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GRB Prompt Emission Simulation

Michael Moss[1,2,3]

[1] *The Department of Physics, The George Washington University, 725 21st NW, Washington, DC 20052, USA*
[2] *Astrophysics Science Division, NASA Goddard Space Flight Center, Greenbelt, MD 20771, USA*
[3] *Sorbonne Université, CNRS, UMR 7095, Institut d'Astrophysique de Paris, 98 bis Arago, 75014 Paris, France*

ABSTRACT

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1. INTRODUCTION

2. JET DYNAMICS

Input parameters for the jet dynamics simulations are listed in Table 1. In this section we provide context for the input parameters and derive some relevant parameter from the input parameters. Following this, we derive the calculated emission parameters (for example, the temperature of the black body). The parameter space of the code can be limited in two ways, either using parameters extracted from fits to real GRB spectra, which we call option (a), or by creating an library of spectra using reasonable values for each parameter, which we call option (b). If the value of the parameter cannot be constrained by observations, then option (a) and (b) are equivalent.

The duration of GRB prompt emission is not well defined. Some GRBs may last only a fraction of a second while others last hundreds of seconds. Additionally, the observed duration does not directly indicate the duration of the wind or central engine, it only provides a hard upper limit. From the observed duration of a GRB, e.g., $\mathrm{T}_{90,\mathrm{obs}}$, we are able to limit the possible duration of the wind,

$$t_{\mathrm{w}} = \frac{\mathrm{T}_{90,\mathrm{obs}}}{1+z} * f_{\mathrm{w}} \tag{1}$$

where z is the redshift of the source and f_{w} is some fraction used to account for the fact that $t_{\mathrm{w}} < \mathrm{T}_{90,\mathrm{obs}}$. We do know the exact value of f_{w}, but we can use a few values, e.g. 0.75, 0.85, 1). If we do not use observations to define the wind in our simulation, we can use various durations ranging from 0.1 sec to 10 sec.

The time resolution of our simulations will be limited by the ejection time between the shells.

The number of shells in the jet can defined by the total duration of the wind, t_{w}, and the ejection time between shells, Δt_{ej},

$$N = \frac{t_{\mathrm{w}}}{\Delta t_{\mathrm{ej}}} \tag{2}$$

The magnetization in the jet is an important parameter which dictates the acceleration and emission processes possible in the jet. If $\sigma = 0$, the jet is in a purely fireball model, meaning the acceleration of the jet is purely thermal and that there can be no magnetic reconnection because there is no magnetic flux in the jet. If on the other hand, $\sigma \geq 1$, the magnetic fields in the jet are strong enough to suppress internal shocks and magnetic reconnection events are expected. In the case of a passive magnetic field that is carried by the outflow without contributing to its acceleration (Spruit et al. ????) the passive magnetization is given

$$\sigma_{\mathrm{passive}} = \frac{1-\epsilon_{\mathrm{th}}}{\epsilon_{\mathrm{th}}} \tag{3}$$

corresponding to a pure and complete thermal acceleration (and indicating no magnetic acceleration). If there is any magnetic acceleration $\sigma < \sigma_{\mathrm{passive}}$.

Similar implications can be obtained from ϵ_{th}, when ϵ_{th} we are in a pure fireball model and the upper limit can be defined as $\epsilon_{\mathrm{th}} < \frac{1}{1+\sigma}$.

Using observations of GRB prompt emission we can define a value of the injected energy, $\dot{E}$. We can use

Corresponding author: Michael Moss
mikejmoss3@gmail.com

the observed isotropic injected energy rate of the non-thermal component in the gamma-ray regime, $\dot{E}_{\gamma,\mathrm{NT,iso}}$, to define the injected energy rate in the jet,

$$\dot{E} = (1+\sigma)\frac{\dot{E}_{\gamma,\mathrm{NT,iso}}}{f_{\mathrm{rad}}\epsilon_{\mathrm{e}}} \tag{4}$$

where f_{rad} is the efficiency of the non-thermal acceleration process, e.g., for internal shocks f_{rad} is determined by the Lorentz factor of two colliding shells, but is typically around a few percent. Instead of using the observed values for a particular value, we can simulate the wind using injection energies ranging from 10^{50} to 10^{54} erg.

We can derive the various energies before acceleration

$$\dot{E}^{init}_{\mathrm{th}} = \epsilon_{\mathrm{th}}\dot{E} \tag{5}$$
$$\dot{E}^{init}_{\mathrm{mag}} = (1-\epsilon_{\mathrm{th}})\dot{E} \tag{6}$$
$$\dot{E}^{init}_{\mathrm{kin}} = 0 \tag{7}$$

and energies after acceleration

$$\dot{E}^{final}_{\mathrm{th}} = 0 \tag{8}$$
$$\dot{E}^{final}_{\mathrm{mag}} = \frac{\sigma}{1+\sigma}\dot{E} \tag{9}$$
$$\dot{E}^{final}_{\mathrm{kin}} = \frac{\dot{E}}{1+\sigma} \tag{10}$$

where $\dot{E}_{\mathrm{th}}$, $\dot{E}_{\mathrm{mag}}$, $\dot{E}_{\mathrm{kin}}$ are the amounts of energy in the form of thermal, magnetic, and kinetic, respectively. We note $\dot{E} = (1-\cos\theta_{\mathrm{j}})\dot{E}_{\mathrm{iso}}$.

2.1. *Shell Distribution*

We begin by assigning the initial Lorentz factor, mass, emission time since jet launch, radius, and status of the N shells in the jet.

The distribution of the shell Lorentz factors can be specified by the user before running the jet dynamics simulation. Currently, there is a step function and a oscillatory function implemented. For the step function, two Lorentz factors must be specified to specify the upper Γ_1 and low steps Γ_2. The the fraction of the total mass with Γ_1 must also be specified, $f = M_1/M_{\mathrm{tot}}$

$$N(\Gamma_1) = \frac{N_{\mathrm{tot}}}{\frac{(1-f)*\Gamma_2}{f*\Gamma_1}+1} \tag{11}$$
$$N(\Gamma_2) = N_{\mathrm{tot}} - N(\Gamma_1) \tag{12}$$
$$\tag{13}$$

For the oscillatory distribution, we adapt the function from Equation 16 in Hascoët et al. (2013), but has been made more general,

$$\Gamma_{\mathrm{i}} = \Gamma_{\mathrm{mean}} * \left(1 + A\cos\left(\pi f\left(1-\frac{i}{N_{\mathrm{tot}}}\right)\right)\right) * e^{-\lambda\frac{i}{N_{\mathrm{tot}}}} \tag{14}$$

where Γ_{mean} is the mean Lorentz, A is the amplitude, f is the frequency, i is the shell number, and λ is the decay constant.

The mass of each shell is defined as

$$M_{\mathrm{i}} = \frac{\bar{\Gamma}}{\Gamma_{\mathrm{i}}} \tag{15}$$
$$\tag{16}$$

where $\bar{\Gamma}$ is the mean Lorentz factor. Currently, this is a dimensionless quantity, later it is multiplied by the $\bar{M}$ to give dimensions, where

$$\bar{M} = \frac{\dot{E}_{\mathrm{iso}}\Delta t_{\mathrm{e}}}{\bar{\Gamma}c^2}\ \mathrm{g} \tag{17}$$

The emission time of each shell can either be supplied as a list by the user or will be calculated if only Δt_{e} is given. The emission time is the $t_{\mathrm{e}} = -\Delta t_{\mathrm{e}} * i$, where i is the shell number. Notice the emission time is negative, this doesn't have a physical reason, its just how I coded it.

The radius of the jet computed as,

$$R_{0,\mathrm{i}} = \beta_{\mathrm{i}} * t_{\mathrm{e,i}}\ \mathrm{cm/c} \tag{18}$$

where $\beta_{\mathrm{i}} = \sqrt{(1-\Gamma^{-2})}$ Notice that these are in units of R/c, for the remainder of this discussion distances will be in units of light seconds. Distances should be multiplied by c in order to obtain distance units. Recall that the values of $t_{\mathrm{e,i}}$ are negative, so these radii will also be negative. We set $R = 0 = R_{0,0}$ and all shells begin at negative distances.

The status of a shell indicates whether it is currently active or not. An active shell is one that is currently propagating through the jet. An inactive shell is one that has not been launched yet or alternatively has already collided.

2.2. *Emission Output Parameters*

2.2.1. *Thermal Shell Dynamics and Emission*

Once we have assigned the initial Radius, Lorentz factor, Mass, and emission time of each shell, we can begin to calculate the dynamics of the shells. We calculate when the shells will cross their respective photospheres and we calculate when/where collisions between the shells will occur.

Table 1. List of input parameters used in the jet dynamic simulations, the typical values or expression and a description of each is provided. The parameter space of the code can be limited in two ways, either using parameters extracted from fits to real GRB spectra, which we call option (a), or by creating an library of spectra using reasonable values for each parameter, which we call option (b). If the value of the parameter cannot be constrained by observations, then option (a) and (b) are equivalent.

Parameter symbol	Typical Values		Description
	(a)	(b)	
$t_{\rm w}$	$\frac{\text{obs duration}}{1+z} * (0.75, 0.85, 1)$ sec	(0.1, 0.3, 1, 3, 10) sec	Duration of the wind.
$\Delta t_{\rm ej}$	(10^{-2}, 10^{-3}) sec		Time Resolution of the simulation, or time between the ejection of each shell.
$\Gamma_{\rm min}$	(50, 100, 200)		Minimum Lorentz factor for any shell.
$\Gamma_{\rm max}$	(2, 4, 6)*$\Gamma_{\rm min}$		Maximum Lorentz factor for any shell.
f_Γ	(0.2, 0.4, 0.6, 0.8)		Fraction of shells which have $\Gamma_{\rm max}$.
σ	(0, 10^{-2}, 10^{-1})		Magnetization of the jet.
$\epsilon_{\rm e}$	(0.1, 1/3)		Fraction of energy stored in electrons.
$\epsilon_{\rm B}$	(10^{-4}, 10^{-3}, 10^{-2}, 10^{-1})		Fraction of energy stored in the magnetic field.
$\epsilon_{\rm Th}$	(10^{-3}, 10^{-2}, 10^{-1}, 1)		Fraction of energy stored as Thermal energy.
$\dot{E}$	$(1+\sigma) * \frac{\dot{E}_{\gamma,\rm NT,iso}}{f_{\rm rad}\epsilon_{\rm e}}$ erg/s	(10^{50},10^{51},10^{52},10^{53},10^{54}) erg/s	Injected energy rate.
ζ	(10^{-3}, 3*10^{-3}, 10^{-2}, 3*10^{-2})		Fraction of electrons which are accelerated.
p	(2.2, 2.5, 2.8, 3.4)		Power law index of electron distribution.
$\theta_{\rm j}$	0.1 rad		Opening angle of the jet.
l	10^6 cm		Opening radius of the jet.

We have found that all the shells pass their respective photospheres much before the first collision between two shells (in the source frame).

The photospheric radius and crossing time of each shell is given by:

$$R_{\rm phot,i} = \frac{0.2\dot{E}_{\rm iso}}{(1+\sigma)8\pi c^4\Gamma_{\rm i}^3} \text{ cm/c} \tag{19}$$

$$t_{\rm phot,i} = \frac{R_{\rm phot} - R_{0,\rm i}}{\beta_{\rm i}} \text{ s} \tag{20}$$

where $R_{0,\rm i}$ is the initial radius of the ith shell.

For all thermal emission events, the arrival time of the photons and duration of the emission is given by

$$t_{\rm a} = (t_{\rm e} - R_{\rm phot})\text{s} \tag{21}$$

$$\Delta t_{\rm obs} = \frac{R_{\rm phot}}{2\Gamma_{\rm r}^2}(1+z) \text{ s} \tag{22}$$

Following the prescription from Hascoët et al. (2013).

$$\Phi = \theta^{-2/3}(R_{\rm phot} * c)^{-2/3}(R_{\rm open})^{2/3}\Gamma^{2/3} \tag{23}$$

$$\approx \dot{E}_{\rm iso}^{-2/3}\Gamma^{8/3} \tag{24}$$

$$T0 = \left(\frac{\dot{E}_{\rm th}^2\theta^2}{4\pi ac R_{\rm open}^2}\right)^{1/4} \text{ K} \tag{25}$$

$$T_{\rm phot,obs} = T0 * \Phi * (1+z) \text{ K} \tag{26}$$

$$L_{\rm phot} = \frac{\theta^2\dot{E}_{\rm th}\Phi}{4} \text{ erg/s} \tag{27}$$

where a is the radiation constant.

Output parameters calculated from the model for thermal emission in Table 2

2.2.2. *Internal Shock Shell Dynamics and Emission*

When considering the collisions between shells, we take each pair of adjacent active shells (shells which have already been launched) and calculate the time till they collide (see Appendix A). Selecting the minimum time until collision, $t_{\rm coll,min}$, we move all shells according to $R_{\rm new,i} = R_{\rm old,i} + ()\beta_{\rm i} * t_{\rm coll,min})$. The two shells that collide will have an initial average Lorentz factor given by

Table 2. Output parameters calculated from the model for thermal emission

Parameter	Description
(1)	(2)
$t_{\mathrm{e,i}}$	The time when shell i crosses the photosphere. Also will be the emission time.
$t_{\mathrm{a,i}}$	The arrival time of photons produced by shell i.
Δt_{i}	Duration width of the emission of shell i.
T_{i}	Temperature of shell i at emission.
L_{i}	Emitted luminosity of shell i.
$R_{\mathrm{phot,i}}$	Radius of the photosphere for shell i.

$$\Gamma_{\mathrm{r}} \sim \sqrt{(\Gamma_{\mathrm{s}}\Gamma_{\mathrm{r}})} \tag{28}$$

for the slow and rapid shells. Once the momentum and energy of the shells has been fully redistributed the final Lorentz factor will be

$$\Gamma_{\mathrm{f}} = \sqrt{(\Gamma_{\mathrm{s}}\Gamma_{\mathrm{r}}\frac{m_1\Gamma_1 + m_2\Gamma_2}{m_1\Gamma_2 + m_2\Gamma_1})} \tag{29}$$

The new mass will be the sum of the two colliding shell masses.

For all collision events, the arrival time of the photons and duration of the emission is given by

$$t_{\mathrm{a}} = (t_{\mathrm{e}} - R_{\mathrm{coll}}) \tag{30}$$

$$\Delta t_{\mathrm{obs}} = \frac{R_{\mathrm{coll}}}{2\Gamma_{\mathrm{r}}^2}(1+z) \tag{31}$$

To calculate the emission from the internal shocks, we follow the procedure detailed in Daigne & Mochkovitch (1998). Assuming synchrotron emission from electrons accelerated in a magnetic field.

The average energy dissipated per proton in a shock is given by

$$\epsilon = (\Gamma_{\mathrm{int}} - 1)m_{\mathrm{p}}c^2 \tag{32}$$

$$\text{where } \Gamma_{\mathrm{int}} = \frac{1}{2}\left[\left(\frac{\Gamma_1}{\Gamma_2}\right)^{1/2} + \left(\frac{\Gamma_2}{\Gamma_1}\right)^{1/2}\right] \tag{33}$$

where Γ_{int} is the Lorentz factor for internal motions in the shocked material.

The comoving proton number density is given by

$$n \approx \frac{\dot{M}}{4\pi r^2\bar{\Gamma}m_{\mathrm{p}}c} \approx \frac{\dot{E}_{\mathrm{kin}}}{4\pi r^2\bar{\Gamma}^2 m_{\mathrm{p}}c^3}\ 1/\mathrm{cm}^3 \tag{34}$$

which allows us to calculate the equipartition magnetic field

$$B_{\mathrm{eq}} \approx (8\pi\epsilon_{\mathrm{B}}n\epsilon)^{1/2}\ (\mathrm{erg/cm^3})^{1/2} \tag{35}$$

where $\epsilon_{\mathrm{B}} \lesssim 1$.

The typical synchrotron energy of the emitting electrons is given by

$$E_{\mathrm{syn}} = 50\frac{\Gamma_{\mathrm{r}}}{300}\frac{B_{\mathrm{eq}}}{1000\mathrm{G}}\left(\frac{\Gamma_{\mathrm{e}}}{100}\right)^2\ \mathrm{eV} \tag{36}$$

This can be placed in the comoving frame

$$E^0_{\mathrm{syn}} = E_{\mathrm{syn}}/\Gamma_{\mathrm{r}}\ \mathrm{eV} \tag{37}$$

For electrons with a large enough Lorentz factor to produce gamma-rays via synchrotron emission, Γ_{e} can be expressed using Bykov & Meszaros (1996) who considered the scattering of electrons by turbulent magnetic field fluctuations and found

$$\Gamma_{\mathrm{e}} \sim \left[\left(\frac{\alpha_{\mathrm{M}}}{\xi}\right)\left(\frac{\epsilon}{m_{\mathrm{e}}c^2}\right)\right]^{-1/(3-\mu)} \tag{38}$$

where α_{M} is the fraction fo the dissipated energy which goes into magnetic field fluctuations, ξ is the fraction of electrons which are accelerated, and μ is the index of the fluctuation spectrum. For simplicity, we have currently used $\Gamma_{\mathrm{e}} = 10^4$.

Although we do not calculate the Inverse Compton (IC) emission in this work, we still must calculate the amount of electrons scattered to higher energies due to Compton scatter. This also requires considering when the particles are in the Klein-Nishina regime. Using the Klein-Nishina cross section occurs when $w >> 1$, where

$$w = \frac{\Gamma_{\rm e} E^0_{\rm syn}}{m_{\rm e} c^2} \approx 33 \frac{B_{\rm eq}}{1000{\rm G}} \left(\frac{\Gamma_{\rm e}}{10^4}\right)^3 \tag{39}$$

In the Klein-Nishina regime, the fraction of electrons which are scattered up to higher energies is given by

$$\alpha_{\rm IC} = \frac{Q_{\rm IC}}{(1+Q_{\rm IC})} \text{ for } w < 1 \tag{40}$$

$$\alpha_{\rm IC} = \frac{Q_{\rm IC}/w}{(1+Q_{\rm IC}/w)} \text{ for } w >> 1 \tag{41}$$

where $Q_{\rm IC}$ is the Compton parameter and can be defined as

$$Q_{\rm IC} = \tau_* \Gamma_{\rm e}^2 \tag{42}$$

(Note: commonly in the literature, the Compton is labeled with a Y) where τ_* is the optical depth of the shell with mass M_* and radius r_* which contains relativistic electrons,

$$\tau_* = \frac{\kappa_{\rm T} M_*}{4\pi r_*^2} \tag{43}$$

where $\kappa_{\rm t}$ is the Thomson opacity. The mass M_* can be estimated

$$M_* = \frac{t_{\rm syn}}{1+Q_{\rm IC}} \dot{M}_{\rm s} hock \tag{44}$$

$$\text{where} \tag{45}$$

$$t_{\rm syn} = 6 * \left(\frac{\Gamma_{\rm e}}{100}\right)^{-1} \left(\frac{B_{\rm eq}}{1000{\rm G}}\right)^{-2} \tag{46}$$

is the synchrotron time of the relativistic electrons and $\dot{M}_{\rm shock}$ is the mass flow rate across the shock, both in the comoving frame of the shocked material. Since the shock moves with a Lorentz factor $\sim \bar{\Gamma}$, $\dot{M}_{\rm shock}$ can be approximated by

$$\dot{M}_{\rm shock} \approx \frac{\dot{M}}{\bar{\Gamma}} \tag{47}$$

$$= \frac{\dot{E}_{\rm iso}}{c^2 \bar{\Gamma}^2} \tag{48}$$

From Equations 43, 43, and 43, we can form the relation

$$Q_{\rm IC} = \frac{\kappa_{\rm T} \dot{M}_{\rm shock} t_{\rm syn} \Gamma_{\rm e}^2}{4\pi r_*^2 (1+Q_{\rm IC})} \tag{49}$$

$$Q_{\rm IC}(1+Q_{\rm IC}) = \frac{\kappa_{\rm T}}{4\pi r_*^2} \frac{\dot{E}_{\rm iso}}{c^2 \bar{\Gamma}^2} \Gamma_{\rm e}^2 t_{\rm syn} \tag{50}$$

$$Q_{\rm IC}(1+Q_{\rm IC}) = 150 \frac{\kappa_{\rm T}}{\pi r_*^2} \frac{\dot{E}_{\rm iso} \Gamma_{\rm e}}{c^2 \bar{\Gamma}^2} \left(\frac{B_{\rm eq}}{1000{\rm G}}\right)^{-2} \tag{51}$$

The fraction of electrons which remain to emit synchrotron emission is simply

$$\alpha_{\rm syn} = 1 - \alpha_{\rm IC} \tag{52}$$

Lastly, the energy dissipated in a collision between two shells is given by

$$e = min(M_1, M_2) c^2 (\Gamma_1 + \Gamma_2 - 2 * \Gamma_{\rm r}) * \epsilon_{\rm e} * \alpha_{\rm syn} \text{ erg} \tag{53}$$

Where we use the smaller mass of the two colliding shells.

Although, this is the emission that occurs from two collisions, two more constraints must be met in order to be observed: (i) the relative velocity between the two shells must be larger than the local sound speed and (ii) the wind must be transparent to the emitted photons. The relative velocity between two layers is given by

$$\frac{v_{\rm rel}}{c} \approx \frac{\Gamma_1^2 - \Gamma_2^2}{\Gamma_1^2 + \Gamma_2^2} \tag{54}$$

where we have adopted a sound speed of $v_{\rm s}/c = 0.1$. Daigne & Mochkovitch (1998) have checked that other choices make little difference in the results since the main contribution to the burst comes from shocks with large differences in the Lorentz factors, e.g., $\frac{\Gamma_{\rm r}}{\Gamma_{\rm s}} \gtrsim 2$.

The transparency of the wind to the emitted photons is

$$\tau = \kappa_{\rm T} \sum_{\rm i > i_{shock}} \frac{m_{\rm i}}{4\pi r_{\rm i}^2} \tag{55}$$

where $m_{\rm i}$ and $r_{\rm i}$ are the mass and radius of shell i. The sum over indices i larger than $i_{\rm shock}$, which corresponds to the colliding shells.

Output parameters calculated from the model for synchrotron emission in Table 3

Table 3. Output parameters calculated from the model for synchrotron emission

Parameter (1)	Description (2)
$t_{\rm e,i}$	Time of the ith collision (source frame).
$t_{\rm a,i}$	The arrival time of photons produced the ith collision.
$\alpha_{\rm syn,i}$	Fraction of energy which went into synchrotron elections for the ith collision.
$B_{\rm eq,i}$	Magnetic field in equipartition for the ith collision.
$\Gamma_{\rm e,i}$	Average Lorentz factor of the electrons in the ith collision.
$E_{\rm syn}$	Typical synchrotron energy in the ith collision.
$\Gamma_{\rm r,i}$	$=\sqrt{\Gamma_{\rm s}\Gamma_{\rm r}}$, initial averaged Lorentz factor of the ith collision.
$e_{\rm diss,i}$	Energy dissipated in the ith collision.
$\Delta t_{\rm i}$	Duration width of the emission of shell i.
$\tau_{\rm i}$	Optical depth at the ith collision.
$v_{\rm rel,i}$	Relative collision speed between the slow and fast shells of the ith collision.

3. SPECTRUM GENERATION

3.1. *Thermal Spectrum*

3.2. *Synchrotron Spectrum*

Similar to how the thermal spectrum is created, the non-thermal spectrum generated from the GRB prompt emission simulations for a particular time-interval is created from the summation of spectra created by each emission event which occurred within time-interval.

The spectrum used for each emission event is therefor an important consideration. A simple first step is to assume a Broken Power Law (BPL) where the low and high energy power law indices are given by $\alpha = -1$ and $\beta \leq -2.5$. The value of alpha is an active discussion within the field and ranges between -0.6 to -1.5. The peak energy of the distribution can be determined by synchrotron energy calculated in the simulation, $E_{\rm synch}$.

Alternatively, if we assume that synchrotron emission is produced by internal shocks events in the jet, we can use a standard definition of the synchrotron spectrum (Sari et al. 1998). The variable parameters in this definition are the minimum Lorentz factor of the electron population and the cyclotron Lorentz factor, $\Gamma_{\rm m}$ and $\Gamma_{\rm c}$, respectively. These can be defined in terms of the parameters used in our simulations.

$$\Gamma_{\rm m} = \frac{p-2}{p-1}\frac{\epsilon_{\rm e}}{\zeta}\frac{m_{\rm p}}{m_{\rm e}}\left(\frac{\epsilon}{c^2}\right) \tag{56}$$

$$\Gamma_{\rm c} = \frac{const}{B^2 t'_{\rm dyn}} \Rightarrow \frac{R}{\Gamma c} \tag{57}$$

where $\epsilon/c^2 = (\Gamma_{\rm int} - 1)$. These Lorentz factors can be directly turned to frequencies, $\nu_{\rm m}$ and $\nu_{\rm c}$, respectively.

4. LIGHT CURVE GENERATION

The light curve of a simulated GRB is generated by taking the sum of all counts in the spectra created for each time bin of the light curve.

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APPENDIX

A. SHELL COLLISION TIME AND RADIUS

Consider two shells traveling along the same axis. We will label the radius, Lorentz factor, more rapid shell with r and the slower shell with s

Consider two shells launched into from a central engine into a collimated relativistic jet. The second shell is launched with a delay of $\Delta t_{\rm e}$. We can describe the position of the shells at a time t with

$$R_1(t) = R_{1,0} + c\beta_1 t \tag{A1}$$

$$R_2(t) = R_{2,0} + c\beta_2 t \tag{A2}$$

where $R_{\rm i,0}$ is the initial radius of the launched shell and $\beta_{\rm i} = v_{\rm i}/c$. Since we are only considering these two shells, we can set the zero position at $R_{2,0}$,

$$R_1(t) = L + c\beta_1 t \tag{A3}$$

$$R_2(t) = c\beta_2 t \tag{A4}$$

where L is the distance between the two shells at the launch of the second shell

$$L = c\beta_1 \Delta t_{\rm e} \tag{A5}$$

$$\rightarrow R_1(t) = c\beta_1(t + \Delta t_{\rm e}) \tag{A6}$$

Alternatively, we could have set the zero to $R_{1,0}$ and found

$$R_1(t) = c\beta_1 t \tag{A7}$$

$$R_2(t) = c\beta_2(t - \Delta t_{\rm e}) \tag{A8}$$

The case where the second shell launched is slower than the first is trivial and results in no collision between the two. In the case where the second shell is more rapid than the first, there will eventually be a collision between the two shells. The time until collision can be written as

$$R_1(t_{\rm coll}) = R_2(t_{\rm coll}) \tag{A9}$$

$$L + c\beta_1 t_{\rm coll} = c\beta_2 t_{\rm coll} \tag{A10}$$

$$\rightarrow t_{\rm coll} = \frac{L}{c(\beta_2 - \beta_1)} \tag{A11}$$

$$t_{\rm coll} = \frac{c\beta_1 \Delta t_{\rm e}}{c(\beta_2 - \beta_1)} \tag{A12}$$

$$t_{\rm coll} = \frac{\beta_1 \Delta t_{\rm e}}{(\beta_2 - \beta_1)} \tag{A13}$$

Substituting this back in the expression for the position we can find the collision radius,

$$R_1(t_{\rm coll}) = L + c\beta_1 t_{\rm coll} \tag{A14}$$

$$= L + c\beta_1 \left(\frac{L}{c(\beta_2 - \beta_1)}\right) \tag{A15}$$

$$= L\left(1 + \frac{\beta_1}{(\beta_2 - \beta_1)}\right) \tag{A16}$$

$$R_{\rm coll} = L\left(\frac{\beta_2}{\beta_2 - \beta_1}\right) \tag{A17}$$

These calculations can be done for more than two shells, but keep in mind when calculating the time and position of the collisions after the initial collision the distance between them (i.e., L) will just be the difference in their positions (not their "initial" separation, but the separation at the time of calculation).

When calculating $R_{\rm coll}$, it may be advantageous to use $R_1(t)$ or $R_2(t)$ because Equation A17 typically leads to significant numerical instabilities for high Lorentz factors.

A.1. *Typical Values*

Let us assume shell 1 and 2 have Lorentz factors $\Gamma_1 = 100$ and $\Gamma_2 = 400$ and that $\Delta t_{\rm e} = 0.002$ seconds. The time and radius of collision would be

$$L = 5.9 * 10^7 \text{ cm} \tag{A18}$$

$$t_{\rm coll} \approx 42.6 \text{ seconds} \tag{A19}$$

$$R_{\rm coll} = 1.2 * 10^{12} \text{ cm} \tag{A20}$$

B. DERIVATIONS OF $\Gamma_{\rm R}$ AND $\Gamma_{\rm INT}$

In a collision between two relativistic shells with Lorentz factors $\Gamma_1 and \Gamma_2$, and masses M_1 and M_2, we can calculate the resulting Lorentz factor of the bulk motion of the merged shells, $\Gamma_{\rm r}$ and the internal motion of the particles, $\Gamma_{\rm int}$. We can find these quantities by adding the four-momentum of each shell. Since the shells are moving radially in the jet, we only need to consider one axis of propagation in the four vector. If we consider the frame of the resulting merged shell we can write the system as

$$\begin{pmatrix} \beta_1\Gamma_1 M_1 c^2 \\ \Gamma_1 M_1 c^2 \end{pmatrix} + \begin{pmatrix} \beta_2\Gamma_2 M_2 c^2 \\ \Gamma_2 M_2 c^2 \end{pmatrix} = \begin{pmatrix} \Gamma_{\rm r} & \beta_{\rm r}\Gamma_{\rm r} \\ \beta_{\rm r}\Gamma_{\rm r} & \Gamma_{\rm r} \end{pmatrix} \begin{pmatrix} 0 \\ \Gamma_{\rm int}(M_1 + M_2)c^2 \end{pmatrix} \tag{B21}$$

This system of equations leads to the relations

$$\Gamma_{\rm r}\Gamma_{\rm i} = \frac{\Gamma_1 M_1 + \Gamma_2 M_2}{M_1 + M_2} \tag{B22}$$

$$\Gamma_{\rm r}\beta_{\rm r}\Gamma_{\rm int}(M_1 + M_2) = \Gamma_1\beta_1 M_1 + \Gamma_2\beta_2 M_2 \tag{B23}$$

Using Eq. B23 and substituting in Eq. B22 we obtian

$$\Gamma_{\rm r}\beta_{\rm r}\Gamma_{\rm int}(M_1 + M_2) = \Gamma_1\beta_1 M_1 + \Gamma_2\beta_2 M_2 \tag{B24}$$

$$\beta_{\rm r}(M_1 + M_2)\frac{\Gamma_1 M_1 + \Gamma_2 M_2}{M_1 + M_2} = \Gamma_1\beta_1 M_1 + \Gamma_2\beta_2 M_2 \tag{B25}$$

$$\beta_{\rm r}(\Gamma_1 M_1 + \Gamma_2 M_2) = \Gamma_1\beta_1 M_1 + \Gamma_2\beta_2 M_2 \tag{B26}$$

$$\beta_{\rm r} = \frac{\Gamma_1\beta_1 M_1 + \Gamma_2\beta_2 M_2}{\Gamma_1 M_1 + \Gamma_2 M_2} \tag{B27}$$

By making the assumption that $\Gamma_1, \Gamma_2 >> 1$, we can approximate $\beta \approx 1 - \frac{1}{2\Gamma^2}$

$$\beta_r \approx 1 - \frac{1}{2\Gamma_r^2} = \frac{\Gamma_1\beta_1 M_1 + \Gamma_2\beta_2 M_2}{\Gamma_1 M_1 + \Gamma_2 M_2} \tag{B28}$$

$$\frac{1}{2\Gamma_r^2} = 1 - \frac{\Gamma_1\beta_1 M_1 + \Gamma_2\beta_2 M_2}{\Gamma_1 M_1 + \Gamma_2 M_2} \tag{B29}$$

$$= \frac{\Gamma_1 M_1 + \Gamma_2 M_2 - \Gamma_1\beta_1 M_1 - \Gamma_2\beta_2 M_2}{\Gamma_1 M_1 + \Gamma_2 M_2} \tag{B30}$$

$$= \frac{\Gamma_1 M_1 + \Gamma_2 M_2 - \Gamma_1(1 - \frac{1}{2\Gamma_1^2})M_1 - \Gamma_2(1 - \frac{1}{2\Gamma_2^2})M_2}{\Gamma_1 M_1 + \Gamma_2 M_2} \tag{B31}$$

$$\frac{1}{2\Gamma_r^2} = \frac{\frac{M_1}{2\Gamma_1} - \frac{M_2}{2\Gamma_2}}{\Gamma_1 M_1 + \Gamma_2 M_2} \tag{B32}$$

$$2\Gamma_r^2 = \frac{\Gamma_1 M_1 + \Gamma_2 M_2}{\frac{M_1}{2\Gamma_1} - \frac{M_2}{2\Gamma_2}} \tag{B33}$$

$$= \frac{\Gamma_1 M_1 + \Gamma_2 M_2}{\frac{1}{2\Gamma_1\Gamma_2}(\Gamma_2 M_1 - \Gamma_1 M_2)} \tag{B34}$$

$$\Gamma_r = \sqrt{\Gamma_1\Gamma_2 \frac{\Gamma_1 M_1 + \Gamma_2 M_2}{\Gamma_2 M_1 - \Gamma_1 M_2}} \tag{B35}$$

$$\tag{B36}$$

We can assume that shock events occur when two shells of approximately equal mass collide, $M_1 \approx M_2 = M$. This simplifies the above expression to

$$\Gamma_r \approx \sqrt{\Gamma_1\Gamma_2} \tag{B37}$$

To find $\Gamma_{\rm int}$ we return to Eq. B22 and continue to use the assumption $M_1 \approx M_2 = M$,

$$\Gamma_{\rm int} = \frac{\Gamma_1 M_1 + \Gamma_2 M_2}{\Gamma_r(M_1 + M_2)} \tag{B38}$$

$$\Gamma_{\rm int} = \frac{M(\Gamma_1 + \Gamma_2)}{2M\Gamma_r} \tag{B39}$$

$$\Gamma_{\rm int} = \frac{1}{2}\left[\left(\frac{\Gamma_1}{\Gamma_2}\right)^{1/2} + \left(\frac{\Gamma_2}{\Gamma_1}\right)^{1/2}\right] \tag{B40}$$

$$\tag{B41}$$

C. SIMULATION TIME CONSIDERATIONS

In this section we make rough estimates of the times required to run all simulations required for a particular parameter space. Each single simulation requires ~ 5 seconds to run.

C.1. *Using observations to help constrain the parameter space*

Using the non-thermal component observed GRB prompt emission spectra, we can help limit the parameter space we must explore during a simulation.

$$[t_w] \quad \times [\Gamma(t)] \quad \times [\zeta] \tag{C42}$$

$$[3] \quad \times [3 \times 3 \times 4] \quad \times [3] = 432 \text{ simulations} \qquad 432 \text{ simulations} \times 5 \text{ sec} \sim 36\text{minutes} \tag{C43}$$

this will need to be done for every time bin used, because $\dot{E}^k_{\rm iso}$ must be changed depending on the observed brightness. If the thermal component is not observed, this part of the simulation can be removed. If the thermal component is observed, the brightness can be used to constrain the value of $\epsilon * (1 + \sigma) \leq 1$.

Table 4. Custom classes and libraries created for the GRB prompt emission simulation code.

Library Name	Description
cosmology.cpp	Defines useful cosmology constants and functions used throughout the code.
utilfuncs.cpp	Defines useful utility functions used throughout the code.
Spectrum.cpp	Defines all necessary variables and functions that pertain to a spectrum object.
LightCurve.cpp	“” light curve object.
TTEs.cpp	“” Time Tagged Event object.
Response.cpp	“” Response object.
FitStats.cpp	Calculates and stores fit statistics values during fitting. Includes read/write methods.
ShellDist.cpp	Defines the functions used to calculate the Lorentz for each shell in the wind.
ModelParams.cpp	Stores the model parameters currently being used. Includes read/write methods.
SynthGRB.cpp	Contains the necessary scripts to simulate GRB prompt emission. Calculates the spectra and light curves.
SynthGRBLibrary.cpp	Creates multiple SynthGRB’s based on user defined parameter space to create a library.
ObsGRB.cpp	Contains methods to store and manipulate observed GRB data.
DataAnalysis.cpp	Contains all methods used for data manipulation and fitting.

C.2. *Creating a library*

To create a library that can be applied to any GRB prompt emission, the parameter space will be much more broad.

$$[t_{\rm w}]\times[\sigma]\times \quad [\dot{E}]\times \quad [\Gamma(t)]\times \quad [\zeta]\times \quad [\epsilon_{\rm th}]\times \quad [\epsilon_{\rm e}]\times \quad [\epsilon_{\rm B}] \tag{C44}$$

$$[5]\times[3]\times \quad [5]\times \quad [3\times 3\times 4]\times \quad [4]\times \quad [4]\times \quad [2]\times \quad [4] = 345,600 \text{ simulations} \tag{C45}$$

$$\tag{C46}$$

$$345,600 \text{ simulations} \times 5 \text{ sec} \sim 20 \text{ days} \tag{C47}$$

This is obviously not feasible.

D. CUSTOM CLASSES/LIBRARIES

Below is a list of the classes and libraries created for the GRB prompt emission simulation code.

E. PACKAGES USED

In Table 5 I list the standard C++ and Python packages used.

Table 5. A list of the standard C++ packages used in the simulation code and Python packages used for plotting.

C++ Package Name	Python Package Name
cmath	matplotlib.pyplot
cstdio	numpy
cstring	os
iostream	scipy.integrate (to calculate luminosity distance)
fstream	
vector	
sstream	