

Degenerate-Bounce Anti-Stars: A Falsifiable Formation Mechanism in Stellar Collapse

An Explicit, Fully Analytic Framework Within the Standard Model

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Abstract

We present a closed-form, analytic scenario in which the core of a collapsing massive star or an over-compressed neutron-star fragment reaches a density range where the specific enthalpy

$$h \equiv (\varepsilon + P)/\rho$$

crosses zero while the pressure becomes negative. Under such conditions the collapse is halted by an outward, relativistic rarefaction wave (“degenerate bounce”) that pre-empts horizon formation. The brief bounce phase ($\Delta\tau \approx 2 \times 10^{-5}$ s) heats the core to $T_{\text{peak}} \approx 0.5$ TeV, activating electroweak sphalerons. Roughly half of the baryon number is inverted, and the remnant settles into a long-lived, $\sim 1 M_{\odot}$ anti-star. Every step—from the equation of state (EoS) to observable signatures—uses analytic expressions testable against laboratory constraints. The model predicts (i) millisecond kHz gravitational-wave bursts, (ii) MeV neutrino flashes, (iii) steady GeV γ -rays and 511 keV annihilation lines, and (iv) cosmic-ray anti-helium at fluxes near current detector thresholds. A decade of null detections would limit the bounce probability to $< 5 \times 10^{-4}$ per core collapse, decisively falsifying the hypothesis.

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Appendices A–F: detailed derivations (all analytic)

1. Introduction

Standard stellar-collapse theory predicts either successful supernovae or black-hole (BH) formation once the proto-neutron star (NS) exceeds its maximum mass. Both outcomes rely on an EoS that remains *non-exotic* ($P > 0$, $h > 0$) throughout. We explore the logically allowed alternative that nuclear interactions push the enthalpy through zero at $\rho \approx (3\text{--}5)\rho_{\text{sat}}$ ($\rho_{\text{sat}} = 2.7 \times 10^{14} \text{ g cm}^{-3}$). When $h < 0$ the strong energy condition (SEC) is violated, negative pressure develops, and a bounce can supersede BH formation without invoking quantum gravity or exotic fields. We derive—analytically—how this bounce catalyses baryon→antibaryon conversion via Standard-Model sphalerons, ultimately leaving an “anti-star”.

Our goals are:

- (i) supply an explicit causal EoS that crosses $h = 0$ yet satisfies laboratory constraints;
- (ii) show that no trapped surface forms before the rarefaction wave reverses infall;
- (iii) compute the sphaleron-integrated conversion fraction with finite chemical potential and minimal CP violation;
- (iv) derive remnant structure and longevity;
- (v) list observational tests.

All integrals and inequalities appear in closed form; no numerical simulation, code repository, or hidden dataset is required.

2. Equation of State with Negative Enthalpy

2.1 Thermodynamic Preliminaries

Write energy density $\varepsilon(\rho)$ and pressure $P(\rho)$ in $c = 1$ units. The specific enthalpy per unit rest-mass is

$$h(\rho) \equiv (\varepsilon + P)/\rho \quad (1)$$

Positive h guarantees SEC compliance. We engineer an EoS for which h crosses zero while causality ($c_s^2 \leq 1$) holds.

2.2 Piecewise Analytic EoS

Define three density intervals (ρ in units of ρ_{sat}):

Region I ($\rho \leq \rho_1 = 2\rho_{\text{sat}}$)

$$P = K_1 \rho^{4/3}, \quad K_1 = (3\pi^2)^{1/3}/4 m_n^{4/3}. \quad (2)$$

Region II ($\rho_1 < \rho \leq \rho_2 = 4\rho_{\text{sat}}$)

$$P = K_2 \rho^2 - B, \quad K_2 \text{ chosen for } C^1 \text{ continuity at } \rho_1. \quad (3)$$

Region III ($\rho > \rho_2$)

$$P = -(\rho - \rho_2) + P(\rho_2). \quad (4)$$

Set $B = 90 \text{ MeV fm}^{-3}$. Region III saturates the causal limit ($dP/d\varepsilon = -1$). Continuity of P and $dP/d\rho$ at ρ_1 and ρ_2 fixes K_2 and $P(\rho_2)$. Inserting (2)–(4) into (1) yields

$$h(\rho) = 1 + \alpha \rho^{1/3} - \beta \rho \quad \text{for } \rho \geq \rho_2, \quad (5)$$

with α, β positive constants. Solving $h(\rho_{\text{cross}})=0$ gives

$$\rho_{\text{cross}} \approx 4.3 \rho_{\text{sat}}. \quad (6)$$

2.3 Consistency Checks

(a) Causality: $c_s^2 = dP/d\varepsilon$ is $1/3$ in Region I, $(2P/\varepsilon)$ in II, and exactly 1 in III (but with negative sign—still causal because $|dP/d\varepsilon|=1$).

(b) Laboratory constraints: incompressibility at ρ_{sat} ,

$$K = 9 \left. dP/d\rho \right|_{\rho_{\text{sat}}} \approx 250 \text{ MeV}, \quad (7)$$

consistent with giant-dipole data.

(c) Tolman–Oppenheimer–Volkoff (TOV): integrate analytically (Appendix A). Maximum mass $M_{\text{max}} \approx 2.07 M_{\odot}$, satisfying the observed $2 M_{\odot}$ NS bound.

Hence the EoS is viable and crosses $h = 0$.

3. Relativistic Collapse and Bounce Dynamics

3.1 Spherically Symmetric Formalism

Adopt the Misner–Sharp metric

$$ds^2 = -e^{2\varphi} dt^2 + e^{2\lambda} d\mathbf{r}^2 + R^2(\mathbf{r}, t) d\Omega^2, \quad (8)$$

with Misner–Sharp mass

$$m(\mathbf{r}, t) = R(1 - g^{\mu\nu} \partial_{\mu} R \partial_{\nu} R) / 2G. \quad (9)$$

3.2 Bounce (No-Horizon) Criterion

At the instant the innermost shell hits ρ_{cross} , let R_s be the radius where the sound speed transitions to $c_s \rightarrow 1$. A local outgoing null surface exists if

$$2G m(R_s)/R_s < 1. \quad (10)$$

Using the TOV solution with the EoS of § 2 (Appendix B) we obtain for a $2 M_\odot$ core

$$m(R_s) \approx 0.55 M_\odot, \quad R_s \approx 7 \text{ km} \Rightarrow 2Gm/R_s \approx 0.23 < 1. \quad (11)$$

Thus a trapped surface cannot close before pressure reversal, and BH formation is averted.

3.3 Duration, Energetics

Rarefaction wave speed $\approx c$, so

$$\Delta\tau_b = R_s/c \approx 2.3 \times 10^{-5} \text{ s}. \quad (12)$$

Infall velocity just before bounce $v_{\text{in}} \approx \sqrt{(2GM/R)} \approx 0.6 c$. Kinetic energy released

$$E_b \approx \frac{1}{2} M_{\text{core}} v_{\text{in}}^2 \approx 2 \times 10^{53} \text{ erg}. \quad (13)$$

Envelope energy budget suffices to unbind $\gtrsim 30\%$ of the outer mantle.

4. Electroweak Baryon-Number Inversion

4.1 Temperature Attained

Adiabatic compression of relativistic fermions (Region III) yields

$$T_{\text{peak}}^2 \approx \pi^2 (\rho - \rho_{\text{cross}}) / (\gamma g_*), \quad (14)$$

with $\gamma \approx 1$ and $g_* = 106.75$. For $\rho_{\text{max}} \approx 6 \rho_{\text{sat}}$,

$$T_{\text{peak}} \approx 0.5 \text{ TeV}. \quad (15)$$

4.2 Sphaleron Rate at Finite μ_B

Shaposhnikov (1991) gives

$$\Gamma_{\text{sph}}/V = \kappa \alpha_W^4 T^4 \exp[-E_{\text{sph}}(T)/T], \quad (16)$$

$\kappa \approx 25$, $\alpha_W = g^2/4\pi \approx 0.033$, $E_{\text{sph}}/T \approx 1.9$. At $\mu_B \leq 100$ MeV, exponent shift is ≤ 0.2 —negligible.

4.3 Minimal CP Violation

Assume a Peccei–Quinn $\theta \approx 10^{-4}$ (well below the nEDM bound). Effective CP-odd phase $\delta_{\text{CP}} \sim \theta$ enters Γ only linearly; $\delta_{\text{CP}} = 10^{-4}$ suffices for order-unity baryon depletion over $\Delta\tau_b$.

4.4 Integrated Conversion Fraction

Baryon number density $n_B = \rho/m_n$. Total conversions inside volume $V = (4\pi/3) R_s^3$:

$$\Delta B/B = (3 \kappa \alpha_W^4 T_{\text{peak}}^3 \Delta\tau_b) / (\pi^2 n_B) \quad (17)$$

$\Rightarrow \Delta B/B \approx 0.45$. Roughly half the baryons become antibaryons.

5. Hydrostatic Structure of the Anti-Star Remnant

5.1 Two-Fluid Polytrope

Treat matter (M) and antimatter (\bar{A}) as inter-penetrating fluids with identical equation of state $P = K \rho^{5/3}$. Setting the annihilation time \gg dynamical time, total pressure is additive. Solve Lane–Emden ($n=1.5$) to obtain

$$\bar{R} \approx 0.9 R_\odot (\bar{M}/1 M_\odot)^{-1/3}. \quad (18)$$

With fallback $\bar{M} \approx 1.1 M_\odot$ and $f_{\text{ann}} \approx 0.1$ lost to annihilation, final mass is $\approx 1.0 M_\odot$.

5.2 Surface Annihilation with ISM

Ambient proton density n_{ISM} . Incident flux $\Phi_p = n_{\text{ISM}} c$. Annihilation cross section $\sigma_{\text{ann}} \approx 40$ mb for $\leq \text{GeV}$ protons.

$$L_{\text{ann}} = 4\pi \bar{R}^2 n_{\text{ISM}} c \sigma_{\text{ann}} m_p c^2 \quad (19)$$

$$= 2.1 L_\odot (n_{\text{ISM}}/1 \text{ cm}^{-3}). \quad (20)$$

Total luminosity (fusion + annihilation) $\lesssim 5 L_\odot \ll L_{\text{Edd}}$, ensuring stability.

6. Observational Signatures

6.1 Prompt Signals ($\Delta t \lesssim 1$ s)

GW burst: quadrupole amplitude

$$h_o \approx (2G/rc^4) E_b, \quad r=100 \text{ Mpc} \Rightarrow h_o \approx 3 \times 10^{-23} \text{ at } f \approx 2 \text{ kHz}. \quad (21)$$

$$\bar{\nu}_e \text{ burst: } E_\nu \approx 3 \times 10^{53} \text{ erg}, \quad T_\nu \approx 80 \text{ MeV} \rightarrow \sim 100 \text{ events in Hyper-K for a Galactic event}. \quad (22)$$

6.2 Steady-State Signals

(i) 511 keV line:

$$\Phi_{511} \approx (2/\pi) L_{\text{ann}}/(4\pi d^2 E_{511}) \approx 1.3 \times 10^{-3} \text{ ph cm}^{-2} \text{ s}^{-1} (d/300 \text{ pc})^{-2}. \quad (23)$$

(ii) π^0 -decay γ -rays (0.1–5 GeV):

$$F_\gamma \approx 10^{-7} \text{ cm}^{-2} \text{ s}^{-1} (d/\text{kpc})^{-2}. \quad (24)$$

(iii) Cosmic-ray anti-helium:

$$\Phi_{\{^3\bar{\text{H}}\}}(1\text{--}10 \text{ GeV}/n) \approx 6 \times 10^{-7} \text{ m}^{-2} \text{ s}^{-1} \text{ sr}^{-1} \text{ GeV}^{-1} (d/300 \text{ pc})^{-2}. \quad (25)$$

(iv) Optical/IR spectrum: $L \approx 3 L_\odot$, hydrogen/helium lines only; metals absent due to surface annihilation.

7. Population Synthesis for the Milky Way

Core-collapse events over 10 Gyr: $N_{\text{CC}} \approx 2 \times 10^8$. Probability of degenerate bounce

$$\varepsilon_{\text{DB}} \approx 3 \times 10^{-3} \quad (26)$$

(from compactness distribution folded with EoS; Appendix C). Surviving anti-stars today

$$\bar{N} \approx \varepsilon_{\text{DB}} N_{\text{CC}} \approx 600. \quad (27)$$

8. Falsifiability Programme

- Gravitational waves: ET / Cosmic Explorer will detect or rule out $h_0 > 10^{-24}$ at 2 kHz; $\geq 10^5$ CCSN over 10 yr gives ε_{DB} sensitivity 10^{-4} .
- INTEGRAL & e-ASTROGAM: non-detection of $\geq 10^{11}$ ph 511 keV from any object within 500 pc limits $\tilde{N} < 50$.
- AMS-02 & GAPS: a single anti-helium event at < 10 GeV/n would strongly favour the model; none in 20 yr $\rightarrow \varepsilon_{\text{DB}} < 5 \times 10^{-4}$.

All tests are near-term, binary, and require no private data.

9. Critical Assumptions and Open Issues

1. EoS tuning: does any *ab initio* chiral-EFT calculation reproduce $h=0$?
2. Multi-D hydrodynamics: could turbulence delay the rarefaction long enough for a horizon?
3. CP phase $\theta \approx 10^{-4}$ awaits next-gen neutron-EDM verification.
4. Long-term annihilation erosion: outer $10^{-3} M_{\odot}$ lost in 10^7 yr; core survives? (Appendix F).

Each uncertainty is experimentally or observationally accessible.

10. Conclusion

We have constructed, from first principles and analytic expressions alone, a consistent pathway from standard nuclear microphysics to macroscopic anti-stars. The scenario (i) obeys all laboratory EoS constraints, (ii) evades horizon formation via a negative-pressure bounce, (iii) converts $O(1)$ baryon fraction to antimatter through Standard-Model sphalerons, (iv) yields a stable $\sim 1 M_{\odot}$ anti-star, and (v) makes concrete, near-term testable predictions. Failure to observe the predicted signals within the next decade will confine ε_{DB} below 5×10^{-4} , effectively falsifying the hypothesis.

Appendices (analytic)

- A. EoS continuity, causality, and TOV mass limit
- B. Derivation of the no-horizon criterion (Eq. 10)
- C. Compactness distribution fold yielding ε_{DB}
- D. Sphaleron rate at finite μ_B and small θ
- E. Cross-section formulae for $p\bar{p}$ annihilation channels
- F. Time-dependent surface ablation by ISM annihilation

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