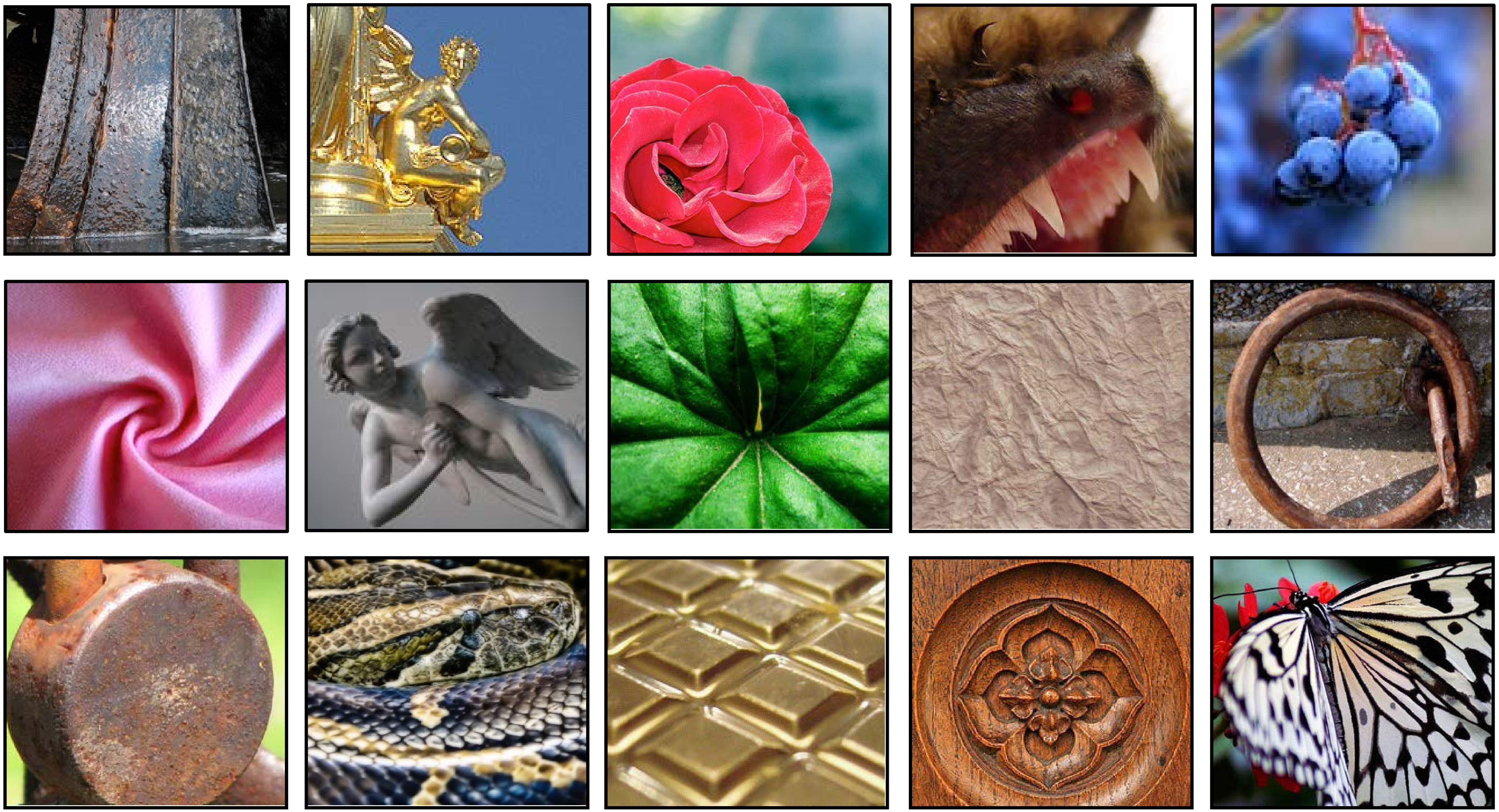
Radiometry and reflectance



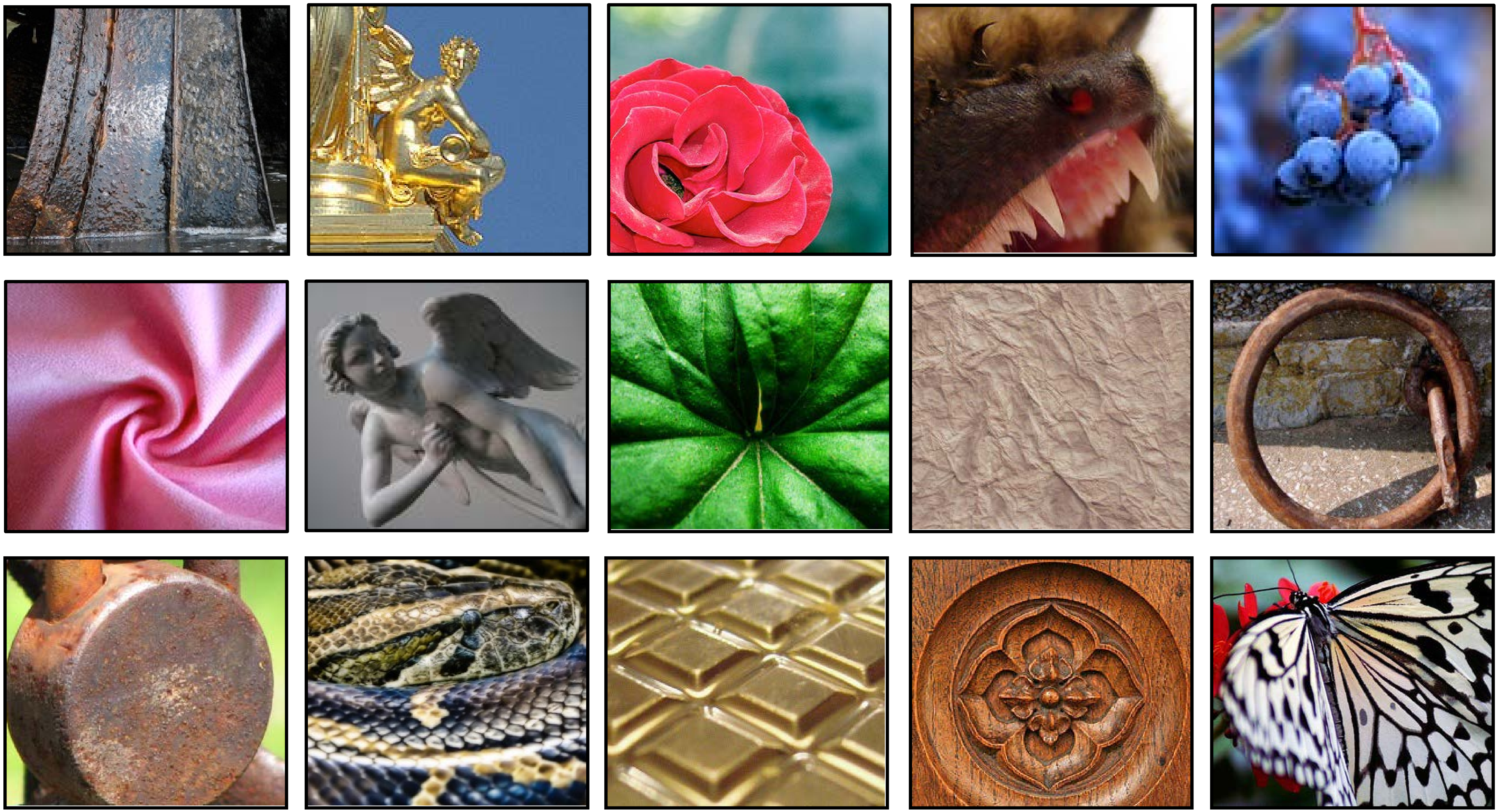
Slide credits: Srinivasa Narasimhan, Todd Zickler, Kayvon Fatahalian, Ioannis Gkioulekas

Overview of today's lecture

- Measuring light and radiometry.
- Reflectance and BRDF.
- Photometric stereo

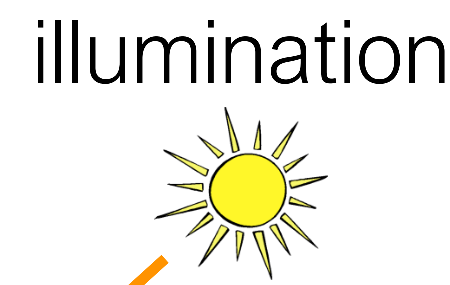
Appearance

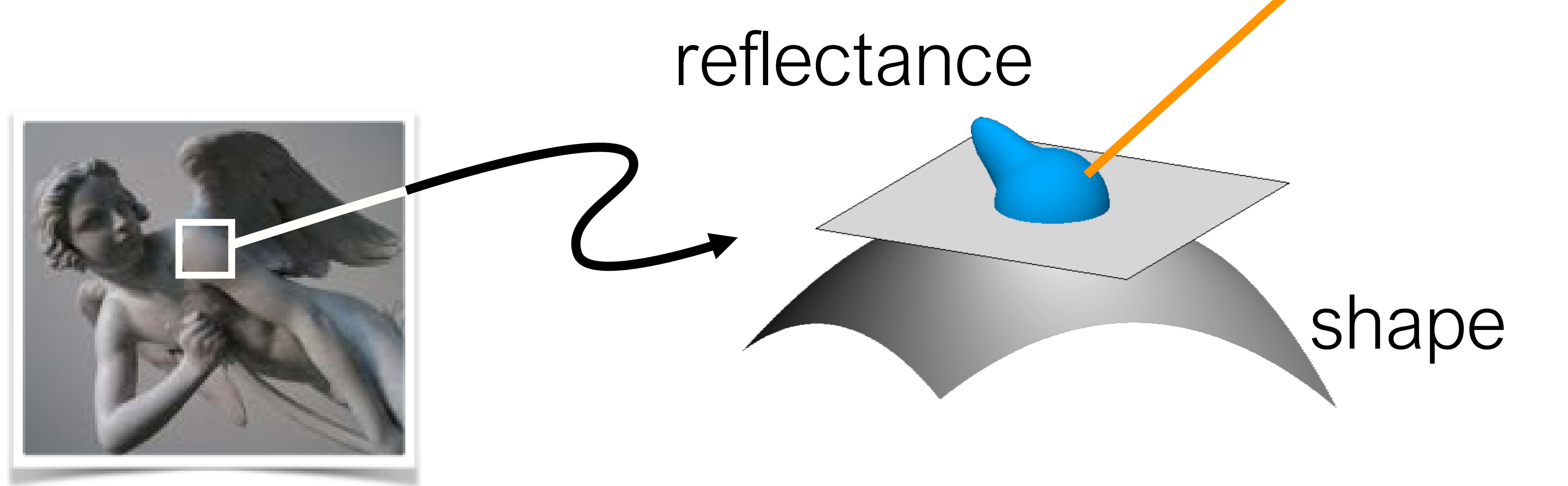
Appearance



“Physics-based” computer vision
(a.k.a “inverse optics”)

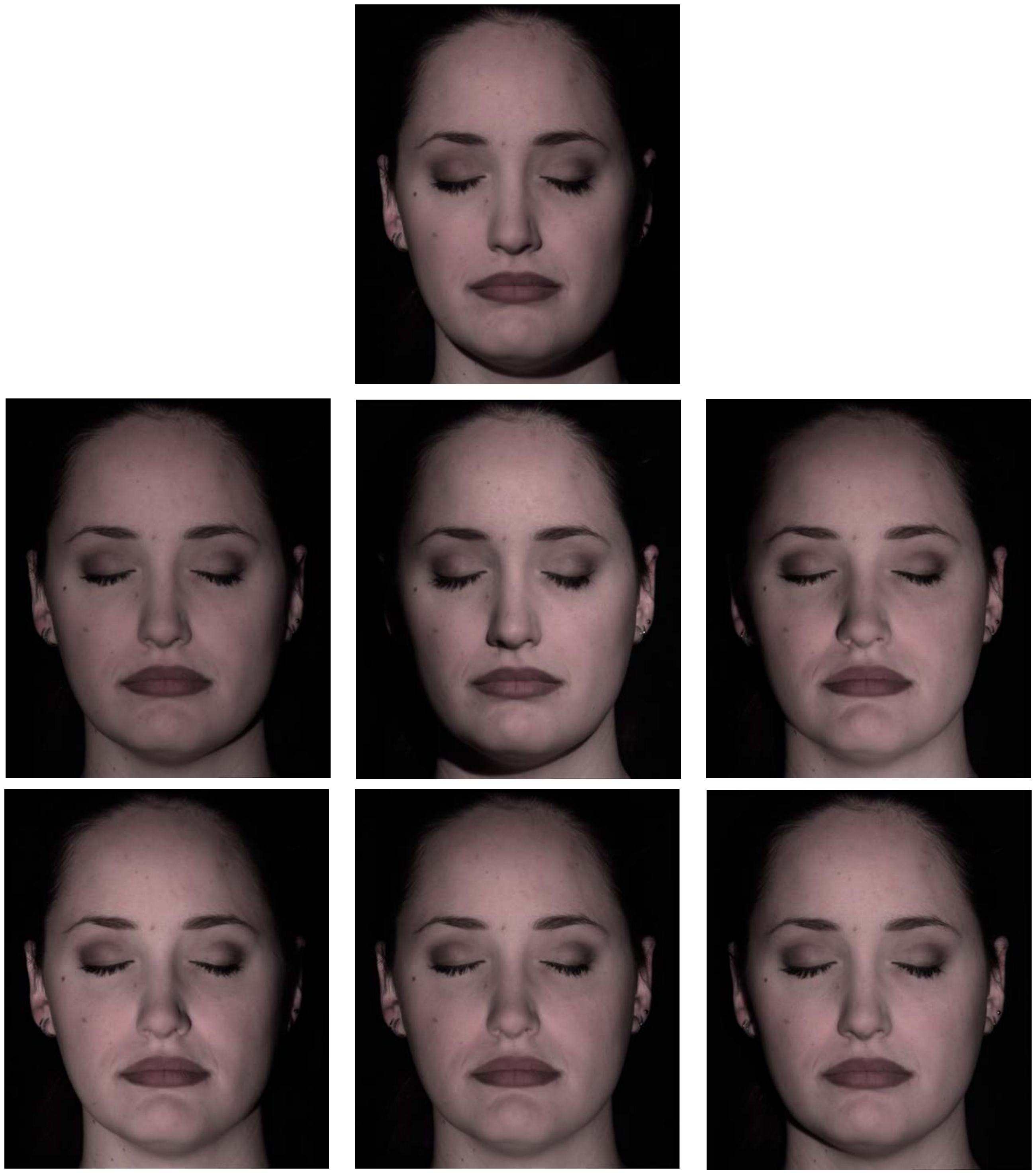
Our challenge: Invent computational representations of shape, lighting, and reflectance that are ***efficient***: simple enough to make inference tractable, yet general enough to capture the world’s most important phenomena





$\mathbf{I} \Longrightarrow$ shape, illumination, reflectance

Example application: Photometric Stereo

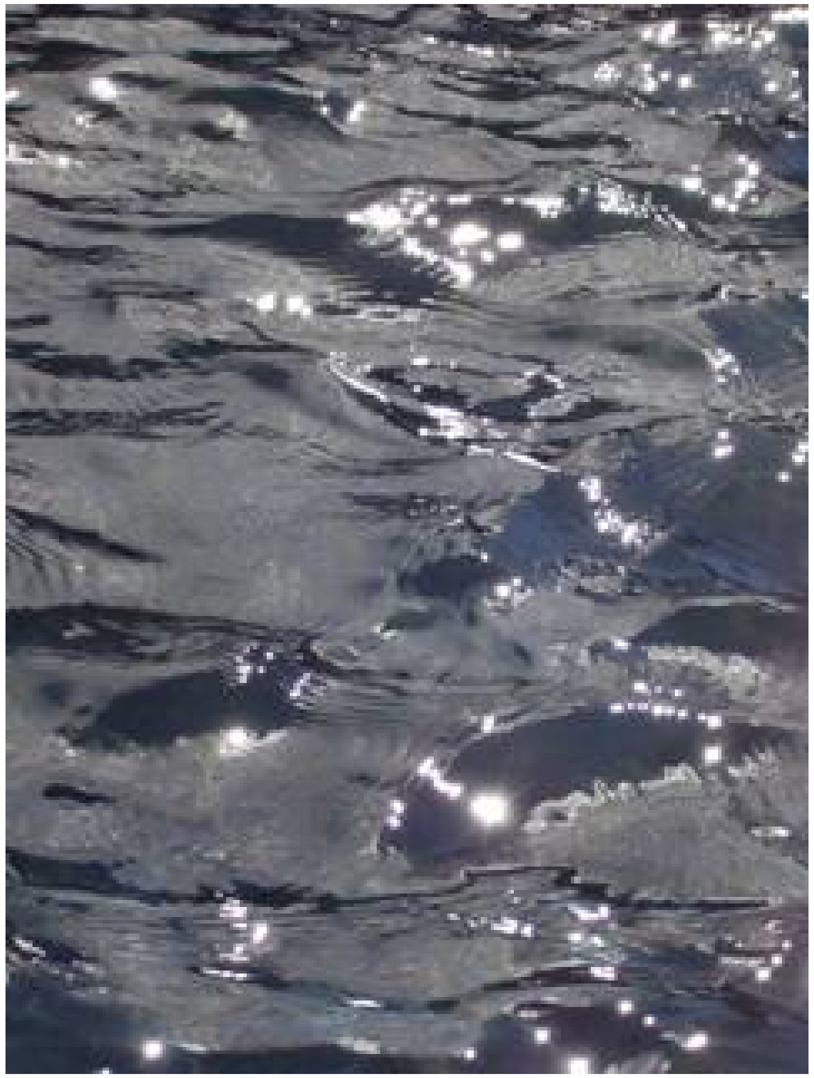


Why study the physics (optics) of the world?

Lets see some pictures!

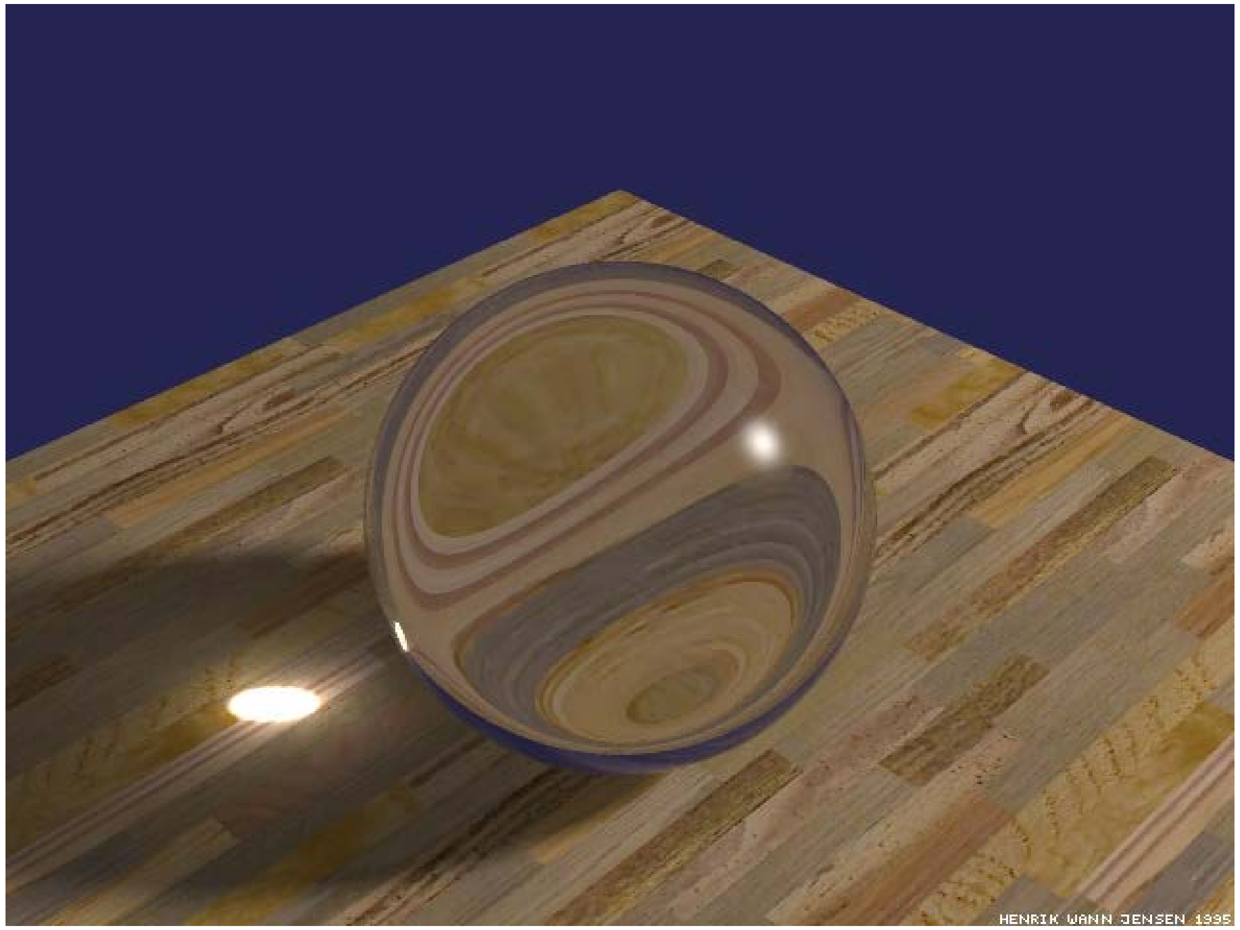
Light and Shadows


Reflections



Refractions

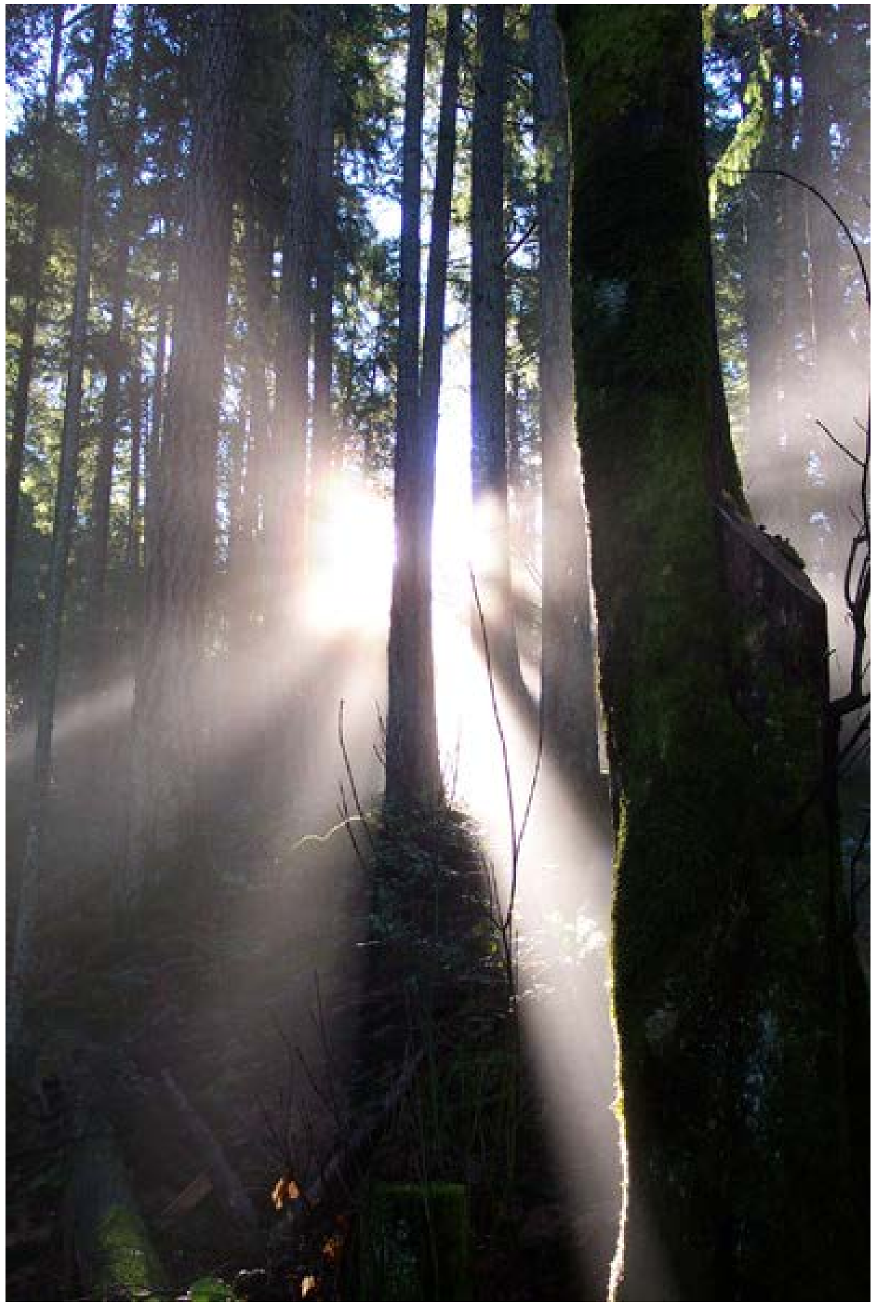
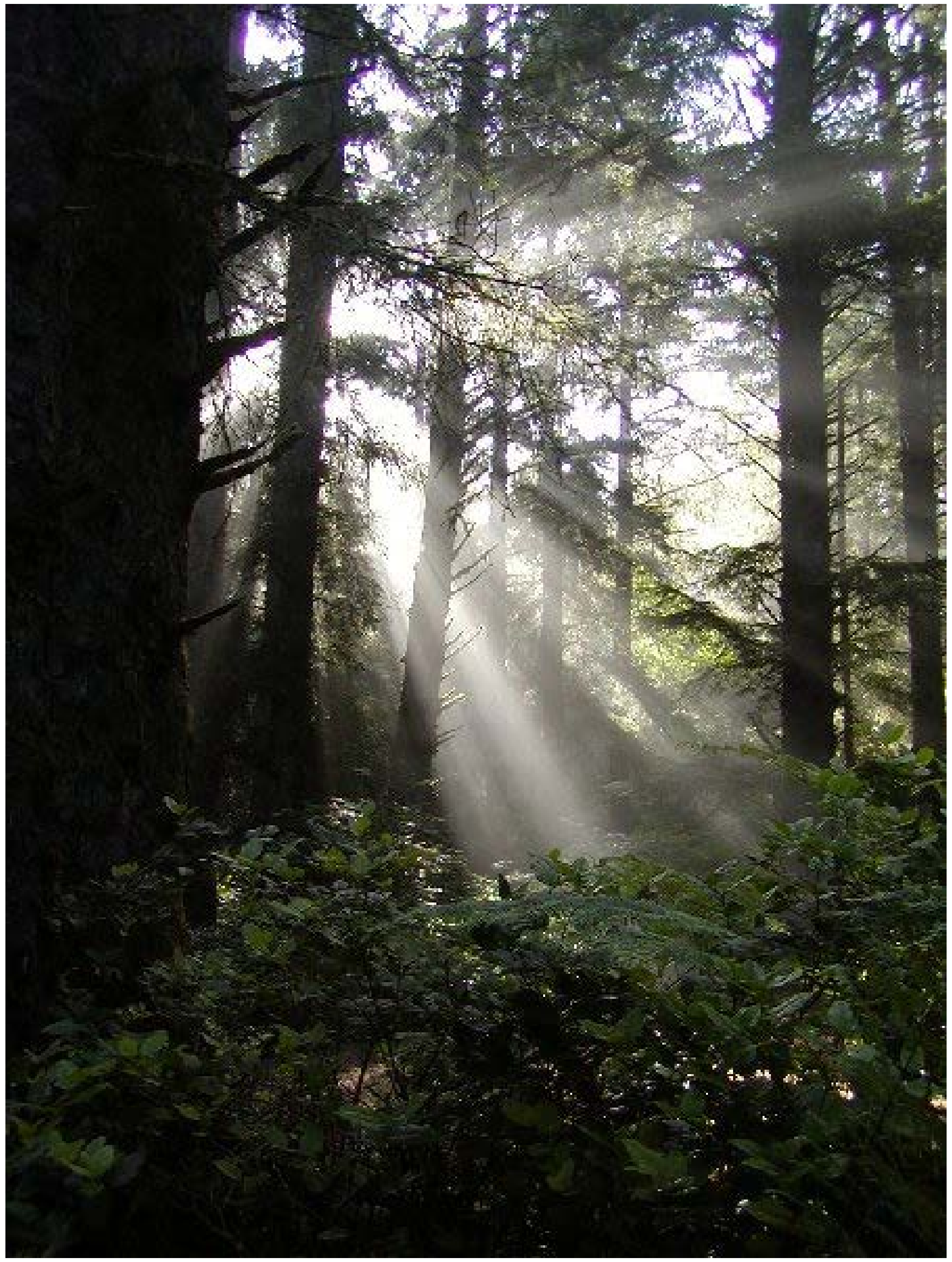

HENRIK WANN JENSEN 1995


HENRIK WANN JENSEN 1995

Interreflections


Mies Courtyard House with Curved Elements

Scattering

More Complex Appearances


RENDERED BY HENRIK WANN JENSEN - 2001


RENDERED BY HENRIK WANN JENSEN - 2001











Material Reflectance

Mirror

Shiny

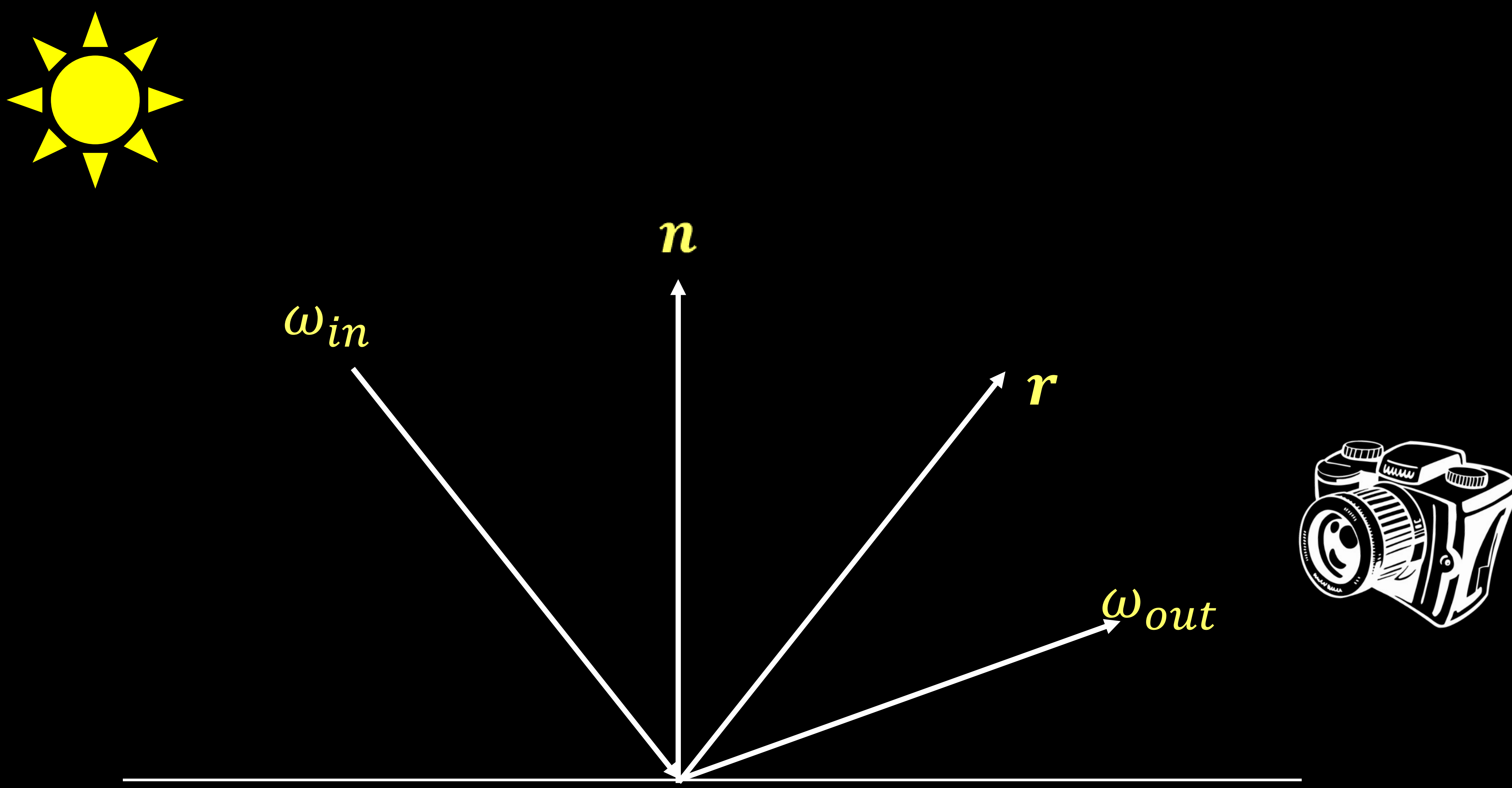
Moderately Glossy

Diffuse

Bidirectional Reflectance Distribution Functions (BRDF)

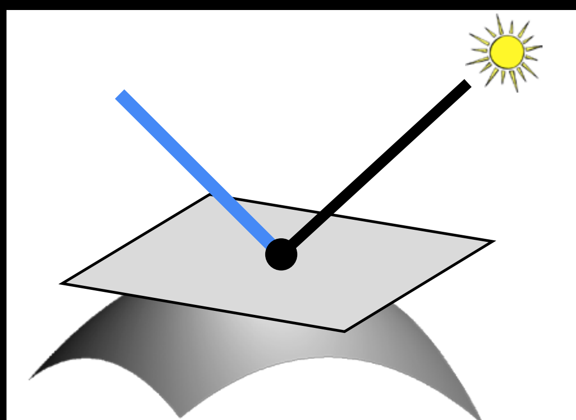
$$f_r(\vec{\omega}_{\text{in}}, \vec{\omega}_{\text{out}})$$

The portion of incoming energy at direction ω_{in}, reflected toward viewing direction ω_{out}

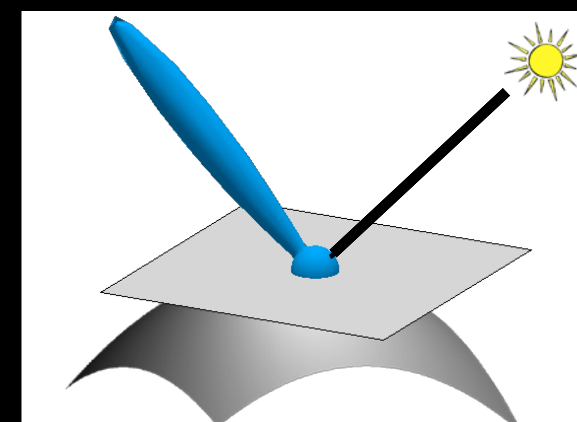


Plotting reflectance

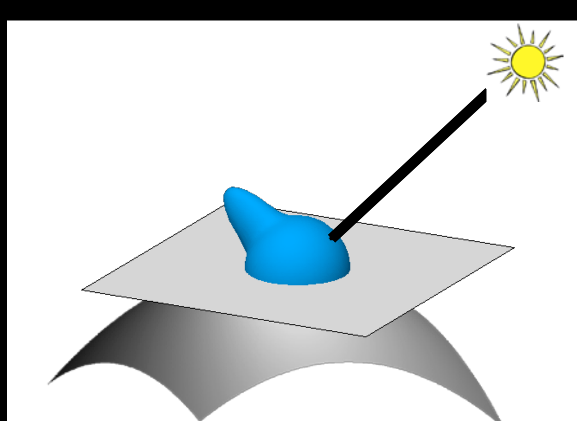
Mirror



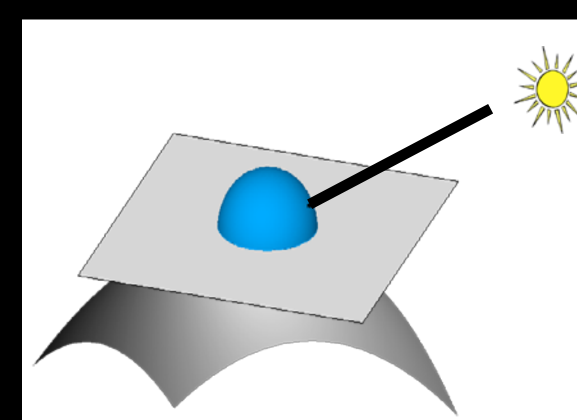
Shiny



Moderately Glossy



Diffuse



Bidirectional Reflectance Distribution Functions (BRDF)

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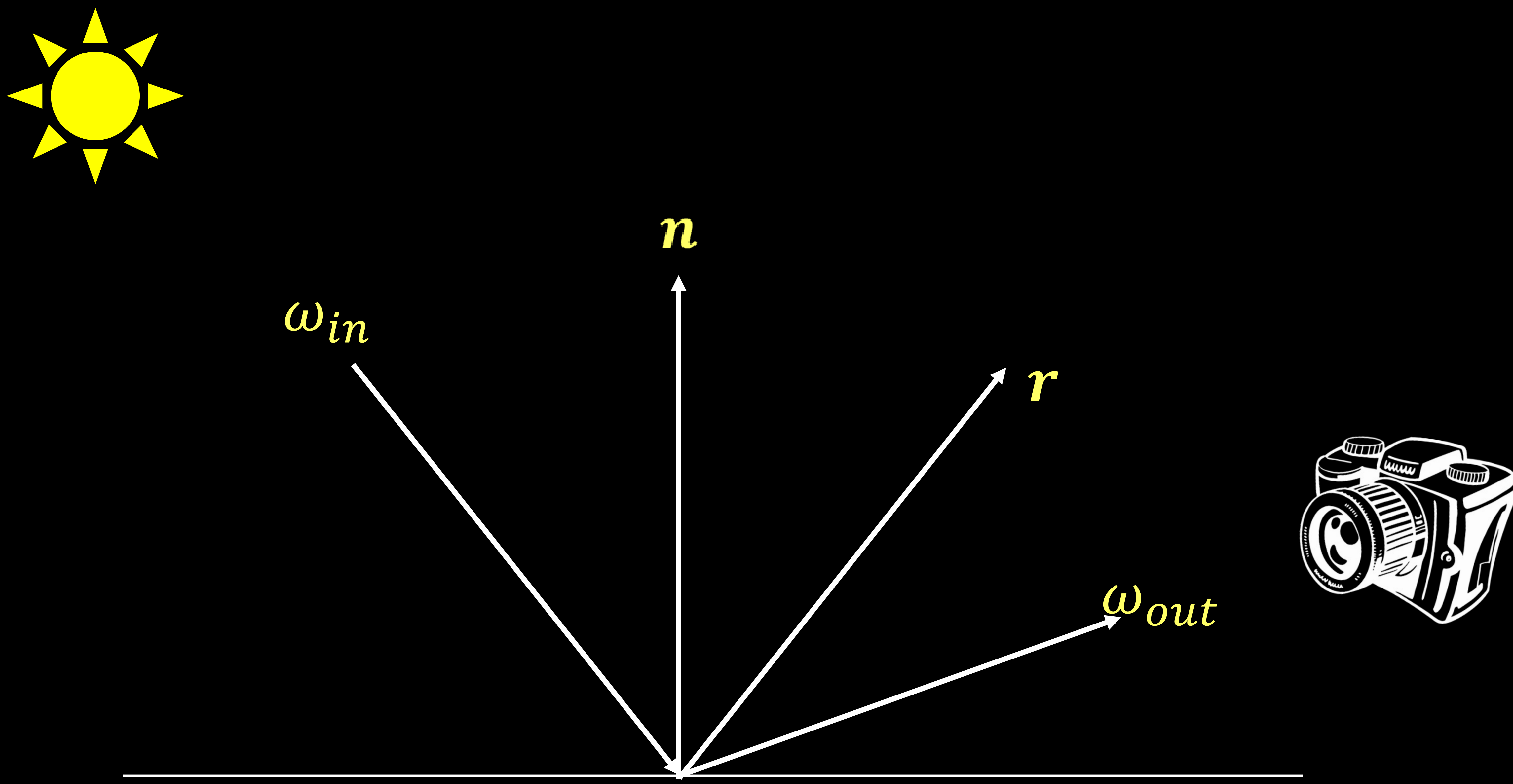
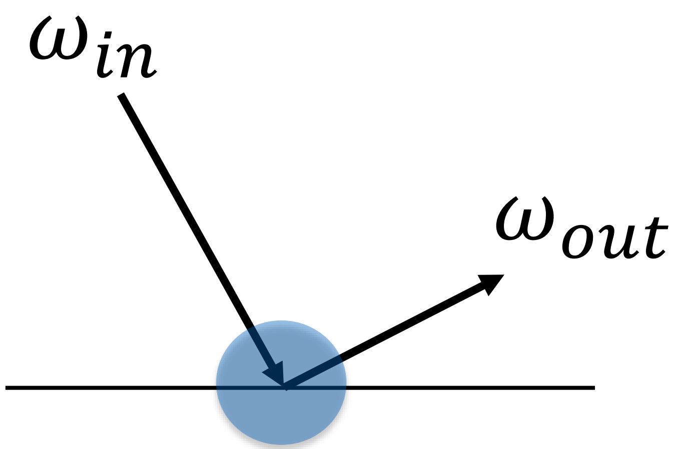
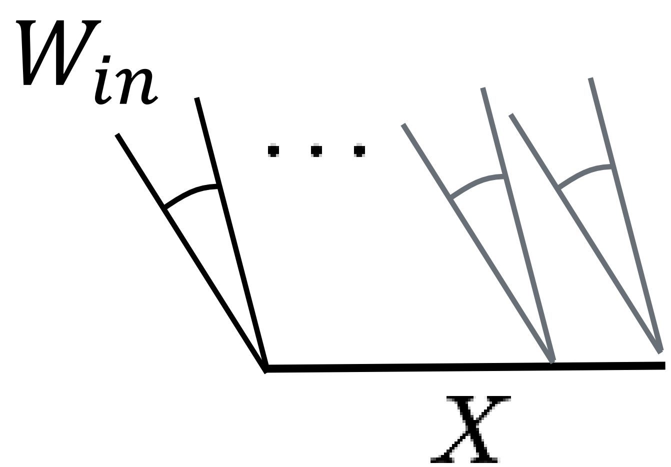


Differential definition of BRDF: What will be involved?

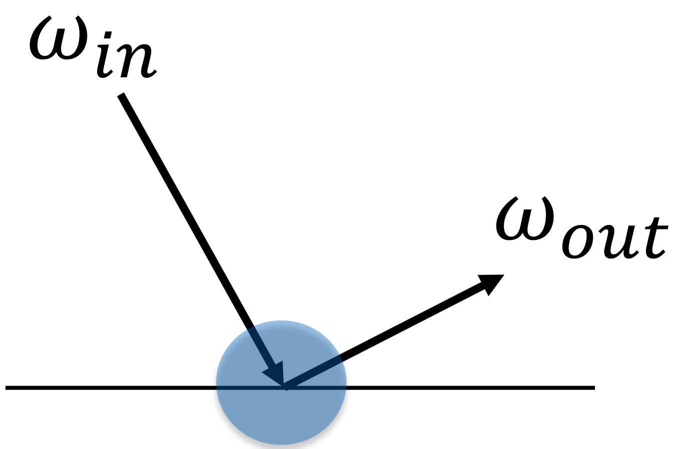


$$f(\omega_{in}, \omega_{out}) = \frac{\text{outgoing energy}}{\text{incoming energy}}$$



$$\frac{()}{\Phi(W_{in}, \mathrm{X})}$$

Differential definition of BRDF: What will be involved?



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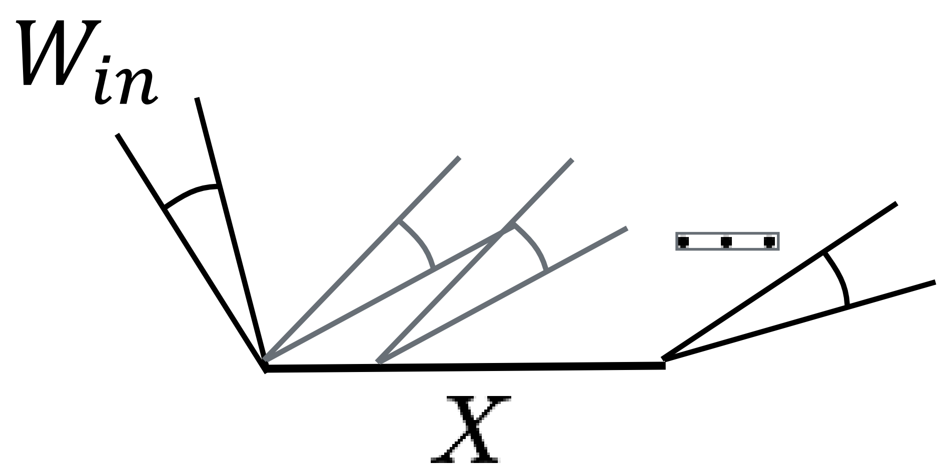
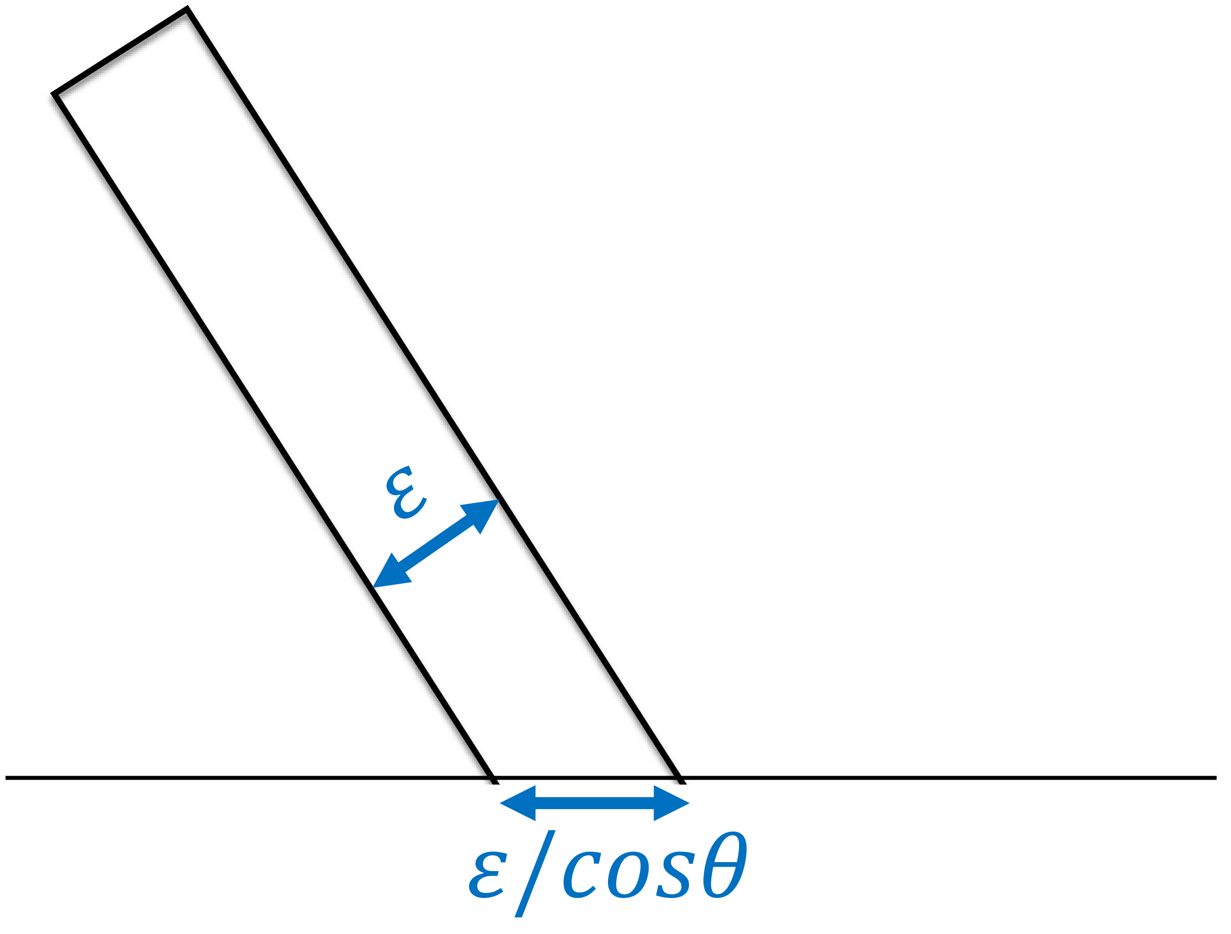
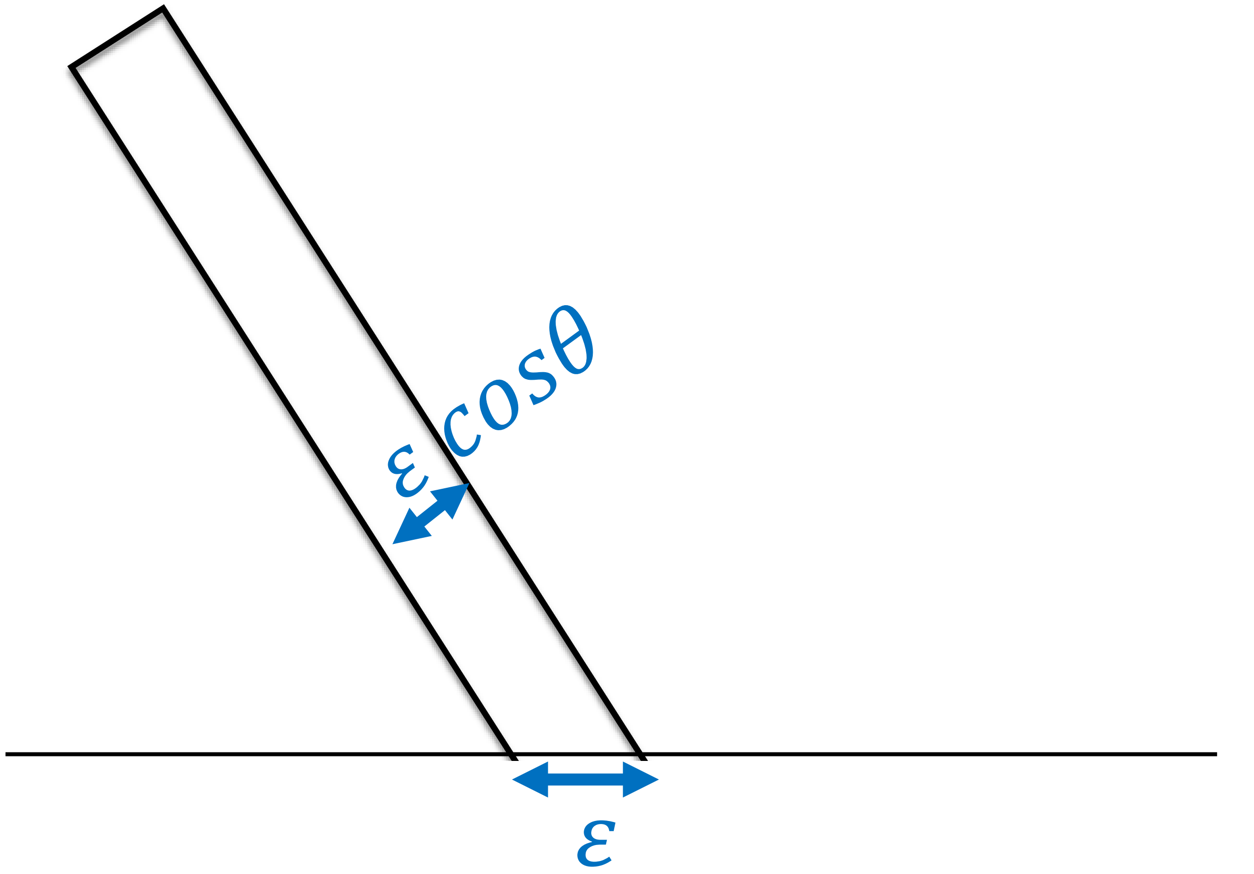


$$\frac{\Phi(W_{out}, X)}{\Phi(W_{in}, X)} \xrightarrow[\substack{|W_{in}| \to 0 \\ |W_{out}| \to 0 \\ |X| \to 0}]{} f(\omega_{in}, \omega_{out})$$

In the next slides: work harder, to define this mathematically

Foreshortening



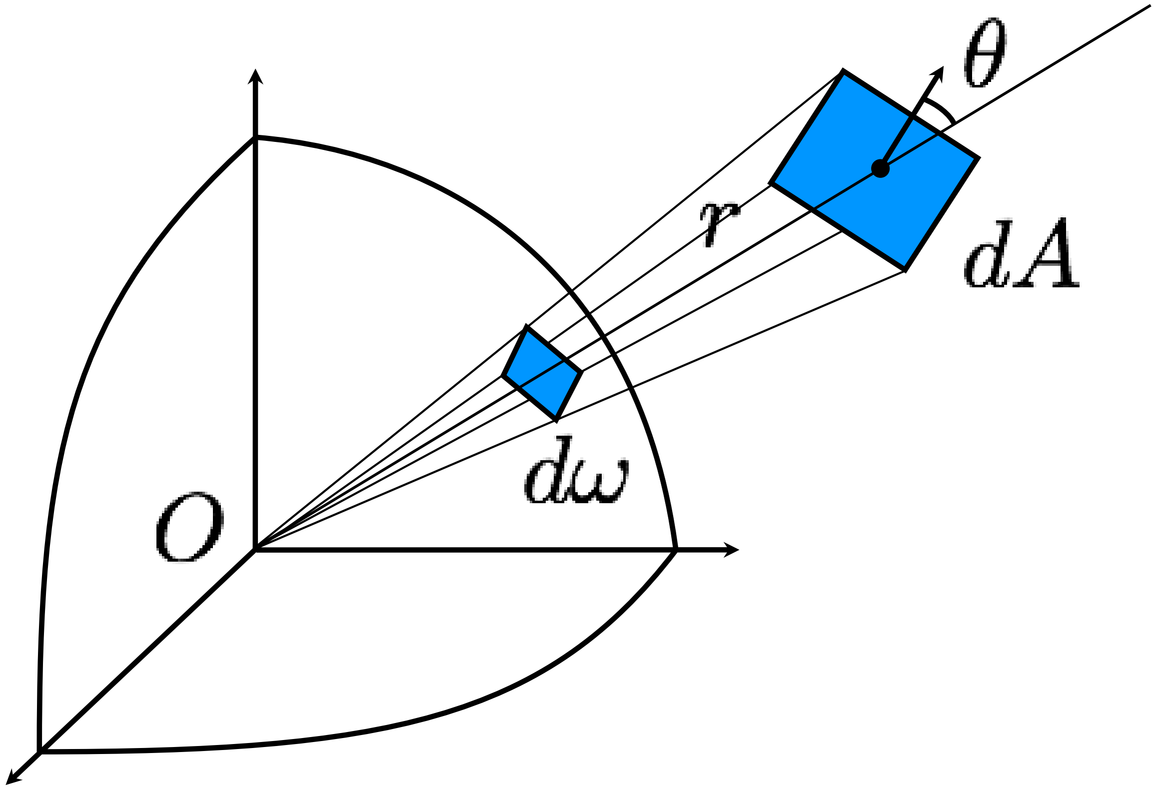


The usual source of confusion

Measuring light and radiometry

Solid angle

- The *solid angle* subtended by a small surface patch with respect to point O is the area of its central projection onto the unit sphere about O



Depends on:

- orientation of patch
- distance of patch

Solid angle

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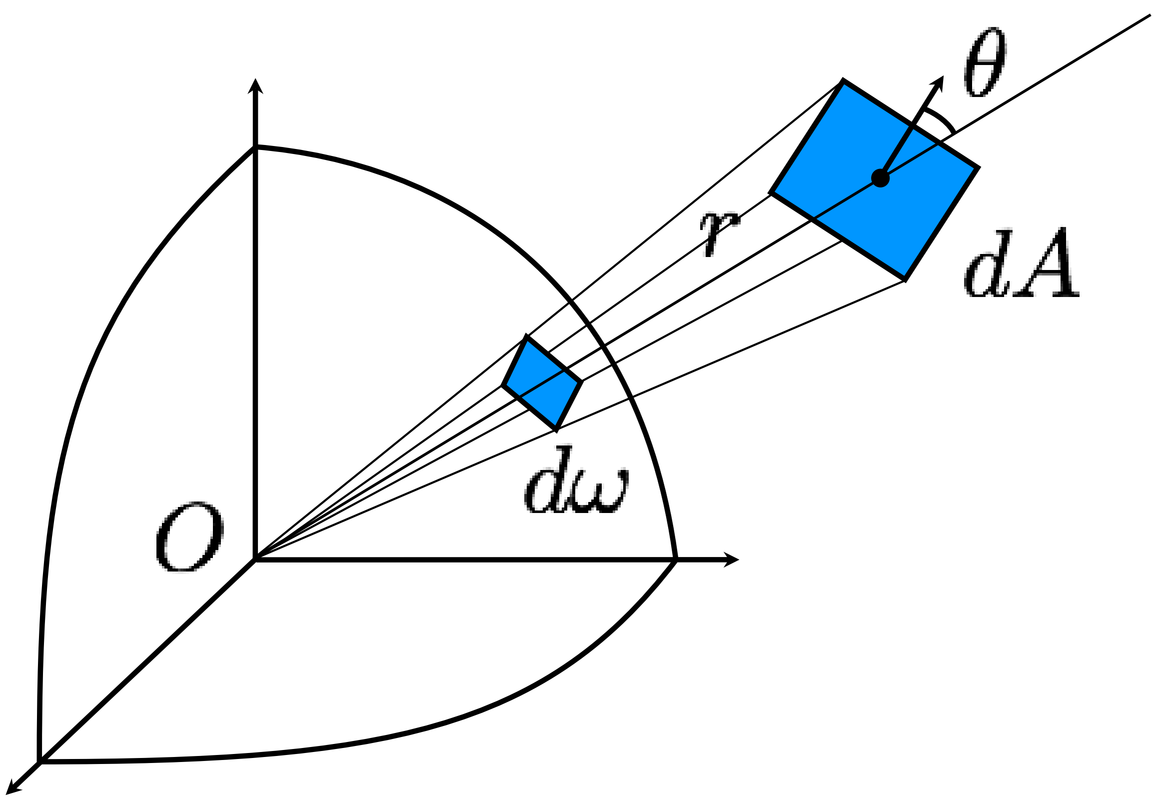


Depends on:

- orientation of patch
- distance of patch

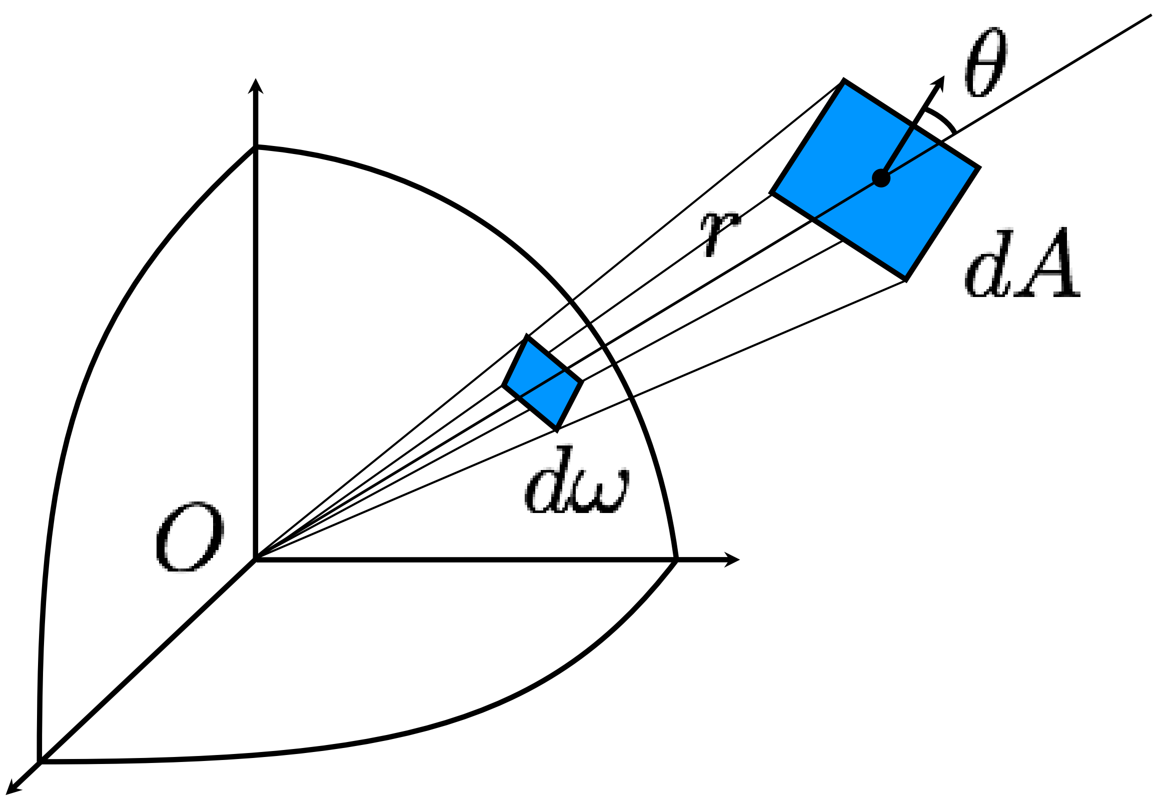
One can show:

$$\boxed{d\omega = \frac{dA\cos\theta}{r^2}}$$

Units: steradians [sr]

Solid angle

- The *solid angle* subtended by a small surface patch with respect to point O is the area of its central projection onto the unit sphere about O



Depends on:

- orientation of patch
- distance of patch

One can show:

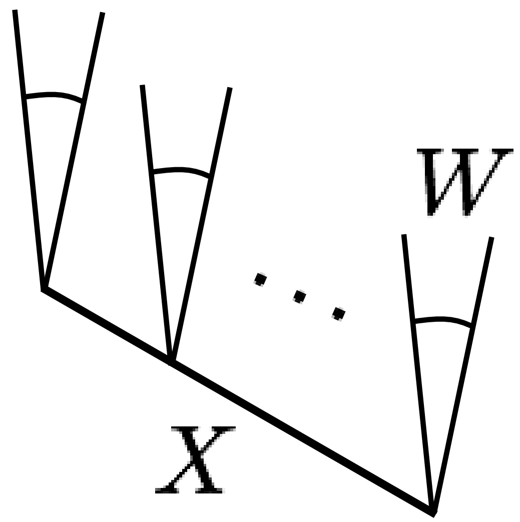
“surface foreshortening”

$$\boxed{d\omega = \frac{dA\cos\theta}{r^2}}$$

Units: steradians [sr]

Quantifying light: flux, irradiance, and radiance

- Imagine a sensor that counts photons passing through planar patch X in directions within angular wedge W
- It measures *radiant flux* [watts = joules/sec]: rate of photons hitting sensor area
- Measurement depends on sensor area |X| and wedge angle |W|

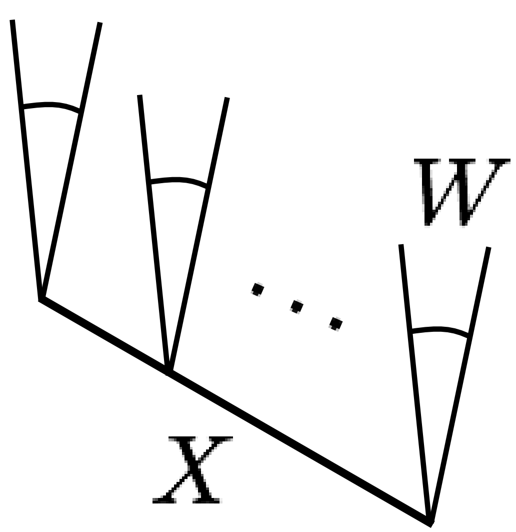


* shown in 2D for clarity; imagine three dimensions

radiant flux $\Phi(W, X)$

Quantifying light: flux, irradiance, and radiance

- *Irradiance*:
  A measure of incoming light that is independent of sensor area |X|
- Units: watts per square meter [W/m^2]



$$\frac{\Phi(W, X)}{|X|}$$

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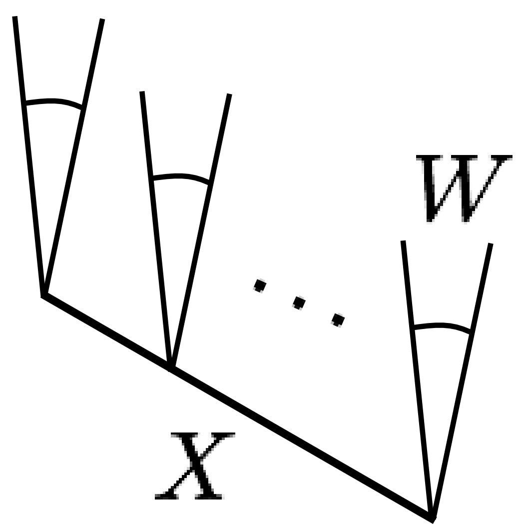
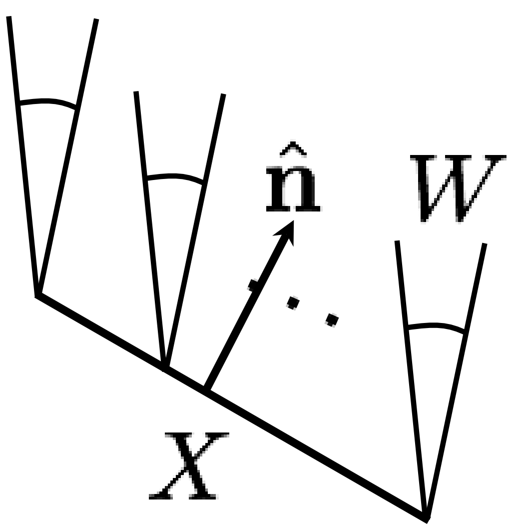


$$\lim_{X \longrightarrow x}$$

$$\frac{\Phi(W, X)}{|X|}$$

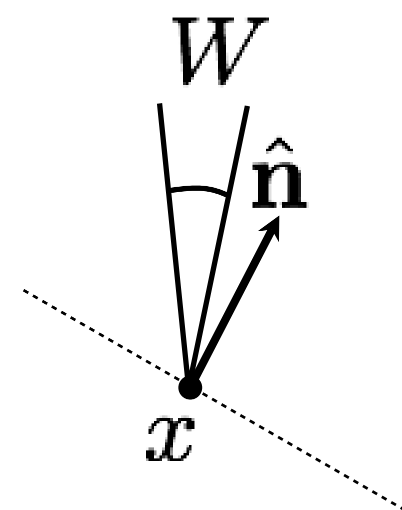
Quantifying light: flux, irradiance, and radiance

- *Irradiance*:
  A measure of incoming light that is independent of sensor area |X|
- Units: watts per square meter [W/m^2]
- Depends on sensor direction normal.



$$\frac{\Phi(W, X)}{|X|}$$

$$\lim_{X \longrightarrow x}$$

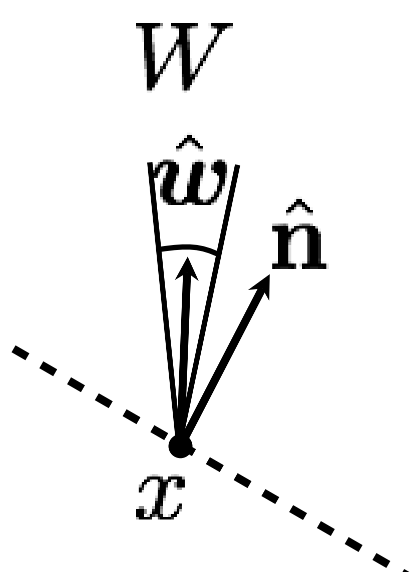


$$E_{\hat{\mathbf{n}}}(W, x)$$

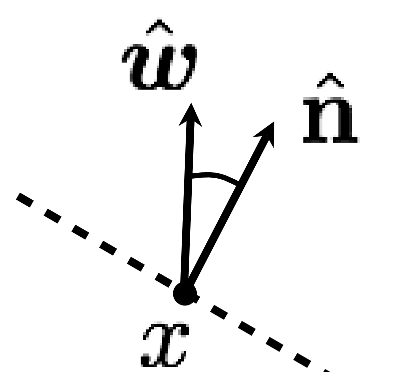
- We keep track of the normal because a planar sensor with distinct orientation would converge to a different limit
- In the literature, notations n and W are often omitted, and values are implied by context

Quantifying light: flux, irradiance, and radiance

- *Radiance*:
  A measure of incoming light that is independent of sensor area |X|, orientation n, and wedge size (solid angle) |W|
- Units: watts per steradian per square meter [$W/(m^2 \cdot sr)$]



$$\lim_{W \longrightarrow \hat{\boldsymbol{w}}}$$



$$\frac{E_{\hat{\mathbf{n}}}(W, x)}{|W|} \qquad L_{\hat{\mathbf{n}}}(\hat{\boldsymbol{\omega}}, x)$$

- Has correct units, but still depends on sensor orientation
- To correct this, convert to measurement that would have been made if sensor was perpendicular to direction ω

Quantifying light: flux, irradiance, and radiance

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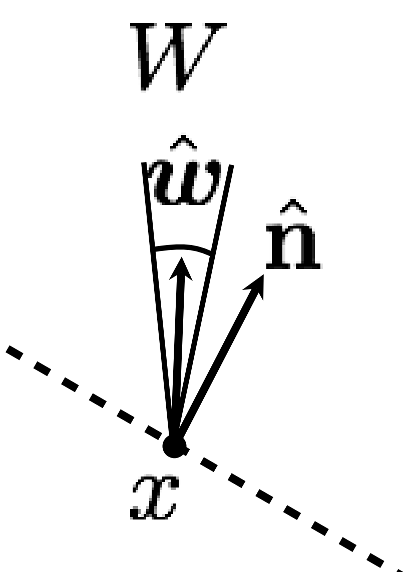

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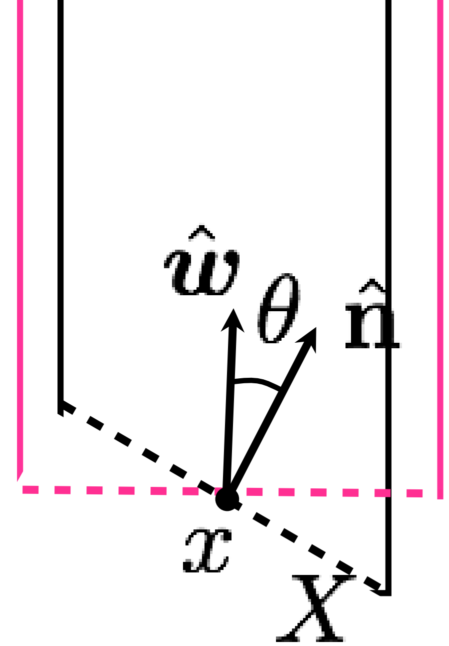

If we rotate segment x to be perpendicular to w (keeping same length) it captures a wider beam

$$\blacksquare = \frac{\blacksquare}{cos\theta}$$

"foreshortened area"

- Has correct units, but still depends on sensor orientation
- To correct this, convert to measurement that would have been made if sensor was perpendicular to direction ω

Quantifying light: flux, irradiance, and radiance

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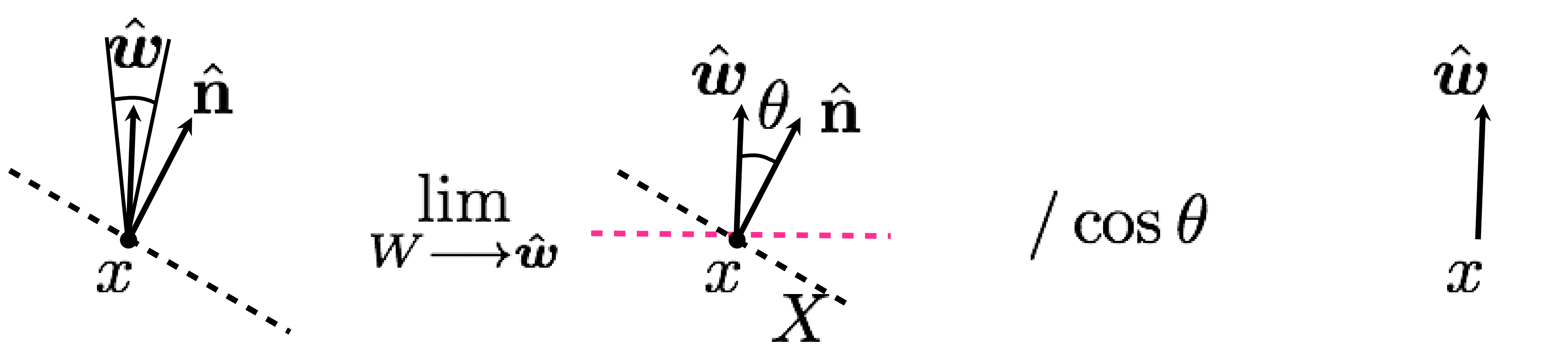
- Units: watts per steradian per square meter [W/(m^2·sr)]

"foreshortened in the direction of travel"



$$\frac{E_{\hat{\mathbf{n}}}(W, x)}{|W|} \qquad L_{\hat{\mathbf{n}}}(\hat{\omega}, x) \qquad L(\hat{\omega}, x)$$

"foreshortened area"

- Has correct units, but still depends on sensor orientation
- To correct this, convert to measurement that would have been made if sensor was perpendicular to direction ω

Temporary summery of radiometric quantities

$$E_{\hat{\mathbf{n}}}(W, x) = \frac{\Phi(W, X)}{|X|}$$

flux

irradiance

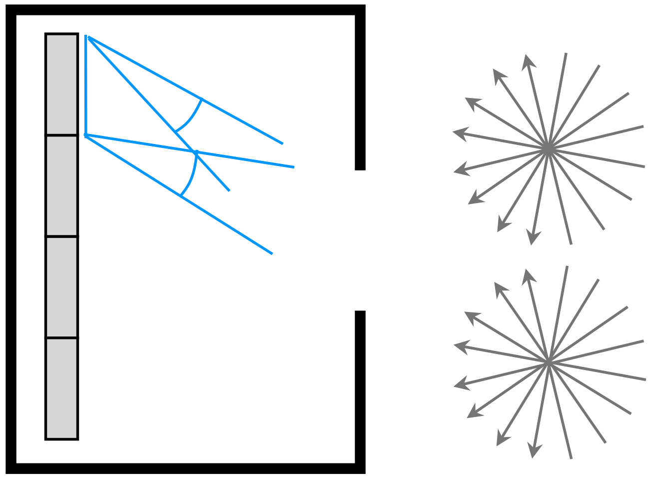
$$\frac{1}{cos\theta} \frac{E_{\hat{n}}(W, x)}{|W|} = L(\widehat{\omega}, x)$$

radiance

Caution: take *limit* at infinitely small X, W

Quantifying light: flux, irradiance, and radiance

- Attractive properties of radiance:
  - Allows computing the radiant flux measured by *any* finite sensor



Quantifying light: flux, irradiance, and radiance

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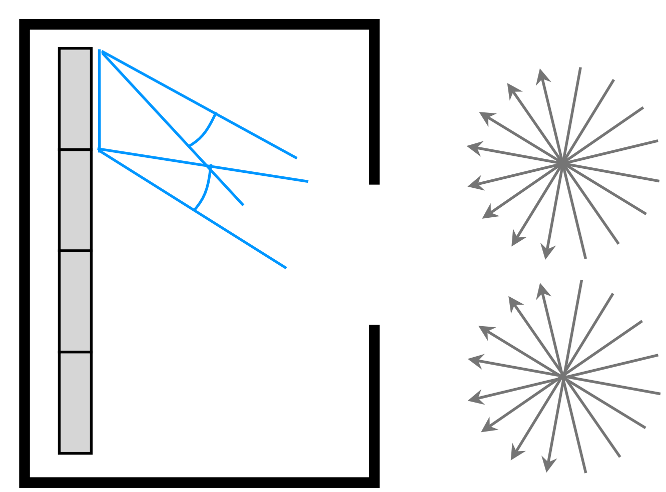
$$\Phi(W, X) = \int_X \int_W L(\hat{\omega}, x) \cos\theta \, d\omega \, dA$$



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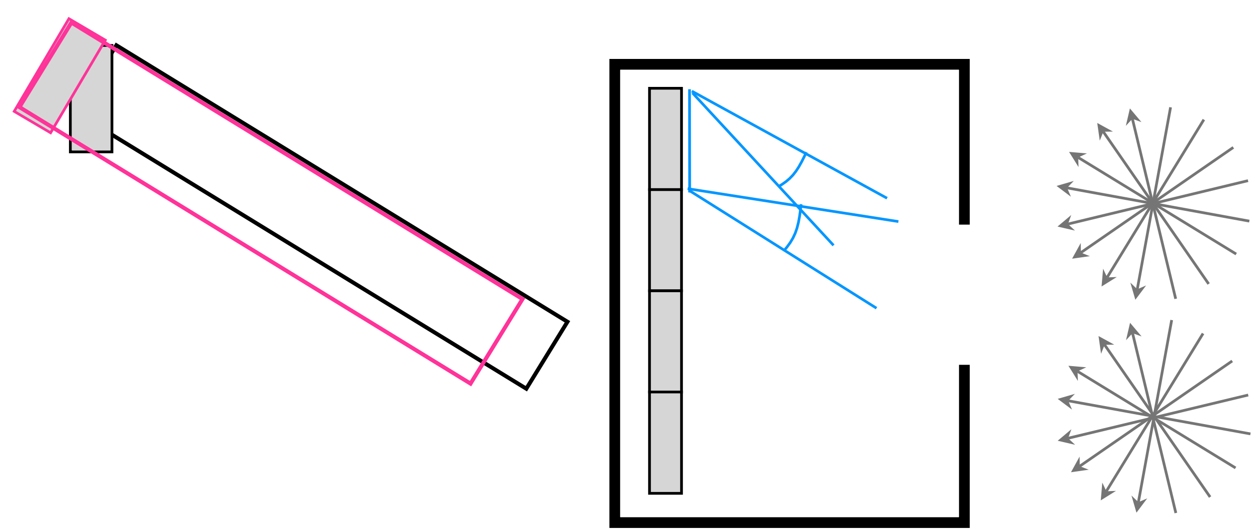
$$\Phi(W, X) = \int_X \int_W L(\hat{\omega}, x) \cos\theta \, d\omega \, dA$$

$$= \int_X E_{\hat{n}}(W, x) \, dA$$

Reminder:

$$E_{\hat{\mathbf{n}}}(W, x) = \frac{\Phi(W, X)}{|X|}$$

$$\frac{1}{cos\theta} \frac{E_{\hat{n}}(W, x)}{|W|} = L(\hat{\omega}, x)$$



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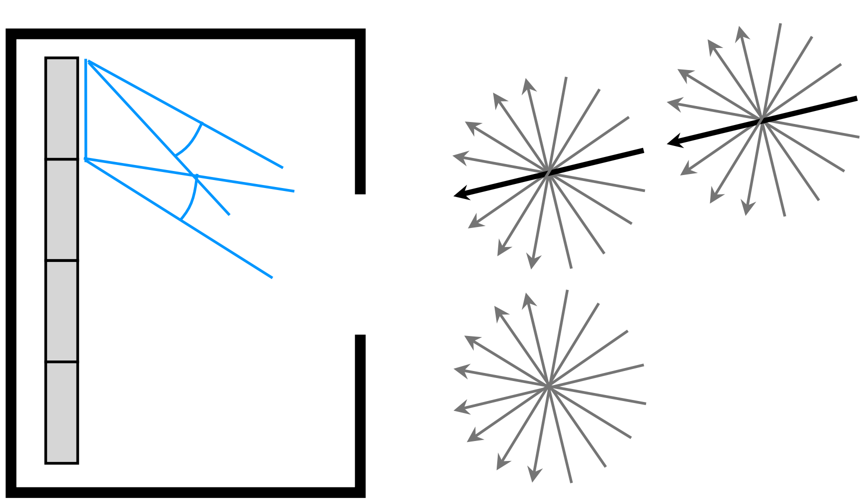
$$\Phi(W, X) = \int_X \int_W L(\hat{\boldsymbol{\omega}}, x) \cos\theta \, d\boldsymbol{\omega} \, dA$$

  - Constant along a ray in free space

$$L(\hat{\boldsymbol{\omega}}, x) = L(\hat{\boldsymbol{\omega}}, x + \hat{\boldsymbol{\omega}})$$



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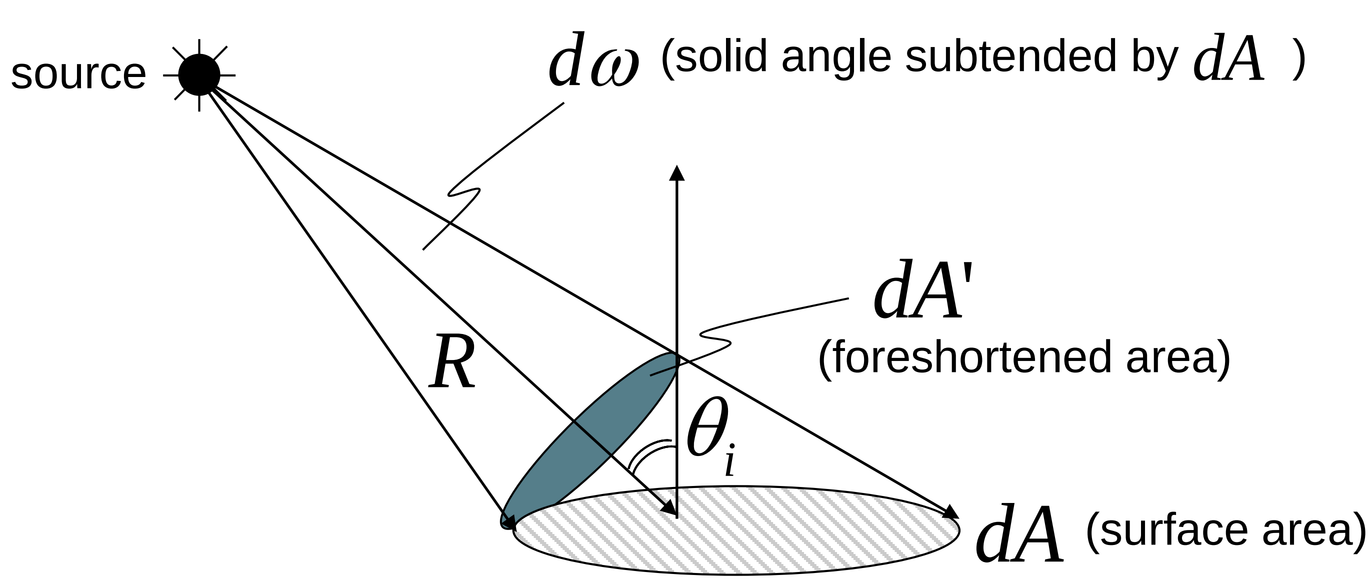
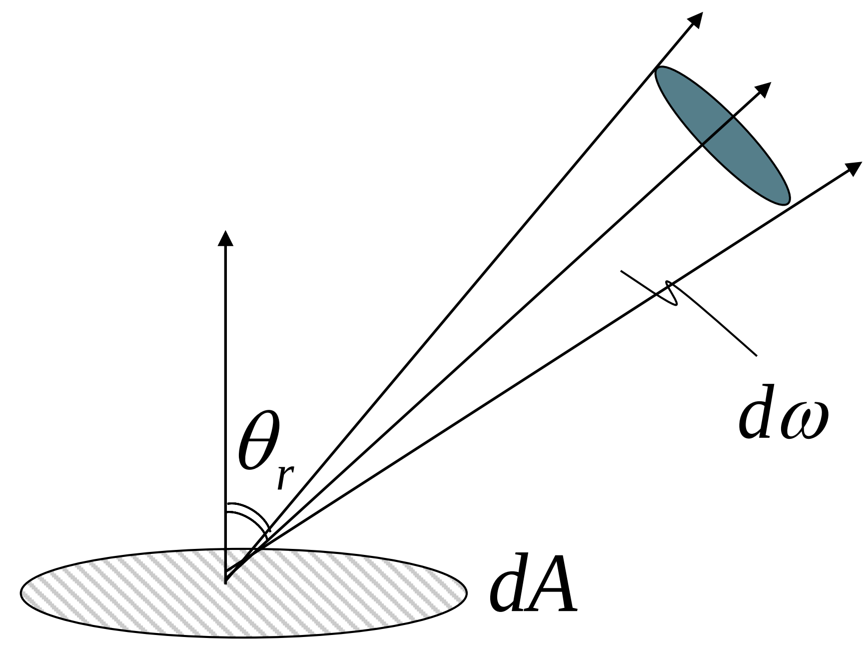
  - Constant along a ray in free space

$$L(\hat{\boldsymbol{\omega}}, x) = L(\hat{\boldsymbol{\omega}}, x + \hat{\boldsymbol{\omega}})$$

  - A camera measures radiance (after a one-time radiometric calibration). So RAW pixel values are proportional to radiance.
    - “Processed” images (like PNG and JPEG) are not linear radiance measurements!!

Radiometric concepts – boring…but, important!





(1) Solid Angle : $d\omega = \dfrac{dA'}{R^2} = \dfrac{dA\cos\theta_i}{R^2}$ (steradian)

What is the solid angle subtended by a hemisphere?

(2) Radiant Intensity of Source : $J = \dfrac{d\Phi}{d\omega}$ (watts / steradian)

Light Flux (power) emitted per unit solid angle

(3) Surface Irradiance : $E = \dfrac{d\Phi}{dA}$ (watts / m)

Light Flux (power) incident per unit surface area.

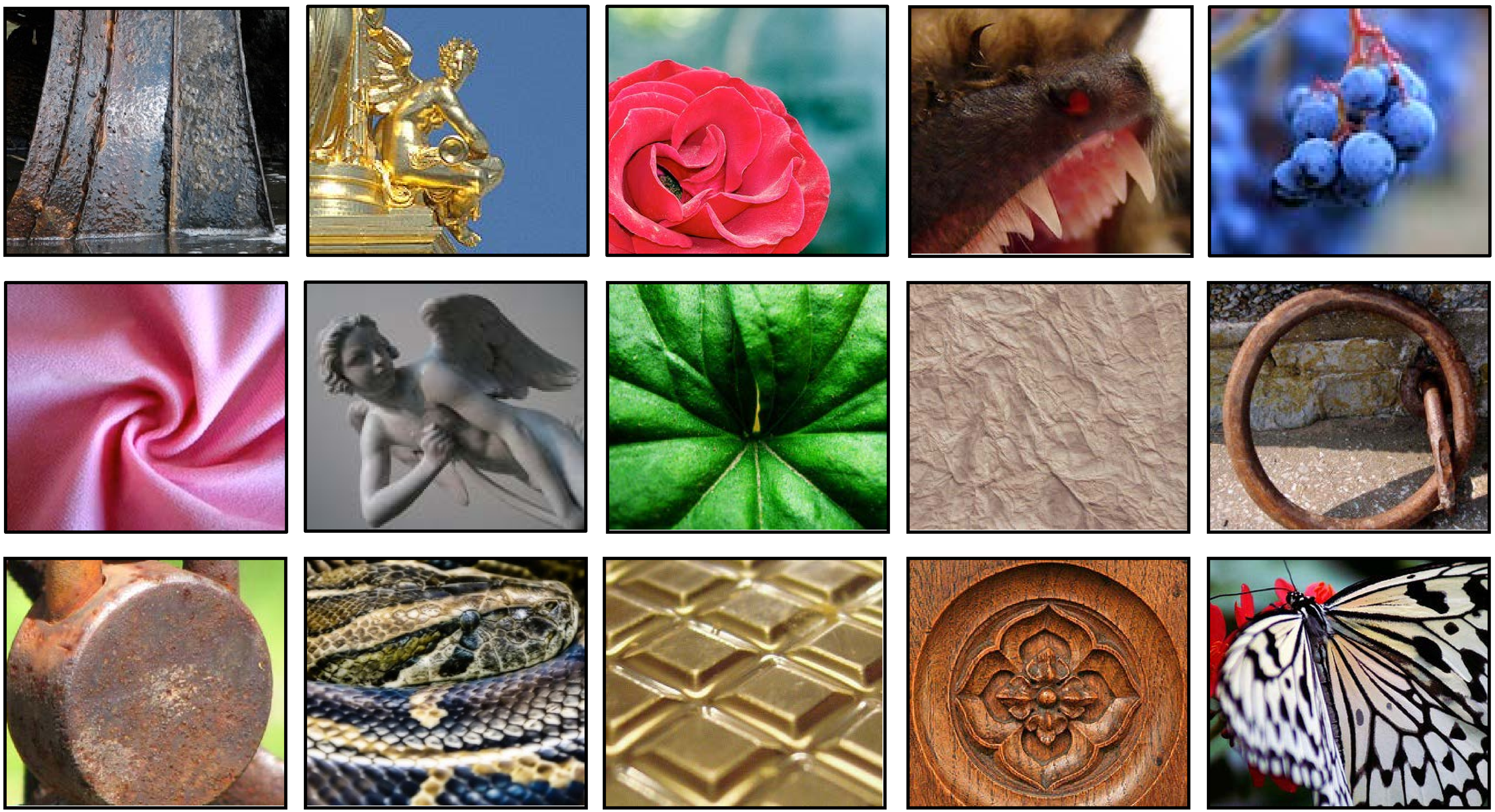
Does not depend on where the light is coming from!

(4) Surface Radiance (tricky) :

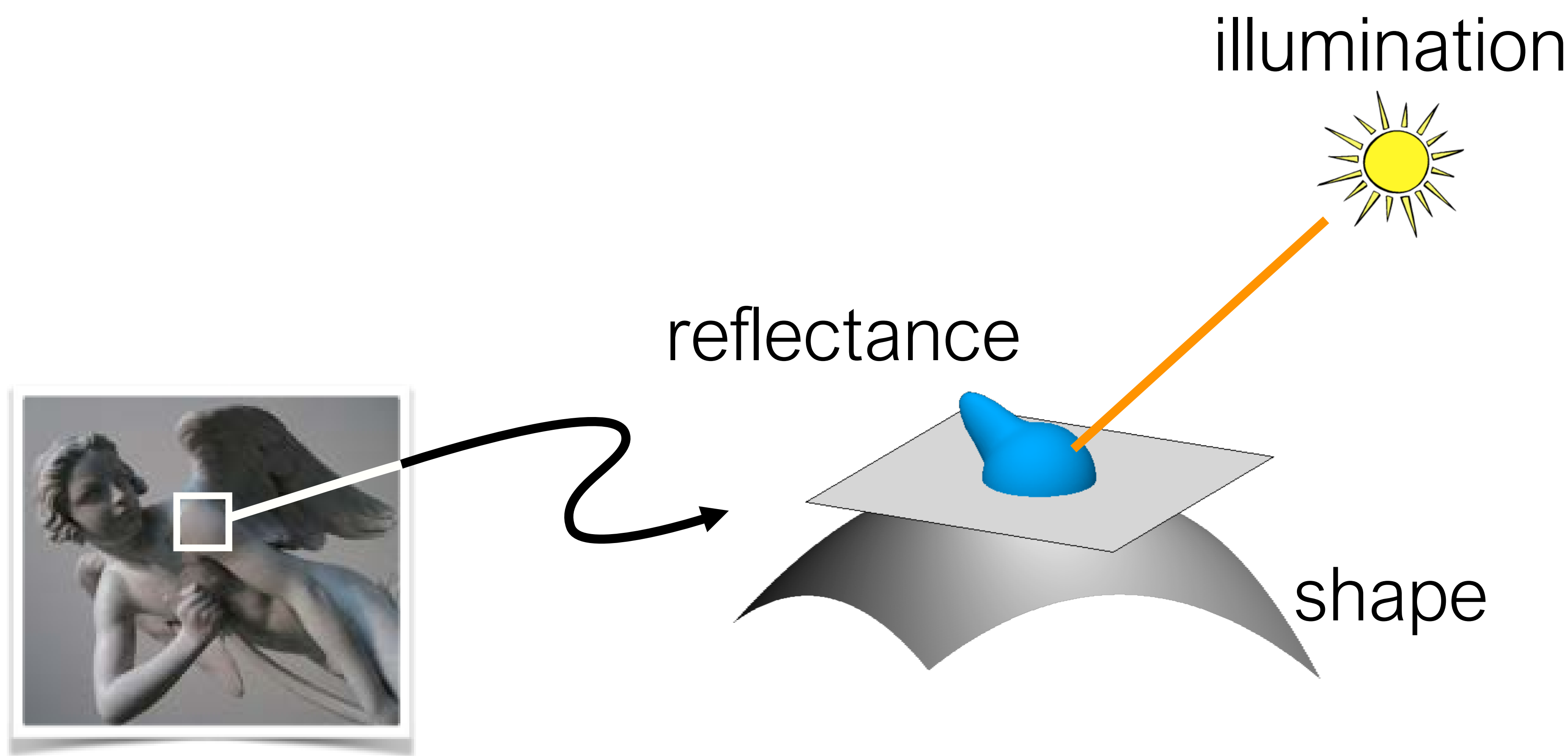
$$L = \frac{d^2\Phi}{(dA\cos\theta_r)\ d\omega}$$ (watts / m^2 steradian)

- Flux emitted per unit foreshortened area per unit solid angle.
- *L* depends on direction θ_r
- Surface can radiate into whole hemisphere.
- *L* depends on reflectance properties of surface.

Appearance



“Physics-based” computer vision (a.k.a “inverse optics”)



$\mathbf{I} \Longrightarrow$ shape, illumination, reflectance

Reflectance and BRDF

Material Reflectance

Mirror

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Plotting reflectance

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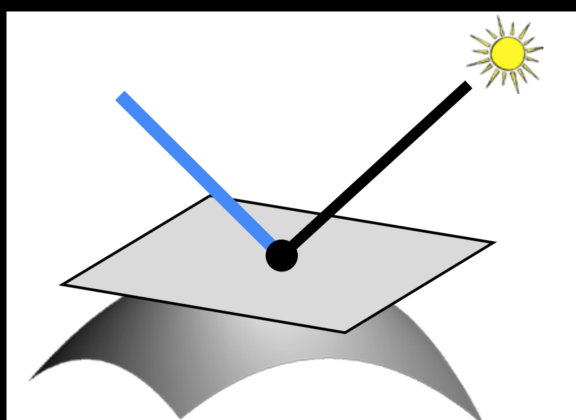
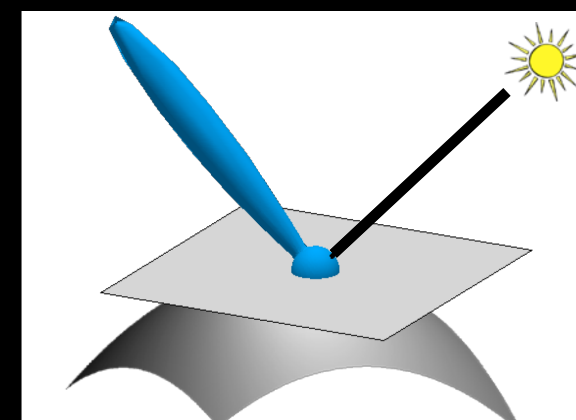



Shiny

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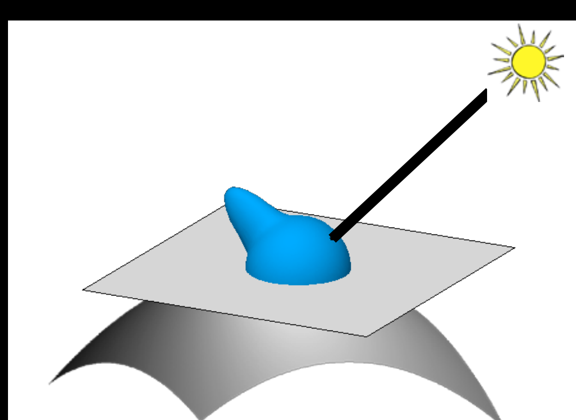


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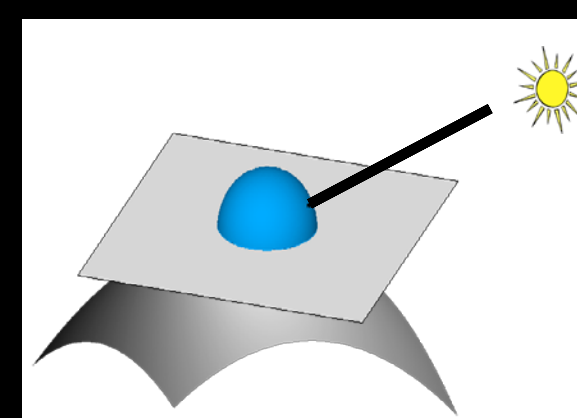


Reflectance

- Ratio of outgoing energy to incoming energy at a single point
- Want to define a ratio such that it:
  - converges as we use smaller and smaller incoming and outgoing wedges
  - does not depend on the size of the wedges (i.e. is intrinsic to the material)

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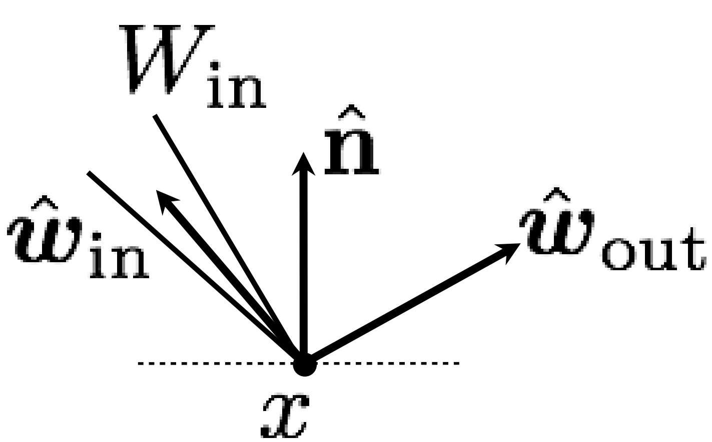


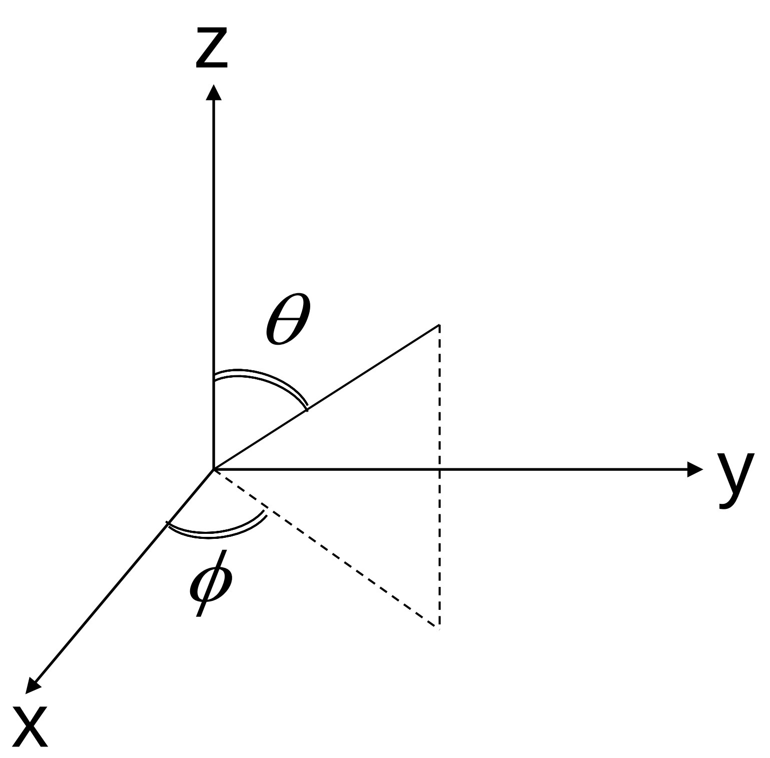
$$\lim_{W_{\text{in}} \longrightarrow \hat{\boldsymbol{w}}_{\text{in}}}$$

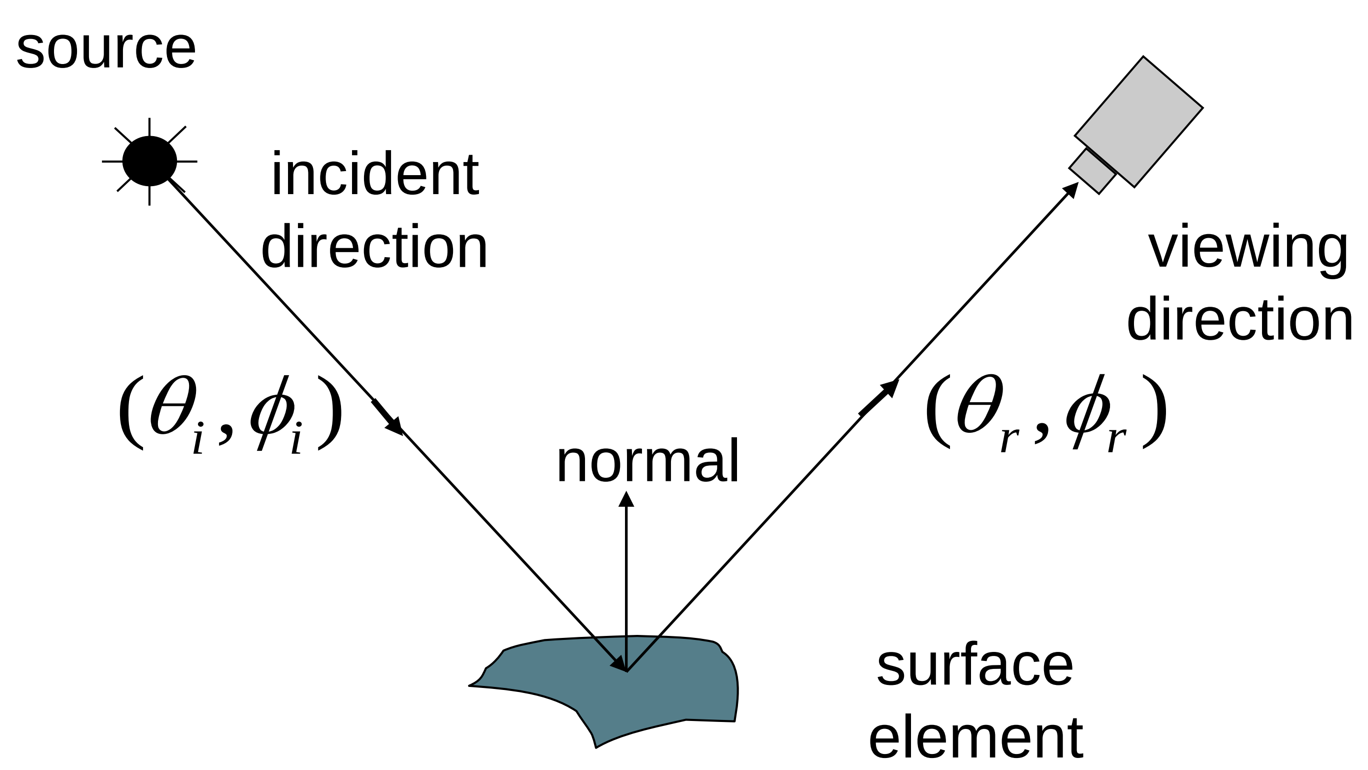
$$f_{x,\hat{\mathbf{n}}}(\hat{\boldsymbol{\omega}}_{\text{in}}, \hat{\boldsymbol{\omega}}_{\text{out}})$$

$$f_{x,\hat{\mathbf{n}}}(W_{\text{in}}, \hat{\boldsymbol{\omega}}_{\text{out}}) = \frac{L^{\text{out}}(x, \hat{\boldsymbol{\omega}}_{\text{out}})}{E^{\text{in}}_{\hat{\mathbf{n}}}(W_{\text{in}}, x)}$$

- Notations x and n often implied by context and omitted; directions \omega are expressed in local coordinate system defined by normal n (and some chosen tangent vector)
- Units: sr^{-1}
- Called Bidirectional Reflectance Distribution Function (BRDF)

BRDF: Bidirectional Reflectance Distribution Function





$E^{surface}(\theta_i, \phi_i)$ Irradiance at Surface in direction (θ_i, ϕ_i)

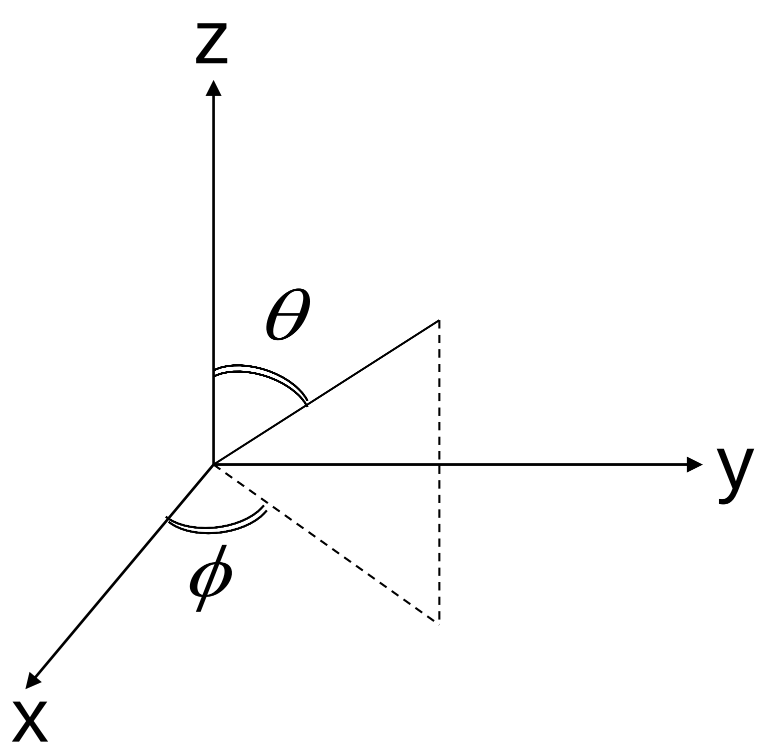
$L^{surface}(\theta_r, \phi_r)$ Radiance of Surface in direction (θ_r, ϕ_r)

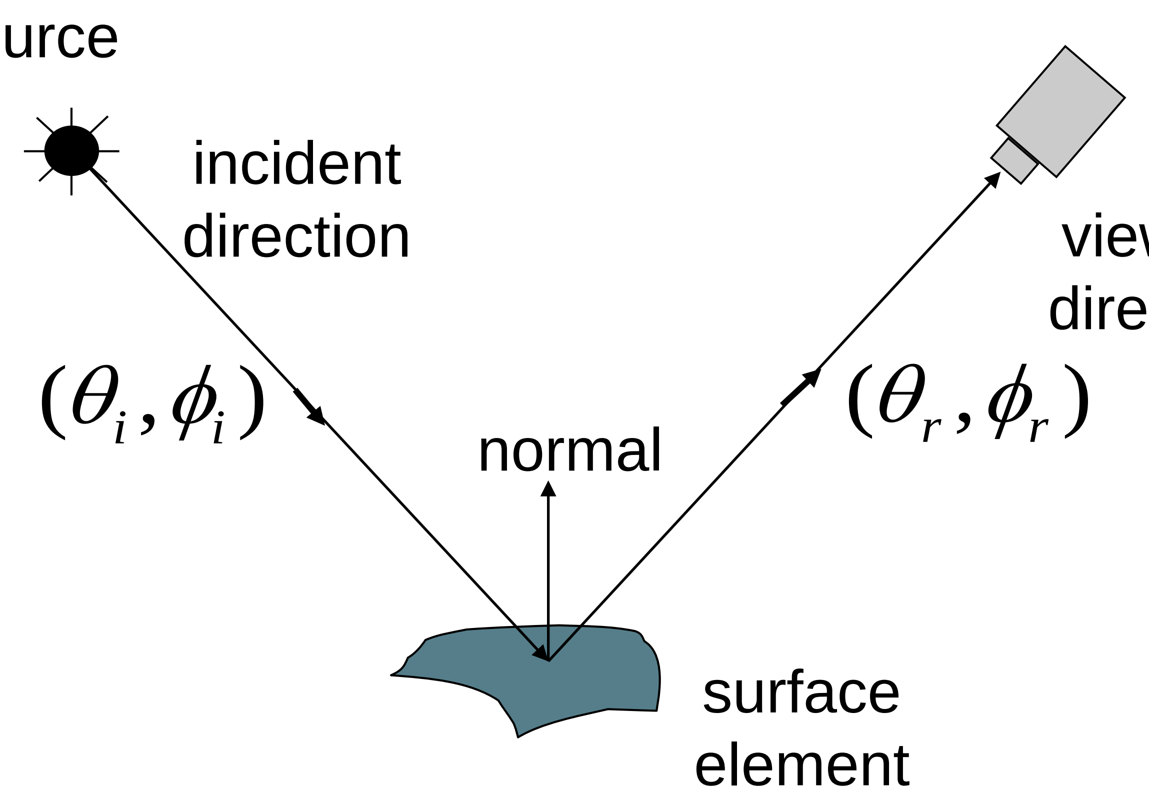
$$\boxed{\text{BRDF} : f(\theta_i, \phi_i; \theta_r, \phi_r) = \frac{L^{surface}(\theta_r, \phi_r)}{E^{surface}(\theta_i, \phi_i)}}$$

Reflectance: BRDF

- Units: sr^{-1}
- Real-valued function defined on the double-hemisphere
- Has many useful properties

Important Properties of BRDFs



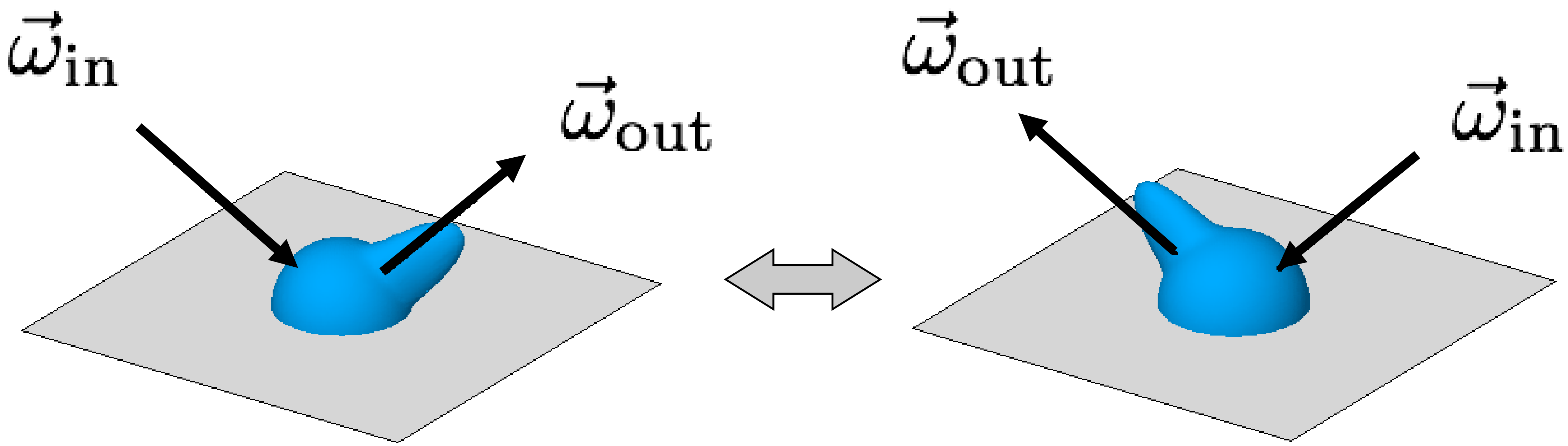


- Conservation of Energy:

$$\forall \hat{\omega}_{\text{in}}, \quad \int_{\Omega_{\text{out}}} f(\hat{\omega}_{\text{in}}, \hat{\omega}_{\text{out}}) \cos\theta_{\text{out}} \, d\hat{\omega}_{\text{out}} \leq 1$$

Why smaller than or equal?

Property: “Helmholtz reciprocity”

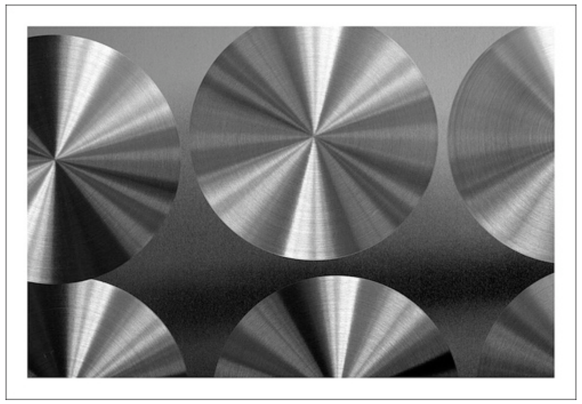


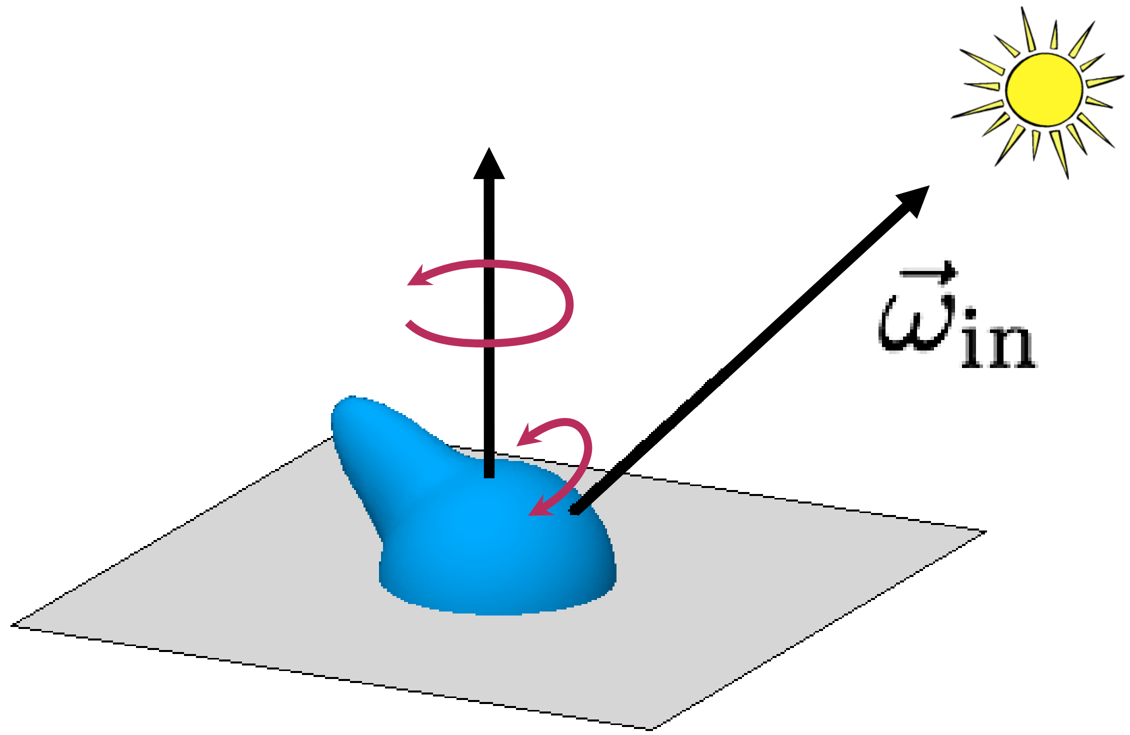
- Helmholtz Reciprocity: (follows from 2nd Law of Thermodynamics)

BRDF does not change when source and viewing directions are swapped.

$$f_r(\vec{\omega}_{\text{in}}, \vec{\omega}_{\text{out}}) = f_r(\vec{\omega}_{\text{out}}, \vec{\omega}_{\text{in}})$$

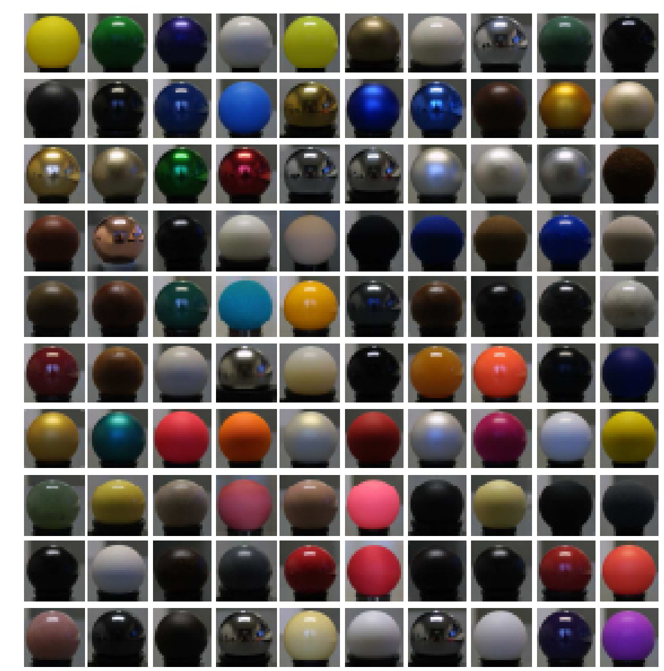
Common assumption: Isotropy





$$f_r(\vec{\omega}_{\mathrm{in}}, \cdot)$$

BRDF does not change when surface is rotated about the normal.



[Matusik et al., 2003]

4D → 3D

$$\boxed{f_r(\cancel{\vec{\omega}_{\mathrm{in}}, \vec{\omega}_{\mathrm{out}}})}$$

Bi-directional Reflectance Distribution Function (BRDF)

Can be written as a function of 3 variables : $f(\theta_i, \theta_r, \phi_i - \phi_r)$

Reflectance: BRDF

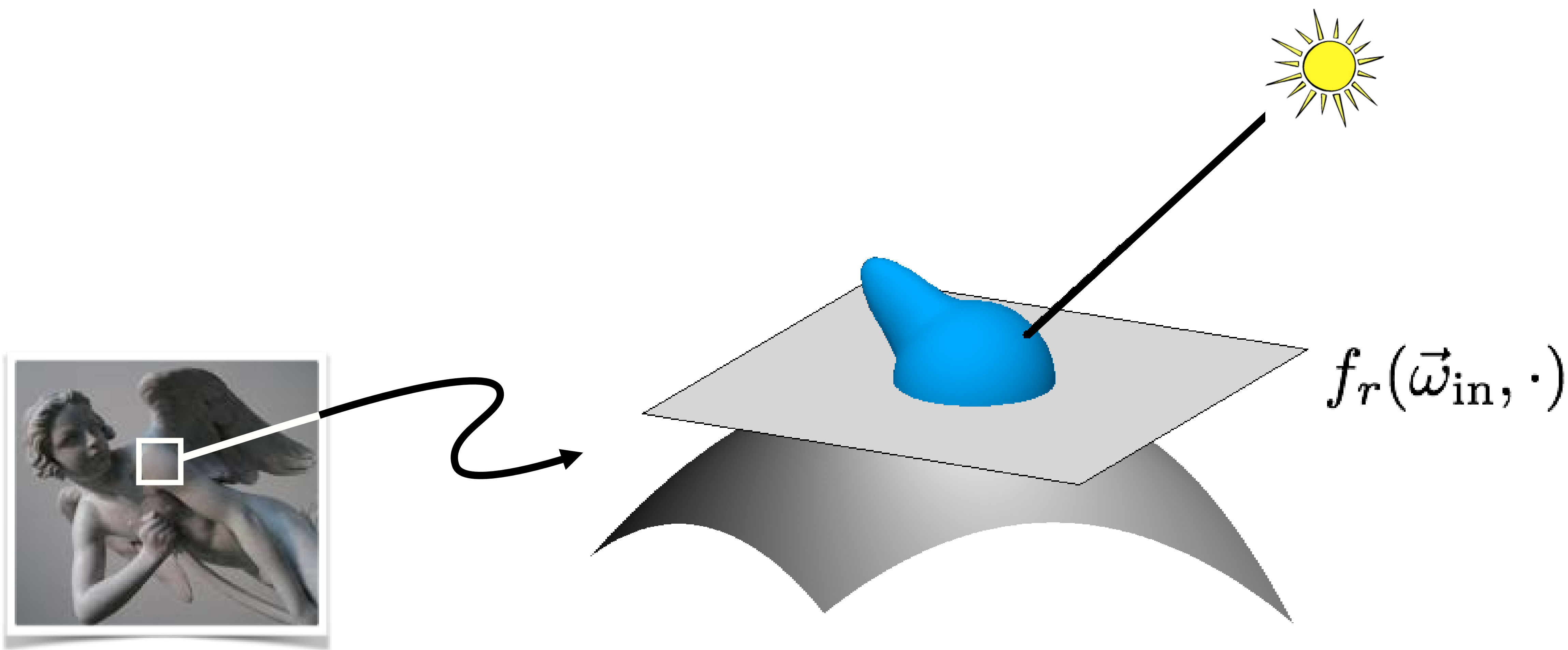
- Units: sr^{-1}
- Real-valued function defined on the double-hemisphere
- Has many useful properties
- Allows computing output radiance (and thus pixel value) for *any* configuration of lights and viewpoint

$$L^{\text{out}}(\hat{\omega}) = \int_{\Omega_{\text{in}}} f(\hat{\omega}_{\text{in}}, \hat{\omega}_{\text{out}}) L^{\text{in}}(\hat{\omega}_{\text{in}}) \cos\theta_{\text{in}} d\hat{\omega}_{\text{in}}$$

reflectance equation

Why is there a cosine in the reflectance equation?

BRDF



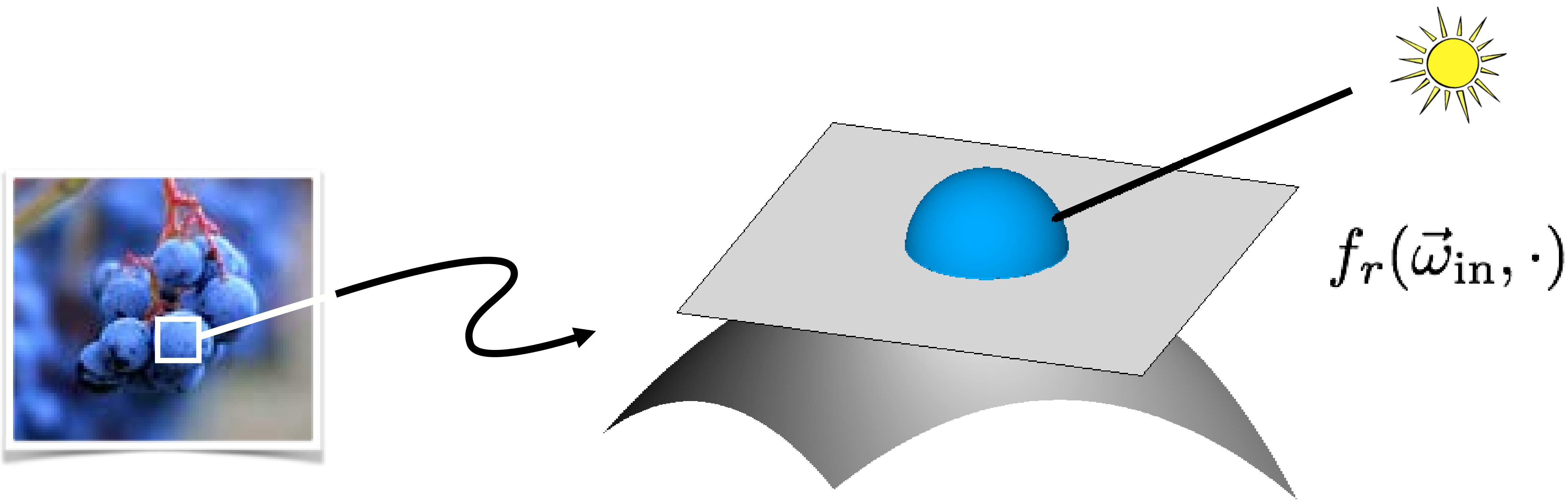
$$\boxed{f_r(\vec{\omega}_{\text{in}}, \vec{\omega}_{\text{out}})}$$

Bi-directional Reflectance Distribution Function (BRDF)

BRDF

Lambertian (diffuse) BRDF: energy equally distributed in all directions

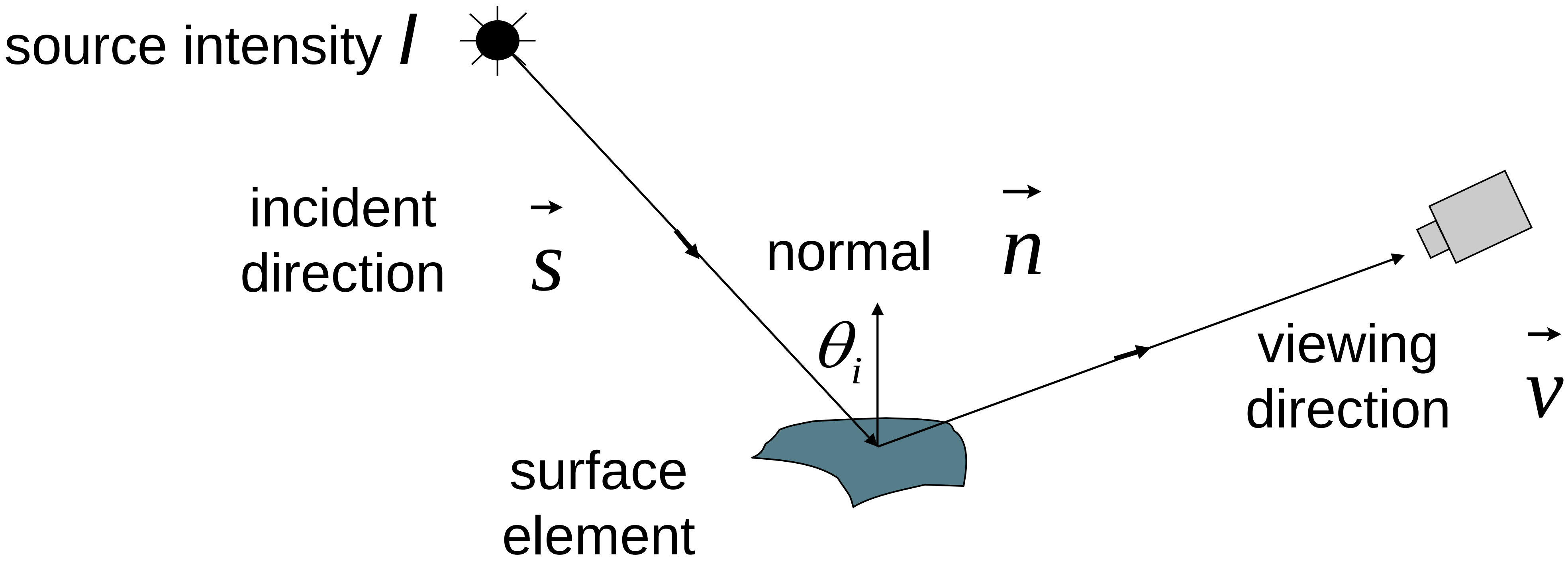
What does the BRDF equal in this case?



$$\boxed{f_r(\vec{\omega}_{\text{in}}, \vec{\omega}_{\text{out}})}$$

Bi-directional Reflectance Distribution Function (BRDF)

Diffuse Reflection and Lambertian BRDF



- Surface appears equally bright from ALL directions! (independent of $\vec{v}$)

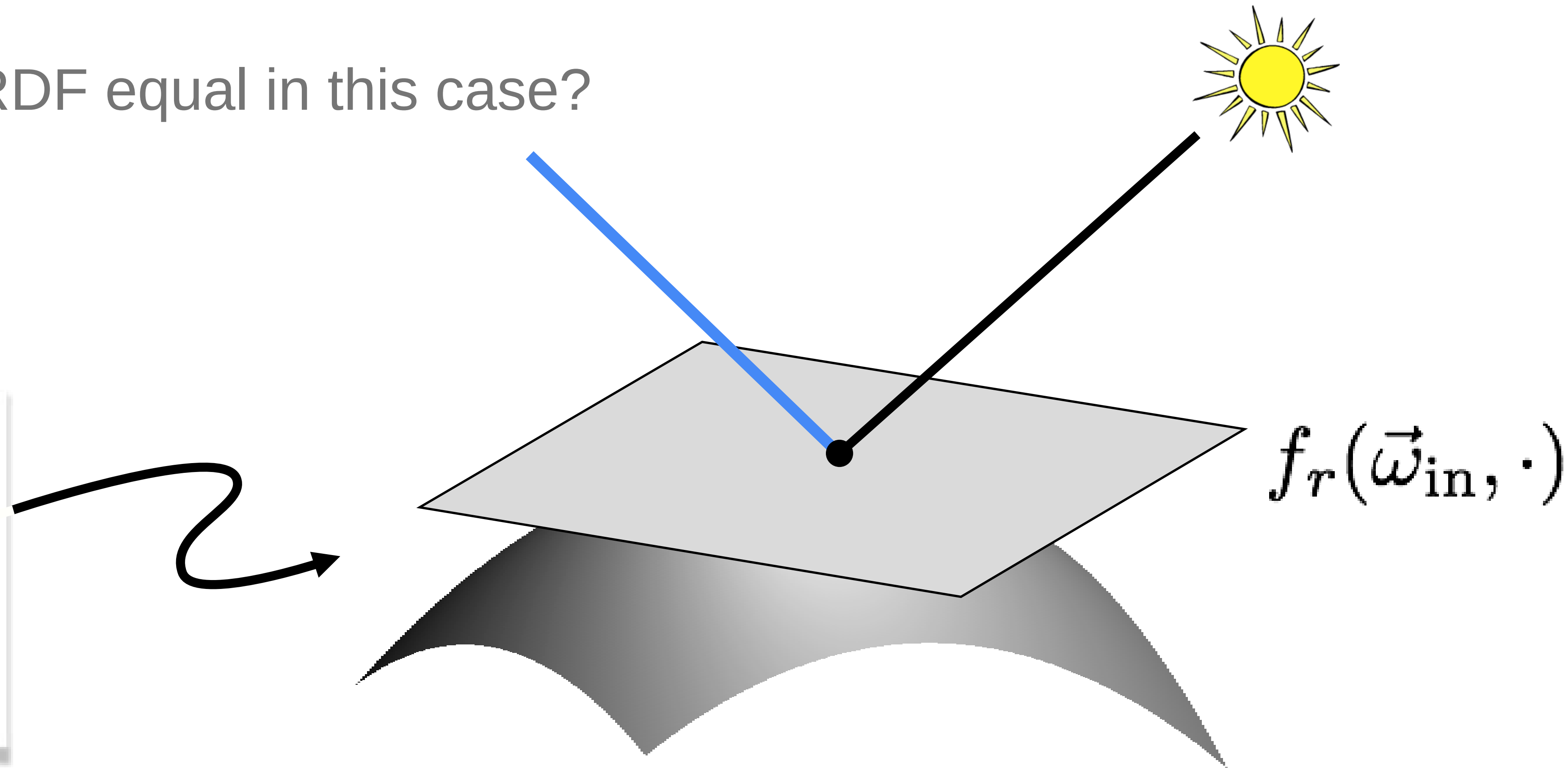
- Lambertian BRDF is simply a constant : $f(\theta_i, \phi_i; \theta_r, \phi_r) = \frac{\rho_d}{\pi}$ albedo

- Most commonly used BRDF in Vision and Graphics!

BRDF

Specular BRDF: all energy concentrated in mirror direction

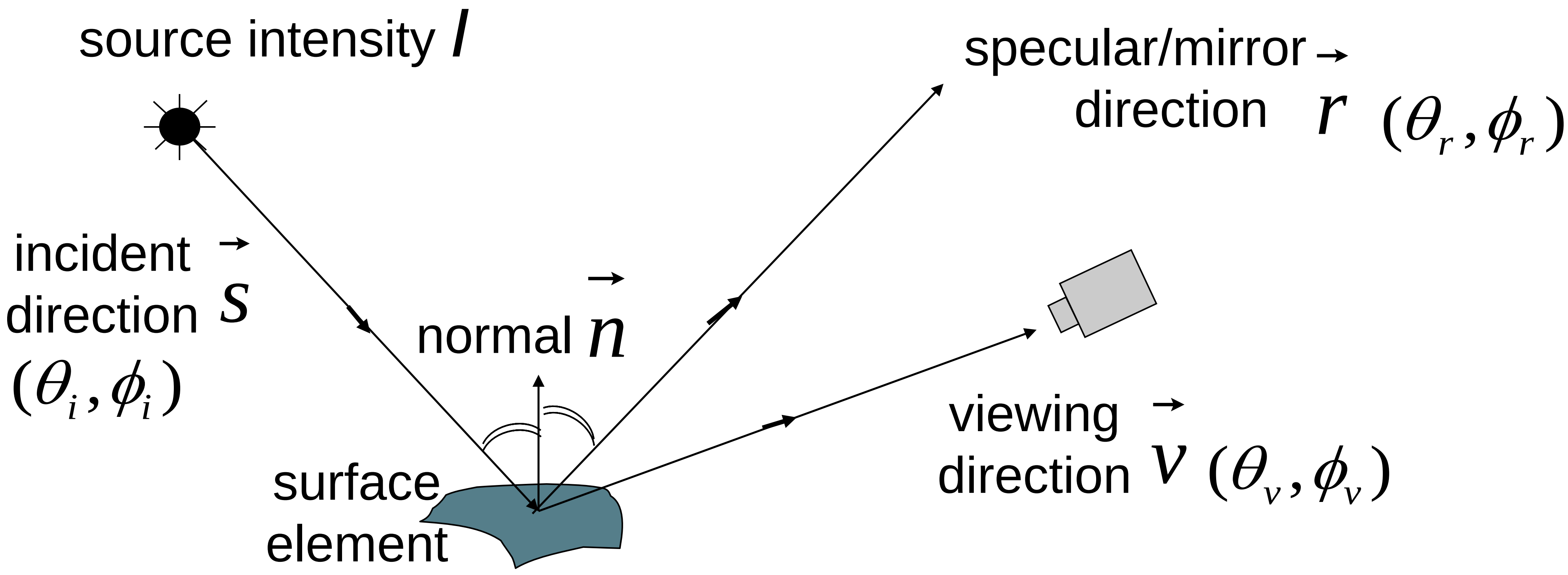
What does the BRDF equal in this case?



$$\boxed{f_r(\vec{\omega}_{\text{in}}, \vec{\omega}_{\text{out}})}$$

Bi-directional Reflectance Distribution Function (BRDF)

Specular Reflection and Mirror BRDF



- Valid for very smooth surfaces.

- All incident light energy reflected in a SINGLE direction (only when $\vec{v} = \vec{r}$).

- Mirror BRDF is simply a double-delta function :

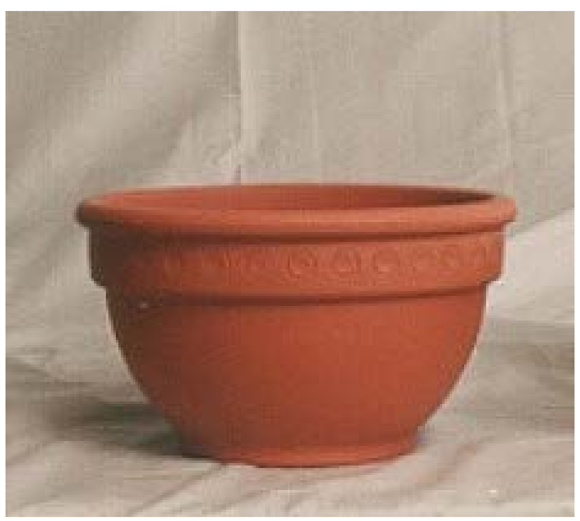
specular albedo

$$f(\theta_i, \phi_i; \theta_v, \phi_v) = \rho_s \ \delta(\theta_i - \theta_v) \ \delta(\phi_i + \pi - \phi_v)$$

Example Surfaces

Body Reflection:

Diffuse Reflection
Matte Appearance
Clay, paper, etc



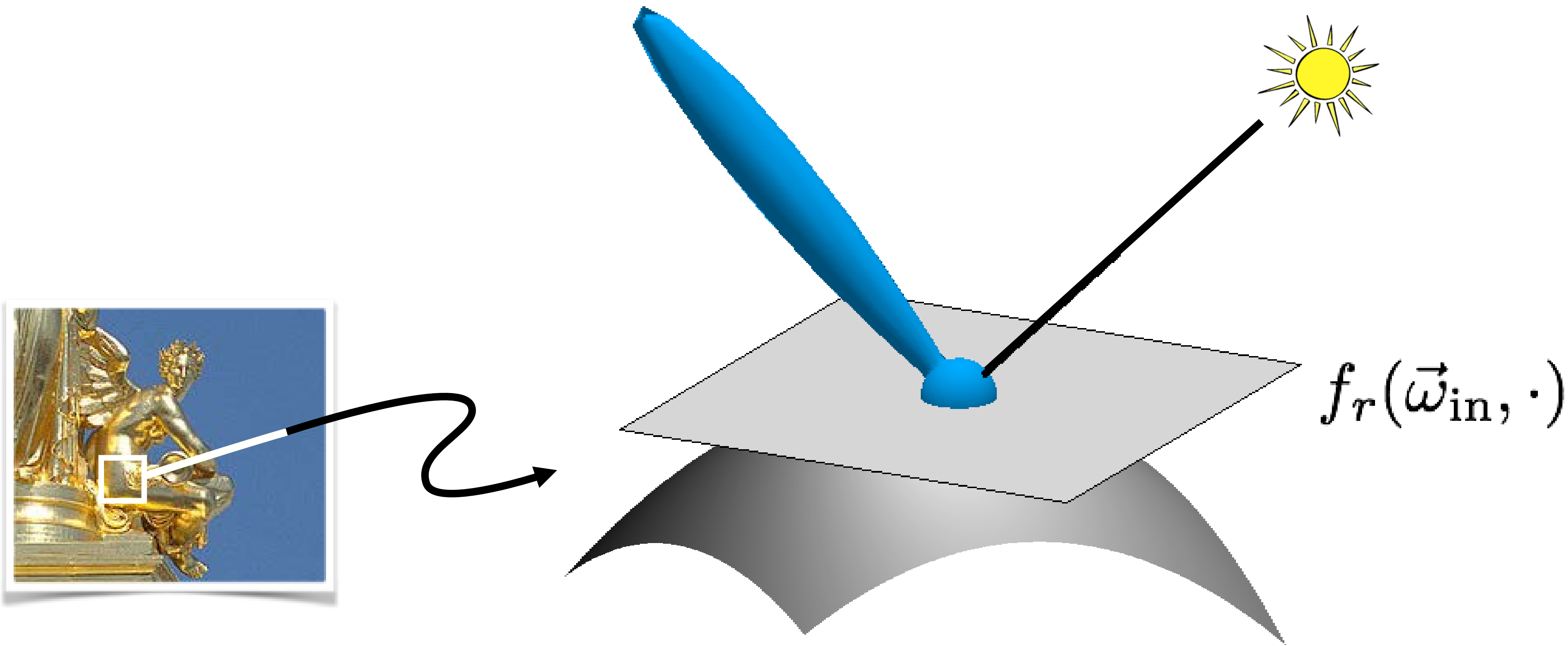
Surface Reflection:

Specular Reflection
Glossy Appearance
Highlights
Dominant for Metals

Many materials exhibit both Reflections:

BRDF

Glossy BRDF: more energy concentrated in mirror direction than elsewhere



$$\boxed{f_r(\vec{\omega}_{\text{in}}, \vec{\omega}_{\text{out}})}$$

Bi-directional Reflectance Distribution Function (BRDF)

Trick for dielectrics (non-metals)

- BRDF is a sum of a Lambertian diffuse component and non-Lambertian specular components
- The two components differ in terms of color and polarization, and under certain conditions, this can be exploited to separate them.

$$f(\vec{\omega}_i, \vec{\omega}_o) = f_d + f_s(\vec{\omega}_i, \vec{\omega}_o)$$

Trick for dielectrics (non-metals)

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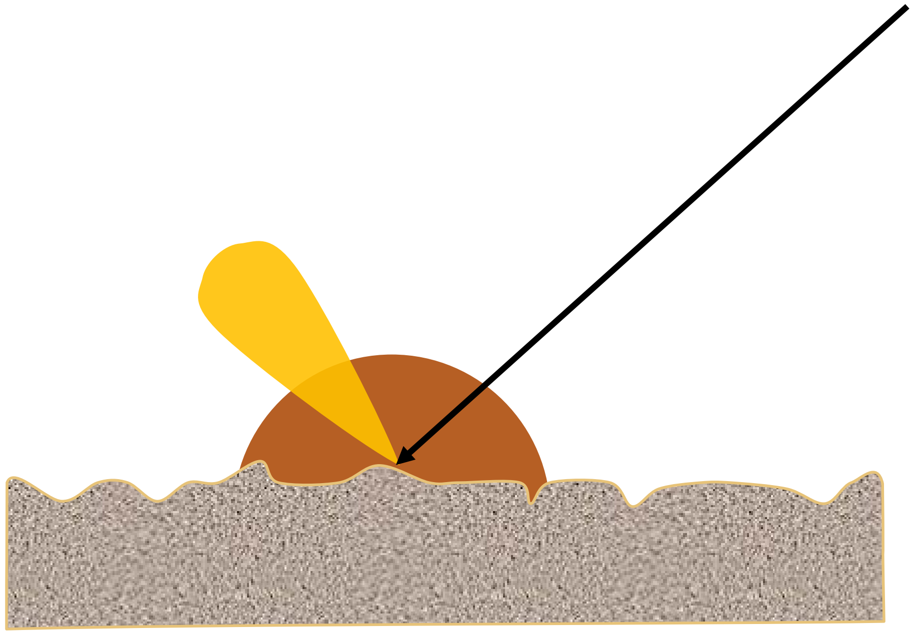
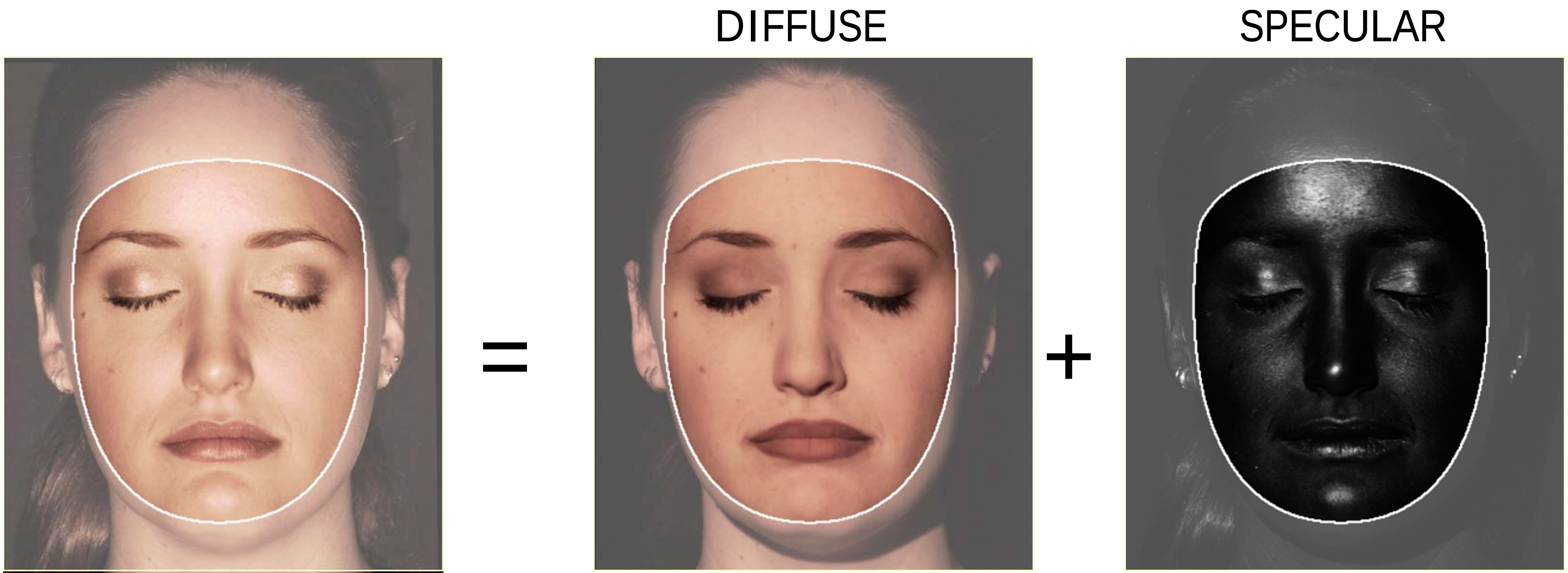
$$f(\vec{\omega}_i, \vec{\omega}_o) = f_d + f_s(\vec{\omega}_i, \vec{\omega}_o)$$

Often called the *dichromatic BRDF*:

- Diffuse term varies with wavelength, constant with polarization
- Specular term constant with wavelength, varies with polarization

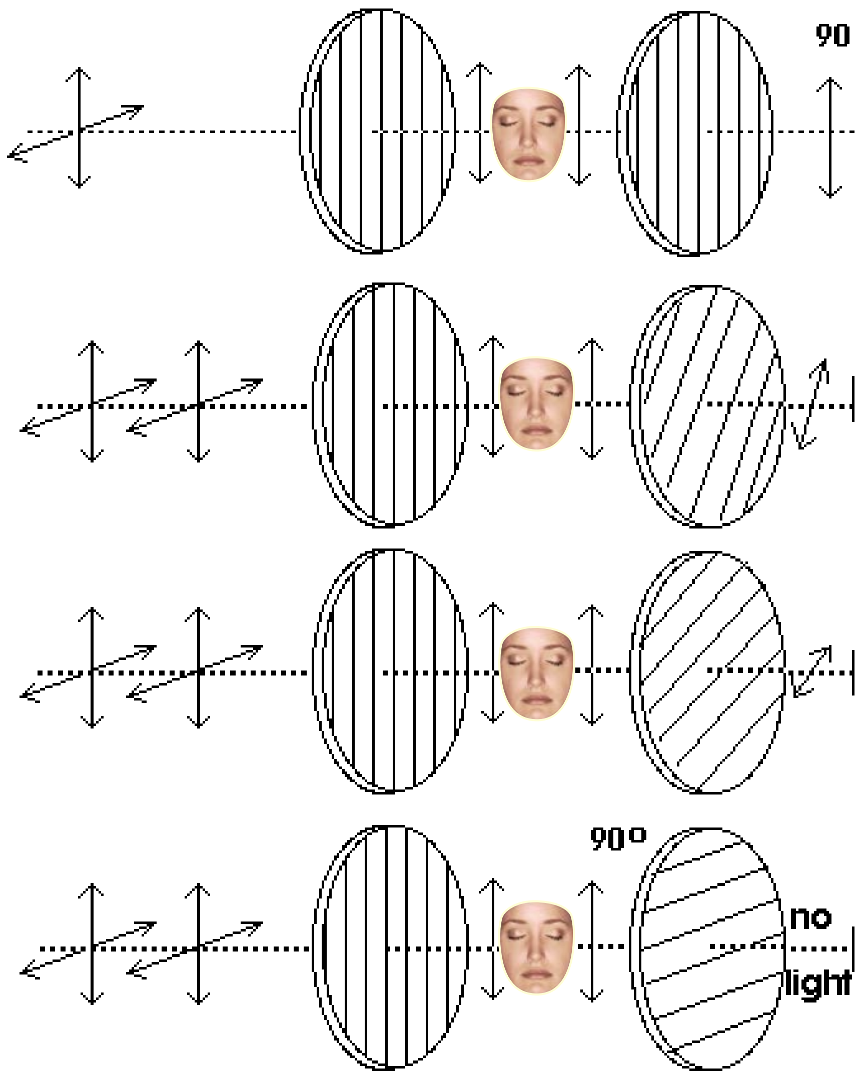


Trick for dielectrics (non-metals)



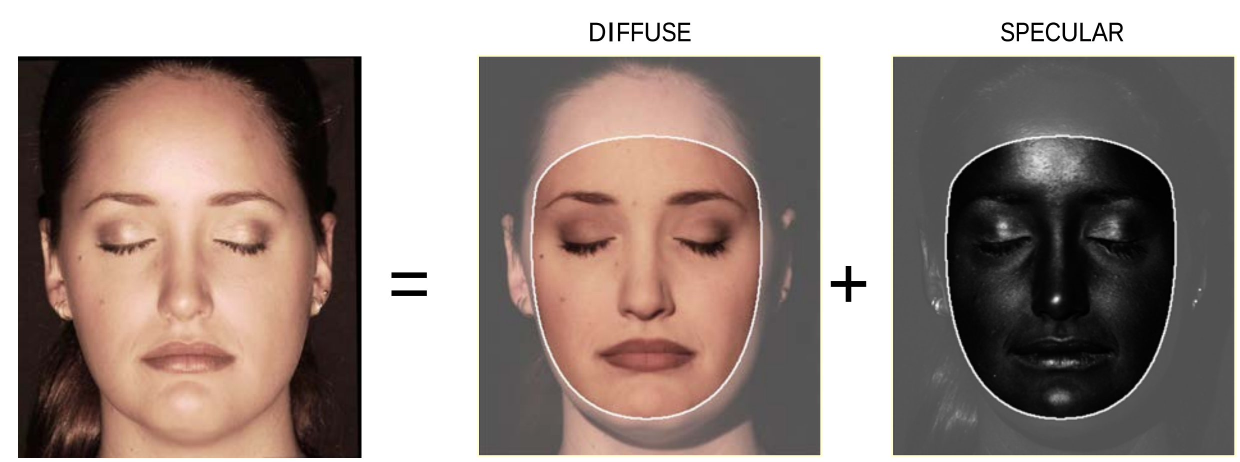
- In this example, the two components were separated using linear polarizing filters on the camera and light source.

Trick for dielectrics (non-metals)

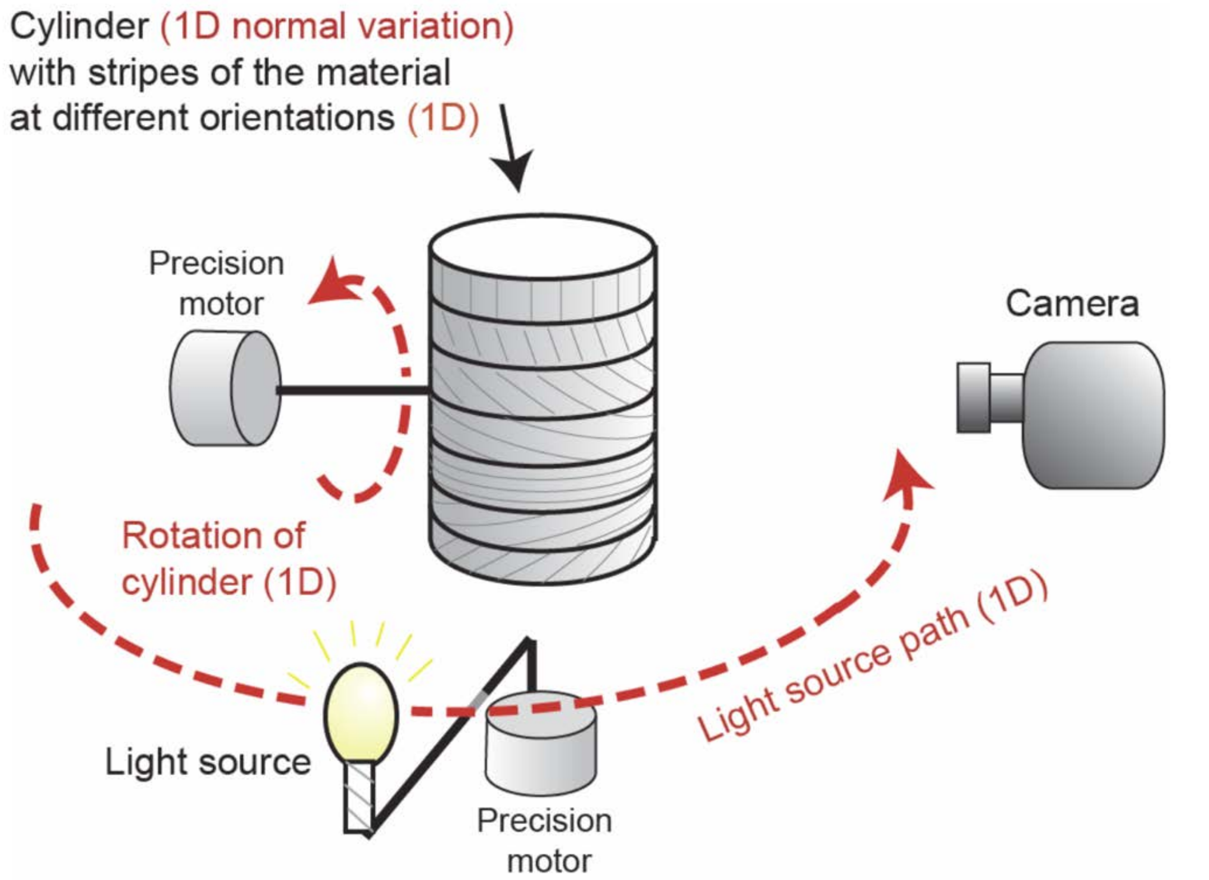


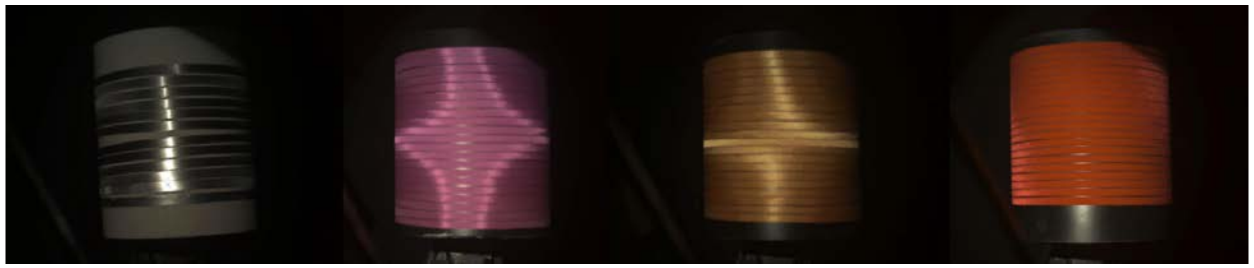
$$I_{\|} = \frac{1}{2} I_{\text{diffuse}} + I_{\text{specular}}$$

$$I_{\perp} = \frac{1}{2} I_{\text{diffuse}}$$

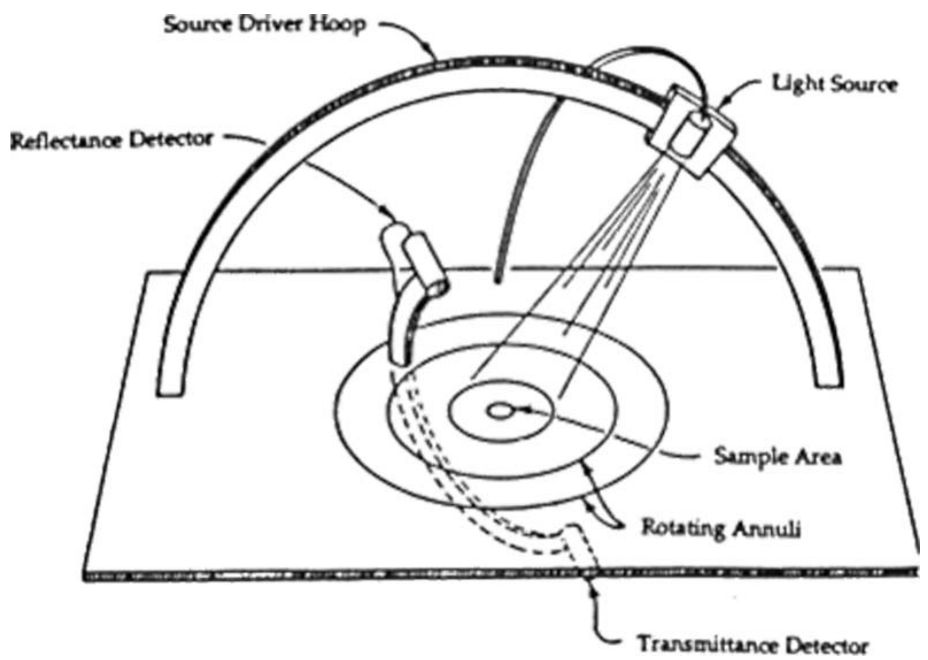


Tabulated 4D BRDFs (hard to measure)





[Ngan et al., 2005]



Gonioreflectometer

Low-parameter (non-linear) BRDF models

- A small number of parameters define the (2D,3D, or 4D) function
- Except for Lambertian, the BRDF is non-linear in these parameters
- Examples:

Where do these constants come from?

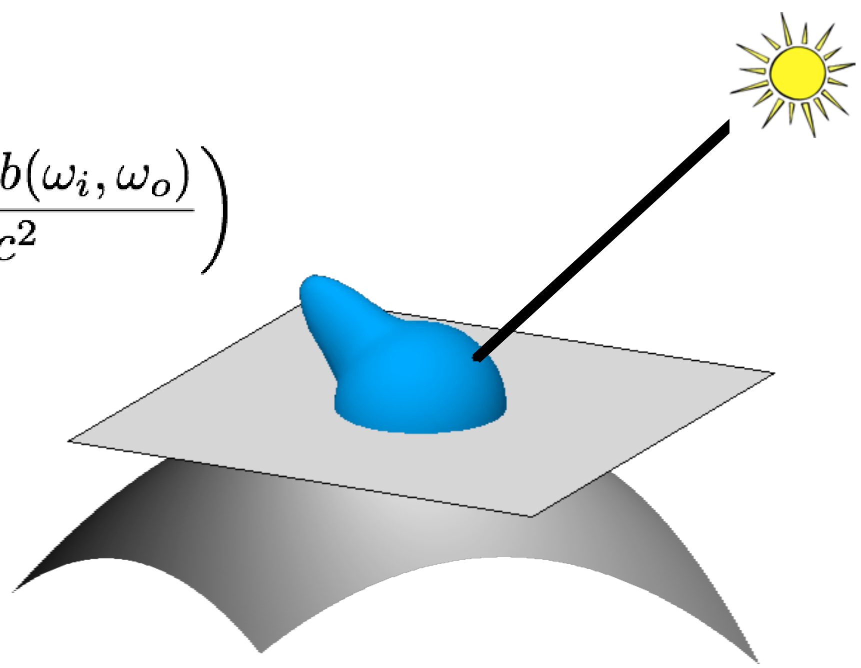
Lambertian: $f(\omega_i, \omega_o) = \dfrac{a}{\pi}$

Phong: $f(\omega_i, \omega_o) = \dfrac{a}{\pi} + b\cos^c\left(2\langle\omega_i, n\rangle\langle\omega_o, n\rangle - \langle\omega_i, \omega_o\rangle\right)$

Blinn: $f(\omega_i, \omega_o) = \dfrac{a}{\pi} + b\cos^c b(\omega_i, \omega_o)$

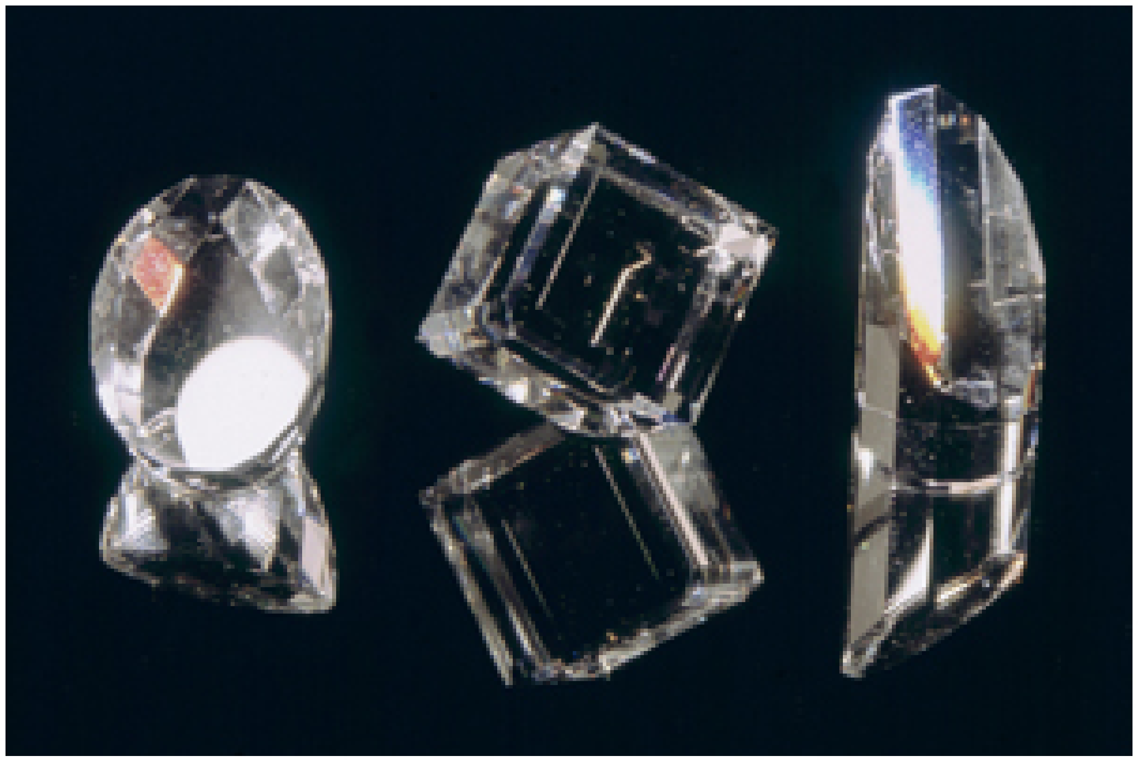
Lafortune: $f(\omega_i, \omega_o) = \dfrac{a}{\pi} + b(-\omega_i^\top A\omega_o)^k$

Ward: $f(\omega_i, \omega_o) = \dfrac{a}{\pi} + \dfrac{b}{4\pi c^2\sqrt{\langle n, \omega_i\rangle\langle n, \omega_o\rangle}} \exp\left(\dfrac{-\tan^2 b(\omega_i, \omega_o)}{c^2}\right)$



α is called the *albedo*

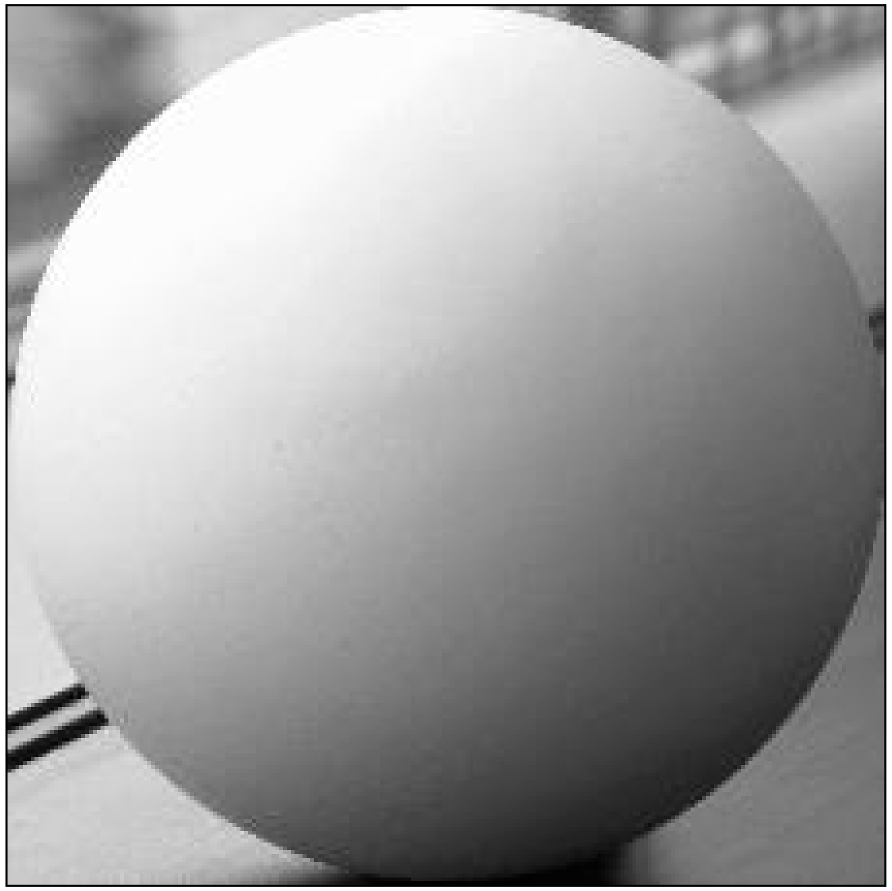
Reflectance that Require Wave Optics





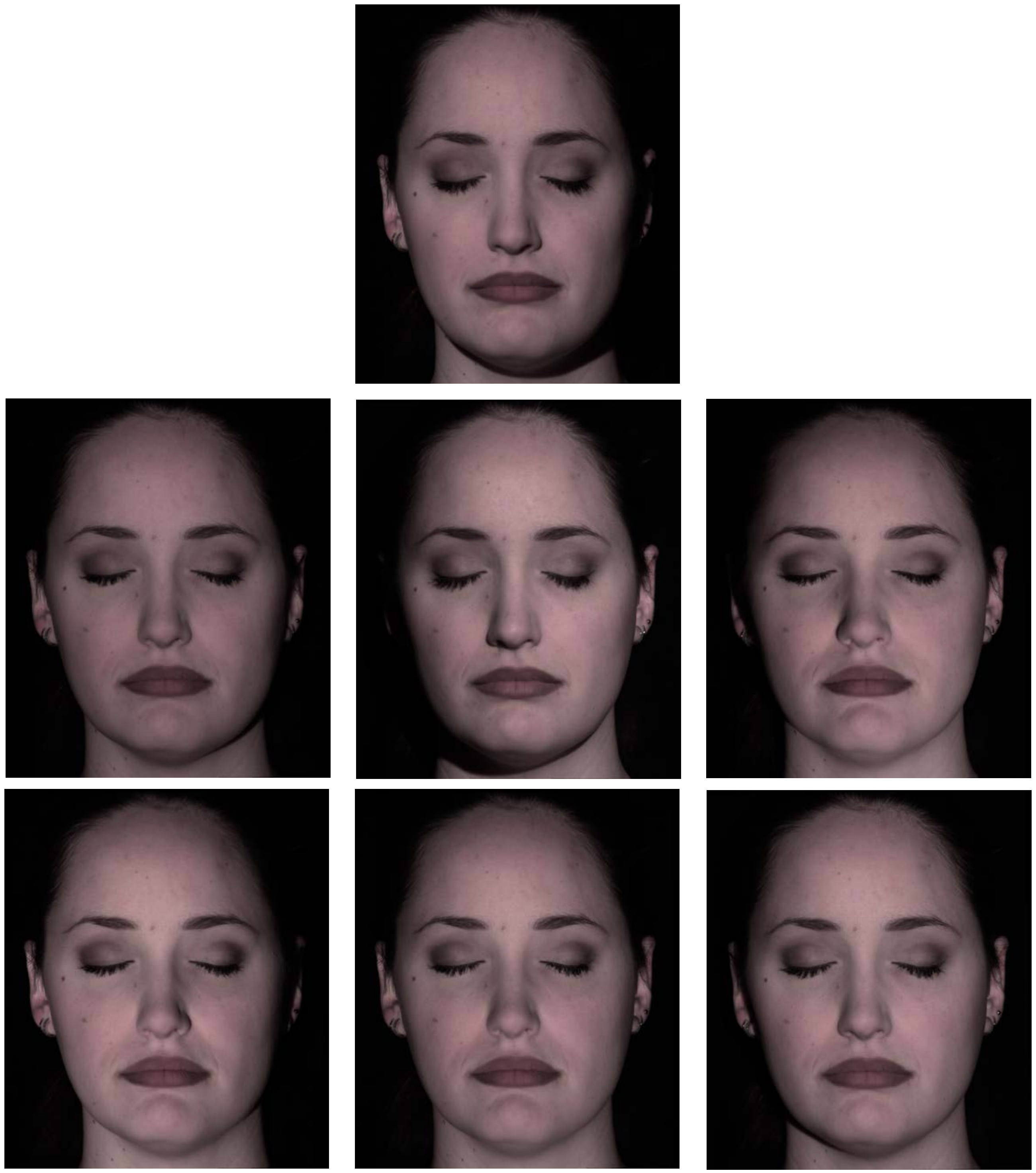

Photometric stereo

Image Intensity and 3D Geometry



- *Shading* as a cue for shape reconstruction
- What is the relation between intensity and shape?

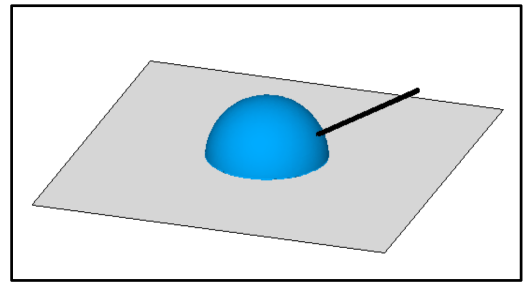
Example application: Photometric Stereo



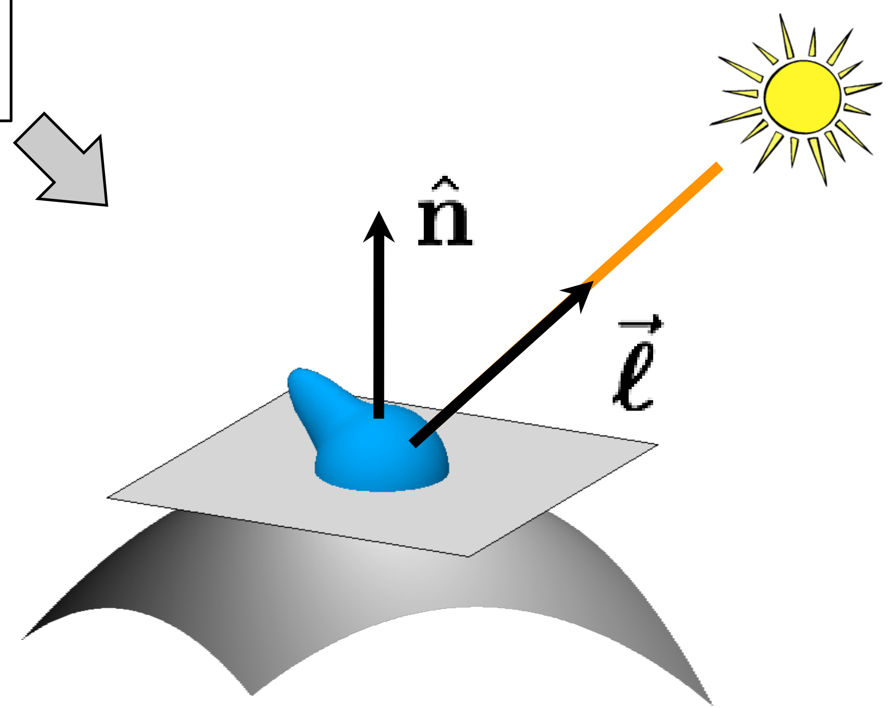
“N-dot-l” shading

ASSUMPTION 1:
LAMBERTIAN

$f(\widehat{\omega}_{in}, :) = a$



ASSUMPTION 2:
DIRECTIONAL LIGHTING

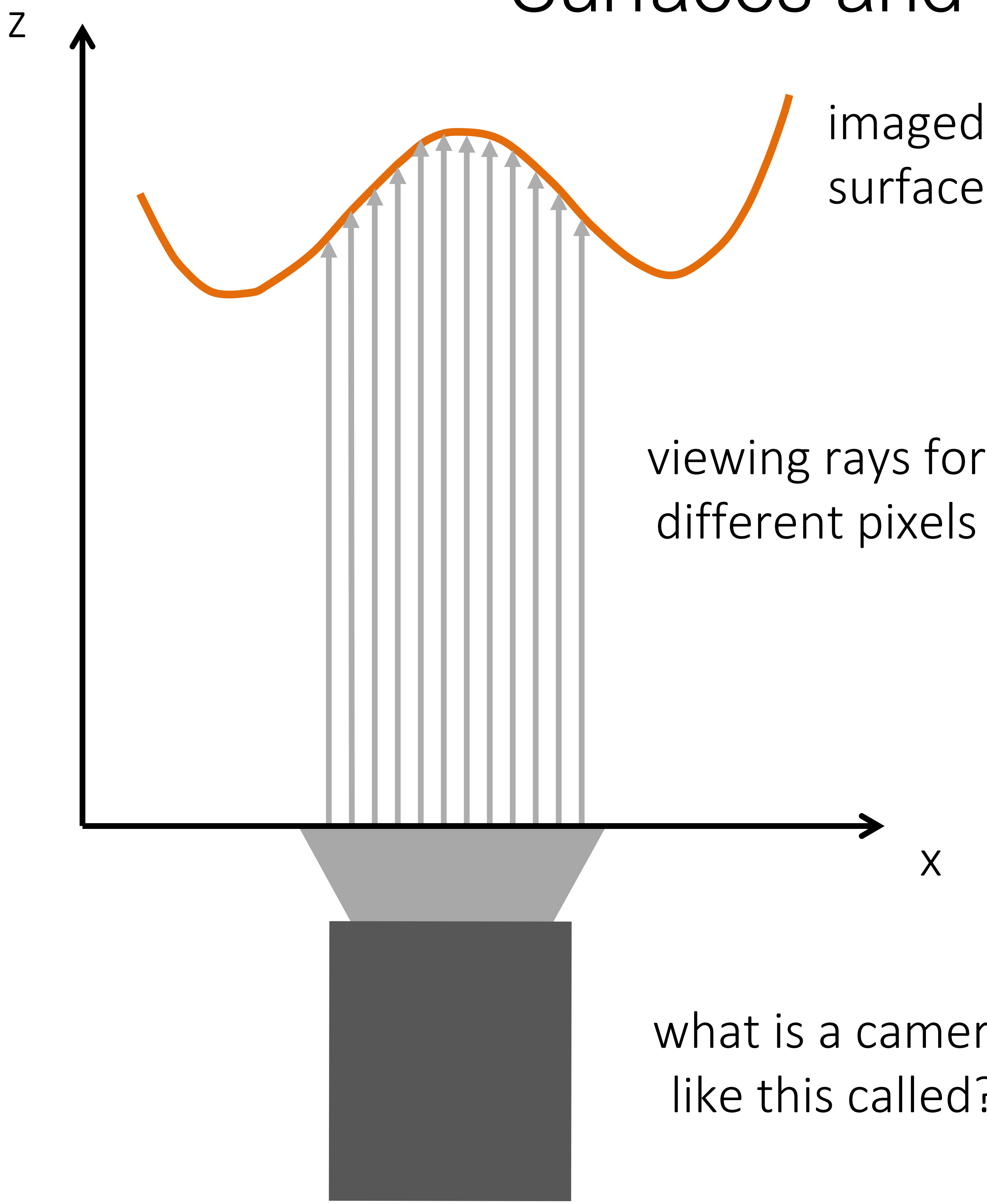


$$L^{\text{out}}(\hat{\boldsymbol{\omega}}) = \int_{\Omega_{\text{in}}} f(\hat{\boldsymbol{\omega}}_{\text{in}}, \hat{\boldsymbol{\omega}}_{\text{out}}) L^{\text{in}}(\hat{\boldsymbol{\omega}}_{\text{in}}) \cos\theta_{\text{in}} d\hat{\boldsymbol{\omega}}_{\text{in}}$$

$$\boxed{I = a\hat{\mathbf{n}}^{\top}\vec{\boldsymbol{\ell}}}$$

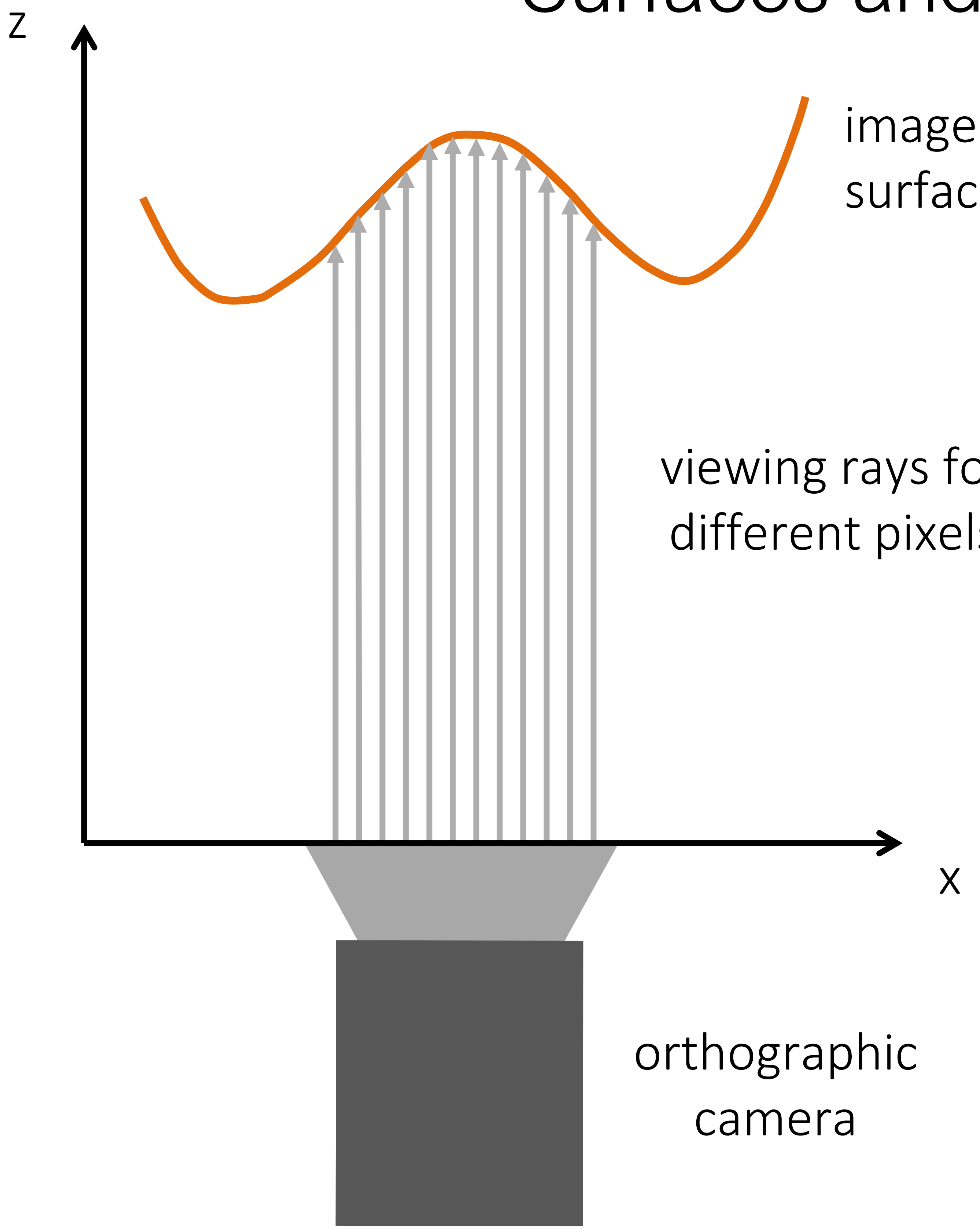
Why do we call these normal “shape”?

Surfaces and normals



what is a camera
like this called?

Surfaces and normals

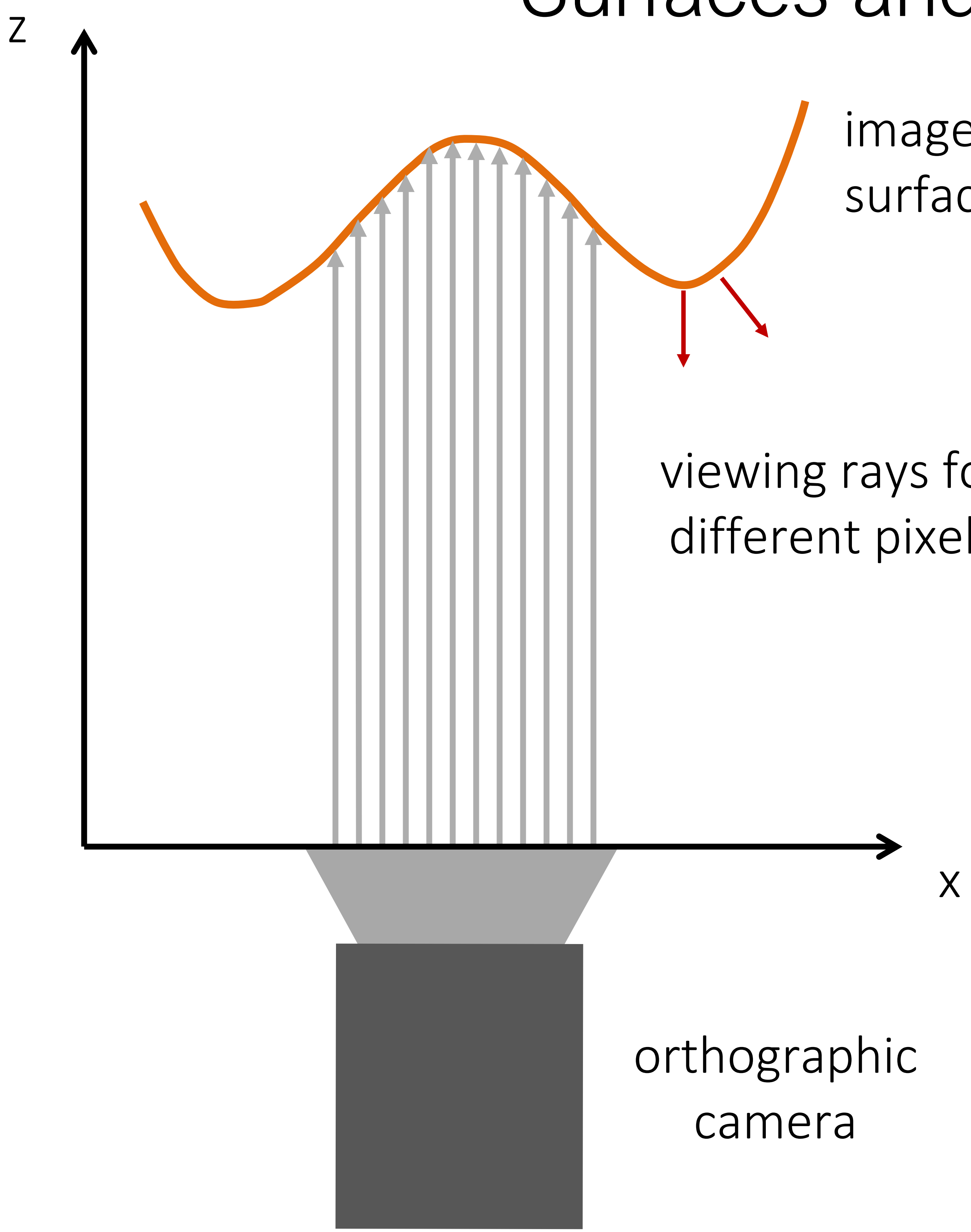


Surface representation as a depth field (also known as Monge surface):

$$\underset{\text{depth at each pixel}}{z} = f(\underbrace{x, y}_{\text{pixel coordinates on image plane}})$$

How does surface normal relate to this representation?

Surfaces and normals



Surface representation as a depth image (also known as Monge surface):

$$\underset{\text{depth at each pixel}}{z} = f(\underbrace{x, y}_{\text{pixel coordinates on image place}})$$

Unnormalized normal:

$$\tilde{n}(x, y) = \left(\frac{df}{dx}, \frac{df}{dy}, -1\right)$$

Actual normal:

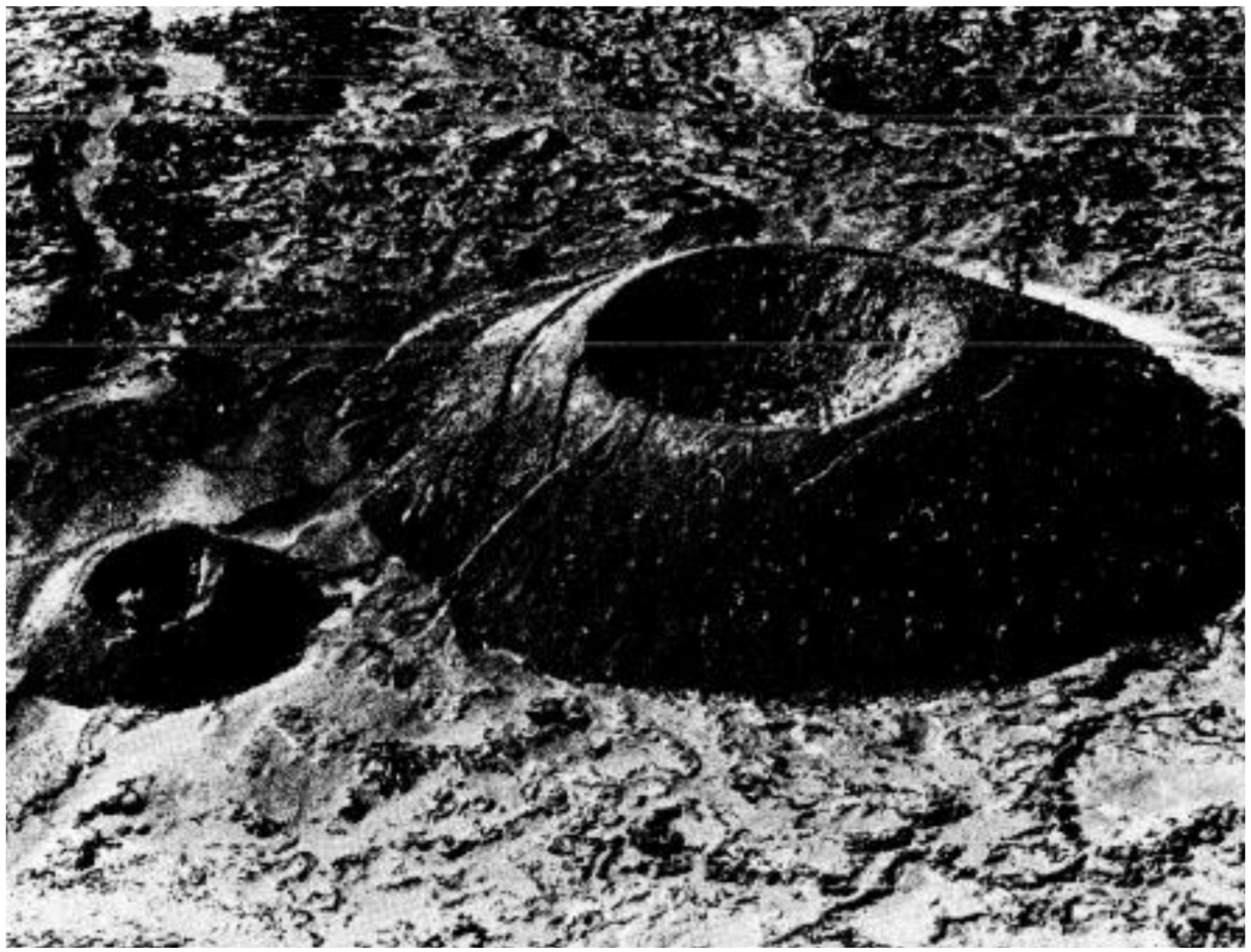
$$n(x, y) = \tilde{n}(x, y) / \|\tilde{n}(x, y)\|$$

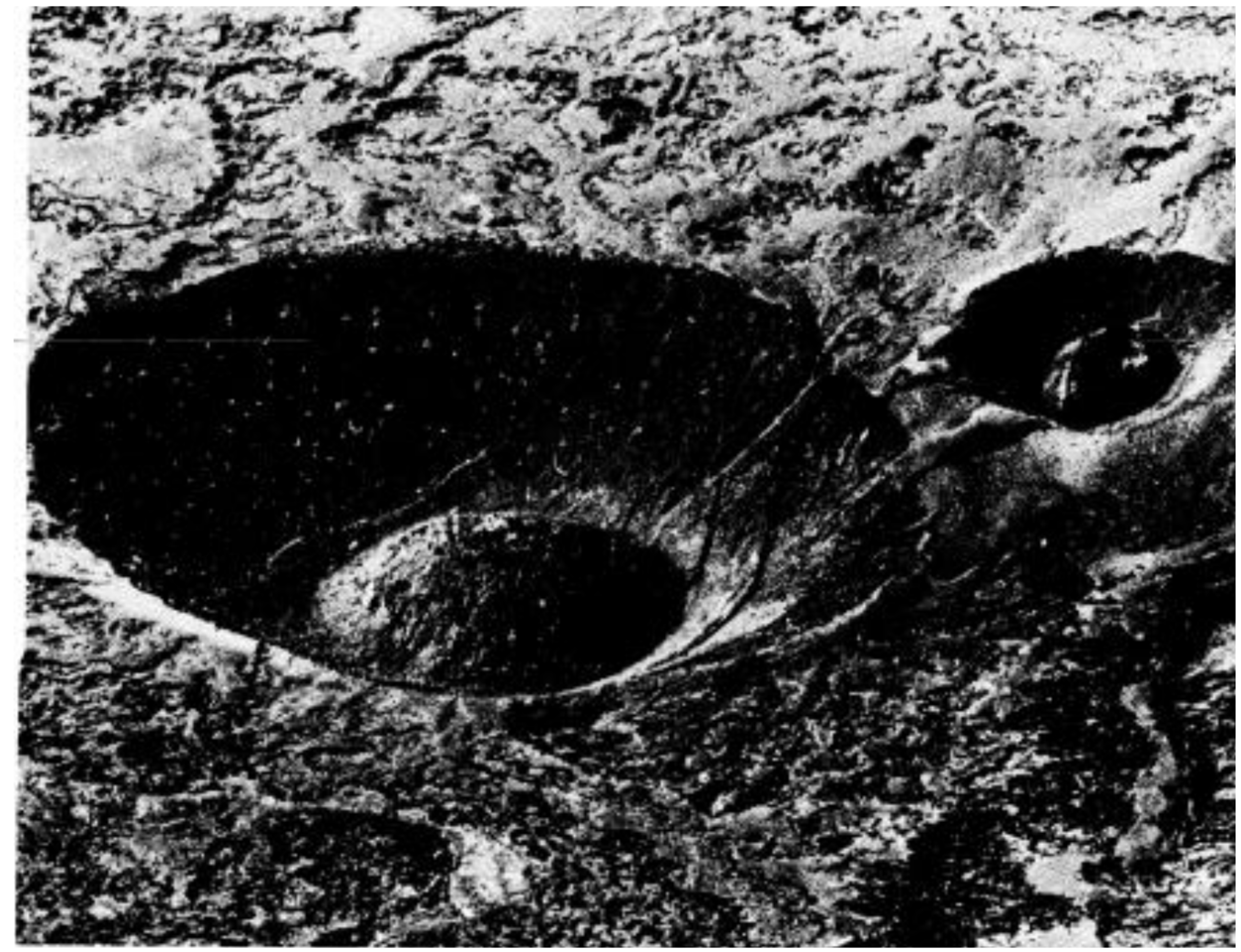
Normals are scaled spatial derivatives of depth image!

Shape from a Single Image?

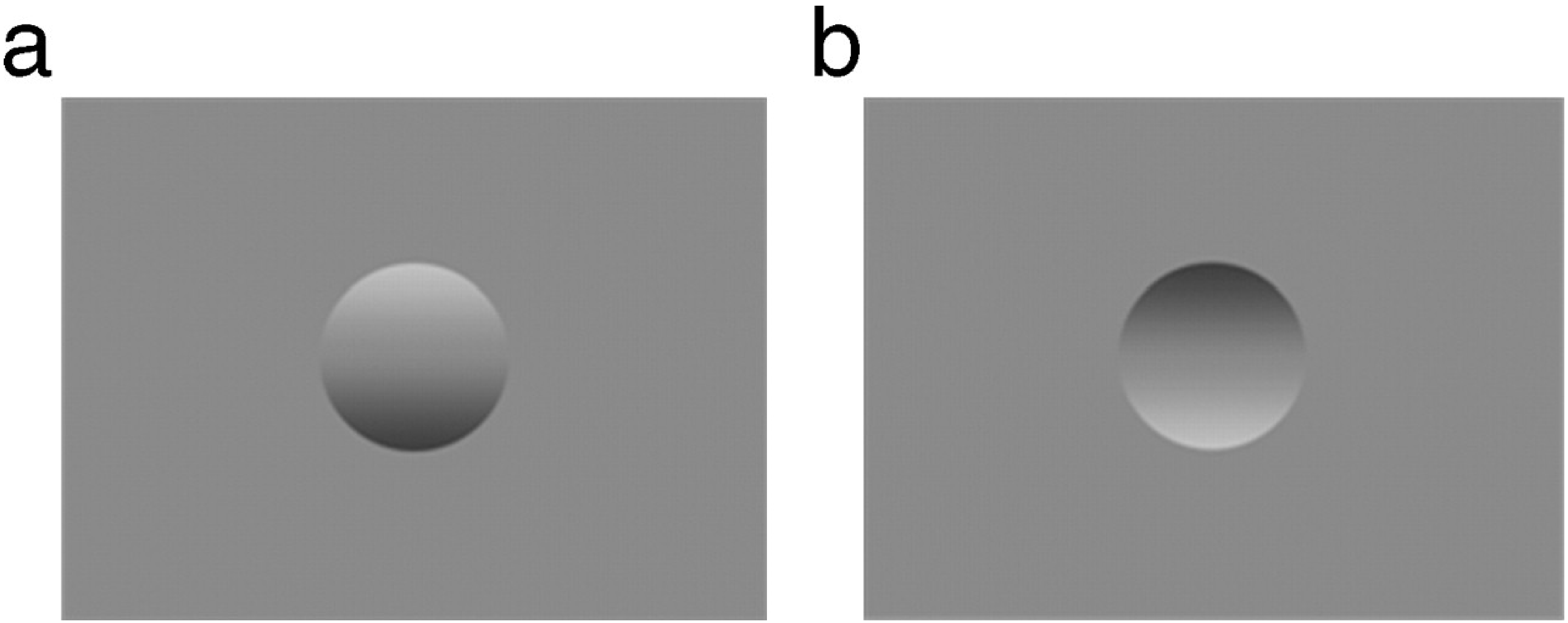
- Given a single image of an object with known surface reflectance taken under a known light source, can we recover the shape of the object?

Human Perception





Examples of the classic bump/dent stimuli used to test lighting assumptions when judging shape from shading, with shading orientations (a) 0° and (b) 180° from the vertical.



Thomas R et al. J Vis 2010;10:6

Journal of VISION

Human Perception

- Our brain often perceives shape from shading.
- Mostly, it makes many assumptions to do so.
- For example:

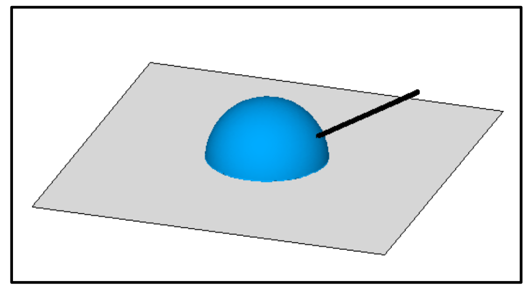
    Light is coming from above (sun).

    Biased by occluding contours.

by V. Ramachandran

Single-lighting is ambiguous

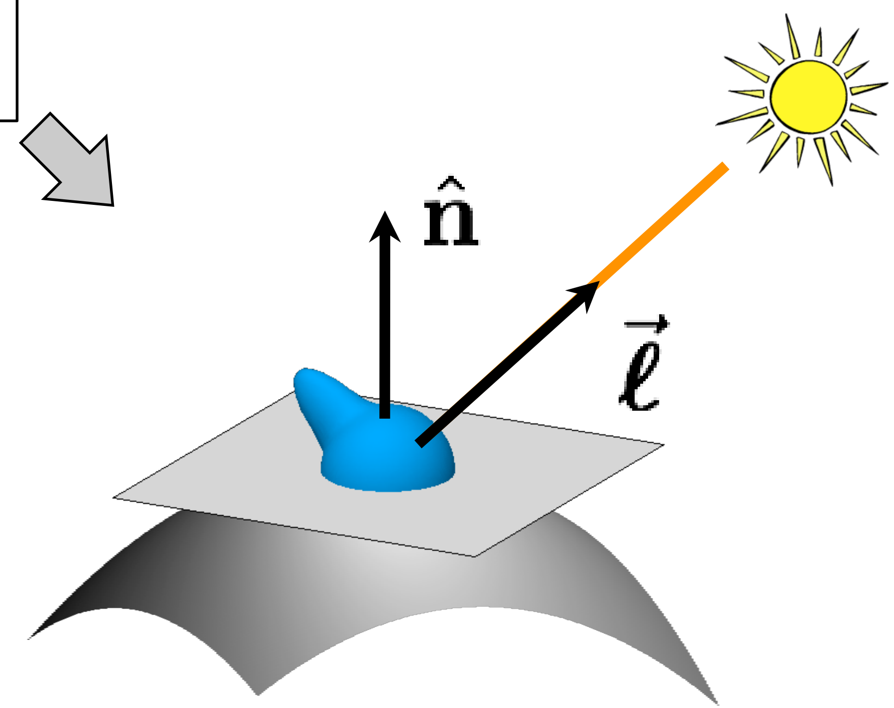
ASSUMPTION 1:
LAMBERTIAN



$$L^{\text{out}}(\hat{\boldsymbol{\omega}}) = \int_{\Omega_{\text{in}}} f(\hat{\boldsymbol{\omega}}_{\text{in}}, \hat{\boldsymbol{\omega}}_{\text{out}}) L^{\text{in}}(\hat{\boldsymbol{\omega}}_{\text{in}}) \cos\theta_{\text{in}} d\hat{\boldsymbol{\omega}}_{\text{in}}$$

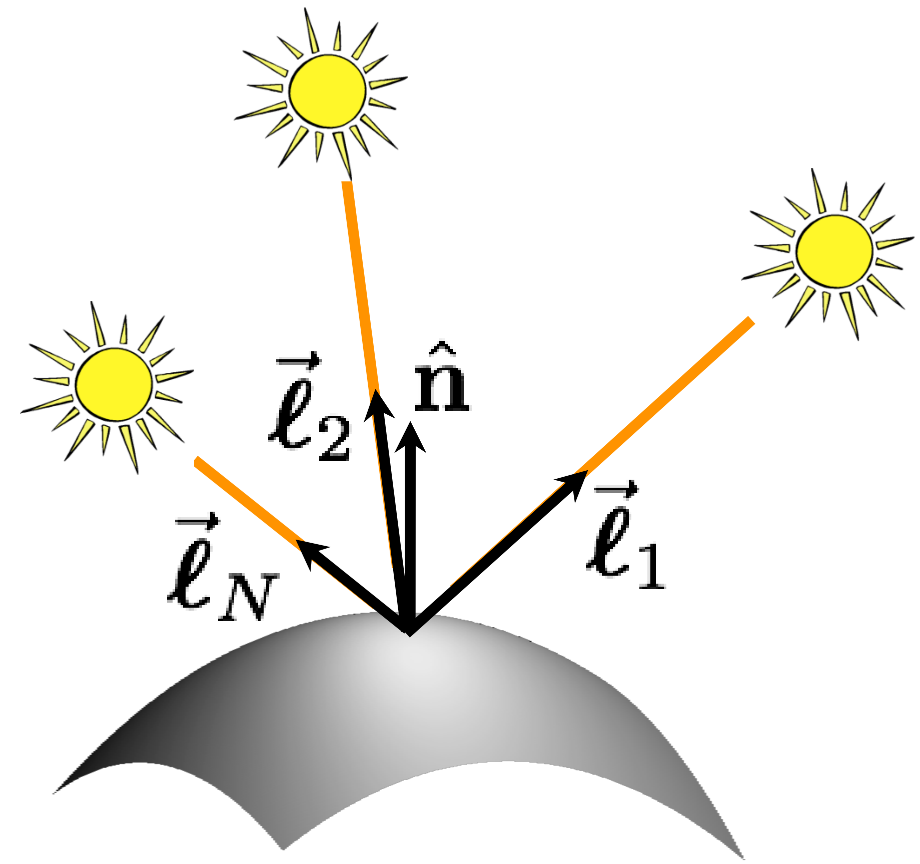
$$\boxed{I = a\hat{\mathbf{n}}^{\top}\vec{\boldsymbol{\ell}}}$$

ASSUMPTION 2:
DIRECTIONAL LIGHTING



Lambertian photometric stereo

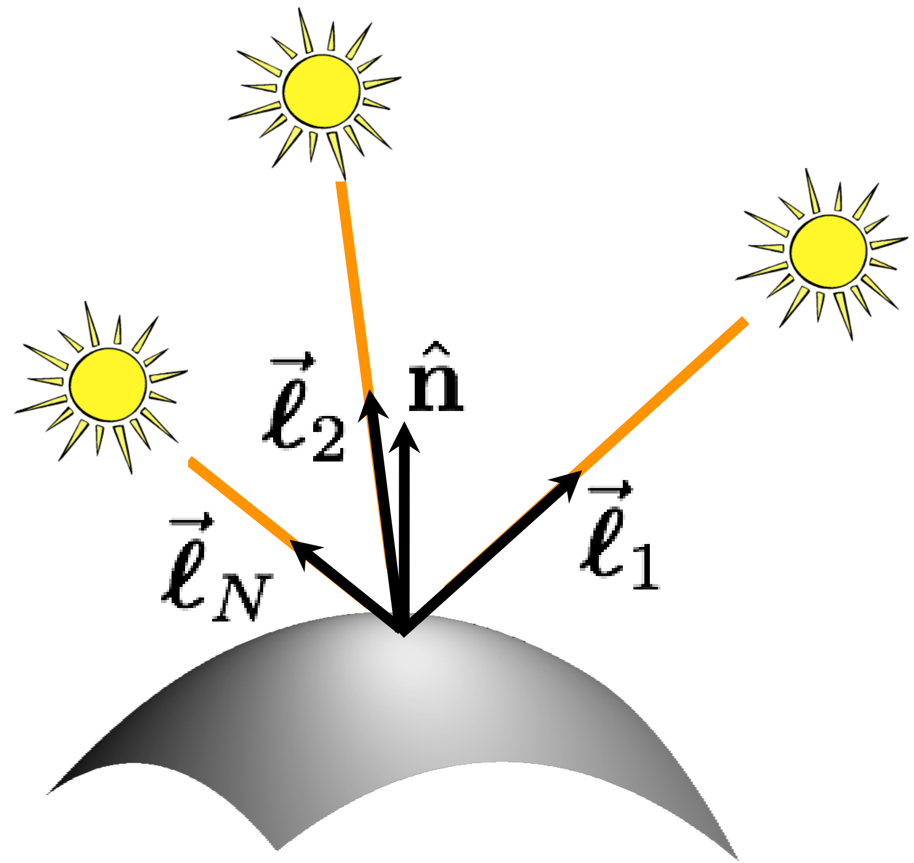
$$
\begin{aligned}
I_1 &= a\hat{\boldsymbol{n}}^\top \vec{\boldsymbol{\ell}}_1 \\
I_2 &= a\hat{\boldsymbol{n}}^\top \vec{\boldsymbol{\ell}}_2 \\
&\vdots \\
I_N &= a\hat{\boldsymbol{n}}^\top \vec{\boldsymbol{\ell}}_N
\end{aligned}
$$



Assumption: We know the lighting directions.

Lambertian photometric stereo

$$
\begin{aligned}
I_1 &= a\hat{\boldsymbol{n}}^\top \vec{\boldsymbol{\ell}}_1 \\
I_2 &= a\hat{\boldsymbol{n}}^\top \vec{\boldsymbol{\ell}}_2 \\
&\vdots \\
I_N &= a\hat{\boldsymbol{n}}^\top \vec{\boldsymbol{\ell}}_N
\end{aligned}
$$



define "pseudo-normal" $\vec{\boldsymbol{b}} \triangleq a\hat{\boldsymbol{n}}$

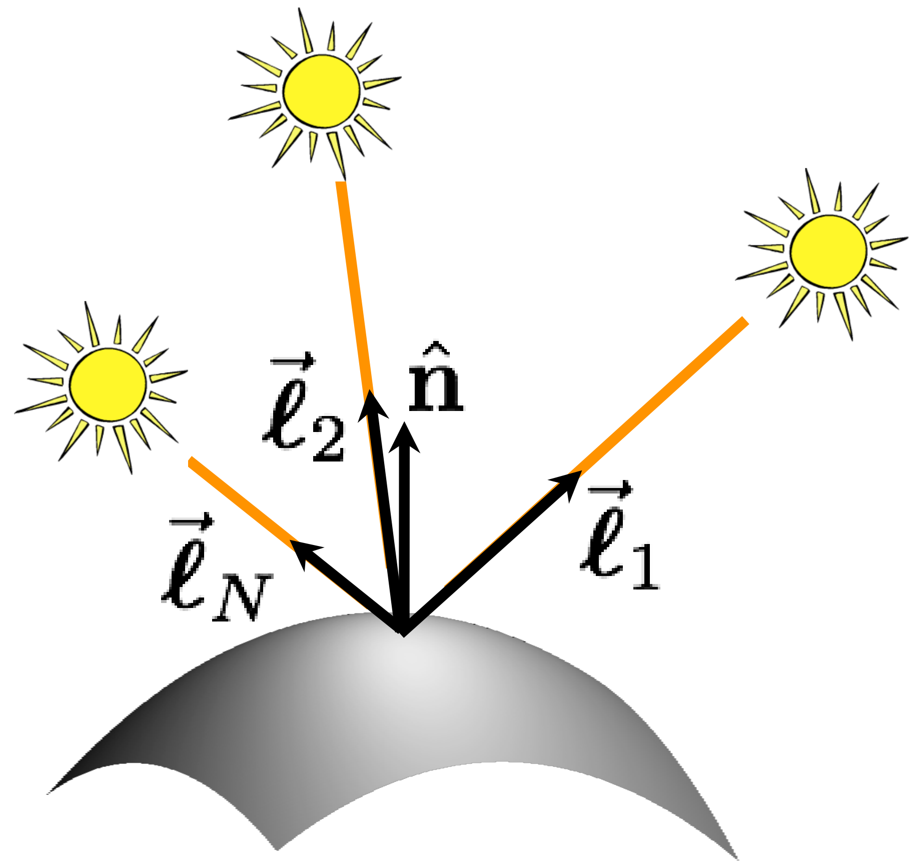
solve linear system for pseudo-normal

$$
\begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_N \end{bmatrix} = \begin{bmatrix} \vec{\boldsymbol{\ell}}_1^\top \\ \vec{\boldsymbol{\ell}}_2^\top \\ \vdots \\ \vec{\boldsymbol{\ell}}_N^\top \end{bmatrix} \begin{bmatrix} \vec{\boldsymbol{b}} \end{bmatrix}
$$

What are the dimensions of these matrices?

Lambertian photometric stereo

$$\boxed{\begin{aligned} I_1 &= a\hat{\boldsymbol{n}}^\top \vec{\boldsymbol{\ell}}_1 \\ I_2 &= a\hat{\boldsymbol{n}}^\top \vec{\boldsymbol{\ell}}_2 \\ &\vdots \\ I_N &= a\hat{\boldsymbol{n}}^\top \vec{\boldsymbol{\ell}}_N \end{aligned}}$$



define "pseudo-normal" $\vec{\boldsymbol{b}} \triangleq a\hat{\boldsymbol{n}}$

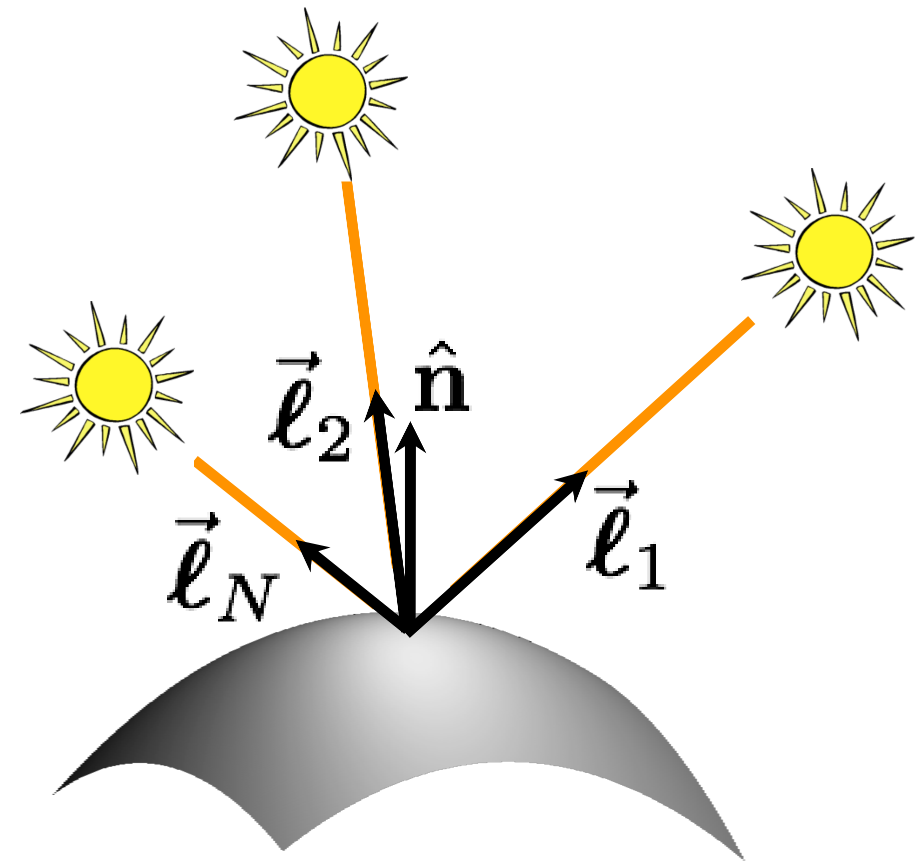
solve linear system for pseudo-normal

$$\begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_N \end{bmatrix}_{N\times 1} = \begin{bmatrix} \vec{\boldsymbol{\ell}}_1^\top \\ \vec{\boldsymbol{\ell}}_2^\top \\ \vdots \\ \vec{\boldsymbol{\ell}}_N^\top \end{bmatrix}_{N\times 3} \begin{bmatrix} \vec{\boldsymbol{b}} \end{bmatrix}_{3\times 1}$$

What are the knowns and unknowns?

Lambertian photometric stereo

$$\begin{aligned} I_1 &= a\hat{\boldsymbol{n}}^\top \vec{\boldsymbol{\ell}}_1 \\ I_2 &= a\hat{\boldsymbol{n}}^\top \vec{\boldsymbol{\ell}}_2 \\ &\vdots \\ I_N &= a\hat{\boldsymbol{n}}^\top \vec{\boldsymbol{\ell}}_N \end{aligned}$$



define “pseudo-normal” $\vec{\boldsymbol{b}} \triangleq a\hat{\boldsymbol{n}}$

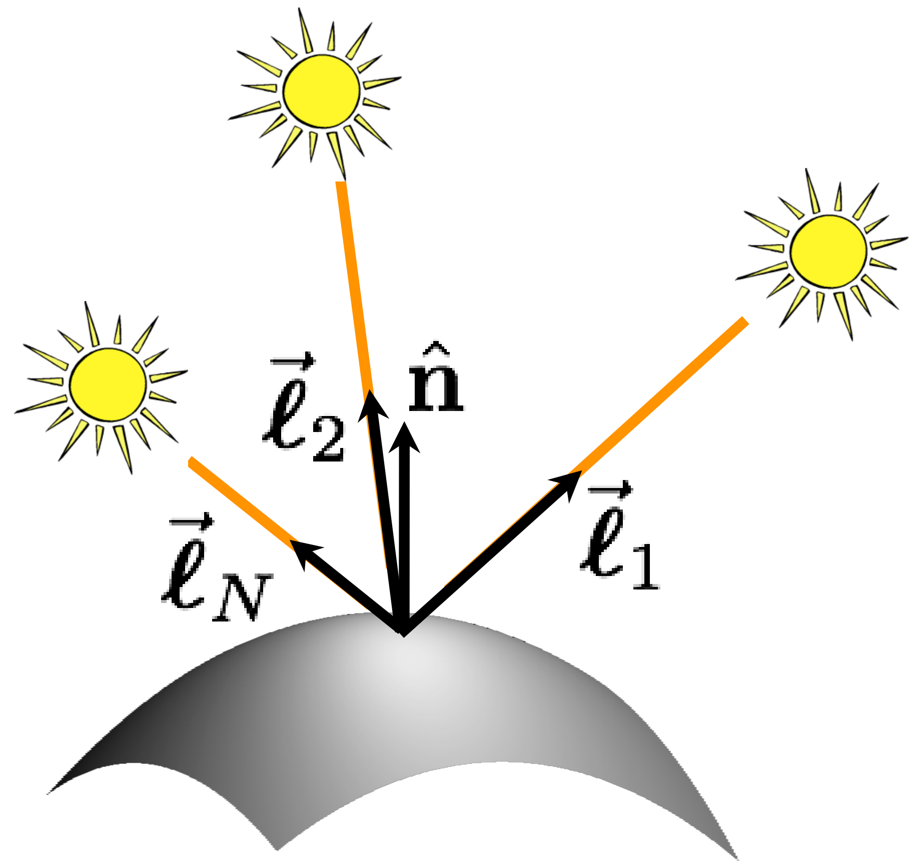
solve linear system for pseudo-normal

$$\begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_N \end{bmatrix}_{N\times 1} = \begin{bmatrix} \vec{\boldsymbol{\ell}}_1^\top \\ \vec{\boldsymbol{\ell}}_2^\top \\ \vdots \\ \vec{\boldsymbol{\ell}}_N^\top \end{bmatrix}_{N\times 3} \begin{bmatrix} \vec{\boldsymbol{b}} \end{bmatrix}_{3\times 1}$$

How many lights do I need for unique solution?

Lambertian photometric stereo

$$
\begin{aligned}
I_1 &= a\hat{\boldsymbol{n}}^\top \vec{\boldsymbol{\ell}}_1 \\
I_2 &= a\hat{\boldsymbol{n}}^\top \vec{\boldsymbol{\ell}}_2 \\
&\vdots \\
I_N &= a\hat{\boldsymbol{n}}^\top \vec{\boldsymbol{\ell}}_N
\end{aligned}
$$



define "pseudo-normal" $\vec{\boldsymbol{b}} \triangleq a\hat{\boldsymbol{n}}$

solve linear system for pseudo-normal

$$
\begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_N \end{bmatrix}_{N\times 1} = \begin{bmatrix} \vec{\boldsymbol{\ell}}_1^\top \\ \vec{\boldsymbol{\ell}}_2^\top \\ \vdots \\ \vec{\boldsymbol{\ell}}_N^\top \end{bmatrix}_{N\times 3} \begin{bmatrix} \vec{\boldsymbol{b}} \end{bmatrix}_{3\times 1}
$$

once system is solved, b gives normal direction and albedo

How do we solve this system?

Solving the Equation with three lights

$$\underbrace{\begin{bmatrix} I_1 \\ I_2 \\ I_2 \end{bmatrix}}_{\underset{3\times 1}{\mathbf{I}}} = \underbrace{\begin{bmatrix} \ell_1^T \\ \ell_2^T \\ \ell_3^T \end{bmatrix}}_{\underset{3\times 3}{L}} \underbrace{a\mathbf{n}}_{\underset{3\times 1}{\widetilde{\mathbf{n}}}}$$

$$\tilde{\mathbf{n}} = L^{-1}\mathbf{I}$$

inverse

$$a = |\tilde{\mathbf{n}}|$$

$$\mathbf{n} = \frac{\tilde{\mathbf{n}}}{|\tilde{\mathbf{n}}|} = \frac{\tilde{\mathbf{n}}}{a}$$

Is there any reason to use more than three lights?

More than Three Light Sources

- Get better SNR by using more lights

$$\begin{bmatrix} I_1 \\ \vdots \\ I_N \end{bmatrix} = \begin{bmatrix} \ell_1^T \\ \vdots \\ \ell_N^T \end{bmatrix} a\mathbf{n}$$

- Least squares solution:

$$\mathbf{I} = L\tilde{\mathbf{n}} \qquad N \times 1 = (N \times 3)(3 \times 1)$$

$$L^T\mathbf{I} = L^T L\tilde{\mathbf{n}}$$

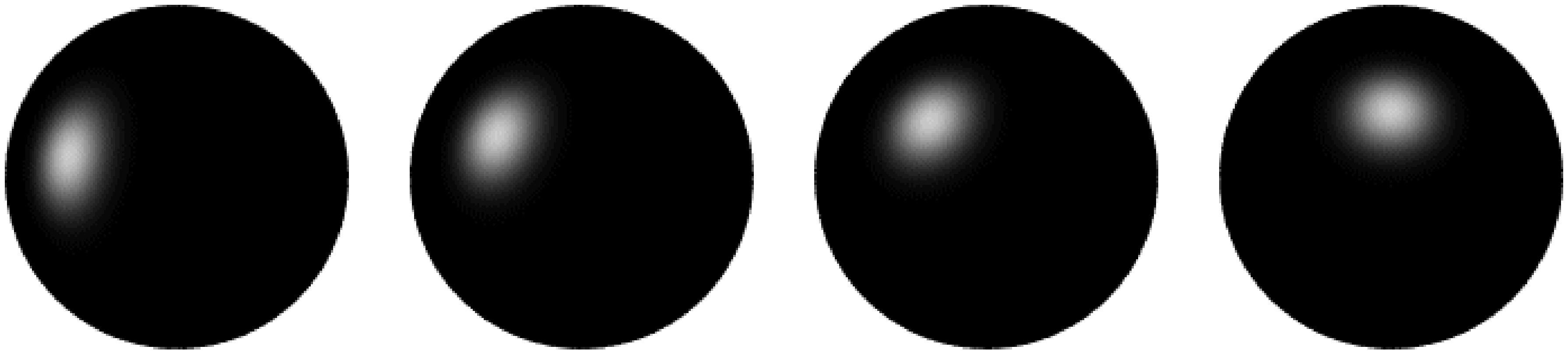
$$\tilde{\mathbf{n}} = \boxed{\left(L^T L\right)^{-1} L^T}\,\mathbf{I}$$

Moore-Penrose pseudo inverse

- Solve for $a, \mathbf{n}$ as before

Computing light source directions

- Trick: place a chrome sphere in the scene



  - the location of the highlight tells you the source direction

Limitations

- Big problems
    - Doesn’t work for shiny things, semi-translucent things
    - Shadows, inter-reflections

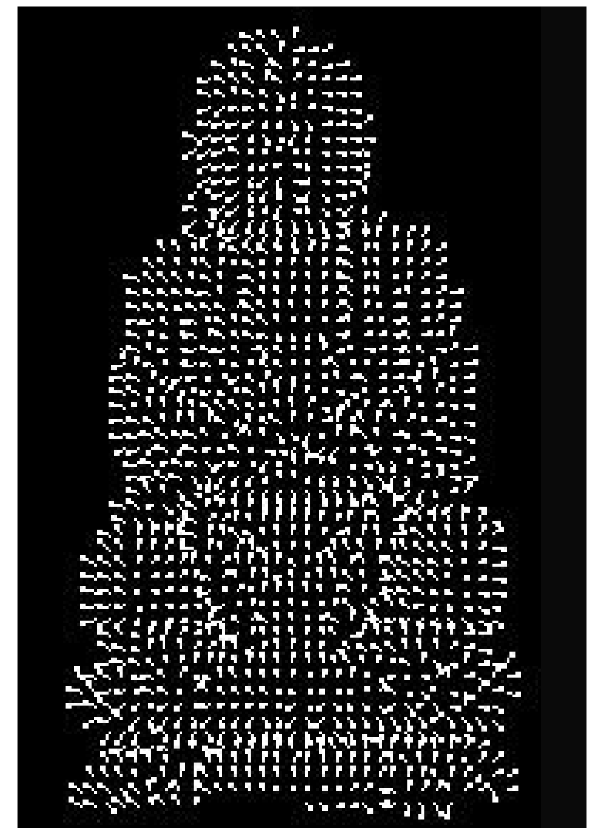
- Smaller problems
    - Camera and lights have to be distant
    - Calibration requirements
        - measure light source directions, intensities
        - camera response function

Depth from normals

- Solving the linear system per-pixel gives us an estimated surface normal for each pixel

Input photo

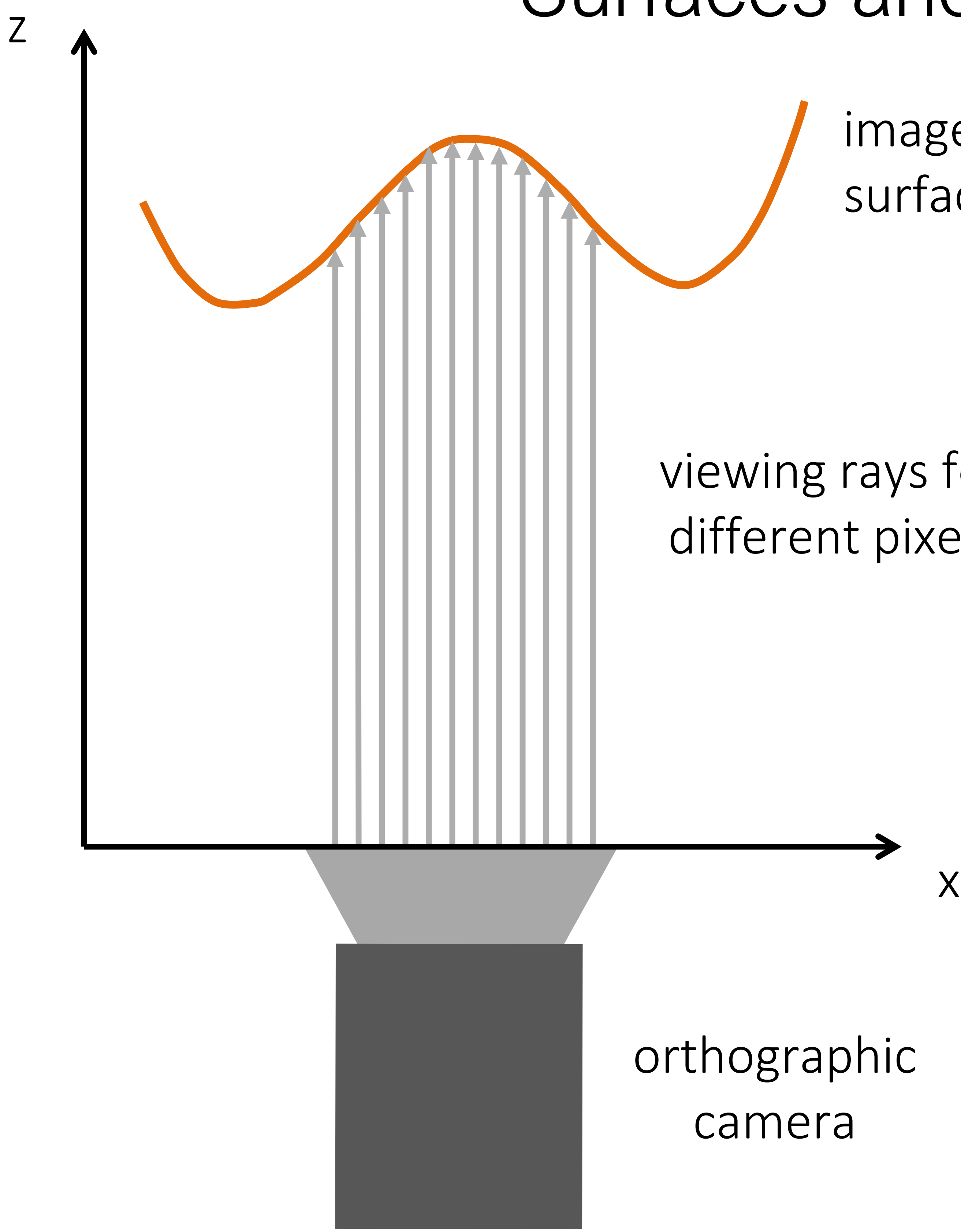
Estimated normals



Estimated normals (needle diagram)

- How can we compute depth from normals?
  - Normals are like the “derivative” of the true depth

Surfaces and normals



Surface representation as a depth image (also known as Monge surface):

$$\overset{\uparrow}{z} = f\underbrace{(x, y)}_{\text{pixel coordinates in image space}}$$

depth at each pixel

Unnormalized normal:

$$\tilde{n}(x, y) = \left(\frac{df}{dx}, \frac{df}{dy}, -1\right)$$

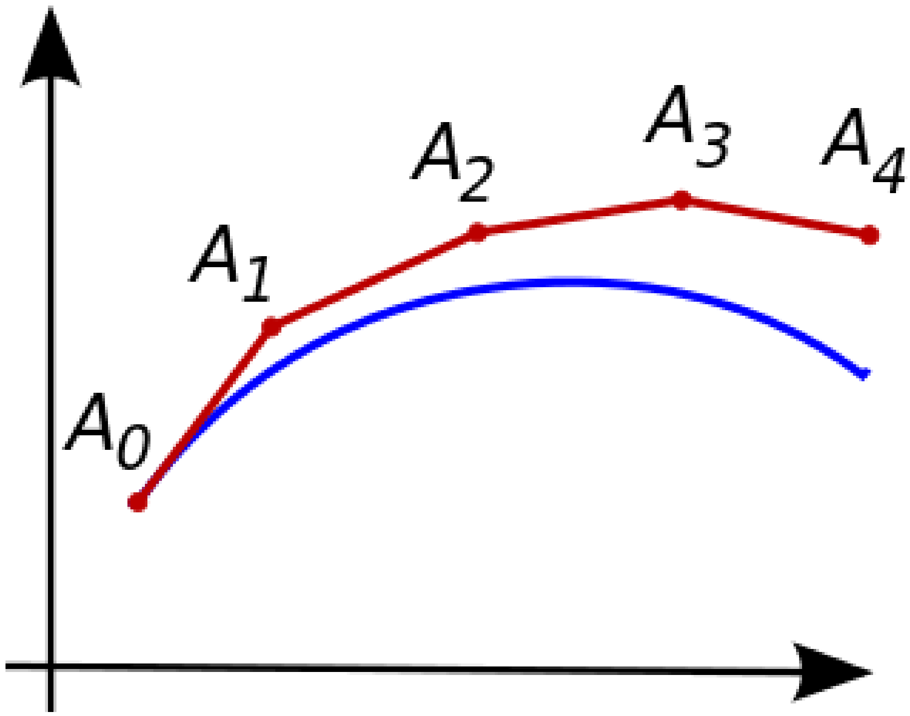
Actual normal:

$$n(x, y) = \tilde{n}(x, y) / \|\tilde{n}(x, y)\|$$

Normals are scaled spatial derivatives of depth image!

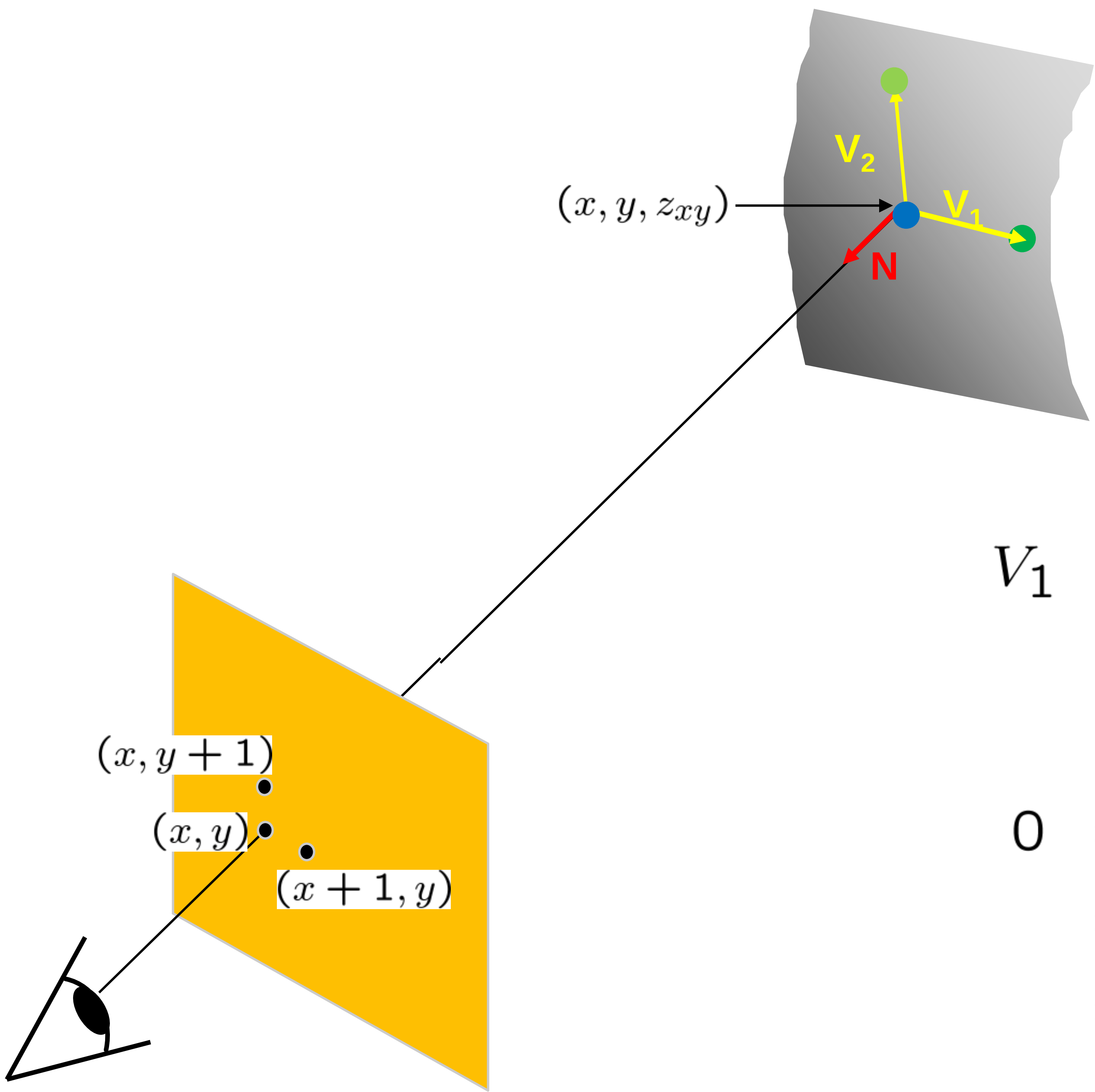
Normal Integration

- Integrating a set of derivatives is easy in 1D
  - (similar to Euler's method from diff. eq. class)



- Could just integrate normals in each column / row separately
- Instead, we formulate as a linear system and solve for depths that *best agree with the surface normals*

Depth from normals



$$\tilde{n}(x,y) = \left(\frac{df}{dx}, \frac{df}{dy}, -1\right)$$
$$n(x,y) = \tilde{n}(x,y)/\|\tilde{n}(x,y)\|$$

$$\begin{aligned} V_1 &= \boxed{(x+1, y, z_{x+1,y})} - \boxed{(x, y, z_{xy})} \\ &= (1, 0, \boxed{z_{x+1,y} - z_{xy}}) \end{aligned}$$

$\frac{df}{dx}$

$$\begin{aligned} 0 &= N \cdot V_1 \\ &= (n_x, n_y, n_z) \cdot (1, 0, z_{x+1,y} - z_{xy}) \\ &= \boxed{n_x} + \boxed{n_z}(\boxed{z_{x+1,y} - z_{xy}}) \end{aligned}$$

known Unknown

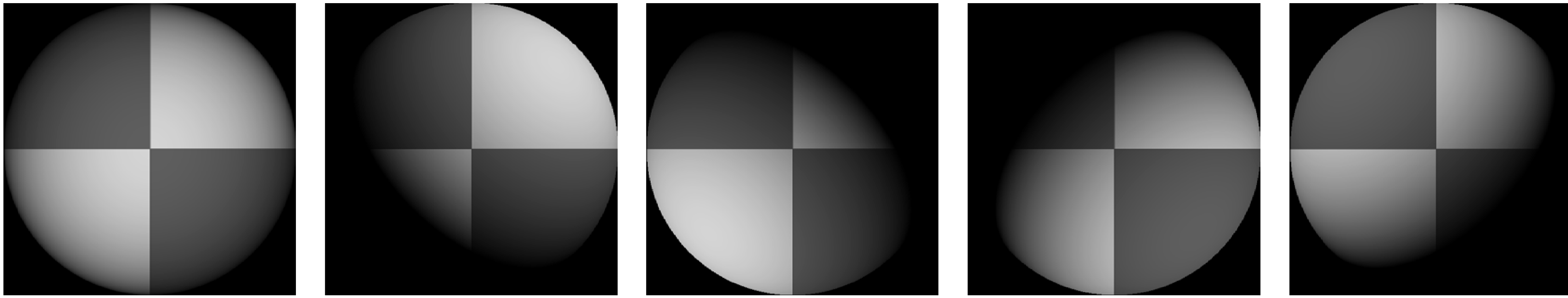
Get a similar equation for **V_2**

- Each normal gives us two linear constraints on z
- compute z values by solving a matrix equation

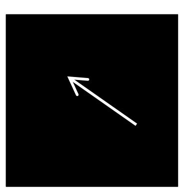
Results

1. Estimate light source directions
2. Compute surface normals
3. Compute albedo values
4. Estimate depth from surface normals
5. Relight the object (with original texture and uniform albedo)

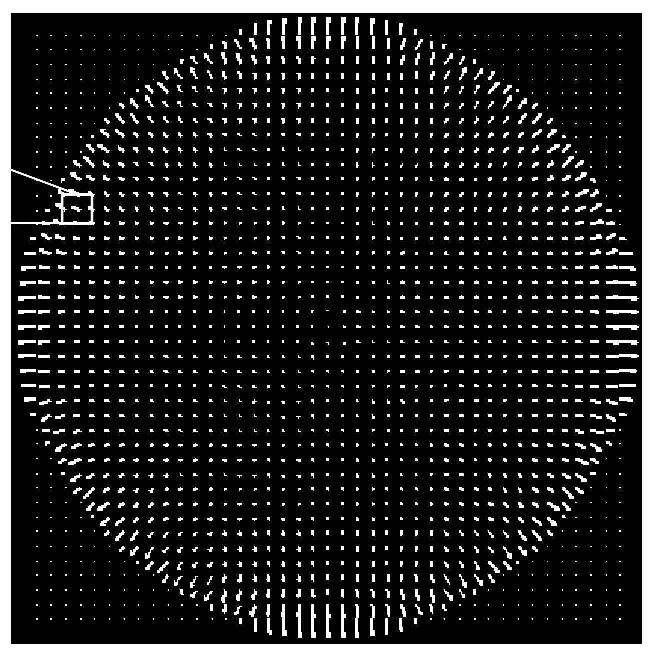
Results: Lambertian Sphere



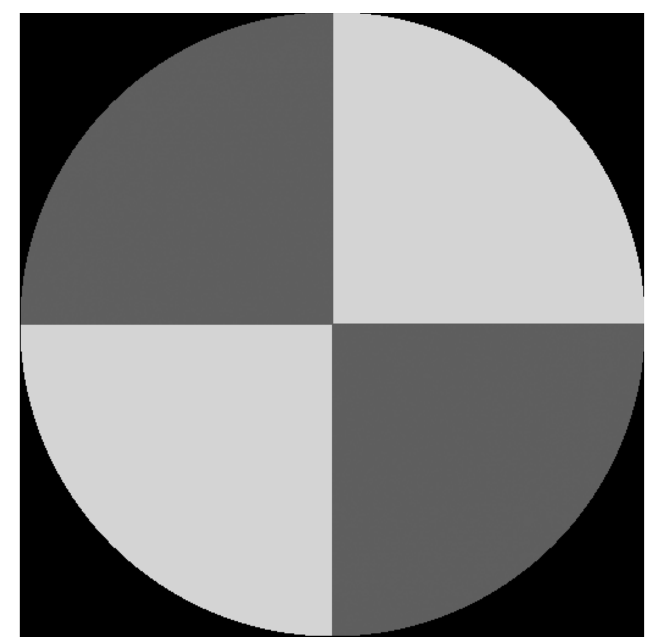
Input Images



Needles are projections of surface normals on image plane



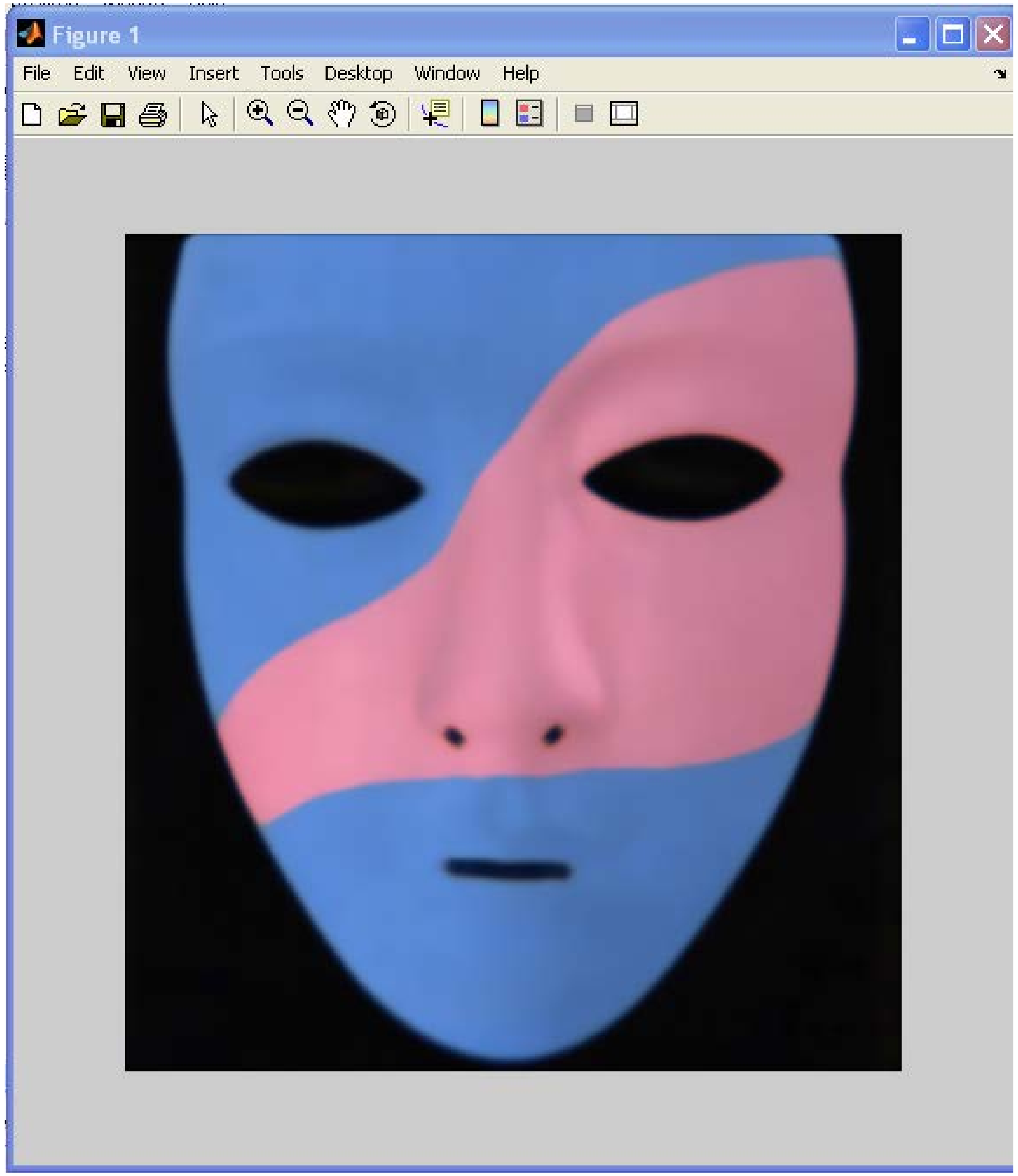
Estimated Surface Normals

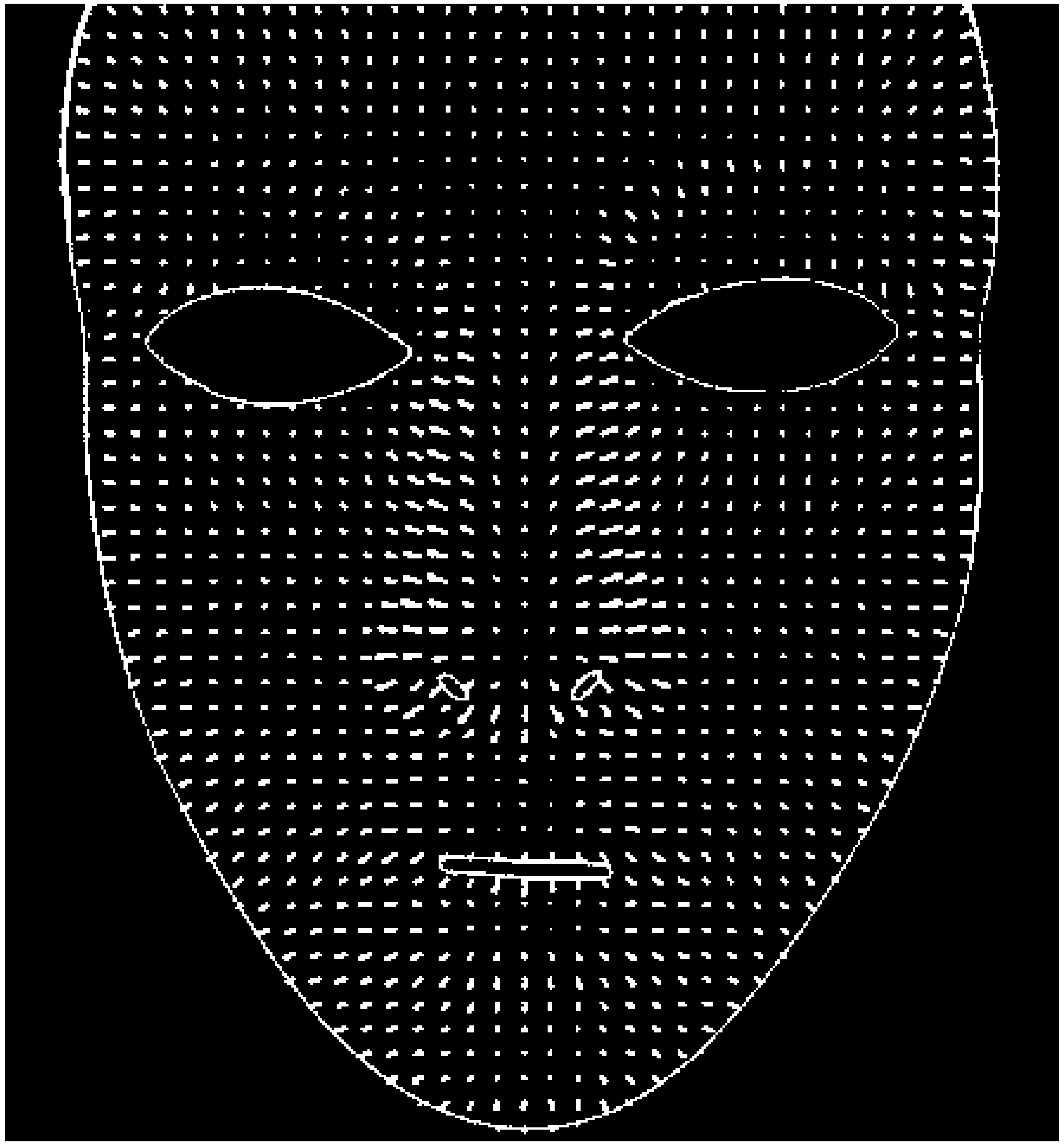


Estimated Albedo

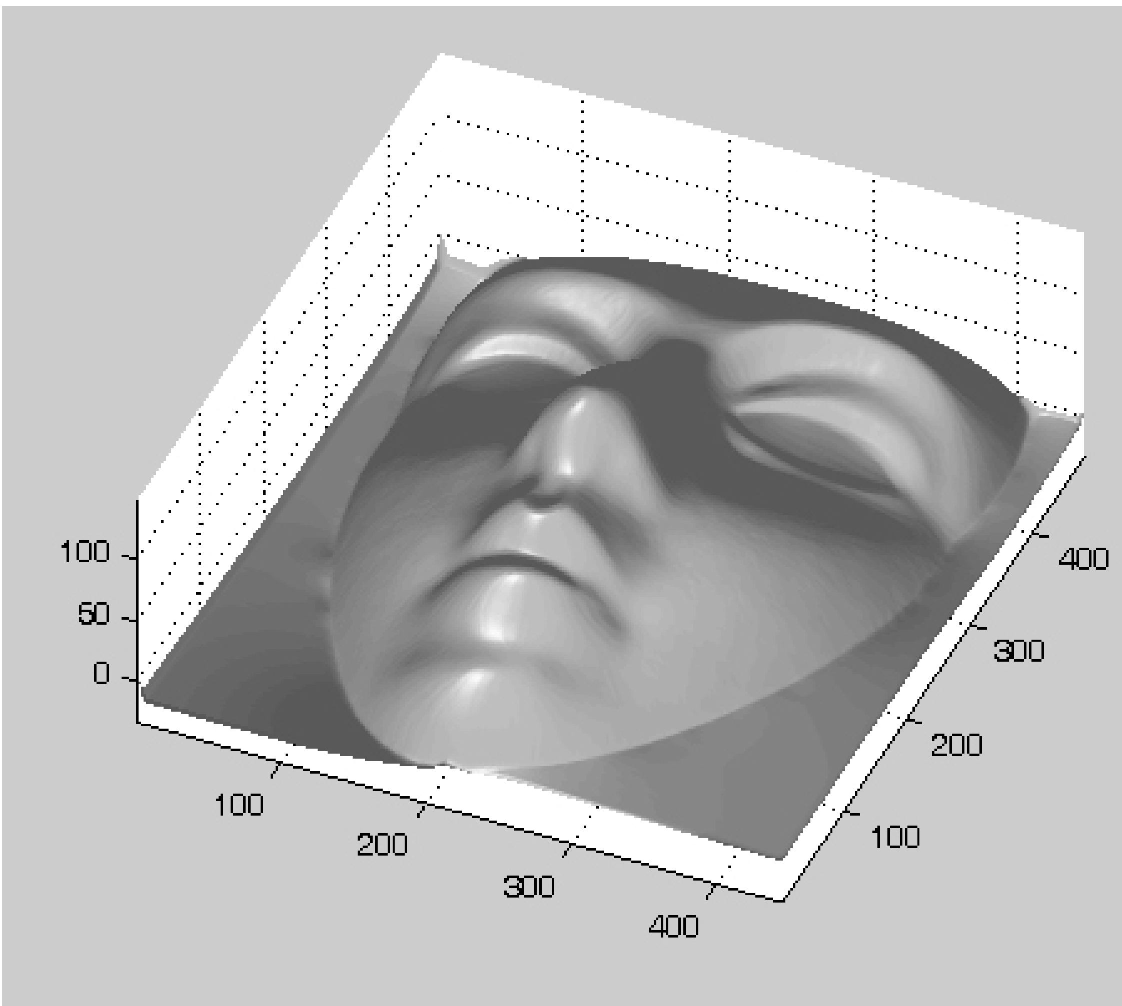
Lambertain Mask

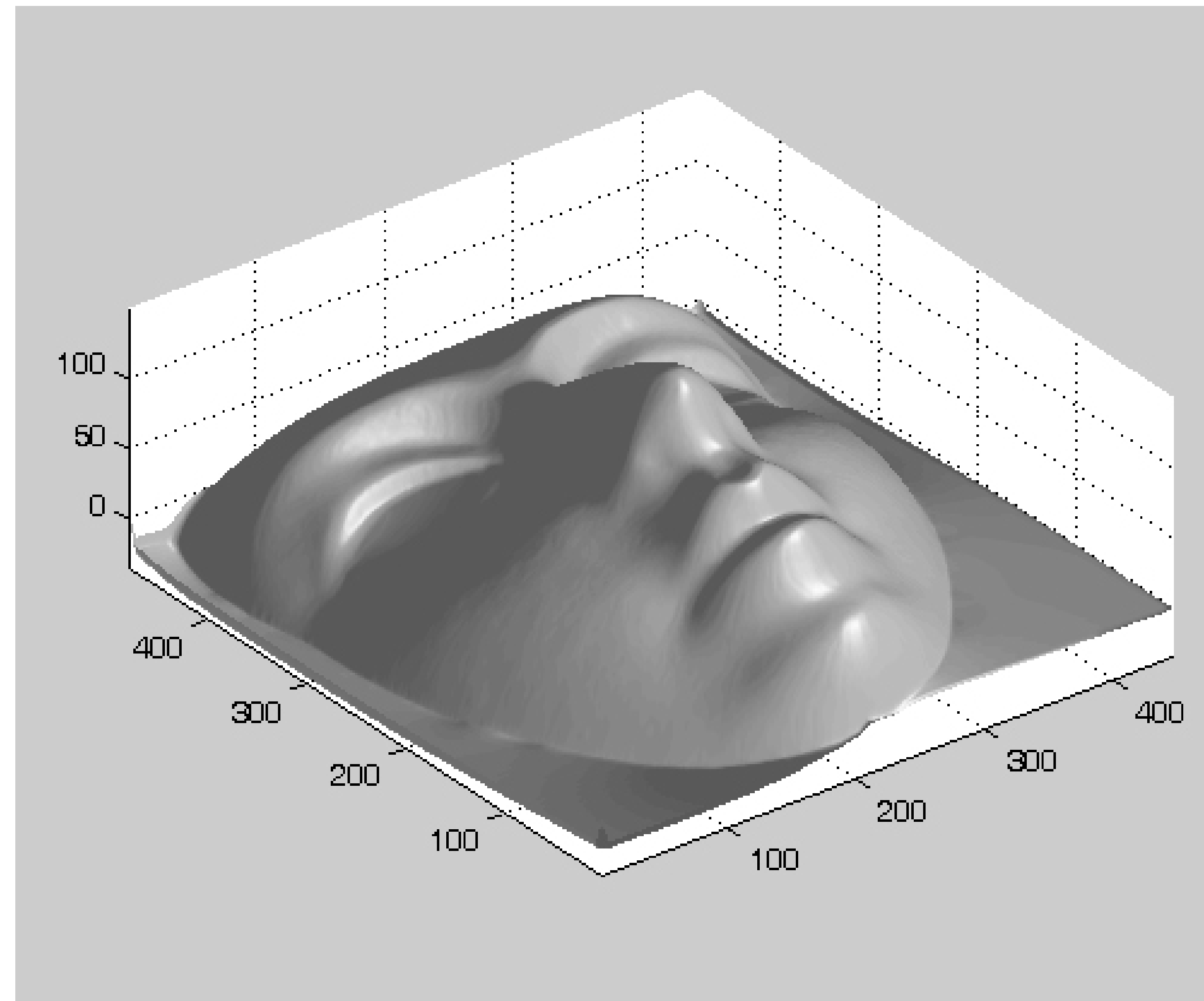
Results – Albedo and Surface Normal





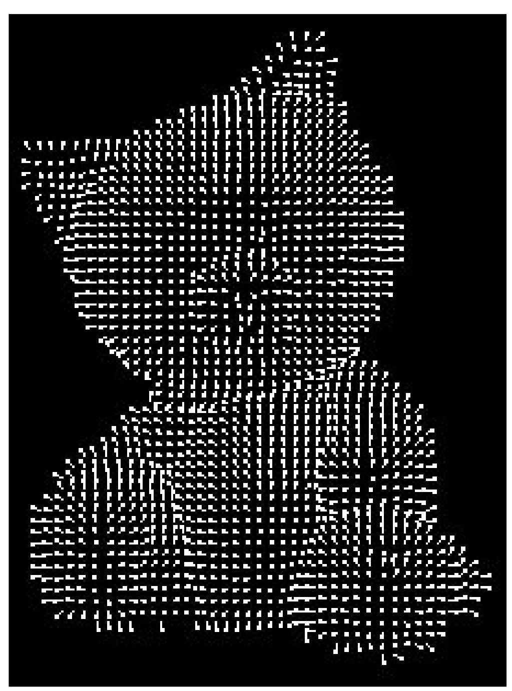
Results – Shape of Mask





Results: Lambertian Toy

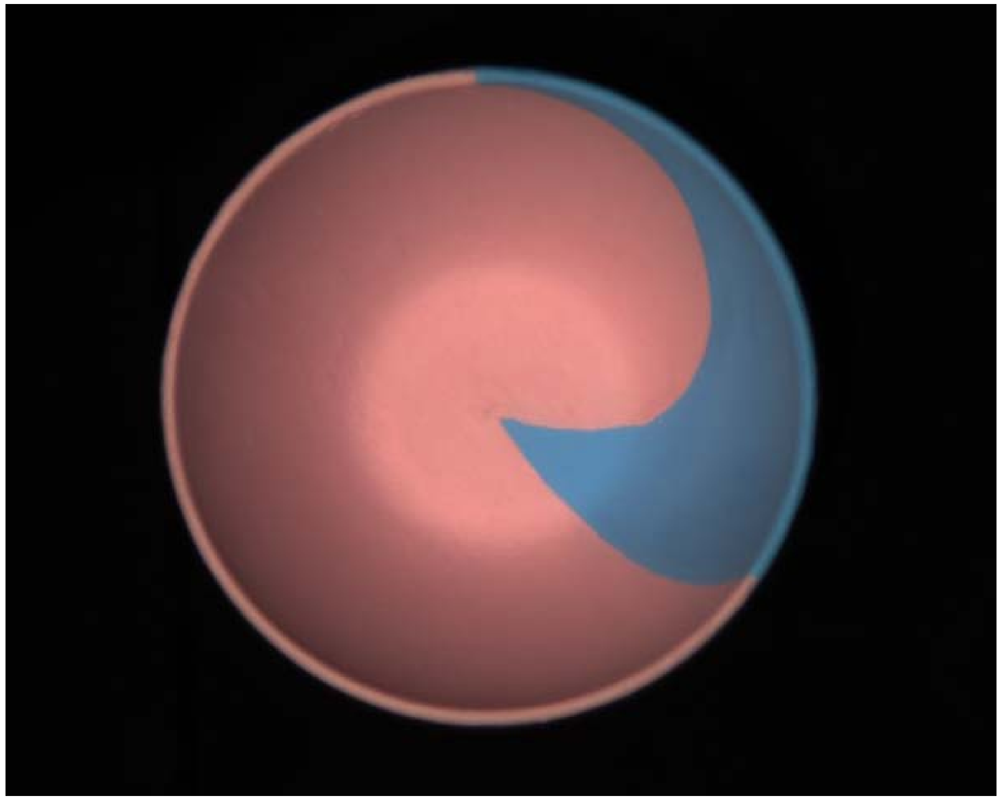
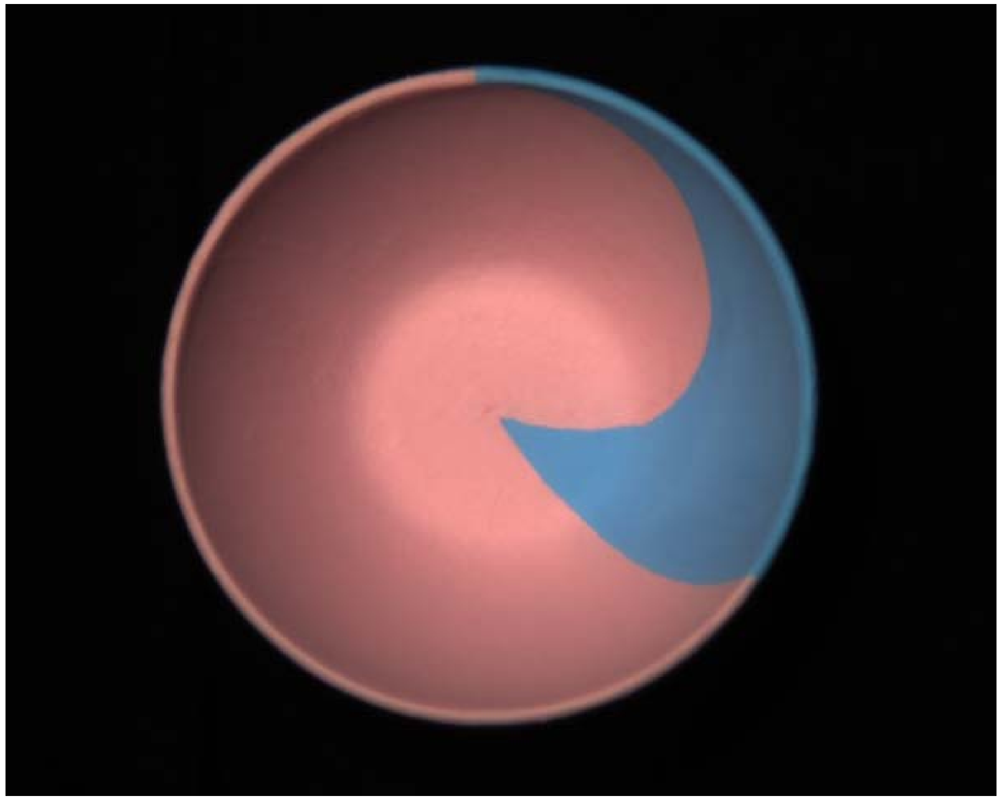
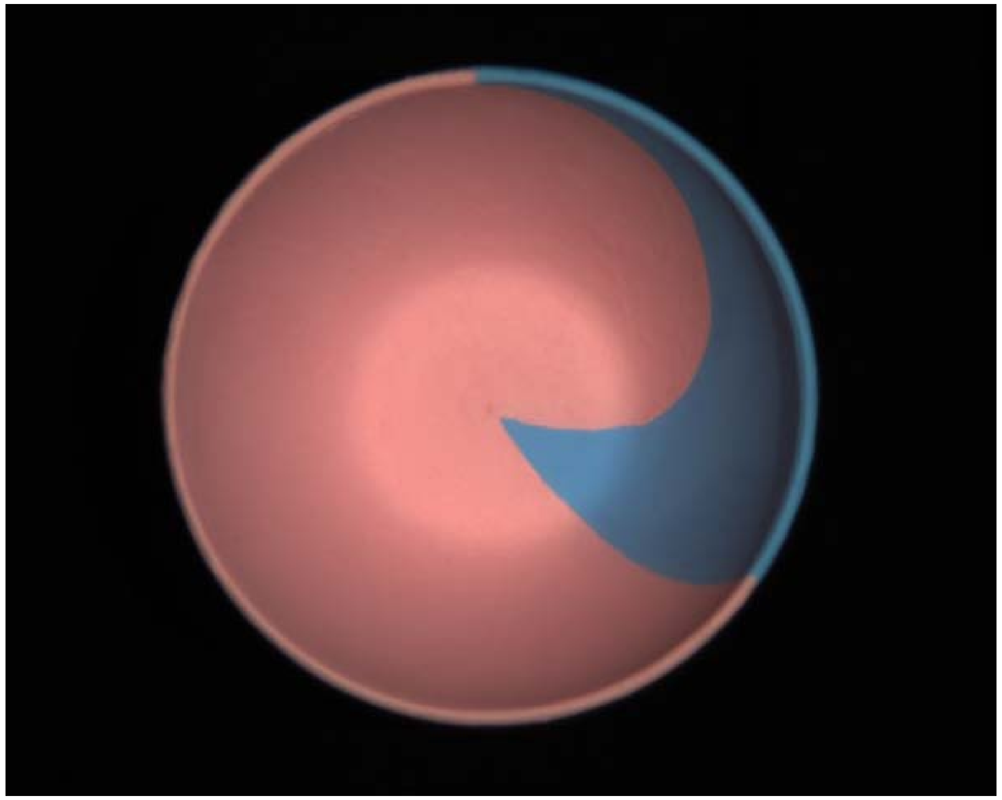
Input Images



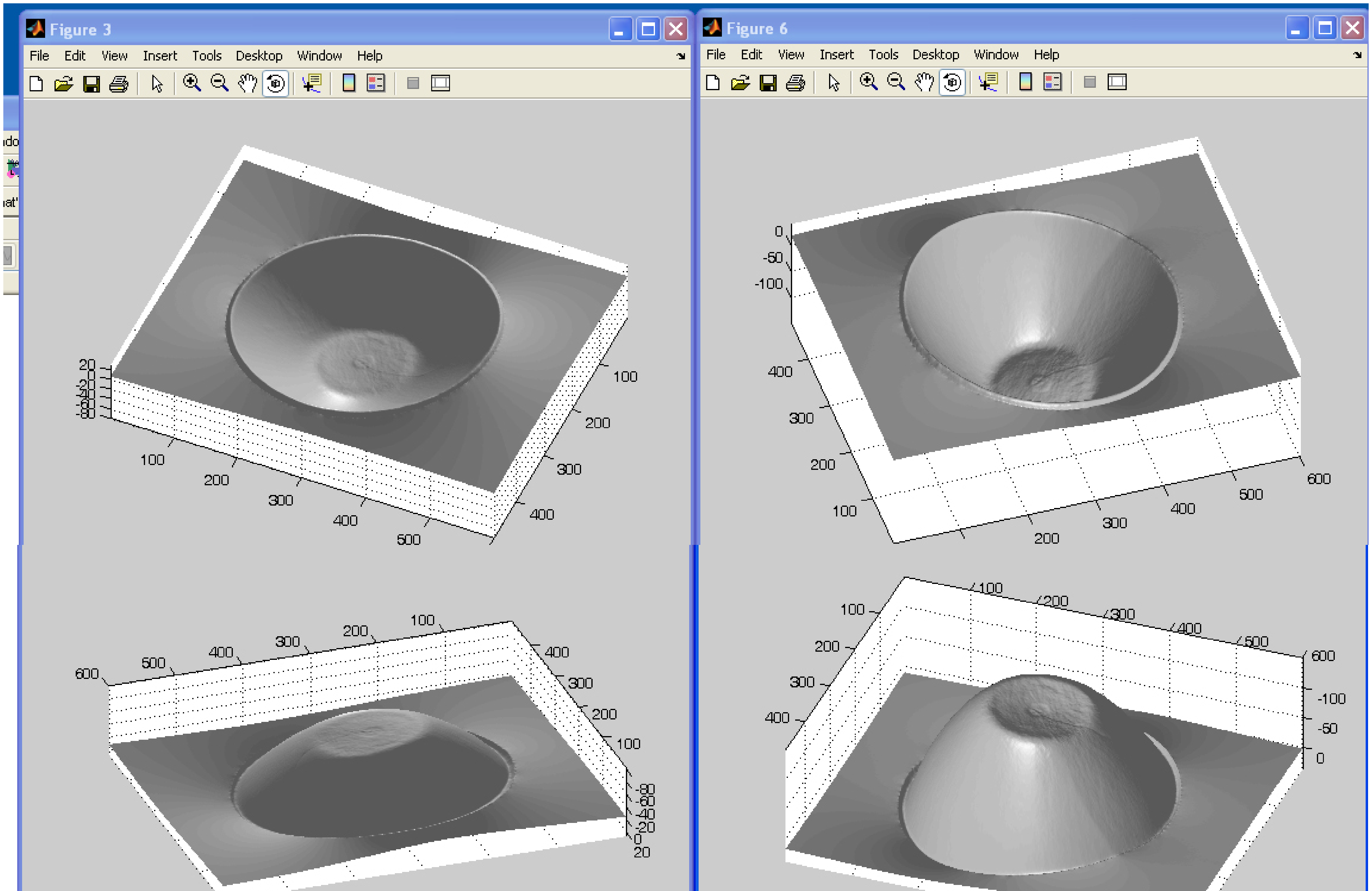
Estimated Surface Normals

Estimated Albedo

Non-idealities: interreflections

Non-idealities: interreflections



Shallow reconstruction (effect of interreflections)

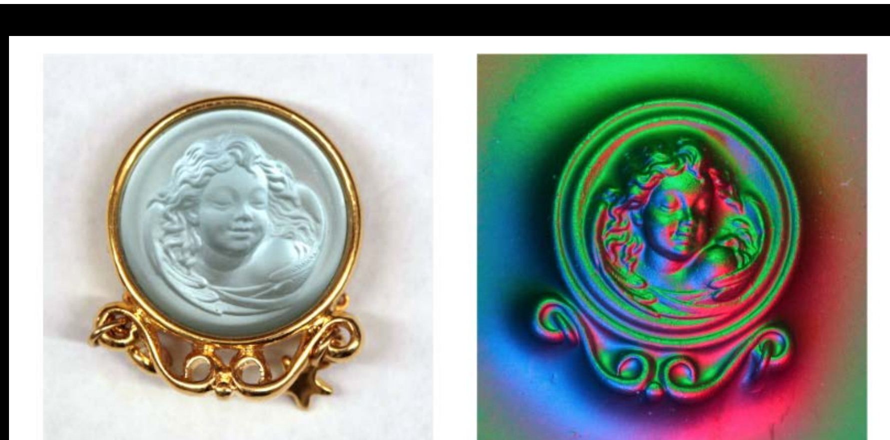
Accurate reconstruction (after removing interreflections)

GelSight 3D Sensor

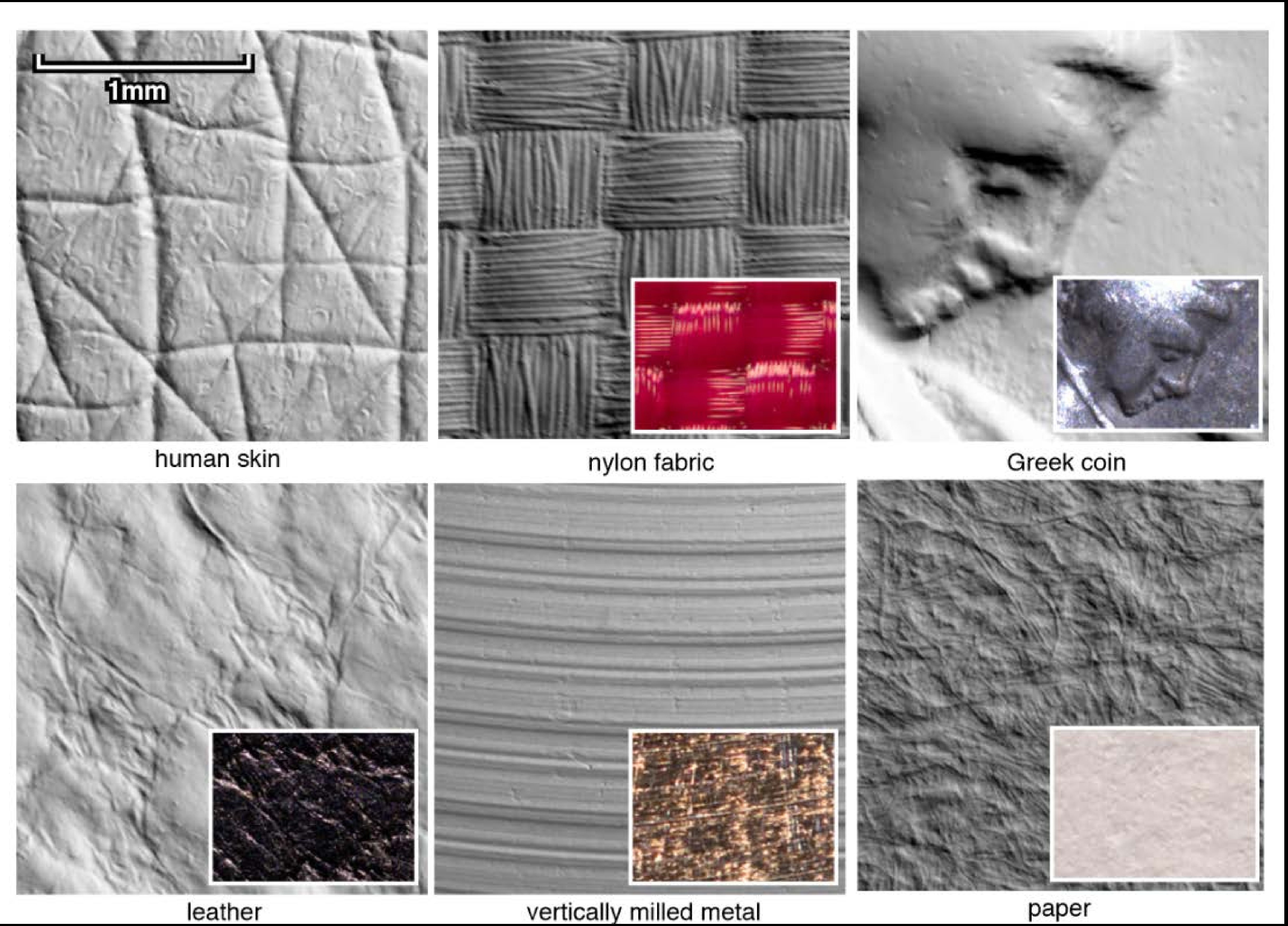
Shape from shading challenge: need to know material reflectance very accurately, and it changes for different objects

Solution: image through a gel slab covered with a reflective skin with known BRDF. Image a relief replica of the surface with the skin reflectance

(a) (b) (c)



3 illumination directions captured simultaneously

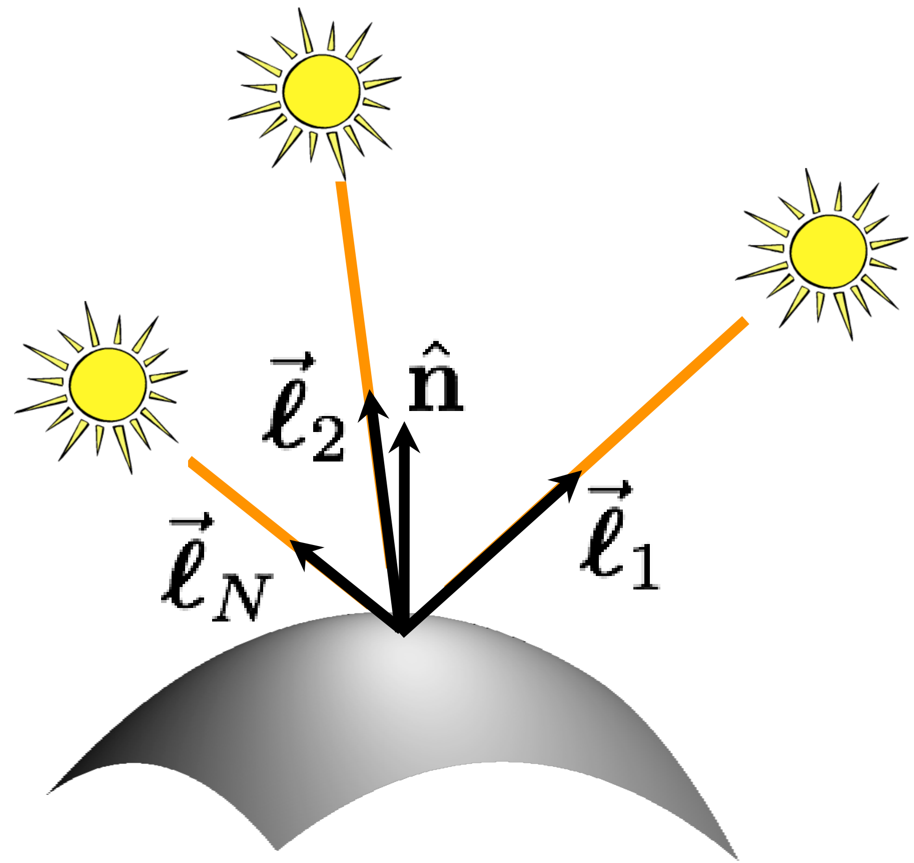

1mm
human skin
nylon fabric
Greek coin
leather
vertically milled metal
paper

What if the light directions are unknown?

Uncalibrated photometric stereo

What if the light directions are unknown?

$$I_1 = a\hat{\boldsymbol{n}}^\top \vec{\boldsymbol{\ell}}_1$$
$$I_2 = a\hat{\boldsymbol{n}}^\top \vec{\boldsymbol{\ell}}_2$$
$$\vdots$$
$$I_N = a\hat{\boldsymbol{n}}^\top \vec{\boldsymbol{\ell}}_N$$



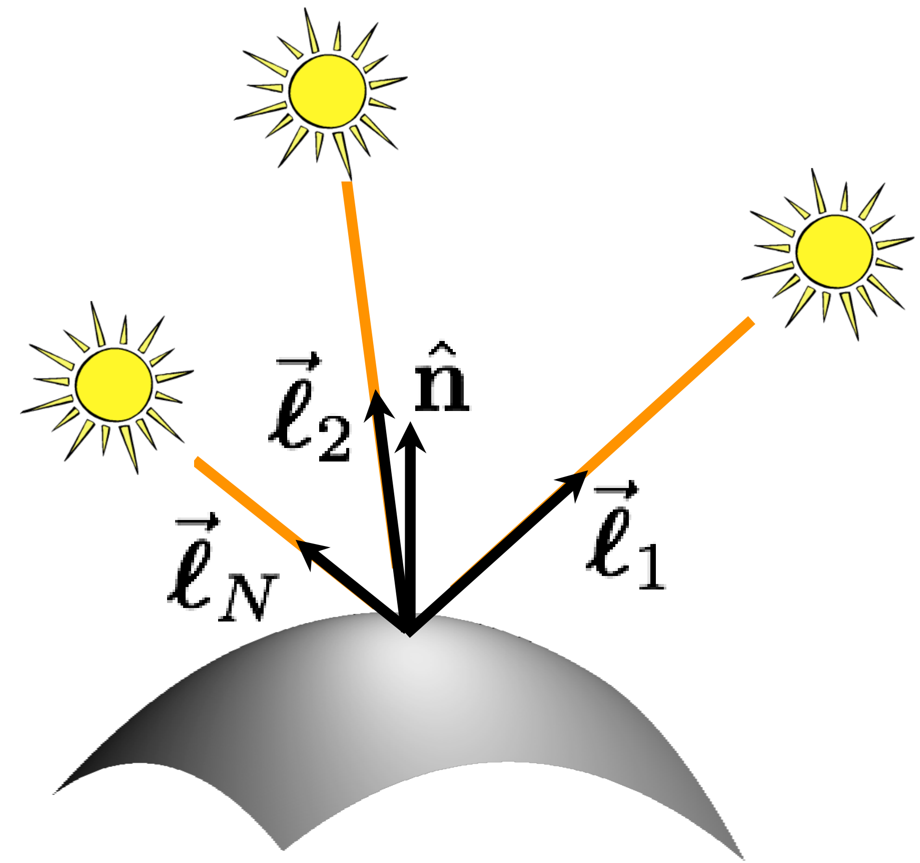
define “pseudo-normal” $\vec{\boldsymbol{b}} \triangleq a\hat{\boldsymbol{n}}$

solve linear system
for pseudo-normal

$$\begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_N \end{bmatrix}_{N\times 1} = \begin{bmatrix} \vec{\boldsymbol{\ell}}_1^\top \\ \vec{\boldsymbol{\ell}}_2^\top \\ \vdots \\ \vec{\boldsymbol{\ell}}_N^\top \end{bmatrix}_{N\times 3} \begin{bmatrix} \vec{\boldsymbol{b}} \end{bmatrix}_{3\times 1}$$

What if the light directions are unknown?

$$
\begin{aligned}
I_1 &= a\hat{\boldsymbol{n}}^\top \vec{\boldsymbol{\ell}}_1 \\
I_2 &= a\hat{\boldsymbol{n}}^\top \vec{\boldsymbol{\ell}}_2 \\
&\vdots \\
I_N &= a\hat{\boldsymbol{n}}^\top \vec{\boldsymbol{\ell}}_N
\end{aligned}
$$



define "pseudo-normal" $\vec{\boldsymbol{b}} \triangleq a\hat{\boldsymbol{n}}$

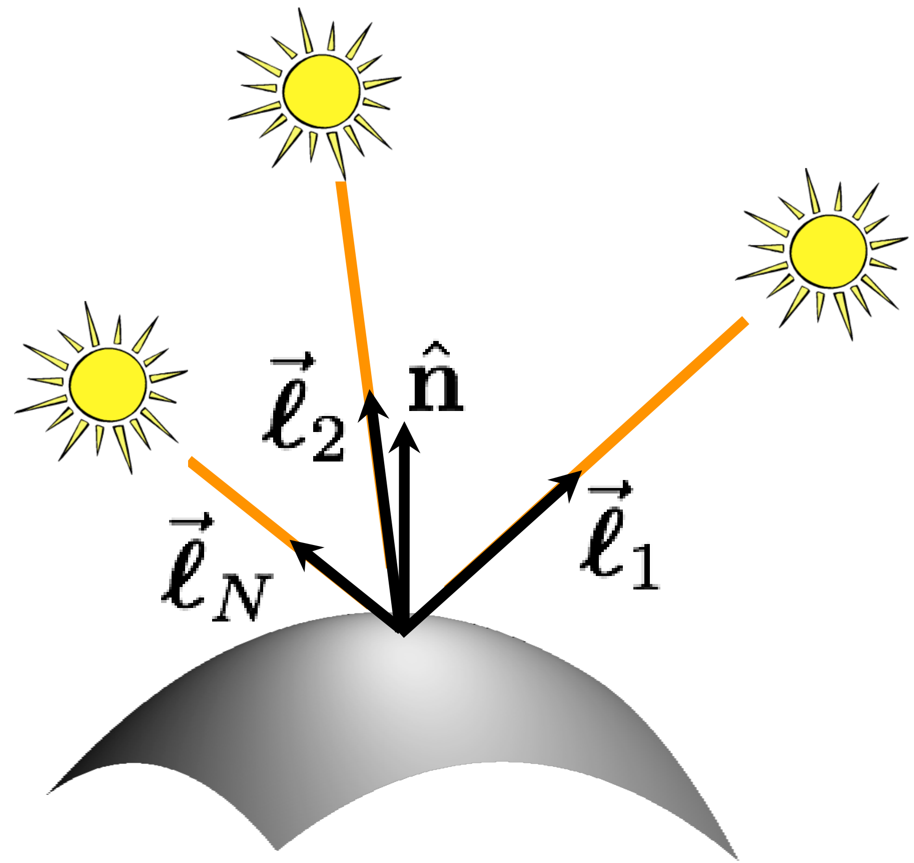
solve linear system for pseudo-normal at each image pixel

$$
\begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_N \end{bmatrix}_{N\times M} = \begin{bmatrix} \vec{\boldsymbol{\ell}}_1^\top \\ \vec{\boldsymbol{\ell}}_2^\top \\ \vdots \\ \vec{\boldsymbol{\ell}}_N^\top \end{bmatrix}_{N\times 3} \begin{bmatrix} B \end{bmatrix}_{3\times M}
$$

M: number of pixels

What if the light directions are unknown?

$$I_1 = a\hat{\boldsymbol{n}}^\top \vec{\boldsymbol{\ell}}_1$$
$$I_2 = a\hat{\boldsymbol{n}}^\top \vec{\boldsymbol{\ell}}_2$$
$$\vdots$$
$$I_N = a\hat{\boldsymbol{n}}^\top \vec{\boldsymbol{\ell}}_N$$



define "pseudo-normal" $\vec{\boldsymbol{b}} \triangleq a\hat{\boldsymbol{n}}$

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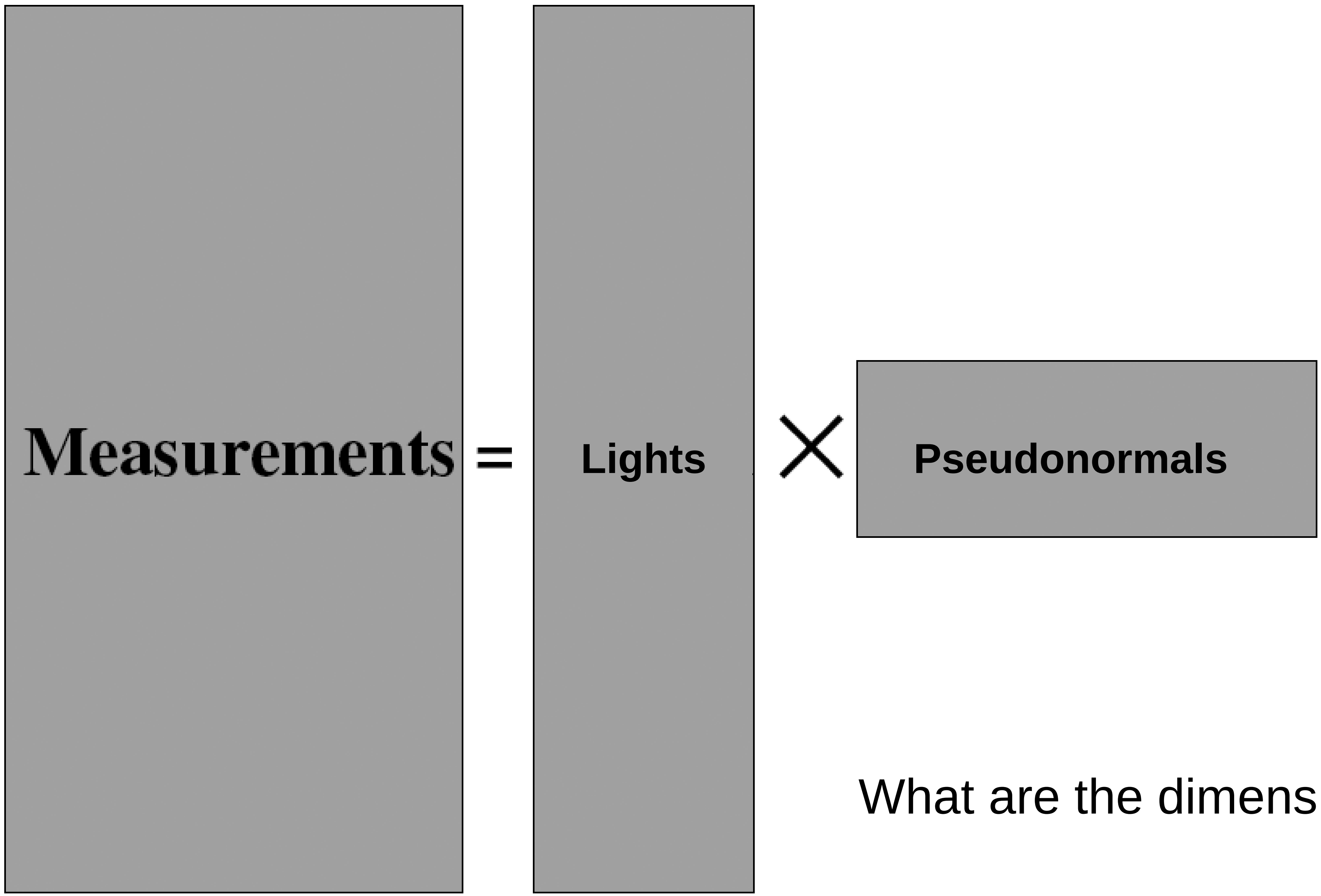
$$\begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_N \end{bmatrix}_{N\times M} = \begin{bmatrix} \vec{\boldsymbol{\ell}}_1^\top \\ \vec{\boldsymbol{\ell}}_2^\top \\ \vdots \\ \vec{\boldsymbol{\ell}}_N^\top \end{bmatrix}_{N\times 3} \begin{bmatrix} B \end{bmatrix}_{3\times M}$$

How do we solve this system without knowing light matrix L?

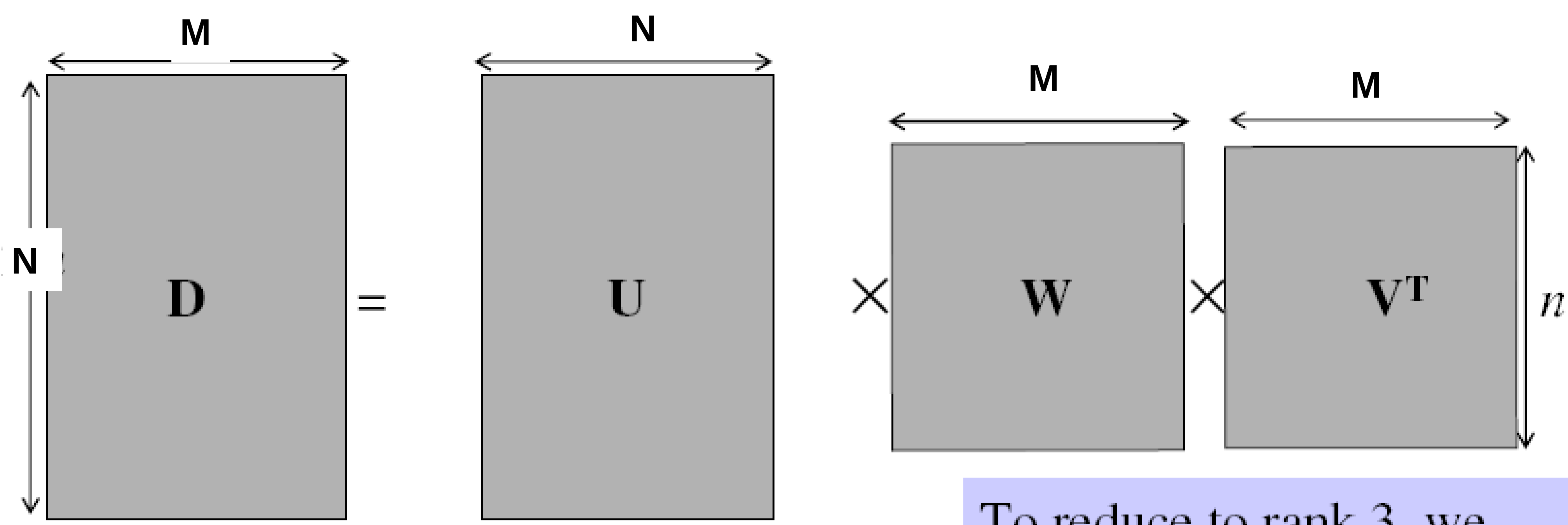
Factorizing the measurement matrix



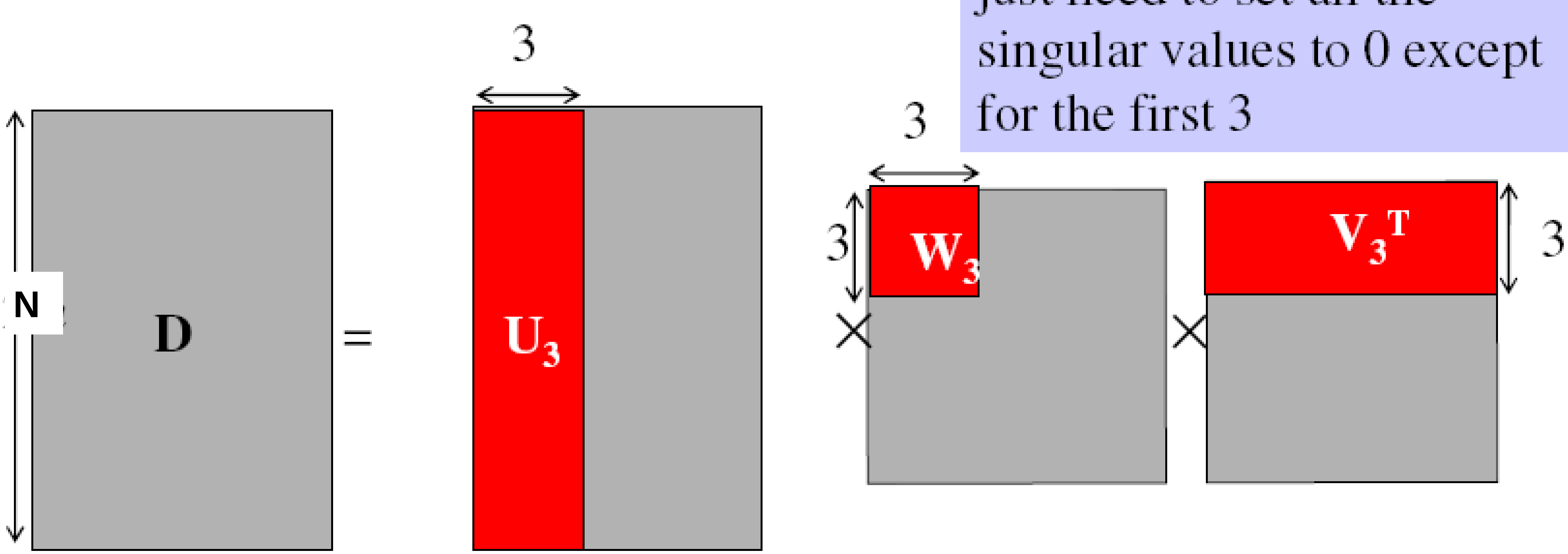
What are the dimensions?

Factorizing the measurement matrix

- Singular value decomposition:



To reduce to rank 3, we just need to set all the singular values to 0 except for the first 3



This decomposition minimizes $|\mathbf{I}\text{-}\mathbf{LB}|^2$

Are the results unique?

Are the results unique?

We can insert any 3x3 matrix Q in the decomposition and get the same images:

$$\mathbf{I} = \mathbf{L}\,\mathbf{B} = (\mathbf{L}\,\mathbf{Q}^{-1})\,(\mathbf{Q}\,\mathbf{B})$$

Are the results unique?

We can insert any 3x3 matrix Q in the decomposition and get the same images:

$$\mathbf{I} = \mathbf{L}\,\mathbf{B} = (\mathbf{L}\,\mathbf{Q}^{-1})\,(\mathbf{Q}\,\mathbf{B})$$

Can we use any assumptions to remove some of these 9 degrees of freedom?

Generalized bas-relief ambiguity

Enforcing integrability

What does the matrix **B** correspond to?

Enforcing integrability

What does the matrix **B** correspond to?

- Surface representation as a depth image (also known as Monge surface):

$$\underset{\substack{\uparrow \\ \text{depth at each pixel}}}{z} = f(\underbrace{x, y}_{\text{pixel coordinates in image space}})$$

- Unnormalized normal:

$$\tilde{n}(x, y) = \left(\frac{df}{dx}, \frac{df}{dy}, -1\right)$$

- Actual normal:

$$n(x, y) = \tilde{n}(x, y) / \|\tilde{n}(x, y)\|$$

- Pseudo-normal:

$$b(x, y) = a(x, y) n(x, y)$$

- Rearrange into 3xN matrix **B**.

Enforcing integrability

What does the integrability constraint correspond to?

Enforcing integrability

What does the integrability constraint correspond to?

- Differentiation order should not matter:

$$\frac{d}{dy}\frac{df(x,y)}{dx} = \frac{d}{dx}\frac{df(x,y)}{dy}$$

- Can you think of a way to express the above using pseudo-normals **b**?

Enforcing integrability

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- Can you think of a way to express the above using pseudo-normals **b**?

$$\frac{d}{dy}\frac{b_1(x,y)}{b_3(x,y)} = \frac{d}{dx}\frac{b_2(x,y)}{b_3(x,y)}$$

Enforcing integrability

What does the integrability constraint correspond to?

- Differentiation order should not matter:

$$\frac{d}{dy}\frac{df(x,y)}{dx} = \frac{d}{dx}\frac{df(x,y)}{dy}$$

- Can you think of a way to express the above using pseudo-normals **b**?

$$\frac{d}{dy}\frac{b_1(x,y)}{b_3(x,y)} = \frac{d}{dx}\frac{b_2(x,y)}{b_3(x,y)}$$

- Simplify to:

$$b_3(x,y)\frac{db_1(x,y)}{dy} - b_1(x,y)\frac{db_3(x,y)}{dy} = b_2(x,y)\frac{db_1(x,y)}{dx} - b_1(x,y)\frac{db_2(x,y)}{dx}$$

Enforcing integrability

What does the integrability constraint correspond to?

- Differentiation order should not matter:

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$$\frac{d}{dy}\frac{b_1(x,y)}{b_3(x,y)} = \frac{d}{dx}\frac{b_2(x,y)}{b_3(x,y)}$$

- Simplify to:

$$b_3(x,y)\frac{db_1(x,y)}{dy} - b_1(x,y)\frac{db_3(x,y)}{dy} = b_2(x,y)\frac{db_1(x,y)}{dx} - b_1(x,y)\frac{db_2(x,y)}{dx}$$

- If $\mathbf{B_e}$ is the pseudo-normal matrix we get from SVD, then find the 3x3 transform $\mathbf{D}$ such that $\mathbf{B}=\mathbf{D}\cdot\mathbf{B_e}$ is the closest to satisfying integrability in the least-squares sense.

Enforcing integrability

Does enforcing integrability remove all ambiguities?

Generalized Bas-relief ambiguity

If **B** is integrable, then:

- **B'=G^{-T}·B** is also integrable for all **G** of the form ($\lambda \neq 0$)

$$G = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \mu & \nu & \lambda \end{bmatrix}$$

- Combined with transformed lights **S'=G·S**, the transformed pseudonormals produce the same images as the original pseudonormals.

- This ambiguity cannot be removed using shadows.

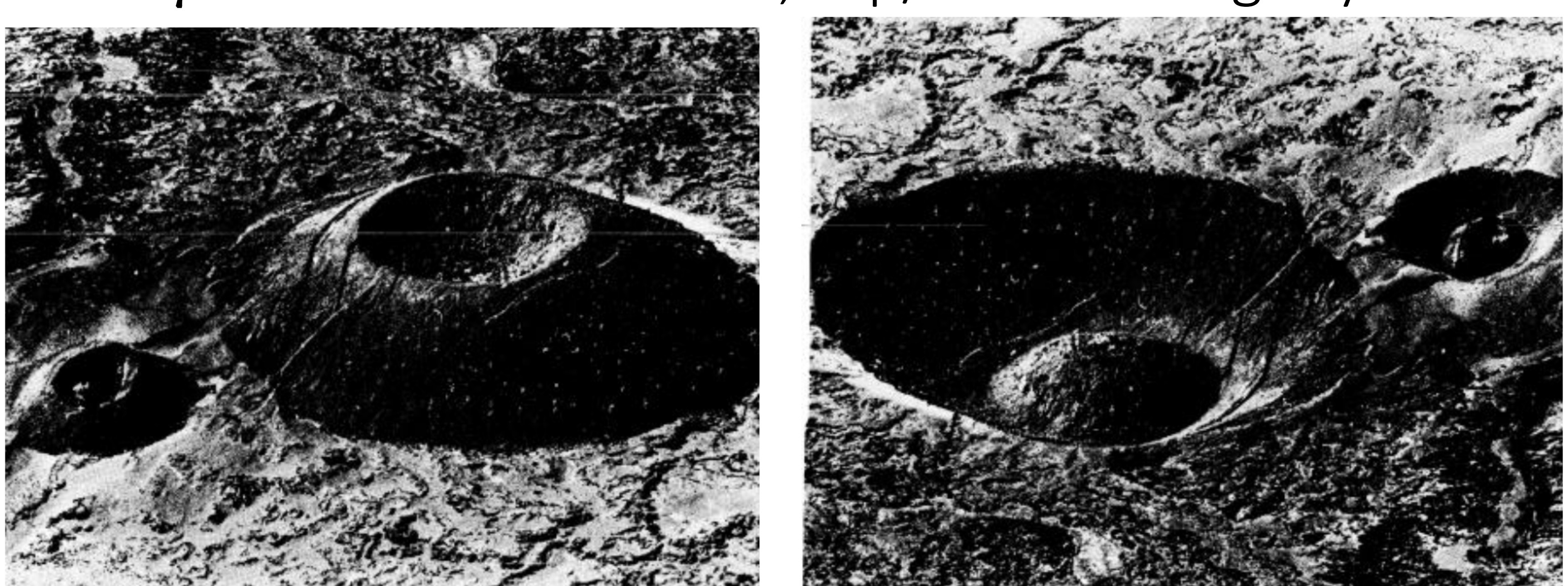
- This ambiguity *can* be removed using interreflections or additional assumptions.

This ambiguity is known as the generalized bas-relief ambiguity.

Generalized Bas-relief ambiguity

When μ = ν = 0, **G** is equivalent to the transformation employed by relief sculptures.

When μ = ν = 0 and $\lambda = +\text{-}1$, top/down ambiguity.



Otherwise, includes shearing.

What assumptions have we made for all this?

What assumptions have we made for all this?

- Lambertian BRDF
- Directional lighting
- No interreflections or scattering

References

Basic reading:

- Szeliski, Section 2.2.
- Gortler, Chapter 21.
  This book by Steven Gortler has a great *introduction* to radiometry, reflectance, and their use for image formation.

Additional reading:

- Arvo, "Analytic Methods for Simulated Light Transport," Yale 1995.
- Veach, "Robust Monte Carlo Methods for Light Transport Simulation," Stanford 1997.
  These two thesis are foundational for modern computer graphics. Among other things, they include a thorough derivation (starting from wave optics and measure theory) of all radiometric quantities and associated integro-differential equations. You can also look at them if you are interested in physics-based rendering.
- Dutre et al., "Advanced Global Illumination," 2006.
  A book discussing modeling and simulation of other appearance effects beyond single-bounce reflectance.
- Weyrich et al., "Principles of Appearance Acquisition and Representation," FTCGV 2009.
  A very thorough review of everything that has to do with modeling and measuring BRDFs.
- Walter et al., "Microfacet models for refraction through rough surfaces," EGSR 2007.
  This paper has a great review of physics-based models for reflectance and refraction.
- Matusik, "A data-driven reflectance model," MIT 2003.
  This thesis introduced the largest measured dataset of 4D reflectances. It also provides detailed discussion of many topics relating to modelling reflectance.
- Rusinkiewicz, "A New Change of Variables for Efficient BRDF Representation," 1998.
- Romeiro and Zickler, "Inferring reflectance under real-world illumination," Harvard TR 2010.
  These two papers discuss the isotropy and other properties of common BRDFs, and how one can take advantage of them using alternative parameterizations.
- Shafer, "Using color to separate reflection components," 1984.
  The paper introducing the dichromatic reflectance model.
- Stam, "Diffraction Shaders," SIGGRAPH 1999.
- Levin et al., "Fabricating BRDFs at high spatial resolution using wave optics," SIGGRAPH 2013.
- Cuypers et al., "Reflectance model for diffraction," TOG 2013.
  These three papers describe reflectance effects that can only be modeled using wave optics (and in particular diffraction).

References

Basic reading:

- Szeliski, Section 2.2.
- Gortler, Chapter 21.

  This book by Steven Gortler has a great *introduction* to radiometry, reflectance, and their use for image formation.

Additional reading:

- Oren and Nayar, “Generalization of the Lambertian model and implications for machine vision,” IJCV 1995.

  The paper introducing the most common model for rough diffuse reflectance.
- Debevec, “Rendering Synthetic Objects into Real Scenes,” SIGGRAPH 1998.

  The paper that introduced the notion of the environment map, the use of chrome spheres for measuring such maps, and the idea that they can be used for easy rendering.
- Lalonde et al., “Estimating the Natural Illumination Conditions from a Single Outdoor Image,” IJCV 2012.

  A paper on estimating outdoors environment maps from just one image.
- Basri and Jacobs, “Lambertian reflectance and linear subspaces,” ICCV 2001.
- Ramamoorthi and Hanrahan, “A signal-processing framework for inverse rendering,” SIGGRAPH 2001.
- Sloan et al., “Precomputed radiance transfer for real-time rendering in dynamic, low-frequency lighting environments,” SIGGRAPH 2002.

  Three papers describing the use of spherical harmonics to model low-frequency illumination, as well as the low-pass filtering effect of Lambertian reflectance on illumination.
- Zhang et al., “Shape-from-shading: a survey,” PAMI 1999.

  A review of perceptual and computational aspects of shape from shading.
- Woodham, “Photometric method for determining surface orientation from multiple images,” Optical Engineering 1980.

  The paper that introduced photometric stereo.
- Yuille and Snow, “Shape and albedo from multiple images using integrability,” CVPR 1997.
- Belhumeur et al., “The bas-relief ambiguity,” IJCV 1999.
- Papadhimitri and Favaro, “A new perspective on uncalibrated photometric stereo,” CVPR 2013.

  Three papers discussing uncalibrated photometric stereo. The first paper shows that, when the lighting directions are not known, by assuming integrability, one can reduce unknowns to the bas-relief ambiguity. The second paper discusses the bas-relief ambiguity in a more general context. The third paper shows that, if instead of an orthographic camera one uses a perspective camera, this is further reduced to just a scale ambiguity.
- Alldrin et al., “Resolving the generalized bas-relief ambiguity by entropy minimization,” CVPR 2007.

  A popular technique for resolving the bas-relief ambiguity in uncalibrated photometric stereo.
- Zickler et al., “Helmholtz stereopsis: Exploiting reciprocity for surface reconstruction,” IJCV 2002.

  A method for photometric stereo reconstruction under arbitrary BRDF.
- Nayar et al., “Shape from interreflections,” IJCV 1991.
- Chandraker et al., “Reflections on the generalized bas-relief ambiguity,” CVPR 2005.

  Two papers discussing how one can perform photometric stereo (calibrated or otherwise) in the presence of strong interreflections.
- Frankot and Chellappa, “A method for enforcing integrability in shape from shading algorithms,” PAMI 1988.
- Agrawal et al., “What is the range of surface reconstructions from a gradient field?,” ECCV 2006.

  Two papers discussing how one can integrate a normal field to reconstruct a surface.