From Trigonometry to Calculus II in a Semester

Checkpoint 1: Review of SOHCAHTOA

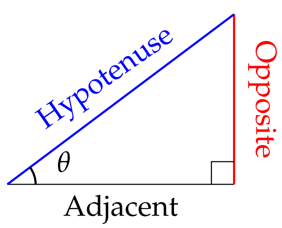
Welcome to the *From Trigonometry to Calculus II in a Semester* series. This series contains study guides of typical high school math courses. Specifically, Trigonometry, Precalculus, Calculus I and Calculus II. If this is the first study guide you see, you are on the right place! The study guides are designed to be able to self-study without a teacher. A tutor might be helpful depending on your learning style but not necessary.

The series of study guides are designed to be able to complete within a semester for a typical high school student. At least, this is my goal, inspired by one of my classmates who claimed that he wanted to self study all these courses in a semester. Anytime you complete ”Checkpoint n”, go to ”Checkpoint $n+1$” unless otherwise noted.

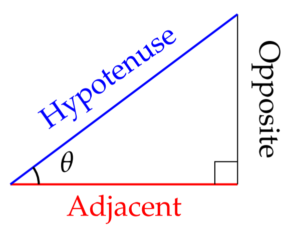
LET”S BEGIN!

In Geometry course, you have learned a mnemonic of trigonometric function called SOHCAH-TOA, which means that

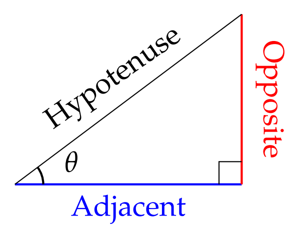
$\sin\theta = \frac{\text{Opposite}}{\text{Hypotenuse}}$



$\cos\theta = \frac{\text{Adjacent}}{\text{Hypotenuse}}$

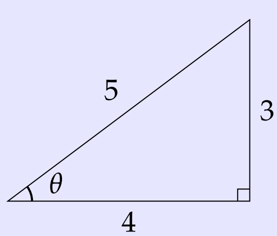


$\tan\theta = \frac{\text{Opposite}}{\text{Adjacent}}$



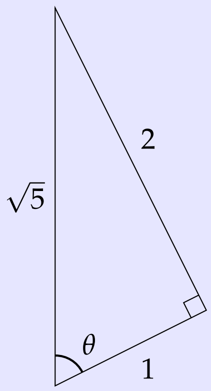
Now let’s try some problems. Figures are not necessarily drawn to scale.

¿1 Find $\sin\theta$ of the following triangle.



$$\text{Opposite} = \underline{\qquad\qquad}$$
$$\text{Hypotenuse} = \underline{\qquad\qquad}$$
$$\sin\theta = \frac{(\qquad\qquad)}{(\qquad\qquad)} = \underline{\qquad\qquad}$$

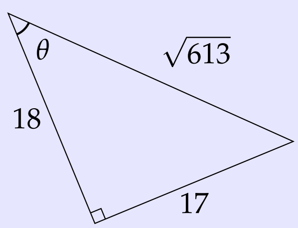
¿2 Find $\tan\theta$ of the following triangle.



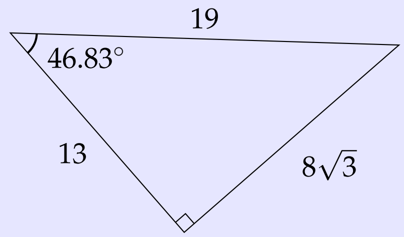
$$\text{Opposite} = \underline{\hspace{2cm}}$$
$$\text{Adjacent} = \underline{\hspace{2cm}}$$
$$\tan\theta = \frac{(\qquad)}{(\qquad)} = \underline{\hspace{2cm}}$$

Now let's try some without my help!

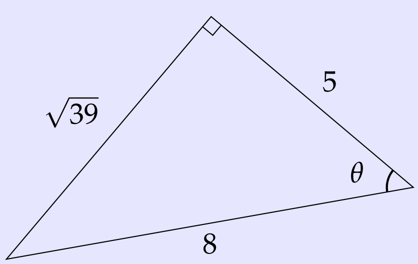
¿3 Find $\cos\theta$ of the following triangle.



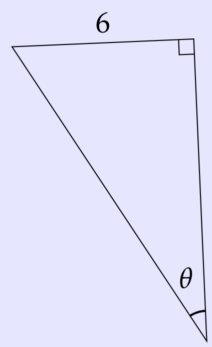
¿5 Find $\cos 46.83^\circ$ using the following triangle.



¿4 Find $\sin\theta$ of the following triangle.



¿6 It is known that $\tan\theta = \frac{3}{5}$. What is the length of the adjacent side of θ?



After reviewing the definition of trigonometric functions, let's figure out some special values of trigonometric functions by completing the following steps:

Step 1: Draw a right triangle with a 30° angle on the right. What is the length of the opposite side of the 30° angle? __________.

Step 2: What is the length of the hypotenuse? ________. If you do it right, the length of the hypotenuse should be twice the length of the opposite side.

Step 3: Use pythagorean theorem ($a^2 + b^2 = c^2$), what is the length of the adjacent side? ________.

Step 4: Now that we get all 3 edges of your right triangle. Use them to calculate the following:

$$\sin 30^\circ = \_\_\_\_\_\_\_\_ \qquad \cos 30^\circ = \_\_\_\_\_\_\_\_ \qquad \tan 30^\circ = \_\_\_\_\_\_\_\_$$

Step 5: We can see that your triangle has a right angle and a 30° angle. What is the degree measure of the other angle? ________.

Step 6: If you get **Step 5** right, you should get that the other angle is 60°. Mark the opposite side, adjacent side and hypotenuse **of the** 60° **angle**, and use them to calculate the following:

$$\sin 60^\circ = \_\_\_\_\_\_\_\_ \qquad \cos 60^\circ = \_\_\_\_\_\_\_\_ \qquad \tan 60^\circ = \_\_\_\_\_\_\_\_$$

Step 7: Draw an isosceles right triangle on the right. Besides the right triangle, the degree measure of the other two angles should be equal. What are they? ________.

Step 8: You should get 45° on the last step. Circle one of your 45° angle, and mark the adjacent side, opposite side and hypotenuse.

Step 9: Your adjacent side and opposite side should have the same length. What are they? ________. Use pythagorean's theorem, what is the hypotenuse? ________.

Step 10: Use the information on **Step 9** to calculate the following:

$$\sin 45^\circ = \underline{\hspace{2cm}} \qquad \cos 45^\circ = \underline{\hspace{2cm}} \qquad \tan 45^\circ = \underline{\hspace{2cm}}$$

Congratulations, you've just calculated some special values of trigonometric functions. If you get everything right, you will get the following values of trigonometric functions:

	30°	45°	60°
$\sin\theta$	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$
$\cos\theta$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$
$\tan\theta$	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$

One mnemonic I learned to memorize these special values is "1,2,3; 3,2,1; 3,9,27", because the values happen to equal the following:

	30°	45°	60°
$\sin\theta$	$\frac{\sqrt{1}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$
$\cos\theta$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{1}}{2}$
$\tan\theta$	$\frac{\sqrt{3}}{3}$	$\frac{\sqrt{9}}{3}$	$\frac{\sqrt{27}}{3}$

I don't use the mnemonic myself though, I just do the steps on the last page when I need a special value of a trigonometric function.

One last thing before we finish: how about 0° and 90°?

If we draw a right triangle with a very small angle (which means that very close to 0°), the opposite will be very close to 0, while the adjacent will be very close to the hypotenuse, so we can get the following special values of trigonometric functions:

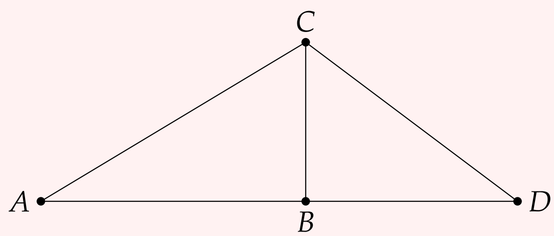
	0°	90°
$\sin\theta$	0	1
$\cos\theta$	1	0
$\tan\theta$	0	undefined

Congratulations, you've just completed **Checkpoint 1**! The next page is some problems that are challenging but meaningful. It will not affect your learning of future checkpoints if you skip these, but they are helpful for you to dig deeper into the concepts, which is an important skill for mathematics.

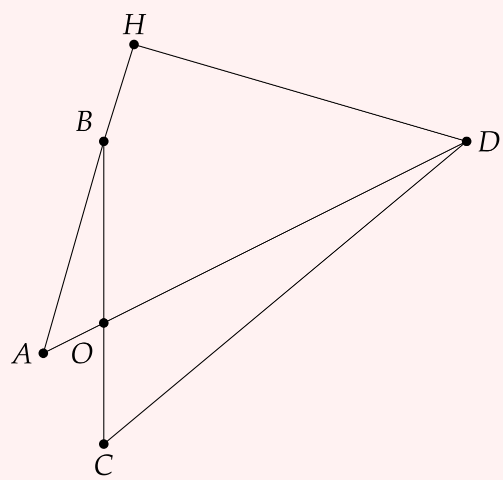
★ 1 Calculate the following expression:

$$\cos 45^\circ - 2\sin 60^\circ + \frac{\tan 60^\circ}{3\tan 30^\circ}.$$

★ 2 As shown in the figure below, triangle ABC and BCD are right triangles, where $\angle ABC = \angle CBD = 90^\circ$. If $AC = 7$, $BD = 4$, and $\sin A = \frac{3}{7}$. Find $\tan D$.



★ 3 As shown in the figure below, segment AD and BC intersect at point O, and $\angle ABO = \angle CDO$. AB was extended to point H such that $\angle AHD = 90^\circ$. It is known that $\tan C = \frac{14}{15}$, and $DH = 7$. Find the length of AH.



★ 4 The parabola $y = x^2 - 4x + 4$ and the line $y = \frac{1}{2}x + 2$ intersect at point A and B, where A is on the left of B. Point C is on the parabola such that $\cos \angle BAC = \frac{4}{5}$. Find the position of C. *(Hint: there are two cases)*