W.H.S Mathematics Presentation

Think about Mathematics and Physics naturally

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Prof. Ramirez's SP 2018 3rd pd. Physics

April 4^{th}, 2018

Notation, Formulas, and Equations

Notation should be pretty standard complex analysis notation[1]. The space of complex numbers is denoted $\mathbb{C}$, by convention, the variable used to denote a function of a complex variable the letter z is used.

$$z := x + iy, \ \forall z \in \mathbb{C}; \quad \Re z = x; \quad \Im z = y; \quad \|z\| = \sqrt{x^2 + y^2};$$

Polar representation of complex numbers requires information about the angle a complex number z makes with the positive $\mathbb{R}$ -axis, for this discussion. The angle θ a complex number z makes is called the argument of the number z:

$$\operatorname{Arg} z = \theta = \arccos \frac{\Re z}{\|z\|} = \arcsin \frac{\Im z}{\|z\|} = \arctan \frac{\Im z}{\Re z} + \gamma;$$

where γ:

$$\gamma \iff \pi, \ \{x < 0, \ y > 0\}, \quad \text{or} \quad \{x < 0, \ y < 0\};$$
$$\gamma \iff 2\pi, \ \{x > 0, \ y < 0\};$$

The complete polar representation of a complex number z below:

$$r = \|z\|; \quad \operatorname{cis} \theta = \cos \theta + i \sin \theta; \quad z = r \operatorname{cis} \theta;$$

These tools are used in the presentation for the proof outlined.

[1] Actual notation used comes from this book: Joseph Bak, Donald J. Newman, 'Complex Analysis', 3rd ed.

Contents

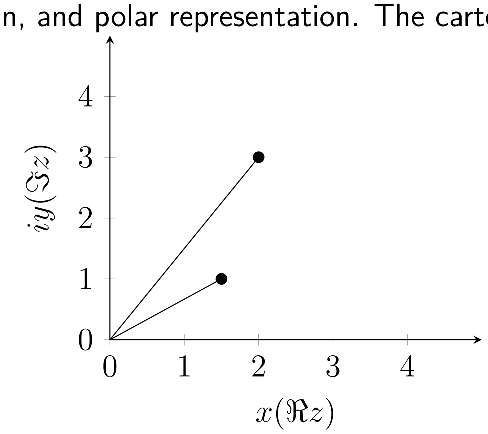
1 Introduction

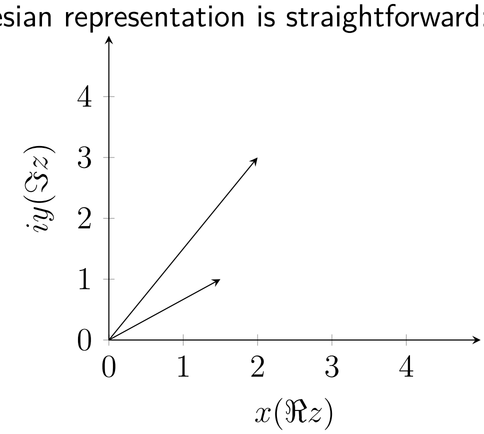
Physics uses mathematical descriptions to model events in the physical world. Consequentially, a firm foundation in mathematics is useful to the modern physicist. Today I am here to explain another way to think about mathematics to facilitate learning more difficult problems.

2 Roots of Complex Numbers

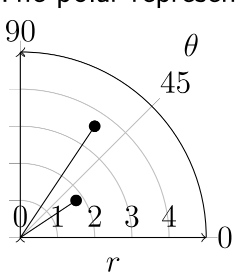
The complex plane imposes interesting results due to the structure of the numbers in the space. Notably, the entire complex plane is isomorphic[2] to the space $\mathbb{R}^2$, the cartesian plane, the complex numbers cannot have an 'traditional' binary order operation, and complex numbers lend themselves to vector equations/operations. All the implications of the structure of the complex numbers are out of scope for now, I want to introduce an interesting problem to solve from the complex plane.

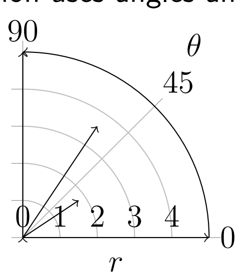
There are two conventional ways of representing the complex numbers in mathematics: cartesian, and polar representation. The cartesian representation is straightforward:





The polar representation uses angles and lengths to represent the same complex numbers.





The type of problem I want to solve is best solved via the polar representation of complex numbers.

[2]Not sure if correct term to use here.