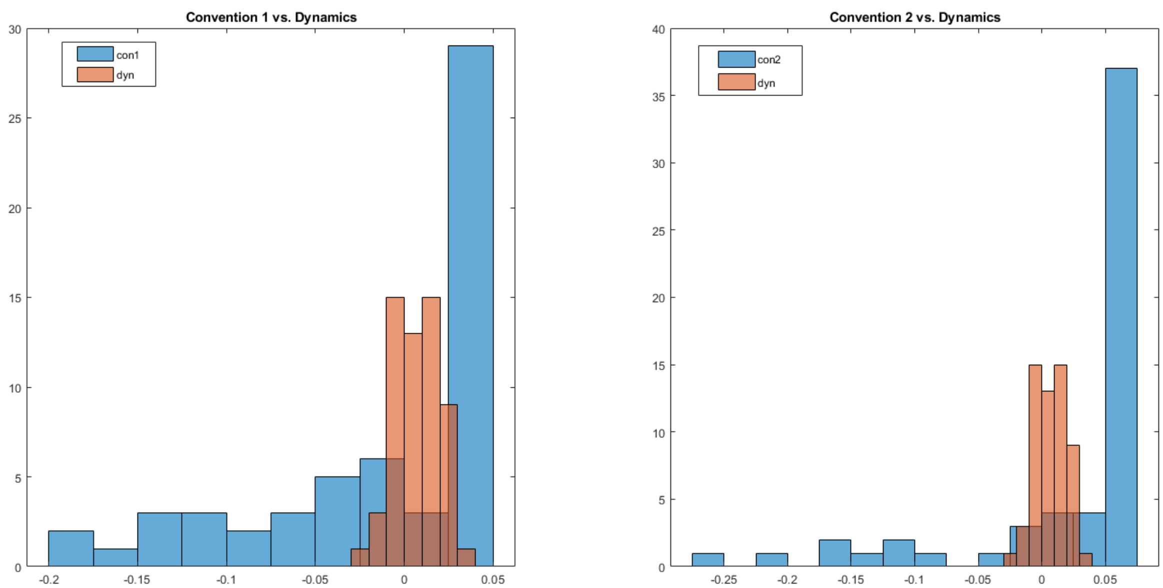

Figure 1: Histogram of return rates between convention strategies and delta hedging

Table 2: Numerical Summary of Quarter Returns of Strategies

	Avg. return	Std.	Min
con1	-1.4%	6.79%	-19.86%
con2	2.56%	8.73%	-27.33%
dyn	0.07%	1.23%	-2.02%

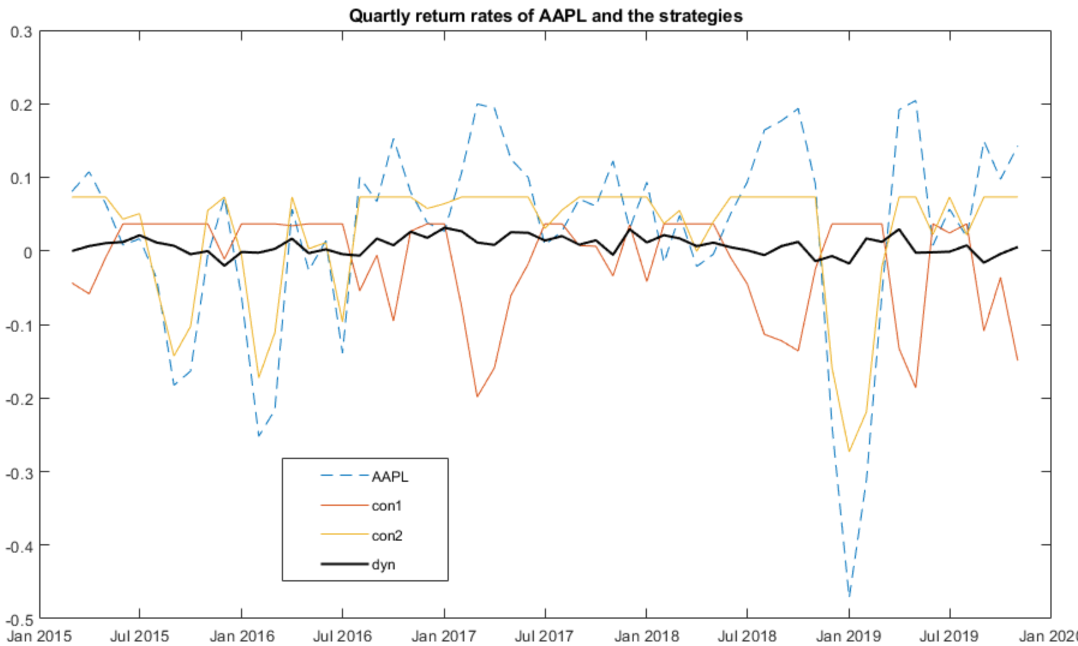

Figure 2: Quarterly return of AAPL and the strategies

Observing the histograms in Figure 1, we can see that the dynamic hedging strategy has a distribution concentrated around the 0.00% and has a lower tail compared with Convention 1 and Convention 2. Looking at Table 2, the dynamic strategy has the lowest standard deviation among the strategies. Also, according to the Figure 2 plotting the quarterly returns corresponding to the time line, we can see that during the most time, the return of the dynamic strategy is bounded by that of the conventional strategies, which demonstrates our dynamic strategy does hedge some risk.

However, noticing that both Convention 1 and Convention 2 has more returns lying around 0.05% in the histogram compared with the dynamic hedging strategy. Moreover, Convention 2 has an average return of 2.56% , which is much larger than the dynamic's 0.07%. As for the conventional strategies, it is partly because in the past five year AAPL stock has generally experienced a rising trend. As we know, the Apple Inc. company is one of the most high-technology company with a rapid growth in the world, and in the past five years, the price of AAPL stock has risen by almost 1.5 times. Convention 2 longs 100 shares of the stock at the beginning of each month, which definitely makes it benefit from the increasing price of the AAPL stock. It is reasonable that in a bull market, the hedging strategy usually cannot beat convention strategies because we trade off the return for less risk by "hedging". Also for Convention 1, since we make the strike price high enough, so the option would not be exercised or even when it was exercised, the cost of borrowing stock would not surplus the payoff of shorting the option.

3.4 Alternative Test

As we see in the last section, the data from real world may contains certain patterns that make the dynamic hedging strategy less compelling when comparing with the convention ones. Therefore, we conduct a alternative test, in which we will use the continuous-time multiplicative model to estimate the stock price in the future. Apply the estimated stock price to implement the both conventional and dynamic strategies. Then we use Monte-Carlo simulation method to get the distribution of the returns. The model estimates the stock price as:

$$S(t_{k+1}) = S(t_k)e^{v\Delta t + \sigma\epsilon(t_k)\sqrt{\Delta t}}$$

$$v = \mu - \frac{1}{2}\sigma^2$$

We use the closing price of the AAPL stock on Nov. 1st, 2019 as our starting price $S(t_0)$, and as before, use one quarter period to construct the strategies. Therefore, the ending price is $S(t_{90})$. Use the Monte-Carlo simulation method to simulate the returns for 10000 times. The results are as following:

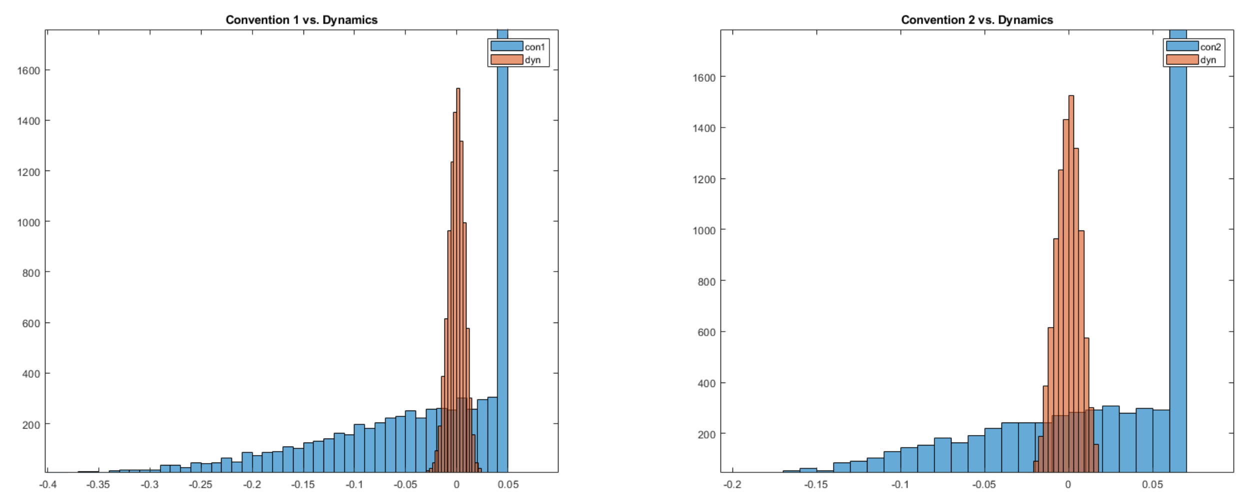

Figure 3: Histogram of return rates between convention strategies and delta hedging

Table 3: Numerical Summary of Quarter Returns of Strategies

	Avg. return	Std.	Min
con1	-2.18%	9.10%	-78.09%
con2	2.05%	6.75%	-35.81%
dyn	0.00%	2.01%	-44.60%

According to this test, we can see that in Figure 3, the histogram demonstrates that the dynamic hedging strategy obviously have lower tails than the two conventional strategies. Also, it beats

Convention 1 in terms of both average return rates and minimum returns. This shows that when assume the stock following the lognormal process (which is also a geometric Brownian motion process), in the long run the dynamic delta strategy should offer us hedging of the risk.

However, we can see that even under this ideal assumption, the dynamic strategy still barely get a positive return, which is far more less than the average return of Convention 2, 2.05%. In the following section, we will try to find the drawbacks and improvement for our dynamic hedging strategy.

4 Further exploration

4.1 Trading Frequency

According to our dynamic hedging strategy, we are estimating the delta and adjust the long stock position on the daily base. For the call option, when the stock price rises, the delta increases, and the value of the call option also increases. As we are shorting the call option, we also need to long more stock to balance with the increasing value of the option. Similarly, when stock price goes down, we sell stocks to balance with the decreasing value of the call option. Therefore, each time we balance the value of the option, we are buying the stock as it price rises and selling the stock as it price falls. Therefore, we may always lose money in this stock trading. Furthermore, we are trading on the daily base, which can accumulate the lost to a very large amount even though the delta has a very small change every day.

Therefore, we want to reduce the trading frequency to a proper amount so that we can emulate the returns of Convention 2 as well as not exposing to too much risk like Convention 2 does. For example, we can test the returns on different frequencies, like every 2 days, every 3 days, and so on. Also, we modify our initial setting to imitate the Convention 2. So the whole strategy becomes:

- Dynamic: Short the call option and buy 100 shares of AAPL stock. Then for every i days, adjust the long position of the stock according to the delta. $i = 1, 2, 3, \ ... \ , T$ (note: $i = T$ means that we don't adjust the long position at all, which is exactly Convention 2).

Then we use both the historical data of AAPL and the Monte-Carlo method (the procedure used in Section: Alternative test) to test the returns for different trading frequency.

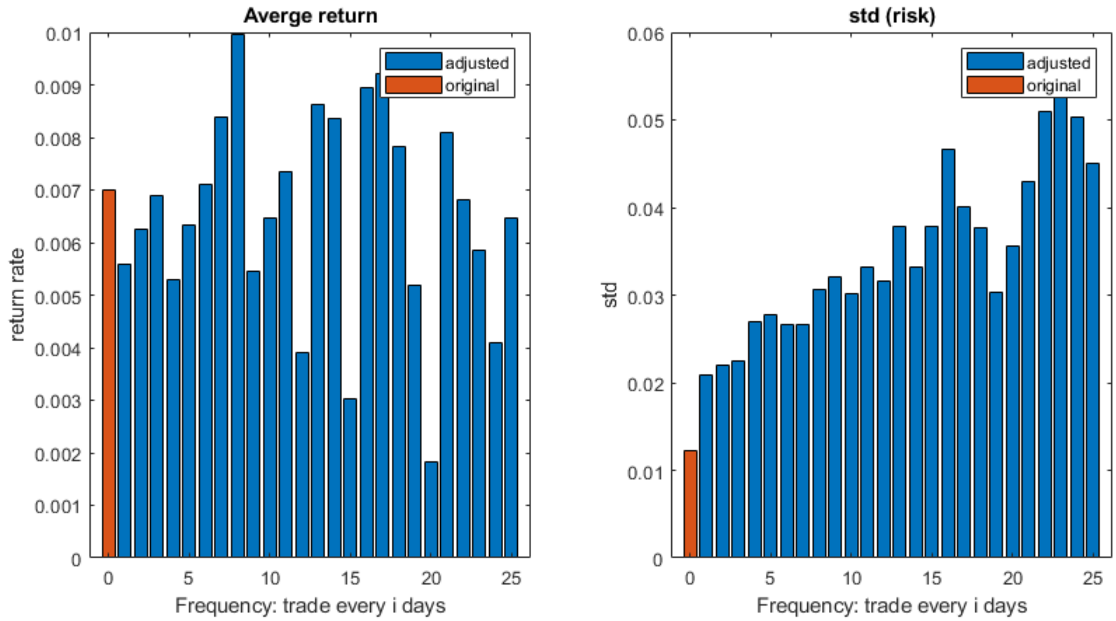

Figure 4: Barplot of Avg. return and std using historical data

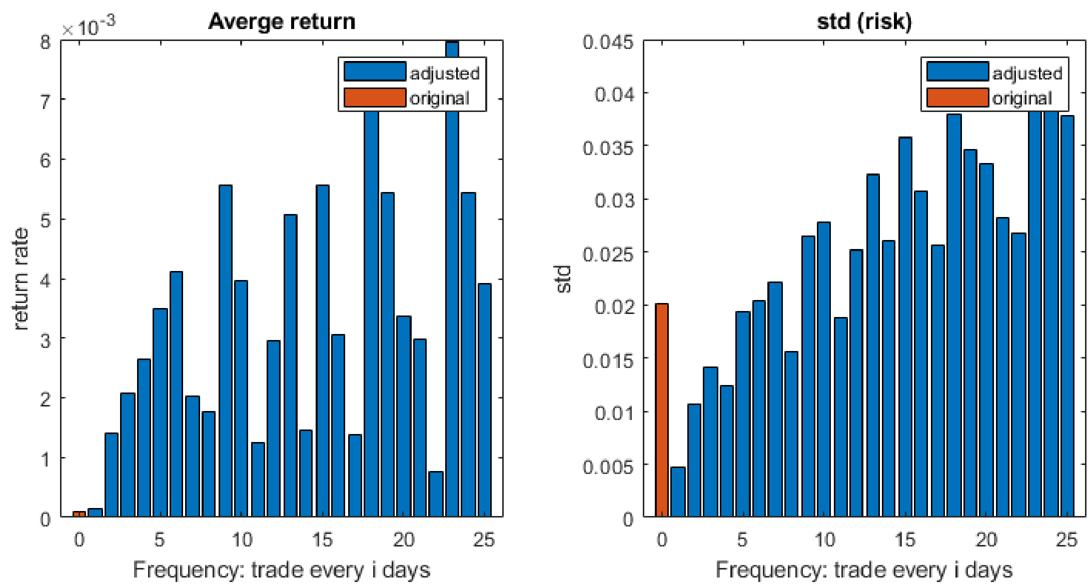

Figure 5: Barplot of Avg. return and std using Monte-Carlo method

From both tests, we can see that by shorting the stock at the beginning and adjusting the long position of the stock less frequently can give us more positive returns than the original one (Only short call option at the beginning and adjust long position on the daily base). As the std plots illustrate, reducing trading frequency does lead to a higher standard deviation, which means more risk. In this case with AAPL, by conclusion from the plots, we can decide to adjust the trading frequency to every 5-8 days, which gives us larger returns and keeps the relatively small risk.

4.2 Special Case Discussion

In Figure 1, it is clear that return rates of dynamic delta hedging always lay between those of con1 and con2, since dynamic hedging method will over-perform con1 when stock price keeps increasing before maturity, or it will over-perform con2 when stock price keeps decreasing before the maturity. However, the three consecutive options issued on Feb-1-19, Apr-1-19 and May-1-19 (matured on Jun-1-19, Jul-1-19 and Aug-1-19) have our dynamic hedging strategy the least preferable compared to 2 other methods.

In Figure 6, we display the daily delta changes in these three periods and compare them with delta changes in other six periods. And we also include a graph of corresponding stock changes of these nine periods in Figure 7.

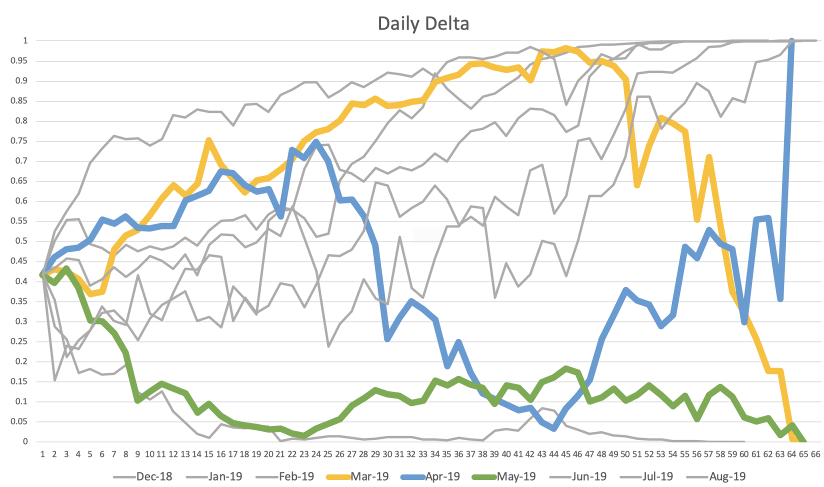

Figure 6: Daily Delta of the Six Picked Options

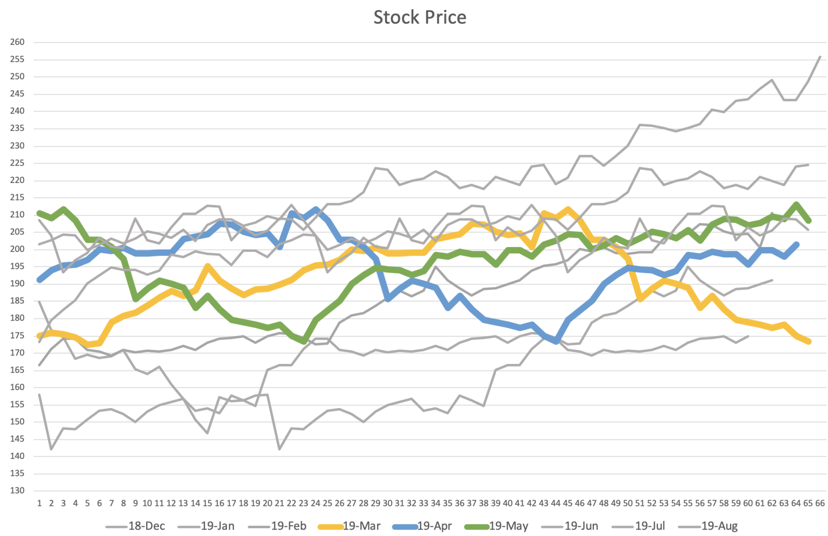

Figure 7: Stock Price of the Six Options

Observing delta of Feb-19, Apr-19 and May-19 in Figure 6, we can see that during these three periods, deltas either first increase then decrease, or first decrease then increase. Each one of them takes a large-scale of vacillation over the whole period, rather than vibrating in low volume and preserving an relatively smooth overall trend, either increase or decrease, like other periods. This abnormal changes in delta vehemently increase our cost when adopting the dynamic delta hedging. Because we are buying more stocks right after the stock price goes up (shown in Figure 7) and selling more stocks right after the stock price goes down. This is a systematic weakness of delta hedging. When confronting big changes in delta, this inherent drawback becomes more apparent.

5 Conclusion and Limitation

In our project, we first test the dynamic delta hedging strategy using the data of AAPL stock and we do see that this hedging strategy can reduce the risk we face. However, trading too frequently may largely reduce the potential returns we could have using the conventional strategies. Especially, when we have a stock like AAPL performing quite good in the past five years, meaning that a conventional investment strategy mostly will give us a better returns than the dynamic hedging strategy. In this case, we may want to reduce the frequency of dynamically hedging to capture more returns caused by the stock price rising. Also, we discuss some special cases in which the dynamic hedging strategy may have an extremely bad returns and lose to all other conventional strategies. Therefore, in our dynamic hedging strategy, a further dynamic adjustment for the trading frequency and a more sophisticated pricing model are needed to improve the return and risk involved with (in some extreme case, we may want to temporarily stop the dynamic hedging and go back to the conventional ones).

Furthermore, there are many limitations of the dynamic hedging strategy discussed in our project. One could be the implied volatility. In our project, we always fix the volatility by calculating standard deviation of the stock with the historical data. However, it is never the case in the reality, and as we all know, the option's prices are largely determined by the volatility. A fixed estimated volatility definitely can lead to the huge error in the estimation of option prices and deltas. A better way may be to acquiring the volatility by observing the price of the option in the real-world market.

Another limitation is that in the real-world market, for individual stocks, only American options are being traded among investor, rather than European options. Therefore, if we want to implement this strategy, we need to find ways to estimate the delta of American options or use index European options to cover our position in the stocks (most market indexes do offer European option trading). Both of these methods needs more complicated analysis and modeling to implement.

In addition, our original plan to discuss the effect of commission cost on our strategy's, since it is obvious that too frequent trading incurs a large amount of commission cost. However, recently, lots of stock brokerage firms in U.S., like Charles Schwab, E*TRADE, and TD Ameritrade, have announced to cancel commission fees for all stock trading. Therefore, I decide not to include commission cost anymore.

6 Codes

```
clear;
set(0, 'defaultTextFontSize',24);
% Import data from text file
% Setup the Import Options
opts = delimitedTextImportOptions("NumVariables", 7);

% Specify range and delimiter
opts.DataLines = [2, Inf];
opts.Delimiter = ",";

% Specify column names and types
opts.VariableNames = ["Date", "Open", "High", "Low", "Close", "AdjClose", "Volume"];
opts.VariableTypes = ["datetime", "double", "double", "double", "double", "double", "double"];
opts = setvaropts(opts, 1, "InputFormat", "yyyy-MM-dd");
opts.ExtraColumnsRule = "ignore";
opts.EmptyLineRule = "read";

% Import the data
M = readtable("AAPL.csv", opts);

% Clear temporary variables
clear opts
```

```
% Setup the Import Options
opts = delimitedTextImportOptions("NumVariables", 7);

% Specify range and delimiter
opts.DataLines = [2, Inf];
opts.Delimiter = ",";

% Specify column names and types
opts.VariableNames = ["Date", "Open", "High", "Low", "Close", "AdjClose", "Volume"];
opts.VariableTypes = ["datetime", "double", "double", "double", "double", "double", "double"];
opts = setvaropts(opts, 1, "InputFormat", "MM/dd/yyyy");
opts.ExtraColumnsRule = "ignore";
opts.EmptyLineRule = "read";

% Import the data
D = readtable("AAPL_D.csv", opts);


% Clear temporary variables
clear opts

%%
R = M.Close(end)/M.Open(1);
R = R^(1/60)-1; % monthly return
V = 0;
for i = 2:60
    r = (M.Close(i)-M.Close(i-1))/M.Close(i-1);
    V = V +(r-R)^2;
end

V  = sqrt(V/59); % volatility


%% Dynamic delta strategy
call = NaN(60,1);
Kc = NaN(60,1);
del = NaN(90,57);
C = NaN(90,57);
SH = NaN(90,57);
DELTA = NaN(60,1);
exe = boolean(zeros(60,1));
dyn = NaN(60,1);
con1 = NaN(60,1);
con2 = NaN(60,1);
QR = (1+R)^3;  % quartly return
```

```
QV = sqrt(3)*V; % quartly volatility
rf = (1+0.02)^(1/4)-1; % quartly risk-free rate
for i = 1:57
    d1 = M.Date(i); % start date
    d2 = M.Date(i+3); % mature date
    row1 = find(D.Date == d1);
    row2 = find(D.Date == d2);
    t = row2-row1; % duration
    T =([0:-1:-t]'+t)/(t);  % time values for black scholes model
    T(end,1) = 0.000001;  % correct the last entry to be non-zero
    S0 = D.Close(row1);
    Kc(i+3) = S0*QR;
    del(1,i) = normcdf((log(D.Close(row1)/Kc(i+3))+(rf+QV^2/2)*T(1))/(QV*sqrt(T(1))));

    call(i+3) = 100*callPrice(S0, Kc(i+3), rf, 1, QV);
    delta = zeros(t+1,1);
    interest = zeros(60,1);
    cost = 0;
    shares = 0;
    for j = 2:t+1
        dlc = (log(D.Close(row1+j-1)/Kc(i+3))+(rf+QV^2/2)*T(j))/(QV*sqrt(T(j)));
        delta(j) = normcdf(dlc);
        del(j,i) = delta(j);
        shares = (delta(j)-delta(j-1))*100;
        SH(j,i) = shares;
        cost = cost+interest(j-1)+ D.Close(row1+j-1)*shares;
        C(j,i) = cost;
        interest(j) = cost*((1+rf)^(1/t)-1);
    end

    cost2 = 100*D.Close(row1)*(1+rf); % cost for con2
    DELTA(i+3) = delta(j);
    if D.Close(row2)<Kc(i+3)
        exe(i) = 0;
        con1(i+3) = call(i+3);
        con2(i+3) = call(i+3)+100*D.Close(row2)-cost2;
        dyn(i+3) = call(i+3)-cost;
    else
        exe(i) = 1;
        con1(i+3) = call(i+3) + 100*(Kc(i+3)-D.Close(row2));
        con2(i+3)= call(i+3)+100*Kc(i+3)-cost2;
        dyn(i+3) = call(i+3)+100*Kc(i+3)-cost;
    end
        % convert to return rates
        dyn(i+3) = dyn(i+3)/(D.Close(row1)*100);
        con1(i+3) = con1(i+3)/(D.Close(row1)*100);
```

```
        con2(i+3) = con2(i+3)/(D.Close(row1)*100);


end

[mean(con1(4:60)),mean(con2(4:60)),mean(dyn(4:60))]
[std(con1(4:60)),std(con2(4:60)),std(dyn(4:60))]
[min(con1(4:60)),min(con2(4:60)),min(dyn(4:60))]

subplot(1,2,1); histogram(con1,'BinWidth',0.025);
hold on
histogram(dyn);
hold off
title('Convention 1 vs. Dynamics');
legend('con1','dyn')

subplot(1,2,2); histogram(con2,'BinWidth',0.025);
hold on
histogram(dyn);
hold off

title('Convention 2 vs. Dynamics');
legend('con2','dyn')


A = table(call,Kc,DELTA,con1,con2,dyn);
table = [M,A];

%% draw the line plot
r = NaN(60,1);
for i=4:60
    r(i) = (table.Open(i)-table.Open(i-3))/table.Open(i);
end

table.r = r;
subplot(1,1,1); plot(table.Date,table.r,'--');
hold on
plot(table.Date, table.con1);
plot(table.Date, table.con2);
plot(table.Date, table.dyn,'LineWidth',1.5,'Color','k');
hold off

legend('AAPL','con1','con2','dyn');
title('Quartly return rates of AAPL and the strategies');

%% alternative test
```

```
    con1 = zeros(10000,1);
    con2 = zeros(10000,1);
    dyn = zeros(10000,1);
for i = 1:10000
    S = zeros(91,1);
    S(1) = M.Close(end);
    T = [90:-1:0]/90;
    T(end) = 0.00001;
    Kc = S(1)*1.03;
    delta = zeros(91,1);
    interest = zeros(91,1);
    call = callPrice(S(1),Kc, rf, T(1),QV);
    v = QR-1/2*QV^2-1;
    cost = 0;
    for j = 2:91
        S(j) = S(j-1)*exp(v/90+normrnd(0,1)*QV/sqrt(90));
        dlc = (log(S(j)/Kc)+(rf+QV^2/2)*T(j))/(QV*sqrt(T(j)));
        delta(j) = normcdf(dlc);
        shares = delta(j)-delta(j-1);
        cost = cost+interest(j-1)+ S(j)*shares;
        interest(j) = cost*((1+rf)^(1/90)-1);
    end

    if S(end)<Kc
        con1(i) = call;
        con2(i) = call+(S(end)-S(1)*(1+rf));
        dyn(i) = call-cost;
    else
        con1(i) = call+(Kc-S(end));
        con2(i) = call+(Kc-S(1)*(1+rf));
        dyn(i) = call+Kc-cost;
    end
        dyn(i) = dyn(i)/(S(1));
        con1(i) = con1(i)/(S(1));
        con2(i) = con2(i)/(S(1));
end

%% draw the histogram
subplot(1,2,1); histogram(con1);
hold on
histogram(dyn);
hold off
title('Convention 1 vs. Dynamics');
legend('con1','dyn')

subplot(1,2,2); histogram(con2);
```

```
hold on
histogram(dyn);
hold off

title('Convention 2 vs. Dynamics');
legend('con2','dyn')

[mean(con1),mean(con2),mean(dyn)]
[std(con1),std(con2),std(dyn)]
[min(con1),min(con2),min(dyn)]


%% frequency analysis 1
Utable = zeros(60,40);
Vtable = NaN(57,40);
QR = (1+R)^3;  % quartly return
QV = sqrt(3)*V; % quartly volatility
rf = (1+0.02)^(1/4)-1; % quartly risk-free rate
for i = 1:57
    d1 = M.Date(i); % start date
    d2 = M.Date(i+3); % mature date
    row1 = find(D.Date == d1);
    row2 = find(D.Date == d2);
    t = row2-row1; % duration
    S0 = D.Close(row1);
    Kc = S0*QR;

    call = 100*callPrice(S0, Kc, rf, 1, QV);
    for p = 1:t/2
        delta = zeros(t+1,1);
        interest = zeros(60,1);
        shares = 100;
        delta(1) = 1;
        cost = shares*D.Close(row1);
            T0 =[t:-p:0]';  % time values for black scholes model
            if(T0(end) ~= 0)
                T = [T0;0.000001];  % correct the last entry to be non-zero
            else
                T0(end) = 0.00001;
                T = T0;
            end
            T1 =flip(T);
            T = T/t;
    for j = 2:length(T1)
        dlc = (log(D.Close(row1+T1(j))/Kc)+(rf+QV^2/2)*T(j))/(QV*sqrt(T(j)));
        delta(j) = normcdf(dlc);
```

```
        shares = (delta(j)-delta(j-1))*100;
        cost = cost+interest(j-1)+ D.Close(row1+T1(j))*shares;
        interest(j) = cost*((1+rf)^(1/t)-1);
    end

    if D.Close(row2)<Kc
        dyn = call-cost;
    else
         dyn= call+100*Kc-cost;
    end
        % convert to return rates
        dyn = dyn/(D.Close(row1)*100);
    Utable(i,p) = dyn;
    end

end

u = NaN(32,1);
for i=1:30
    u(i) = mean(Utable(:,i));
    s(i) = std(Utable(:,i));
end

subplot(1,2,1);bar(1:25, u(1:25)); title('Averge return'); xlabel('Frequency: trade every i days');
hold on
bar(0,0.007); legend('adjusted','original')
hold off
subplot(1,2,2);bar(1:25, s(1:25)); title('std (risk)'); xlabel('Frequency: trade every i days'); yla
hold on
bar(0,0.0123); legend('adjusted','original')
hold off
%% Frequency analysis 2
%  alternative test
    Utable = NaN(1000,45);
for i = 1:1000
    S = zeros(91,1);
    S(1) = M.Close(end);
    for p = 1:45

            T0 =[90:-p:0]';  % time values for black scholes model
            if(T0(end) ~= 0)
                T = [T0;0.000001];  % correct the last entry to be non-zero
            else
                T0(end) = 0.00001;
                T = T0;
            end
```

```
            T1 =flip(T);
            T = T/90;
    Kc = S(1)*QR;
    delta = zeros(91,1);
    delta(1) = 1;
    interest = zeros(91,1);
    call = callPrice(S(1),Kc, rf, T(1),QV);
    shares = 1;
    v = QR-1/2*QV^2-1;
    cost = S(1);
    for j = 2:91
        S(j) = S(j-1)*exp(v/90+normrnd(0,1)*QV/sqrt(90));
    end
    for j = 2:length(T)
        dlc = (log(S(T1(j))/Kc)+(rf+QV^2/2)*T(j))/(QV*sqrt(T(j)));
        delta(j) = normcdf(dlc);
        shares = delta(j)-delta(j-1);
        cost = cost+interest(j-1)+ S(T1(j))*shares;
        interest(j) = cost*((1+rf)^(1/90)-1);
    end

    if S(end)<Kc
        dyn = call+delta(length(T))*S(end)-cost;
    else
        dyn = call+delta(length(T))*Kc-cost;
    end
        dyn = dyn/(S(1));
        Utable(i,p) = dyn;
    end
end
%%
for i=1:25
    u(i) = mean(Utable(:,i));
    s(i) = std(Utable(:,i));
end

subplot(1,2,1);bar(1:25, u(1:25)); title('Averge return'); xlabel('Frequency: trade every i days');
hold on
bar(0,0.0001); legend('adjusted','original')
hold off
subplot(1,2,2);bar(1:25, s(1:25)); title('std (risk)');\\
xlabel('Frequency: trade every i days'); ylabel('std');
hold on
bar(0,0.0201); legend('adjusted','original')
hold off
%%
```

```
function [call] = callPrice(S0, Kc, rf, T, s)

d1c = (log(S0/Kc) + (rf+s^2/2)*T)/(s*sqrt(T)); d2c = d1c-s*sqrt(T);
call = S0*normcdf(d1c)-Kc*exp(-rf*T)*normcdf(d2c);

end
```