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Deriving the Luminosity of the Photosphere

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1. DERIVATION

This follows from the work of Hascoët et al. (2013).

The thermal energy injected into the outflow at the origin of the jet can be obtained from

$$\dot{E}_{\rm th} = \epsilon_{\rm th}\dot{E} = \epsilon_{\rm th}\frac{\Omega}{4\pi}\dot{E}_{\rm iso} = aT_0^4 cS_0 \tag{1}$$

where $\dot{E}$ is the total power injected into the outflow, $\dot{E}_{\rm iso}$ is the isotropic equivalent total injected power, $\epsilon_{\rm th}$ is the fraction of energy stored in thermal form, T_0 is the temperature at the origin of the flow, $S_0 = \pi\ell^2$ is the cross section of the outflow at the origin, ℓ is the initial jet radius, $a = 4\sigma/c$ is the radiation constant, σ is the Stephan-Boltzmann constant, c is the speed of light, and fraction of solid angle is $\frac{\Omega}{4\pi} \simeq \frac{\theta^2}{4}$, where θ is the jet opening angle.

The radius at which the material becomes transparent to photons and releases thermal radiation is known as the photosphere,

$$\begin{aligned} R_{\rm ph} &\simeq \frac{\kappa\dot{M}}{8\pi c\Gamma^2} \\ &\simeq \frac{\kappa\frac{\dot{E}}{c^2\Gamma}}{8\pi c\Gamma^2} = \frac{\kappa\dot{E}}{8\pi c^3\Gamma^3} \\ &\simeq 2.9\times10^3\frac{\kappa_{0.2}\dot{E}_{\rm iso,53}}{(1+\sigma)\Gamma_2^3}\ {\rm cm} \end{aligned} \tag{2}$$

where κ ($\kappa_{0.2}$ in units of 0.2 cm^2 g^{-1}) is the material opacity, Γ (Γ_2 in units of 100) the Lorentz factor of the shell, and σ is the magnetization of the material.

Assuming no dissipation takes place below the photosphere, mass and entropy conservation lead to the expressions

$$\beta\Gamma\rho S(R) = Const. \tag{3}$$

$$\frac{T}{\rho^{1/3}} = Const. \tag{4}$$

where ρ is the comoving density, $S(R) = \pi\theta^2R^2$ is the surface perpendicular to the flow at a radius R, $\beta = v/c \sim 1$ where v is the velocity of the flow, and $\Gamma = (1-\beta^2)^{-1/2}$. Equating the two above expressions leads to

$$\beta\Gamma T^3 S(R) = const. \tag{5}$$

allowing us the define the temperature at a radius R to be

$$T(R) \simeq T_0 \times (\theta^{-2/3}R^{-2/3}\ell^{2/3}\Gamma^{-1/3}) \tag{6}$$

Using Equations 1, 2, and 6, the luminosity of the thermal radiation released at the photosphere can be expressed as

$$\begin{aligned} L_{\rm th} &= \Gamma^2 aT^4(R_{\rm ph})cS(R_{\rm ph}) \\ &= \dot{E}_{\rm th}\times(\theta^{-2/3}R_{\rm ph}^{-2/3}\ell^{2/3}\Gamma^{2/3}) \\ &= \epsilon_{\rm th}\dot{E}\theta^{-2/3}\ell^{2/3}\left(\frac{\kappa\dot{E}}{8\pi c^3\Gamma^3}\right)^{-2/3}\Gamma^{2/3} \\ &= \epsilon_{\rm th}c^2\dot{E}^{1/3}\left(\frac{8\pi\ell}{\kappa\theta}\right)^{2/3}\Gamma^{8/3} \end{aligned} \tag{7}$$

So, we can see that the photospheric luminosity is strongly dependent on only the Lorentz factor of the material in the outflow.

Questions:

1. Why is there a factor of Γ^2 in Equation 7?

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2. In the equation for R˙ph (Eq 9 in Hascoët et al. 2013), why is $\dot{M}$ associated with the isotropic equivalent energy and not the beaming corrected? (Note in Eq 2 here I write the beaming correct, but this leads to incorrect results.)

REFERENCES

Hascoët, R., Daigne, F., & Mochkovitch, R. 2013, A&A, 551, A124, doi: 10.1051/0004-6361/201220023