GRB Jet Simulations: External Shock Considerations

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1. CIRCUM-BURST MEDIUM

The circum-burst medium is determined by the properties and behavior of the GRB progenitor. A naive assumption would be to let the circum-burst medium be similar to the interstellar medium, a constant density medium. Alternatively, a Wolf-Rayet star may be assumed as the progenitor to the GRB in question. Wolf-Rayet stars are believed to have strong winds emanating from the surface of the star, injecting particles into the circum-stellar medium.

For either assumption, the circum-burst medium will only begin at the progenitor's stellar surface, we can use the approximate radius of a Wolf-Rayet star, i.e., $R_{\rm ext,0} \sim R_\odot = 6.9 \times 10^{10}$ cm.

The general definition of the density profile can be given as,

$$\rho(r) = \frac{\rho_0}{r^{\rm k}} \tag{1}$$

Where ρ_0 is the normalization of the density and k describes the profile of the density. The density profiles are both defined in more detail below.

1.1. *Constant Density Medium*

To specify a constant density medium, we set $k = 0$. In this way, the density will remain equal to the value of ρ_0. If we let $\rho_0 = m_{\rm p} n_0$ where $n_0 \sim 1$ cm^{-3} is the particle density. The circum-burst medium is given by,

$$\rho(r) = m_{\rm p} n_0 \tag{2}$$
$$= 1.67 \times 10^{-24} \text{ g cm}^{-3} \tag{3}$$

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1.2. *Wind Density Medium*

In the case of the wind-blown medium, we set $k = 2$. If we assume a stellar wind mass-loss rate $\dot{M}_{\rm w}$ and a terminal velocity v_∞ we can write the normalization of the density as $\rho_0 = \dot{M}_{\rm w} A_* / 4\pi v_\infty$ g cm^{-1} where A_* is a factor to adjust the stellar mass-loss rate and usually takes a value near unity. If we assume the values $\dot{M}_{\rm w} = 10^{-5} M_\odot$ yr^{-1} and $v_\infty = 1000$ km s^{-1}, the circum-burst density can be expressed as

$$\rho(r) = \frac{\dot{M}_{\rm w}}{v_\infty} \frac{A_*}{4\pi r^2} \tag{4}$$

$$= \frac{5 \times 10^{11} A_*}{r^2} \text{ g cm}^{-3} \tag{5}$$

$$\rho(R_{\rm ext,0}) \approx 1.5 \times 10^{-9} \text{ g cm}^{-3} \tag{6}$$

2. EXTERNAL SHOCKS

To represent the forward and reverse shocks and the discontinuity between them, we implement a two-shell model. The two shells will share the lorentz factor $\Gamma_{\rm ext}$ and have a total mass $M_{\rm ext} = M_{\rm FS} + M_{\rm RS}$, where $M_{\rm FS}$ and $M_{\rm RS}$ are the masses of the forward shock shell and reverse shock shell, respectively. Two processes will affect this discontinuity: (i) it will sweep up mass from the external medium (i.e., the forward shock) and (ii) an internal shell ejected by the compact object will collide with the discontinuity from behind (i.e., the reverse shock).

2.1. *The Forward Shock (FS)*

Forward shocks will occur when a mass $m_{\rm ex}$ is swept up by the discontinuity as it collides with the circum-burst medium. We can define $m_{\rm ex}$ as a fraction q of a the un-boosted mass $M_{\rm ext}$,

$$m_{\rm ex} = q \frac{M_{\rm ext}}{\Gamma_{\rm ext}} \tag{7}$$

where $q << 1$ and is on the order of 10^{-2}. We can write the explicit integral form,

$$m_{\mathrm{ex}} = q\frac{M_{\mathrm{ext}}}{\Gamma_{\mathrm{ext}}} = \int_{\mathrm{R}_1}^{\mathrm{R}_2} 4\pi r^2 \rho(r) dr \tag{8}$$

$$= \int_{\mathrm{R}_1}^{\mathrm{R}_2} 4\pi r^2 \frac{\rho_0}{r^{\mathrm{k}}} dr \tag{9}$$

where $\rho(r)$ is the density of the external medium (see discussion in section 1).

2.1.1. FS Dynamics

We can calculate the radius at which a mass m_{swept} will be swept up by a shell over a distance $R = (R_2 - R_1)$. The explicit integral can be evaluated by specifying the density profile of the circum-burst medium. If we assume a constant density (i.e., $k = 0$),

$$m_{\mathrm{swept}} = \int_{\mathrm{R}_1}^{\mathrm{R}_2} 4\pi r^2 \frac{\rho_0}{r^0} dr \tag{10}$$

$$= 4\pi\rho_0 \int_{\mathrm{R}_1}^{\mathrm{R}_2} r^2 dr \tag{11}$$

$$= \frac{4}{3}\pi\rho_0(R_2^3 - R_1^3) \tag{12}$$

$$R_2^3 = \frac{3}{4\pi}\frac{m_{\mathrm{swept}}}{\rho_0} + R_1^3 \tag{13}$$

$$(R_2 - R_1) = \left(\frac{3}{4\pi}\frac{m_{\mathrm{swept}}}{\rho_0} + R_1^3\right)^{1/3} - R_1 \tag{14}$$

Alternatively, if we assume a wind medium (i.e., $k = 2$),

$$m_{\mathrm{swept}} = \int_{\mathrm{R}_1}^{\mathrm{R}_2} 4\pi r^2 \frac{\rho_0}{r^2} dr \tag{15}$$

$$= 4\pi\rho_0 \int_{\mathrm{R}_1}^{\mathrm{R}_2} dr \tag{16}$$

$$= 4\pi\rho_0(R_2 - R_1) \tag{17}$$

$$(R_2 - R_1) = \frac{m_{\mathrm{swept}}}{4\pi\rho_0} \tag{18}$$

Using the velocity of the external shock we can calculate how long it will take to sweep up a mass m_{ex},

$$t_{\mathrm{sweep}} = \frac{(R_2 - R_1)}{\beta c} \tag{19}$$

$$t_{\mathrm{sweep,const}} = \frac{1}{\beta c}\left(\left(\frac{3}{4\pi}\frac{m_{\mathrm{swept}}}{\rho_0} + R_1^3\right)^{1/3} - R_1\right) \tag{20}$$

$$t_{\mathrm{sweep,wind}} = \frac{1}{\beta c}\left(\frac{m_{\mathrm{swept}}}{4\pi\rho_0}\right) \tag{21}$$

By conserving energy momentum, we obtain the resulting Lorentz factor obtain from a forward shock collision,

$$\Gamma'_{\mathrm{ext}} = \left[\frac{(M_{\mathrm{RS}} + M_{\mathrm{FS}}\Gamma_{\mathrm{FS,int}})\Gamma_{\mathrm{ext}}^2 + m_{\mathrm{ex}}\Gamma_{\mathrm{ext}}}{(M_{\mathrm{RS}} + M_{\mathrm{FS}}\Gamma_{\mathrm{FS,int}}) + 2m_{\mathrm{ex}}\Gamma_{\mathrm{ext}}}\right]^{1/2} \tag{22}$$

and also the resulting internal Lorentz factor of the forward shock,

$$\Gamma'_{\mathrm{FS,int}} = \frac{(M_{\mathrm{RS}} + M_{\mathrm{FS}}\Gamma_{\mathrm{FS,int}})\Gamma_{\mathrm{ext}} + m_{\mathrm{ex}} - M_{\mathrm{RS}}\Gamma'_{\mathrm{ext}}}{(M_{\mathrm{FS}} + m_{\mathrm{ex}})\Gamma'_{\mathrm{ext}}} \tag{23}$$

The value of $\Gamma'_{\mathrm{FS,int}} \sim \Gamma_{\mathrm{ext}}$. It should be noted that we assume here that the material in the burst environment is at rest.

2.1.2. FS Emission

The average energy dissipated per proton in a shock between two layers can be prescribed as,

$$e_{\mathrm{int}} = (\Gamma_{\mathrm{FS,int}} - 1)m_{\mathrm{p}}c^2 \tag{24}$$

$$e_* = \frac{e_{\mathrm{int}}}{m_{\mathrm{p}}} \tag{25}$$

Which can be used to calculate the typical Lorentz factor of an electron in the shocked region (see Γ_{e} Prescription),

$$\Gamma_{\mathrm{e}} = \Gamma_{\mathrm{m}} = \frac{p - 2}{p - 1}\frac{\epsilon_{\mathrm{e}}}{\zeta_{\mathrm{FS}}}\frac{m_{\mathrm{p}}}{m_{\mathrm{e}}}\frac{e_*}{c^2} \tag{26}$$

where p is the power law index of the electron distribution, ϵ_{e} is the fraction of energy in electrons, and ζ_{FS} is the fraction of the electron population which is accelerated (taken to be $\lesssim 1$ in the forward shock).

Assuming the equipartition, the magnetic field in the forward shock region is given by,

$$B_{\mathrm{eq}} = (8\pi\epsilon_{\mathrm{B}}\rho e_*)^{1/2} \tag{27}$$

Using Γ_{ext}, B_{eq}, and Γ_{e} we can calculate the typical energy of an electron in the shock (in the observer frame),

$$E_{\mathrm{syn}} = \Gamma_{\mathrm{ext}}B_{\mathrm{eq}}\Gamma_{\mathrm{e}}^2\frac{hq_{\mathrm{e}}}{2\pi m_{\mathrm{e}}c} \tag{28}$$

$$E_{\mathrm{syn}} \approx 50\frac{\Gamma_{\mathrm{ext}}}{300}\frac{B_{\mathrm{eq}}}{1000\ \mathrm{G}}\left(\frac{\Gamma_{\mathrm{e}}}{100}\right)^2\ \mathrm{eV} \tag{29}$$

We can calculate the fraction of energy going in to Synchrotron, α_{synch} or Inverse Compton electrons, α_{IC},

$$\alpha_{\text{IC}} = \frac{\tau_* \Gamma_e^2 / w}{1 + \tau_* \Gamma_e^2 / w} \tag{30}$$

$$\alpha_{\text{synch}} = 1 - \alpha_{\text{IC}} \tag{31}$$

where w indicates whether the electrons are in the Thompson regime ($w < 1$) or the Klein-Nishina limit ($w >> 1$),

$$w = \frac{\Gamma_e}{\Gamma_{\text{ext}}} \frac{E_{\text{syn}}}{m_e c^2} \tag{32}$$

When $w < 1$ we set w to unity in the relation for α_{IC}. $\tau_* \Gamma_e^2$ is the Compton parameter, where

$$\tau_* = \frac{\kappa_{\text{T}} M_*}{4\pi R_*^2} \tag{33}$$

is the optical depth of a shell with mass M_* and at a radius R_*.

Why can I not simply evaluate τ_* from above? Why is M_* not the same as the mass of a shell colliding?

We can estimate M_* as

$$M_* = \frac{t_{\text{syn}}}{1 + \tau_* \Gamma_e^2} \dot{M}_{\text{shock}} \tag{34}$$

where

$$t_{\text{syn}} = 6 \left(\frac{\Gamma_e}{100} \right)^{-1} \left(\frac{B_{\text{eq}}}{1000 \text{ G}} \right)^{-2} \text{ s} \tag{35}$$

is the time for an synchrotron electron with Lorentz factor of Γ_e in a magnetic field of strength B_{eq} to efficiently radiate (in the comoving shock frame). $\dot{M}_{\text{shock}}$ is the mass flow rate across the shock (in the comoving shock frame) and can be estimated as

$$\dot{M}_{\text{shock}} \approx \frac{\bar{M}}{\bar{\Gamma}} \tag{36}$$

From Eq. 33, 34, 35, and 36 we obtain an explicit expression for the Compton parameter,

$$\tau_* \Gamma_e (1 + \tau_* \Gamma_e) = \frac{300 \kappa_{\text{T}}}{2\pi} \frac{\Gamma_e}{R_*^2} \left(\frac{B_{\text{eq}}}{1000 \text{ G}} \right)^{-2} \frac{\dot{E}_{\text{kin}}}{\bar{\Gamma}^2 c^2} \tag{37}$$

The dissipated energy in a forward shock collision is calculated as

$$E_{\text{diss}} = \epsilon_e \alpha_{\text{synch}} c^2 \times \tag{38}$$

$$((M_{\text{RS}} + M_{\text{FS}} \Gamma_{\text{FS,int}}) \Gamma_{\text{ext}} \tag{39}$$

$$+ m_{\text{ex}} \tag{40}$$

$$- (M_{\text{RS}} + m_{\text{ex}} + M_{\text{FS}} \Gamma_{\text{FS,int}}) \Gamma'_{\text{ext}}) \tag{41}$$

$$L_{\text{diss}} = \frac{E_{\text{diss}}}{t_{\text{var}}} = \frac{E_{\text{diss}}}{(R_{\text{FS}}/2c^2 \Gamma'^2_{\text{ext}})} \tag{42}$$

Should I use Γ'_{int} and Γ_{int} for E_{diss}?

where R_{FS} is the radius at which the forward shock occurs.

2.1.3. FS Spectrum

From the above physical parameters we can calculate the emission of the forward shock. We assume the synchrotron emission is the dominant emission process in the forward shock. The synchrotron spectrum can be approximated by a broken power law of the form,

$$N(E) = \begin{cases} \frac{1}{E_{\text{syn}}} * \left(\frac{E}{100 \text{ keV}} \right)^{\alpha} & E \leq E_{\text{syn}} \\ \frac{1}{E_{\text{syn}}} * \left(\frac{E}{100 \text{ keV}} \right)^{\alpha - \beta} \left(\frac{E}{100 \text{ keV}} \right)^{\beta} & E > E_{\text{syn}} \end{cases}$$

In order to replicate the shape of the synchrotron count spectrum, the value of β for is given by $\beta = -\frac{p}{2} - 1$, where p is the power law index of the electron population. On the other hand, the value of α depend on whether the synchrotron emission is in the fast-cooling or slow-cooling regime (i.e., efficient or inefficient radiation). In the fast-cooling regime $\alpha = -1.5$ and in the slow cooling regime $\alpha = -(p+1)/2$.

We must calculate the frequency which corresponds to the minimum Lorentz factor of the electron population, ν_{m}, and the cyclotron frequency, ν_{c}, in order to determine if the synchrotron emission is in the fast-cooling ($\nu_{\text{c}} < \nu_{\text{m}}$) or slow-cooling ($\nu_{\text{m}} < \nu_{\text{c}}$) regime. The minimum Lorentz factor of the electron population is also the typical Lorentz factor, Γ_e, which we calculated in Eq. 51. From this we find ν_e in the source frame

$$\nu_e = B_{\text{eq}} \Gamma_e^2 \frac{q_e}{2\pi m_e c} \tag{43}$$

$$\tag{44}$$

The cyclotron frequency is given by

$$\nu_c = \frac{q_e B_{\text{eq}}}{2\pi m_e} \tag{45}$$

The FS is expected to be in the slow-cooling regime for the majority of its emission. Only if the density of the external medium is extremely high, perhaps close to the star in a wind-density medium, is the FS expected to be in the fast-cooling regime.

2.2. *The Reverse Shock (RS)*

The reverse shock shares many of the same considerations as the forward shock described above, yet a difference arises in their micro-physical parameters at the shock front discontinuity. The reverse shock is due to internal shells catching up and ramming the shock front from behind, because of this it is assumed the reverse shock region shares the same micro-physical parameters as the jet.

2.2.1. *RS Dynamics*

We calculate the time until a collision between the leading shell of the internal ejecta and the shock front as,

$$t_{\mathrm{RS,coll}} = \frac{(R_{\mathrm{RS}} - R_{\mathrm{ej}})}{c(\beta_{\mathrm{ej}} - \beta_{\mathrm{RS}})} \tag{46}$$

where R_{RS} and R_{ej} are the radii of the reverse shock and the ejecta shell, respectively. We can use the relation $\beta \approx 1 - \frac{1}{2\Gamma^2}$ to calculate β_{ej} and β_{RS} using Lorentz factor of the leading shell and reverse shock, respectively.

When the two shells collide we can calculate resulting Lorentz factor of the two shells,

$$\Gamma'_{\mathrm{ext}} = \sqrt{\Gamma_{\mathrm{ext}}\Gamma_{\mathrm{ej}}} \left(\frac{(M_{\mathrm{RS}} + M_{\mathrm{FS}}\Gamma_{\mathrm{FS,int}})\Gamma_{\mathrm{ext}} + m_{\mathrm{ej}}\Gamma_{\mathrm{ej}}}{(M_{\mathrm{RS}} + M_{\mathrm{FS}}\Gamma_{\mathrm{FS,int}})\Gamma_{\mathrm{ej}} + m_{\mathrm{ej}}\Gamma_{\mathrm{ext}}} \right)^{1/2} \tag{47}$$

and also the resulting internal Lorentz factor,

$$\Gamma'_{\mathrm{RS,int}} = \frac{1}{2} \left[\left(\frac{\Gamma_{\mathrm{ext}}}{\Gamma_{\mathrm{ej}}} \right)^{1/2} + \left(\frac{\Gamma_{\mathrm{ej}}}{\Gamma_{\mathrm{ext}}} \right)^{1/2} \right] \tag{48}$$

2.2.2. *RS Emission*

Using the resulting Lorentz factors, we can calculate the energy dissipated per proton as a result of the collision,

$$e_{\mathrm{int}} = (\Gamma'_{\mathrm{RS,int}} - 1)m_{\mathrm{p}}c^2 \tag{49}$$

$$e_* = \frac{e_{\mathrm{int}}}{m_{\mathrm{p}}} \tag{50}$$

Similar to Eq. 51, we can calculate Γ_{e} in the Reverse shock,

$$\Gamma_{\mathrm{e}} = \Gamma_{\mathrm{m}} = \frac{p-2}{p-1} \frac{\epsilon_{\mathrm{e}}}{\zeta_{\mathrm{RS}}} \frac{m_{\mathrm{p}}}{m_{\mathrm{e}}} \frac{e_*}{c^2} \tag{51}$$

In this case ζ is no longer taken to be close to unity and is instead similar to the value found within the jet medium, i.e., $\zeta \approx 10^{-2}$, this is needed to produce radiation in the MeV regime. A small value of ζ indicates that only a small number of electrons are accelerated, this in turn allows the electrons to be accelerated to higher energies, because the energy is being distributed over a smaller number of electrons. When the electrons cool, they produce radiation in the MeV regime.

The prescriptions for B_{eq}, E_{syn}, and α_{syn} are the same for both the FS and RS. The only difference between the two is the comoving density. The density of in the reverse shock is the density of the internal jet medium which can be expressed as

$$\rho(R)_{\mathrm{RS}} = \frac{\dot{E}_{\mathrm{kin}}}{4\pi R^2 \Gamma_{\mathrm{ext}}^2 c^3} \tag{52}$$

The total dissipated energy of a reverse shock collision can be calculated as

$$E_{\mathrm{diss}} = \epsilon_{\mathrm{e}}\alpha_{\mathrm{synch}}c^2 \times \tag{53}$$

$$((M_{\mathrm{RS}} + M_{\mathrm{FS}}\Gamma_{\mathrm{FS,int}}) * \Gamma_{\mathrm{ext}} \tag{54}$$

$$+ m_{\mathrm{ej}}\Gamma_{\mathrm{ej}} \tag{55}$$

$$- (M_{\mathrm{RS}} + m_{\mathrm{ej}} + M_{\mathrm{FS}}\Gamma_{\mathrm{FS,int}})\Gamma'_{\mathrm{ext}}) \tag{56}$$

$$L_{\mathrm{diss}} = \frac{E_{\mathrm{diss}}}{t_{\mathrm{var}}} = \frac{E_{\mathrm{diss}}}{(R_{\mathrm{RS}}/2c^2\Gamma_{\mathrm{ext}}'^2)} \tag{57}$$

2.2.3. *RS Spectrum*

Similar to the FS, the RS is assumed to be the result of synchrotron emission. The RS is expected to be solely in the fast-cooling regime.

APPENDIX

A. LORENTZ FACTOR AFTER FIRST COLLISION

Here we calculate the expected Lorentz factor of the FS after the first external shock collision event. We assume the outermost shell has a Lorentz factor $\Gamma = 100$ and an internal Lorentz factor of $\Gamma_{\mathrm{int}} = 1$ before the collision.

In our code, we reduce numerical instabilities by treating the mass of each shell as a fraction of the average shell mass where

$$M_{\mathrm{i}} = \frac{\dot{E}_{\mathrm{iso}} * \Delta t_{\mathrm{ej,i}}}{\Gamma_{\mathrm{i}} * c^2} \tag{A1}$$

$$\Rightarrow \frac{M_{\mathrm{s}}}{M_{\mathrm{ave}}} = \frac{\Gamma_{\mathrm{ave}}}{\Gamma_{\mathrm{s}}} \tag{A2}$$

By using this prescription, we can write the masses of the FS and RS shells as $M_{\mathrm{FS}} = M_{\mathrm{RS}} = 1$. If we use Eq. 8 with $q = 0.01$ we find the mass of the external medium swept up to be $m_{\mathrm{ex}} = 10^{-4}$. Using Eq. 22, we can calculate the resulting Lorentz factor after the first external shock collision,

$$\Gamma'_{\mathrm{ext}} = \left(\frac{(M_{\mathrm{RS}} + M_{\mathrm{FS}}\Gamma_{\mathrm{int}})\Gamma^2 + m_{\mathrm{ex}}\Gamma_{\mathrm{ext}}}{(M_{\mathrm{RS}} + M_{\mathrm{FS}}\Gamma_{\mathrm{int}}) + 2m_{\mathrm{ex}}\Gamma_{\mathrm{ext}}} \right) \tag{A3}$$

$$= \left(\frac{(1+1)100^2 + 10^{-4}}{(1+1) + 2 \times 10^{-4}} \right) \tag{A4}$$

$$\approx \left(\frac{2 \times 100^2}{2} \right) \tag{A5}$$

$$\approx (10^4)^{1/2} \tag{A6}$$

$$\Gamma'_{\mathrm{ext}} \approx 10^2 \tag{A7}$$