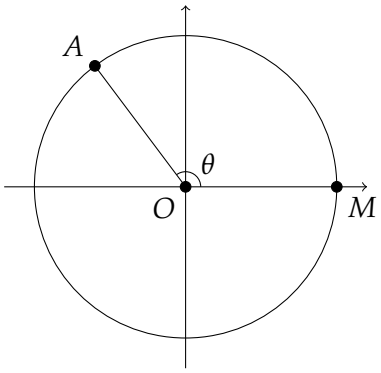
From Trigonometry to Calculus II in a Semester

Checkpoint 4: One Small Detail, Solve Trig Equation and Basic Trig Identity

1 One Small Detail

So far using our unit circle definition of trig functions, we have defined all trig functions from 0 to 2π (or 360° as in degree mode, we will use radians more often and you should try to get familiar with it). However, there is still a problem: How about other values, such as $\sin 300\pi$ or $\cos\left(-\frac{\pi}{2}\right)$?



Still, pull out our unit circle, and let M at $(1,0)$ be the intersection of the circle and the positive x-axis. We can see that if we rotate segment OM counterclockwise with an angle of θ, we get OA.

Also, if we rotate segment OM counterclockwise for a full circle (2π radians) and then rotate an extra angle of θ, we also get OA. In this case, we rotated an angle of $2\pi+\theta$ and still get the same OA. Therefore, we say $\sin(2\pi+\theta)=\sin\theta$, and $\cos(2\pi+\theta)=\cos\theta$.

Same, if we rotate it for multiple full circles, either counterclockwise or clockwise (in this case we rotate a negative angle) and rotate an extra angle of θ, we can still get the same OA. Therefore, we say

$$\sin\theta=\sin(2\pi+\theta)=\sin(4\pi+\theta)=\sin(6\pi+\theta)=\cdots$$

$$\sin\theta=\sin(-2\pi+\theta)=\sin(-4\pi+\theta)=\sin(-6\pi+\theta)=\cdots$$

and the same goes for cos.

Example Problem: What is the value of $\sin\left(\frac{20}{3}\pi\right)$?

Solution: $\sin\left(\frac{20}{3}\pi\right)=\sin\left(6+\frac{2}{3}\pi\right)=\sin\left(\frac{2}{3}\pi\right)=\frac{\sqrt{3}}{2}$.

2 Solve Trig Equations