Derivations of the Normalization Constant

We begin by defining the photon spectrum function. Let's first look at a Synchrotron spectrum as defined by Equation 34 in Daigne and Mochkovitch 1998,

$$\frac{dn}{dE} = N \frac{e}{E_{\text{syn}}} \left(\frac{E}{E_{\text{syn}}} \right)^x \tag{1}$$

Where e is the dissipated energy of the emission, E_{syn} is the typical synchrotron energy of the emission, and N is a normalization constant for the spectrum (which we compute in this derivation).

This spectral function is normalized by setting the constraint:

$$\int_0^\infty E \frac{dn}{dE} dE = e \tag{2}$$

From here we can find N:

$$\int_0^\infty E \frac{dn}{dE} dE = e \tag{3}$$

$$\int_0^\infty EN \frac{e}{E_{\text{syn}}} \left(\frac{E}{E_{\text{syn}}} \right)^x dE = e \tag{4}$$

$$\int_0^\infty EN \frac{1}{E_{\text{syn}}} \left(\frac{E}{E_{\text{syn}}} \right)^x dE = 1 \tag{5}$$

$$N \int_0^\infty \left(\frac{E}{E_{\text{syn}}} \right)^{x+1} dE = 1 \tag{6}$$

$$N = \frac{1}{\int_0^\infty \left(\frac{E}{E_{\text{syn}}} \right)^{x+1} dE} \tag{7}$$

Application to Light Curves

To find the luminosity in the band (E_1, E_2) due to the spectrum, we must divide the total counts from the emission, A, by the duration of the emission, Δt:

$$L = A/\Delta t \tag{8}$$

$$\text{where,} \tag{9}$$

$$A = \int_{(1+\mathrm{z})\mathrm{E}_1}^{(1+z)E_2} \frac{dn}{dE} dE = \int_{(1+\mathrm{z})\mathrm{E}_1}^{(1+z)E_2} N \frac{e}{E_{\text{syn}}} \left(\frac{E}{E_{\text{syn}}} \right)^x dE \tag{10}$$

$$= \int_{(1+\mathrm{z})\mathrm{E}_1}^{(1+z)E_2} \frac{1}{\left[\int_0^\infty \left(\frac{E}{E_{\text{syn}}} \right)^{x+1} dE \right]} \frac{e}{E_{\text{syn}}} \left(\frac{E}{E_{\text{syn}}} \right)^x dE \tag{11}$$

$$\tag{12}$$

Thermal Spectrum

$$\frac{dn}{dE} \approx NL_{\mathrm{therm}} \left(\frac{E}{4k_{\mathrm{B}}T}\right)^{-0.4} * e^{\frac{-E}{4k_{\mathrm{B}}T}} \tag{13}$$