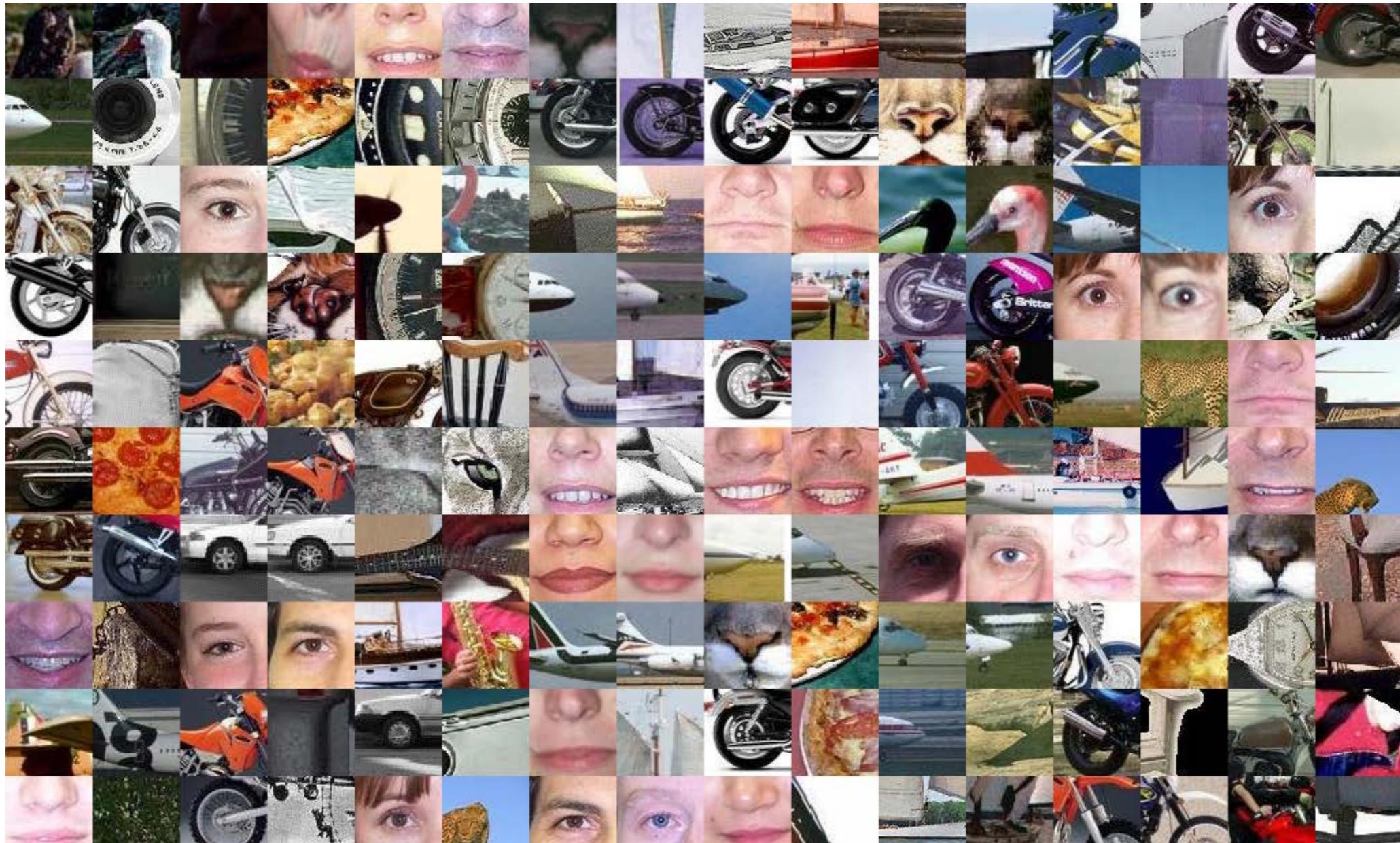
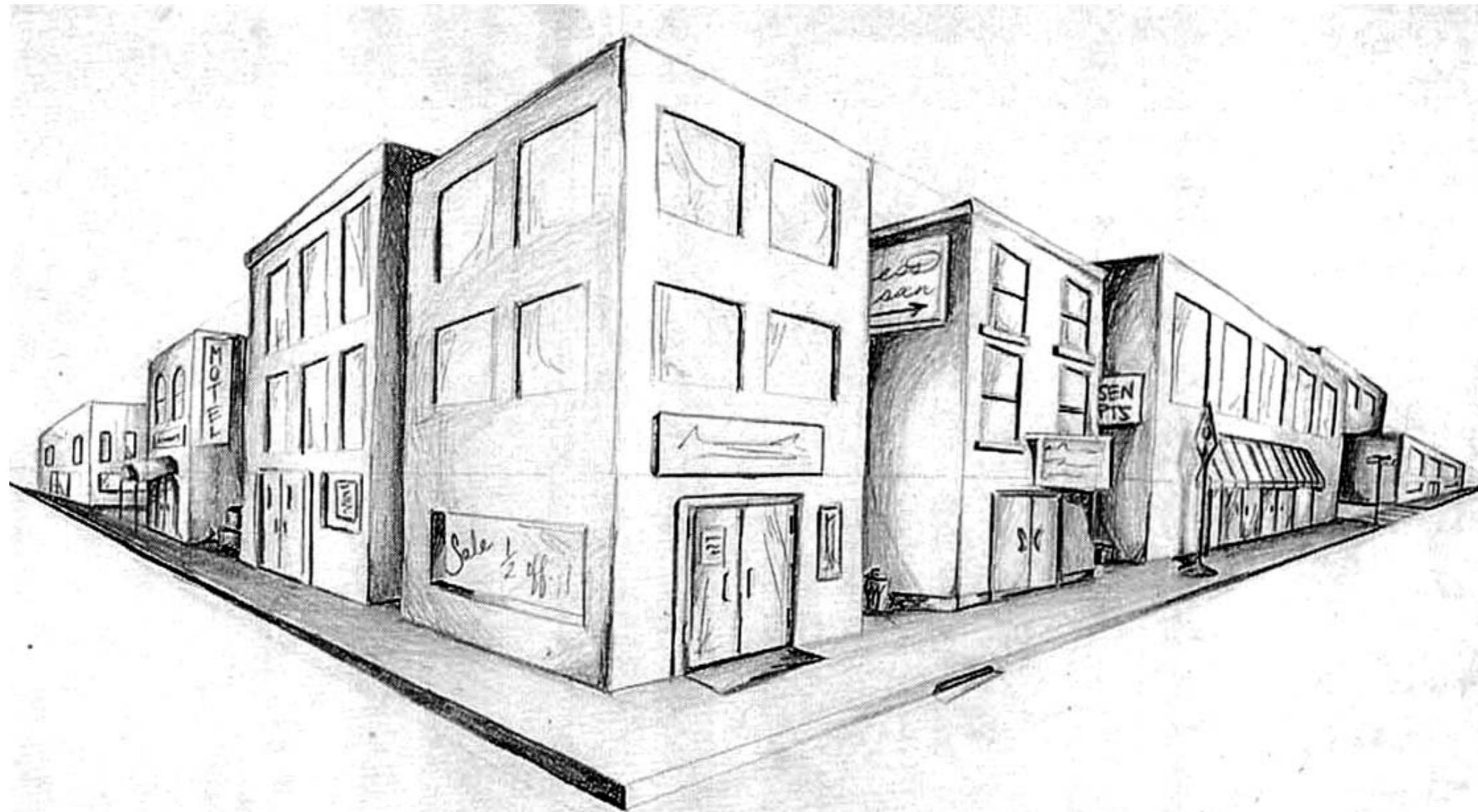


# Feature detectors and descriptors



Slide Credits: Ioannis Gkioulekas, Kris Kitani, Fredo Durand, James Hays

# Corner detector



# Why detect corners?

# Why detect corners?

Image alignment (homography, fundamental matrix)

3D reconstruction

Motion tracking

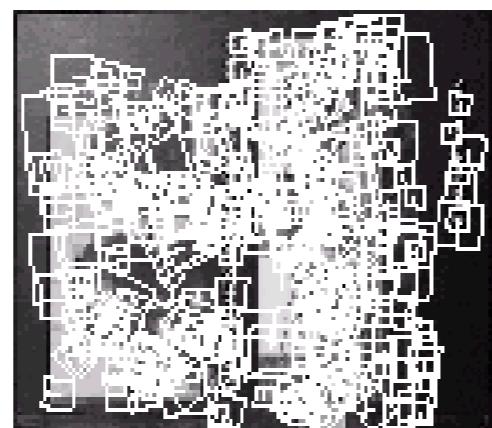
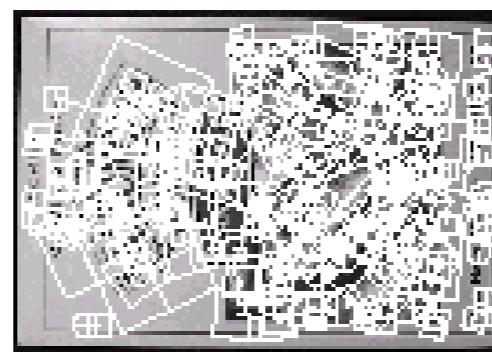
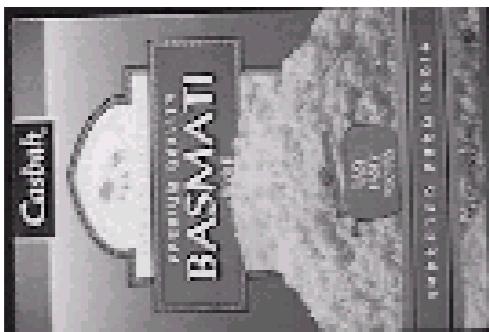
Object recognition

Indexing and database retrieval

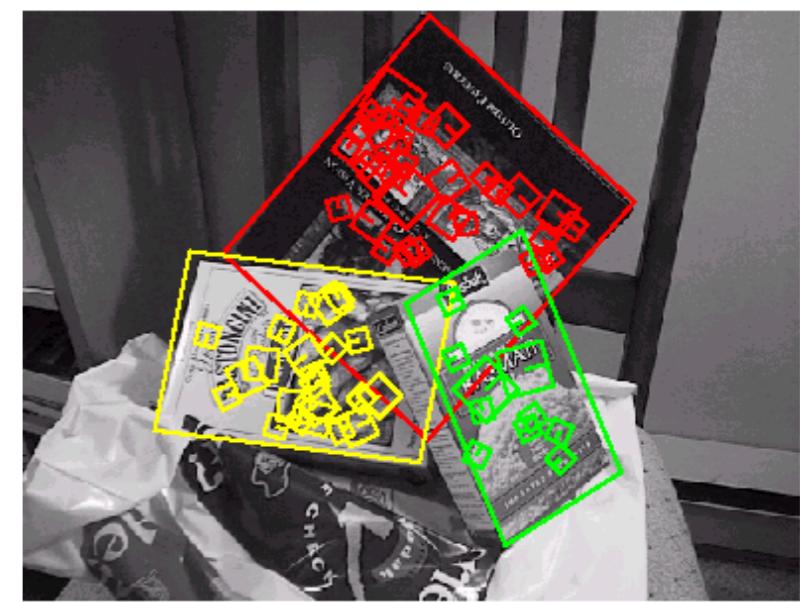
Robot navigation

# Planar object instance recognition

Database of planar objects



Instance recognition



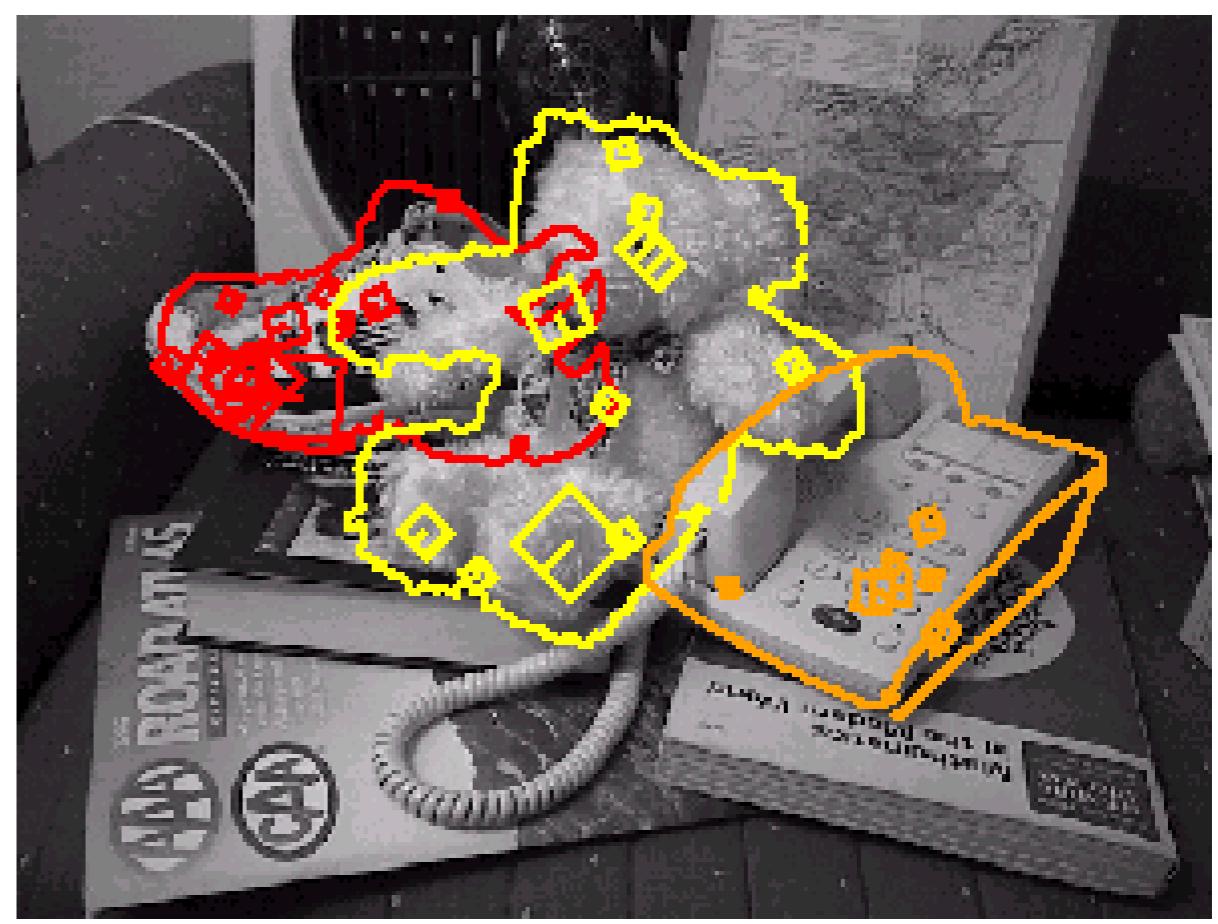
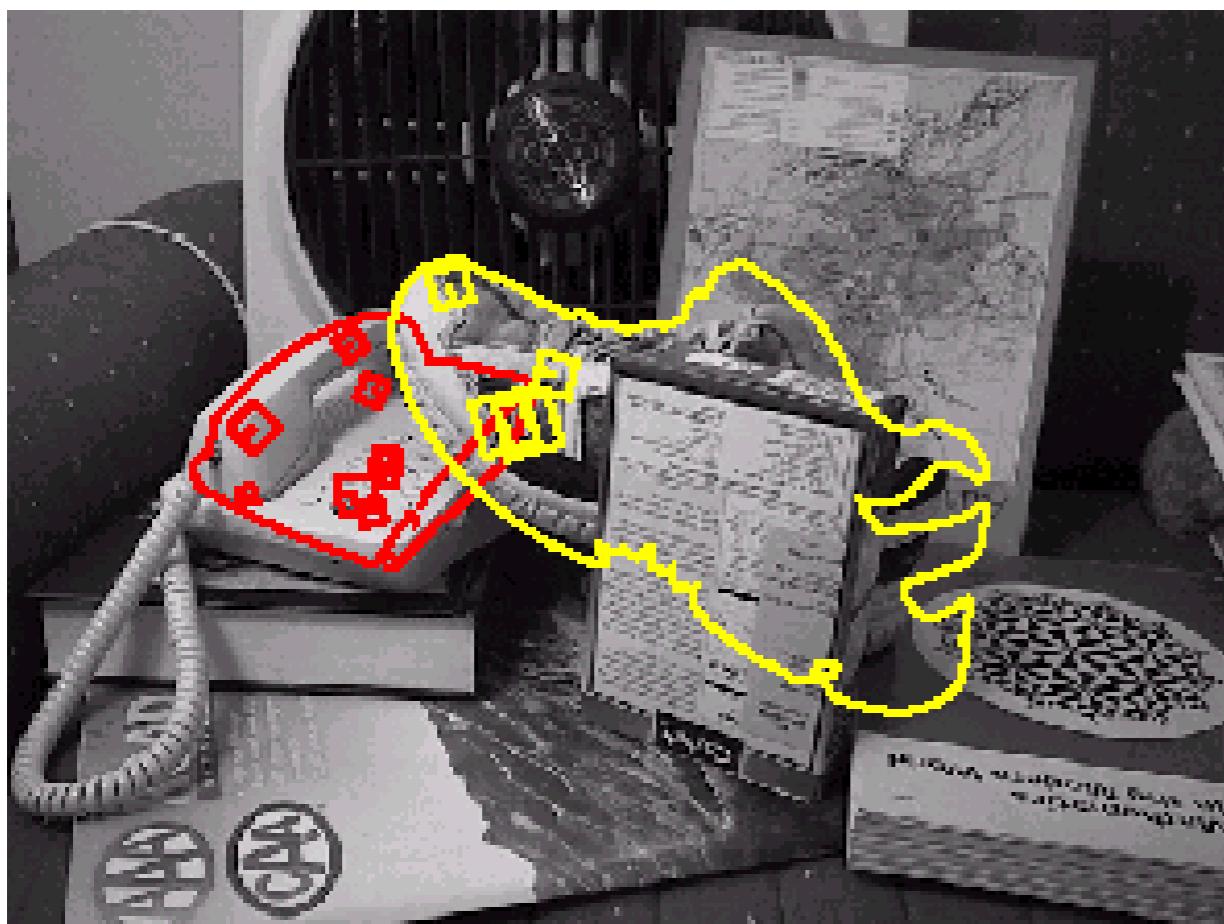
# 3D object recognition

Database of 3D objects



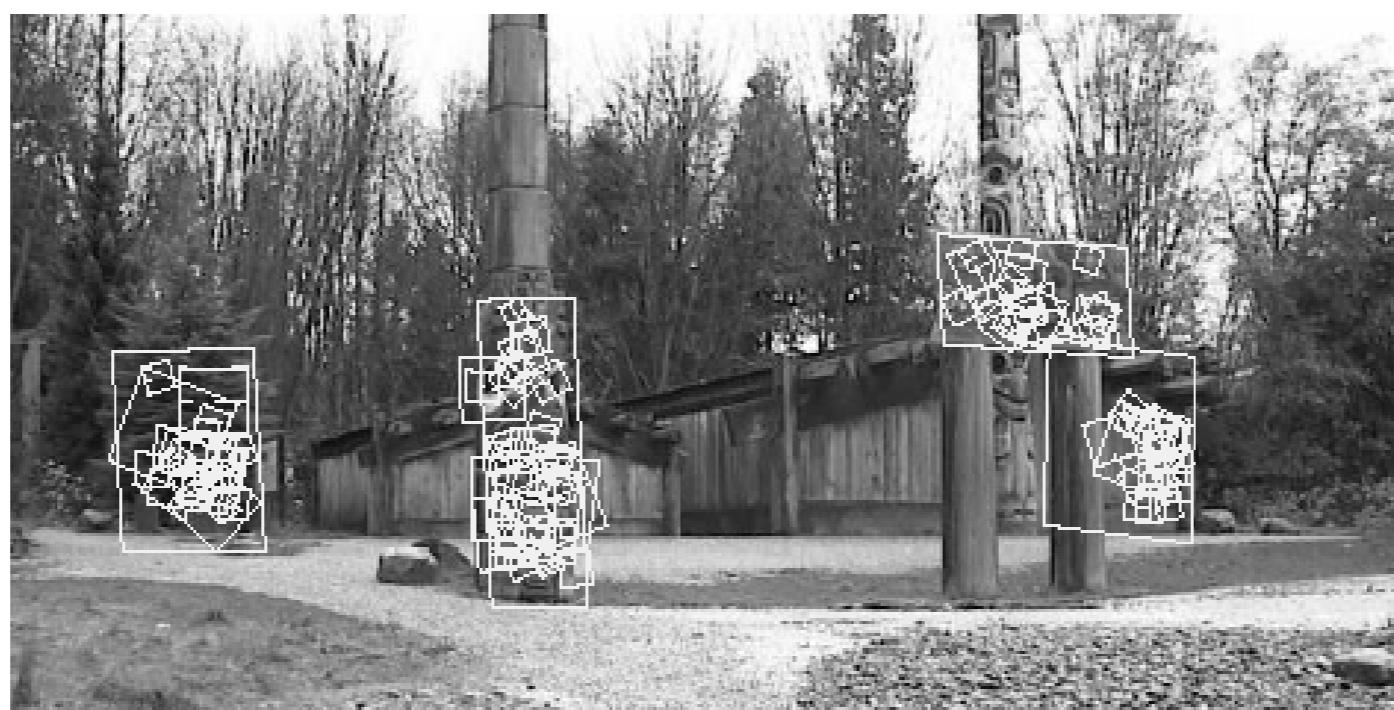
3D objects recognition



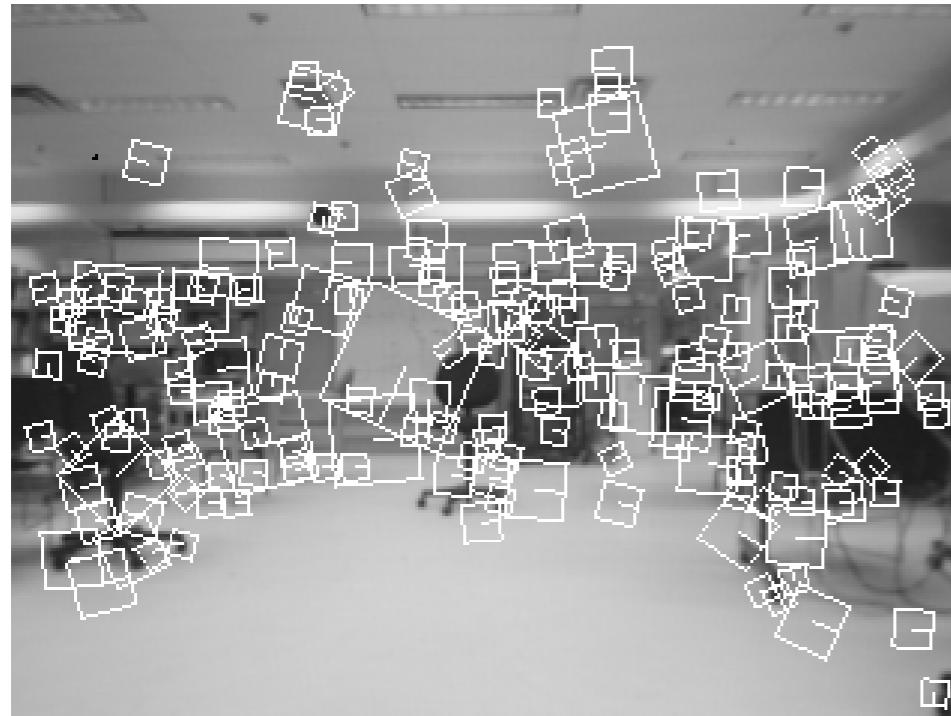


Recognition under occlusion

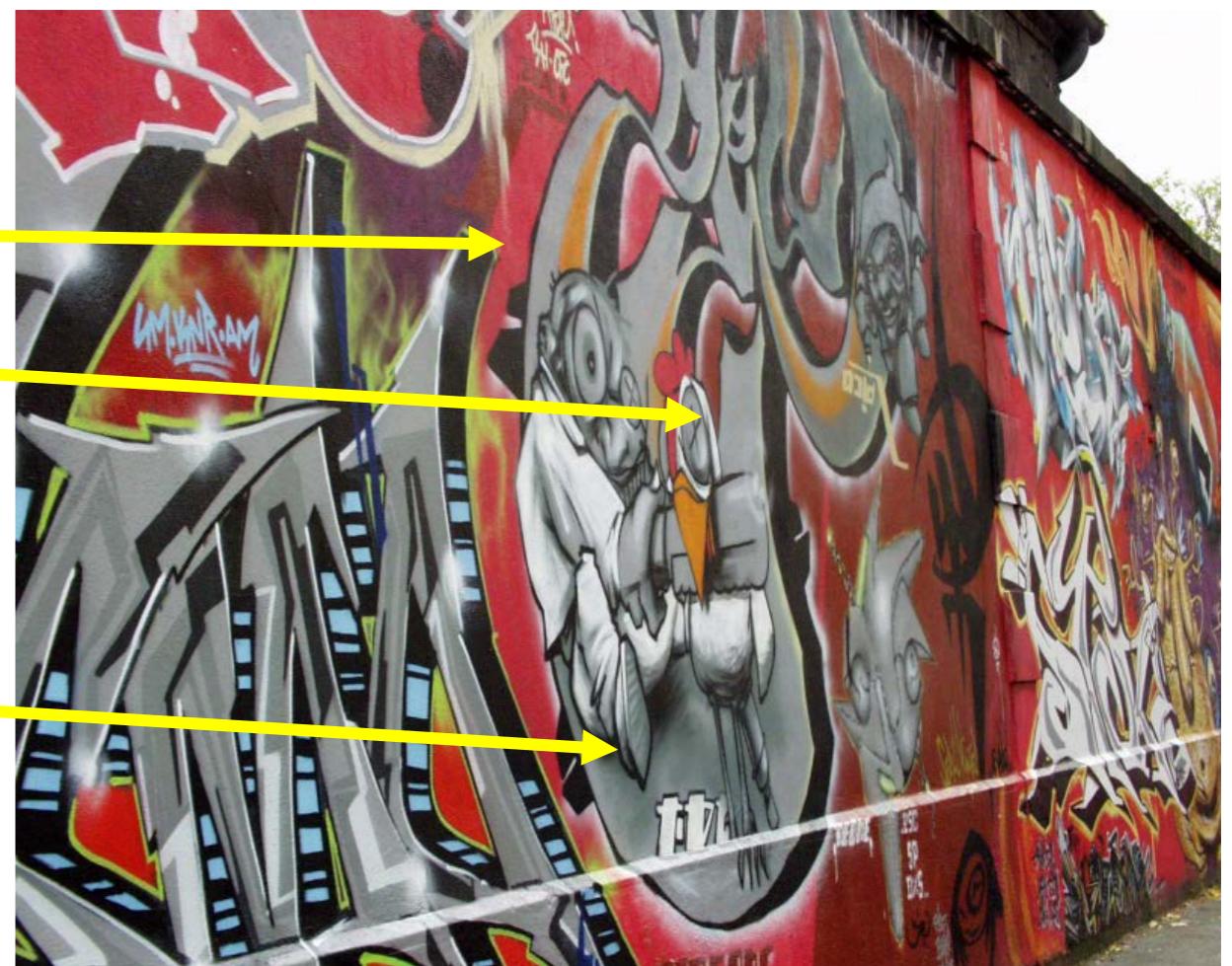
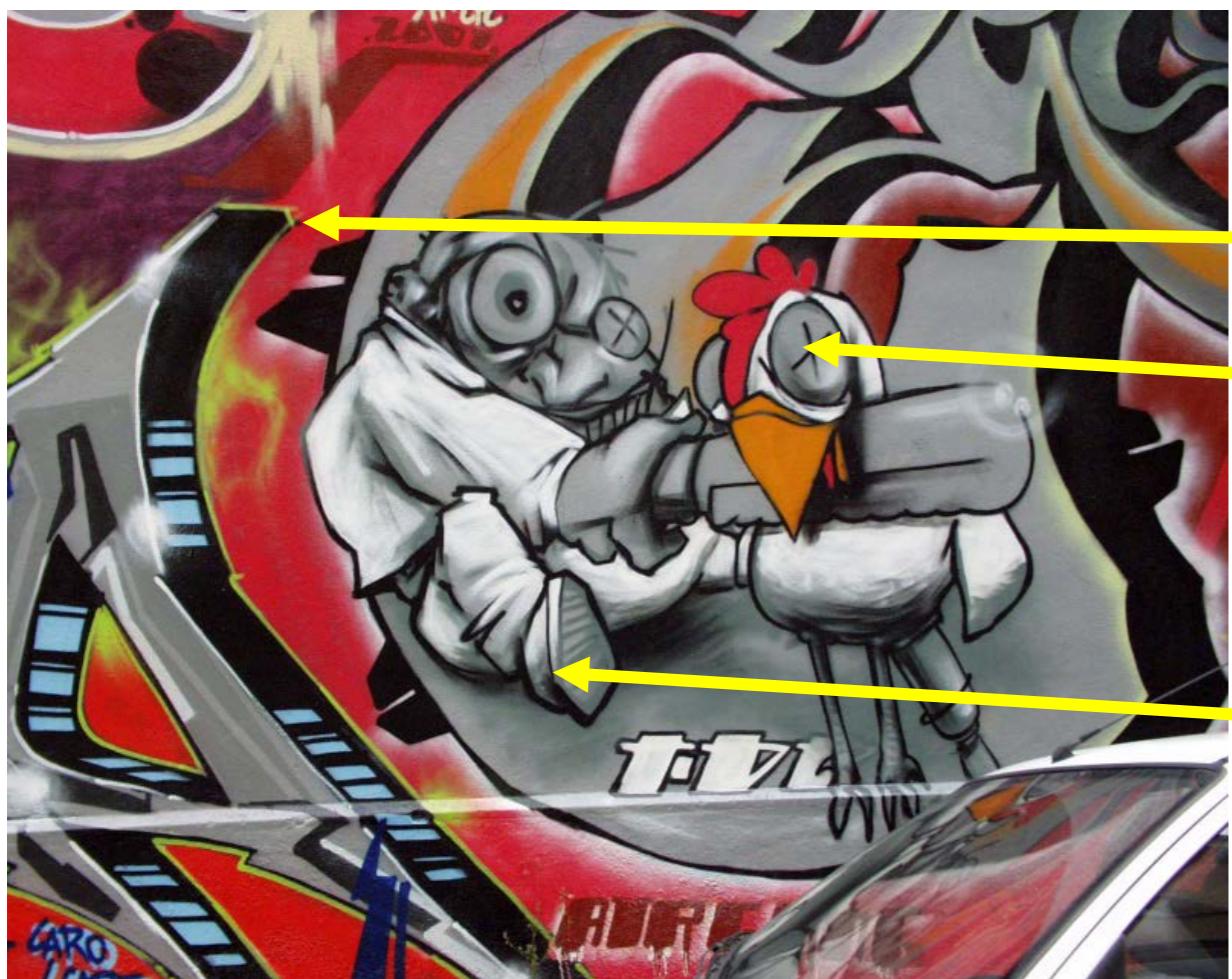
# Location Recognition

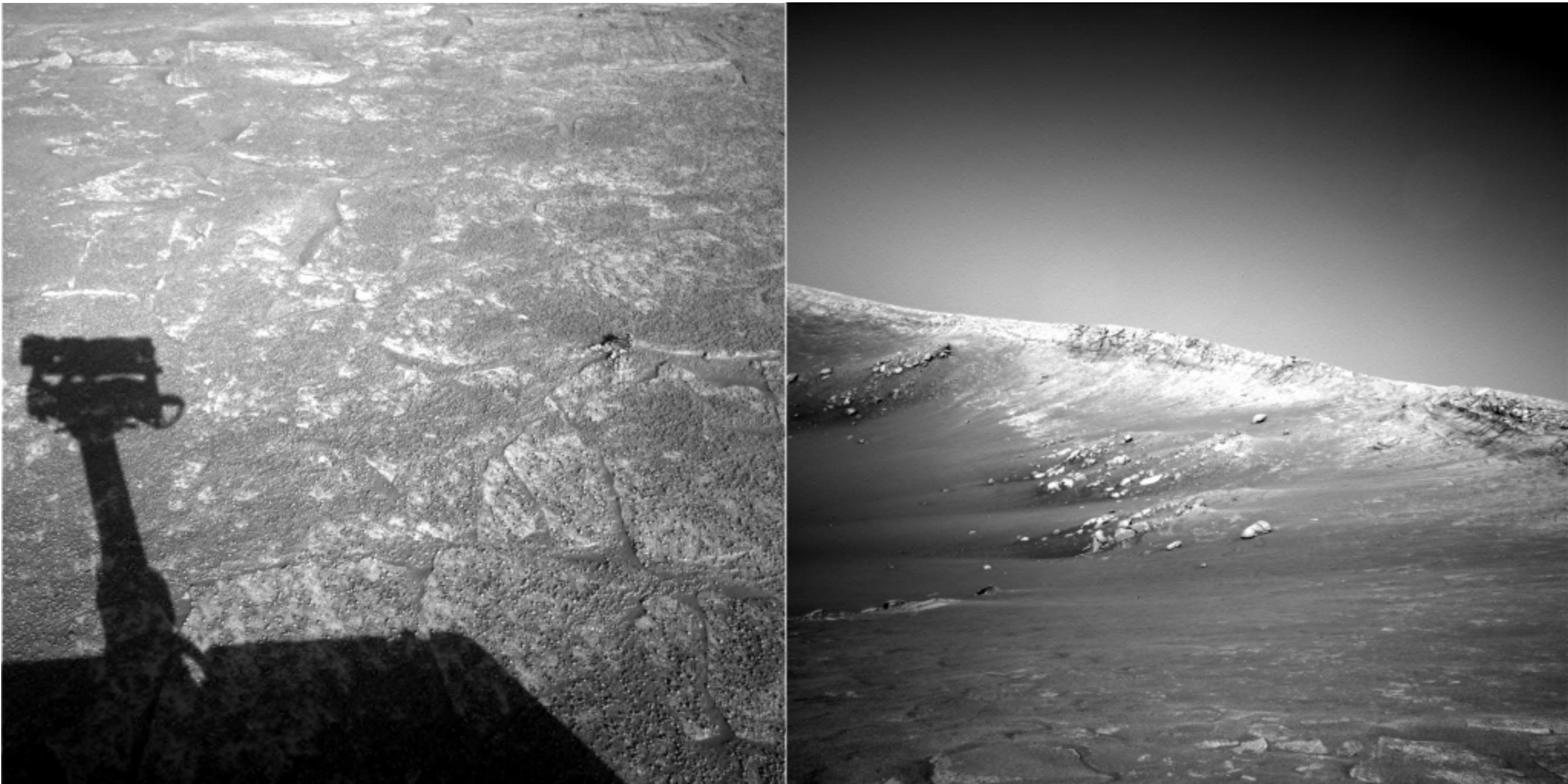


# Robot Localization



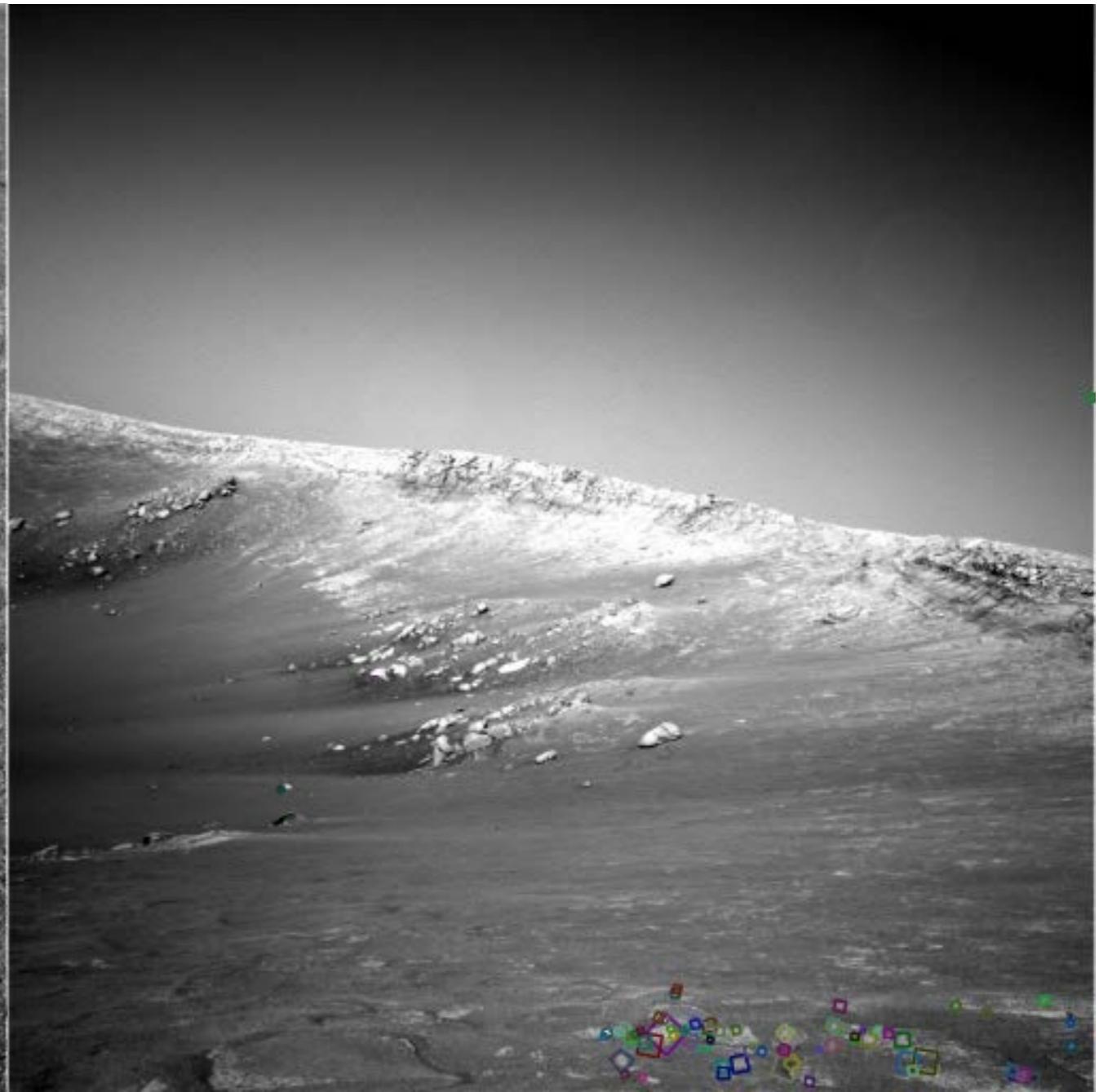
# Image matching

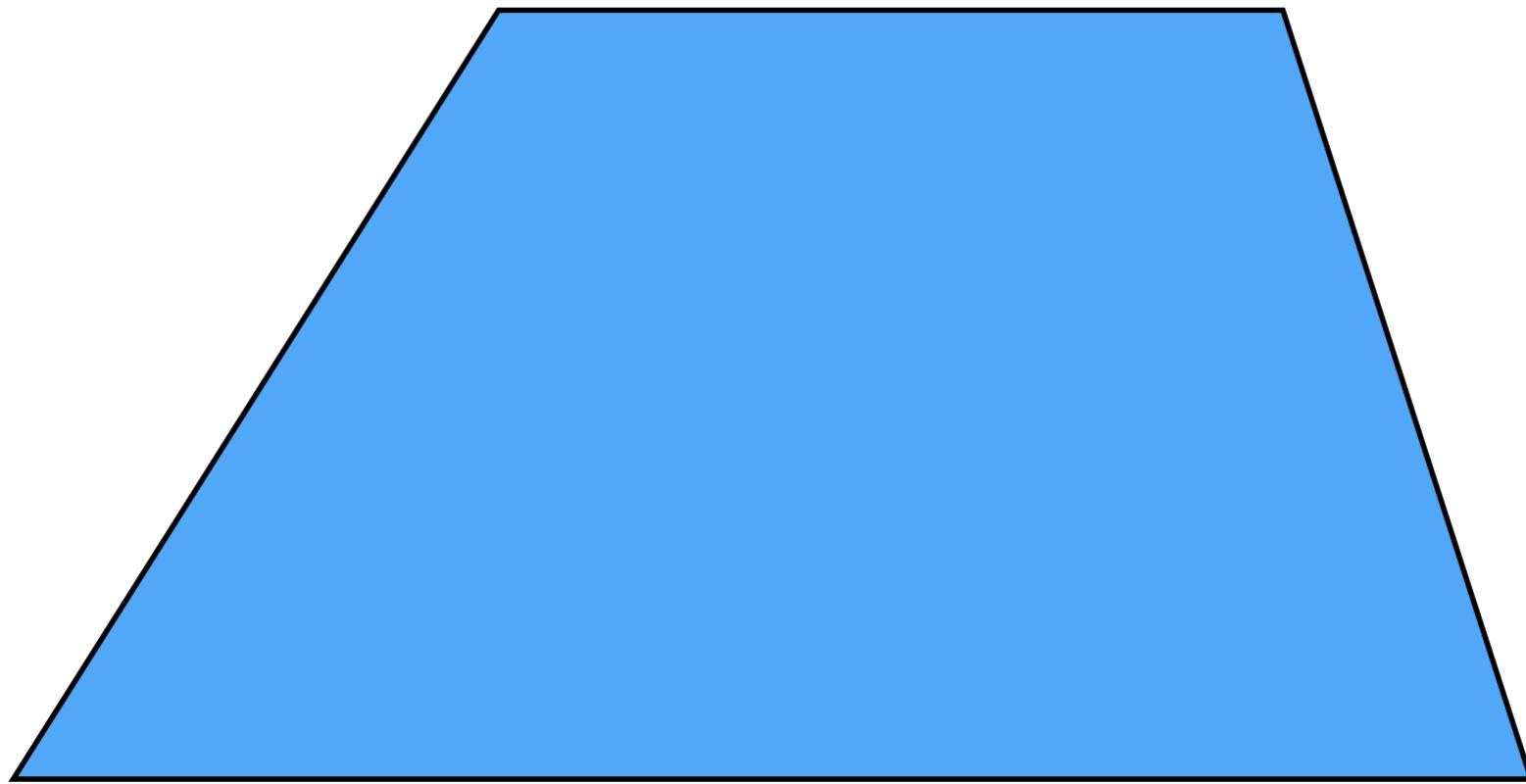




NASA Mars Rover images

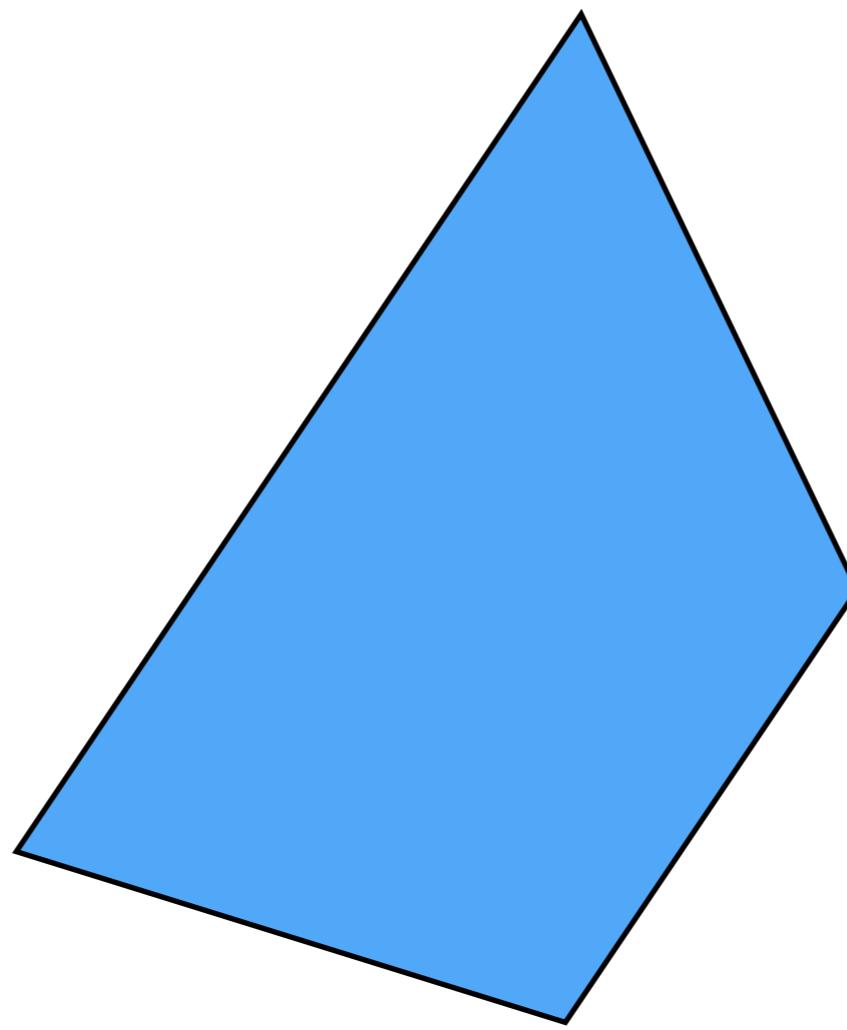
*Where are the corresponding points?*





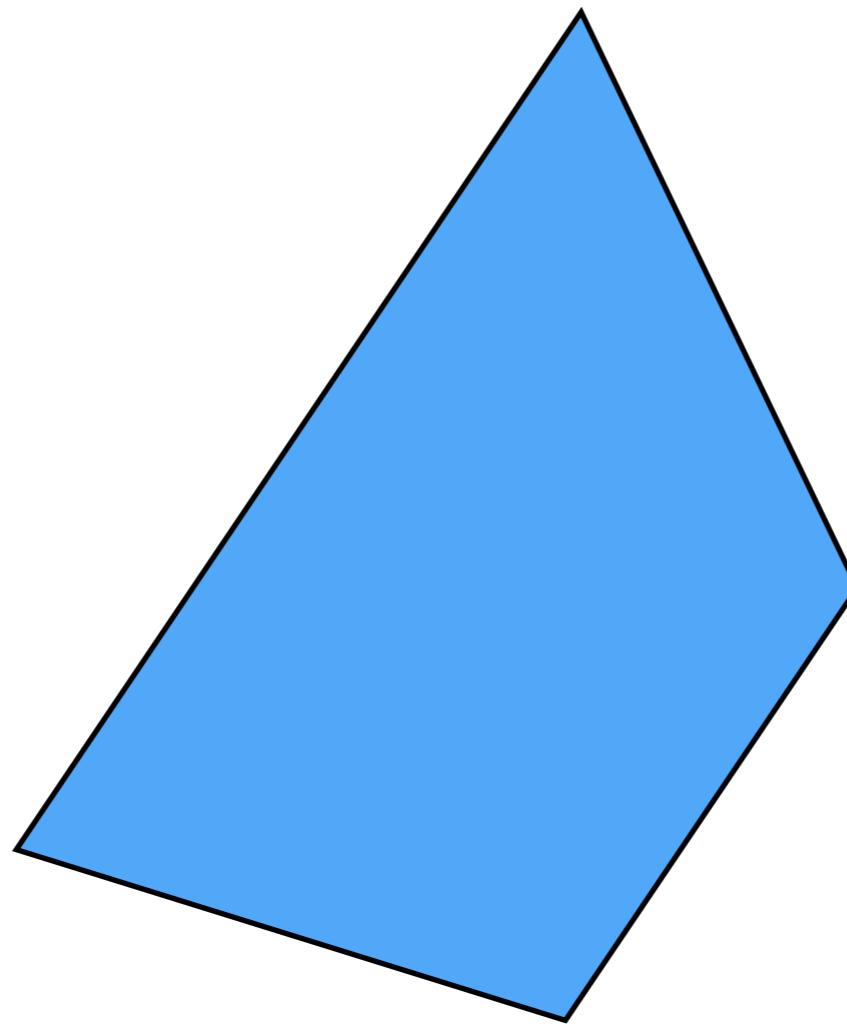
Pick a point in the image.  
Find it again in the next image.

*What type of feature would you select?*



Pick a point in the image.  
Find it again in the next image.

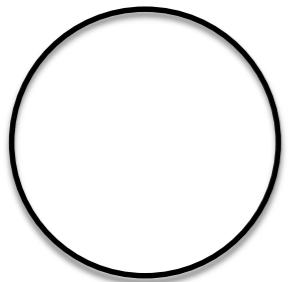
*What type of feature would you select?*



Pick a point in the image.  
Find it again in the next image.

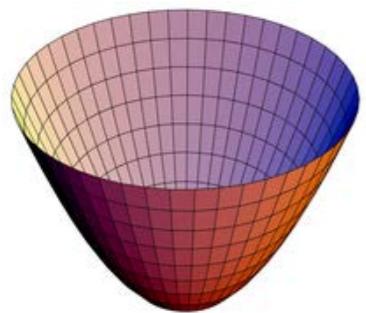
*What type of feature would you select?*  
a corner

Reminder I:  
Visualizing quadratics



Equation of a circle

$$1 = x^2 + y^2$$



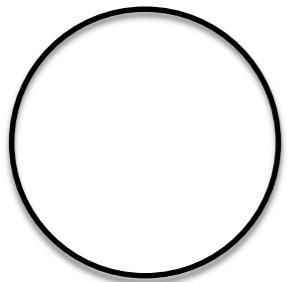
Equation of a ‘bowl’ (paraboloid)

$$f(x, y) = x^2 + y^2$$

*If you slice the bowl at*

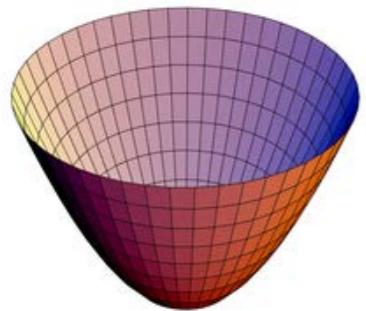
$$f(x, y) = 1$$

*what do you get?*



Equation of a circle

$$1 = x^2 + y^2$$



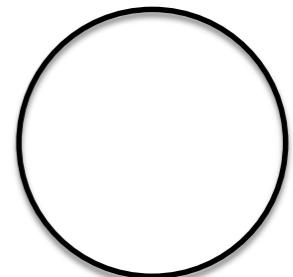
Equation of a ‘bowl’ (paraboloid)

$$f(x, y) = x^2 + y^2$$

*If you slice the bowl at*

$$f(x, y) = 1$$

*what do you get?*



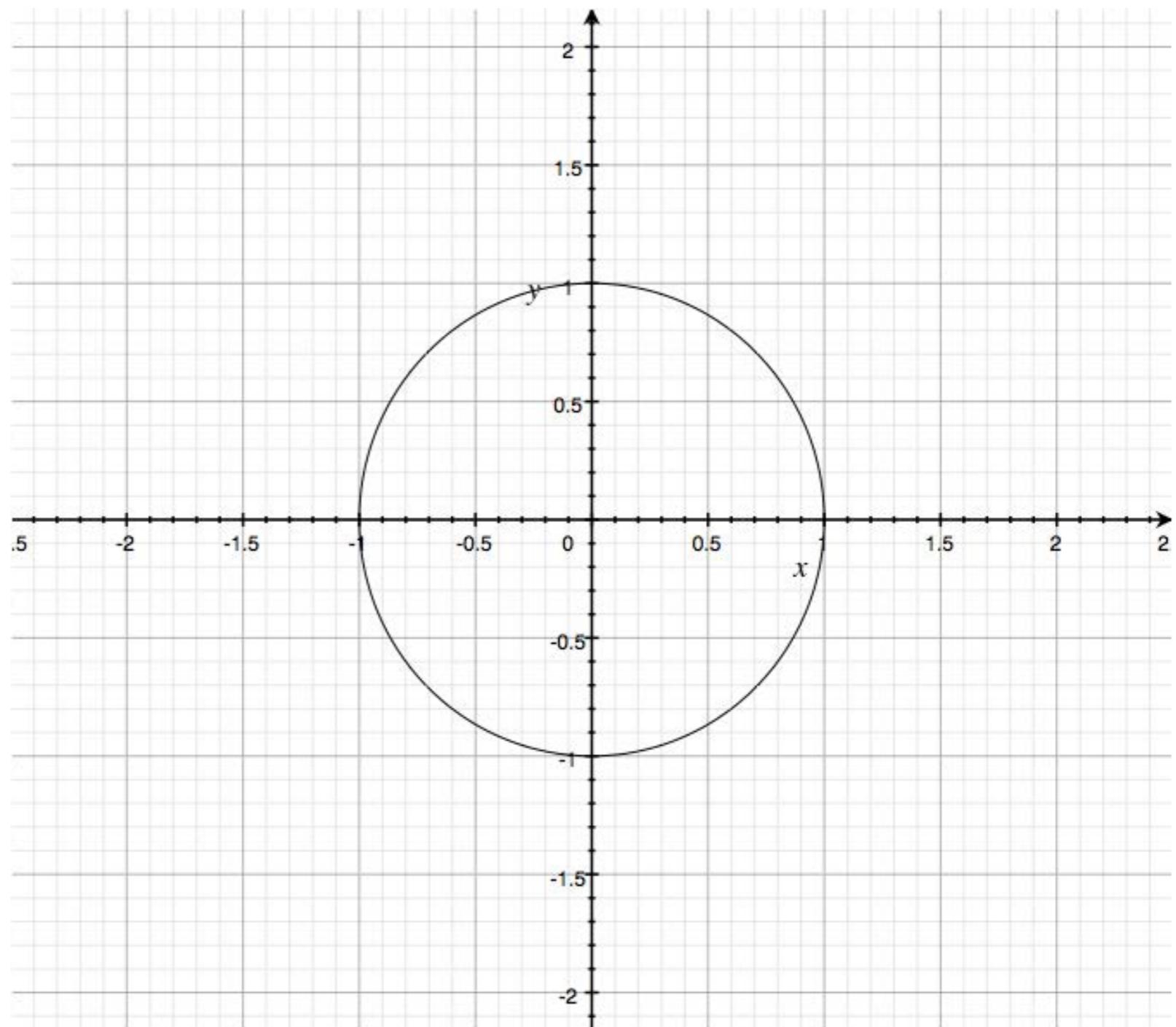
$$f(x, y) = x^2 + y^2$$

can be written in matrix form like this...

$$f(x, y) = \begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$f(x, y) = \begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

‘sliced at 1’



*What happens if you **increase** coefficient on  $x$ ?*

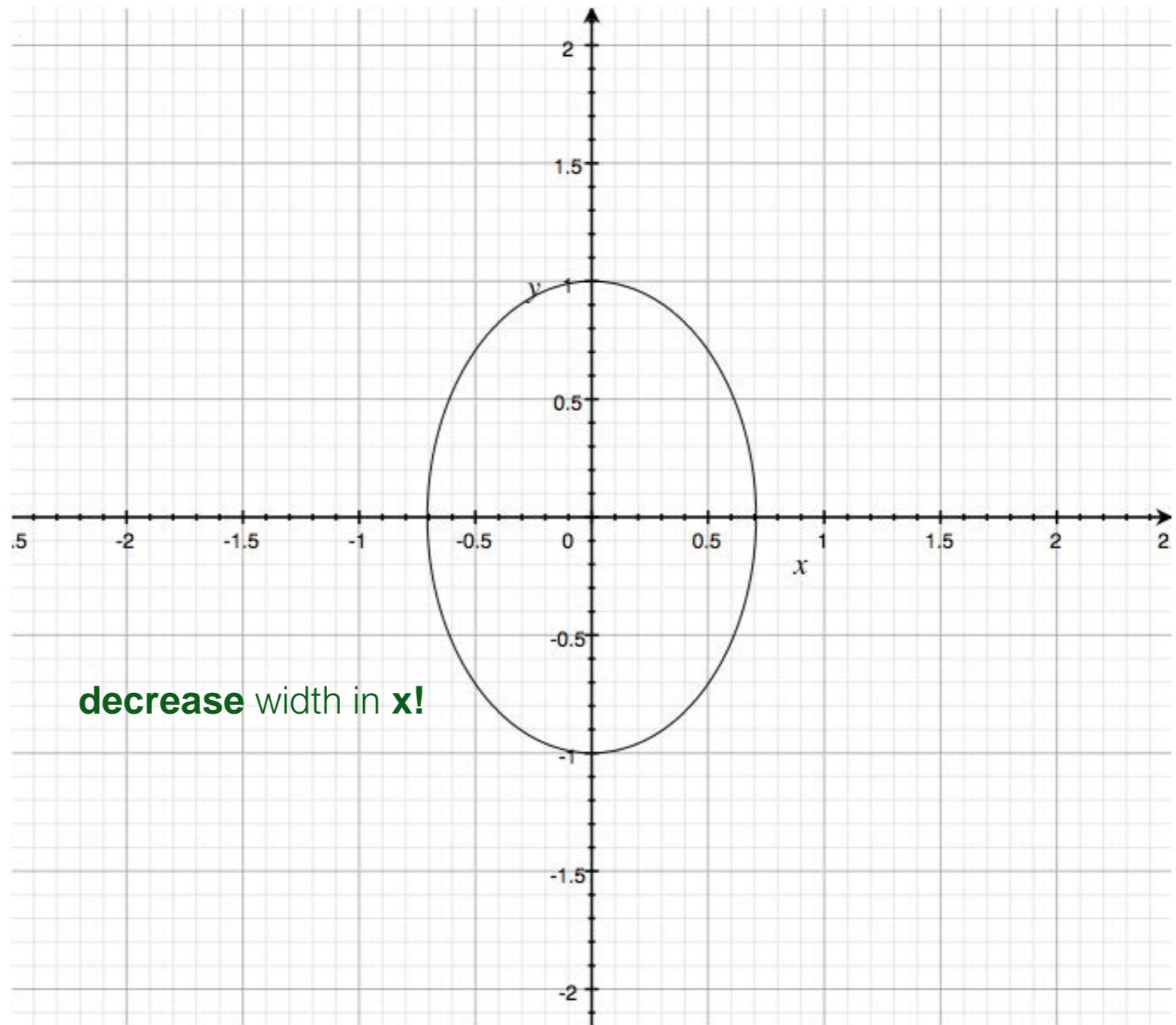
$$f(x, y) = \begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

and slice at 1

*What happens if you **increase** coefficient on **x**?*

$$f(x, y) = \begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

and slice at 1



*What happens if you **increase** coefficient on  $y$ ?*

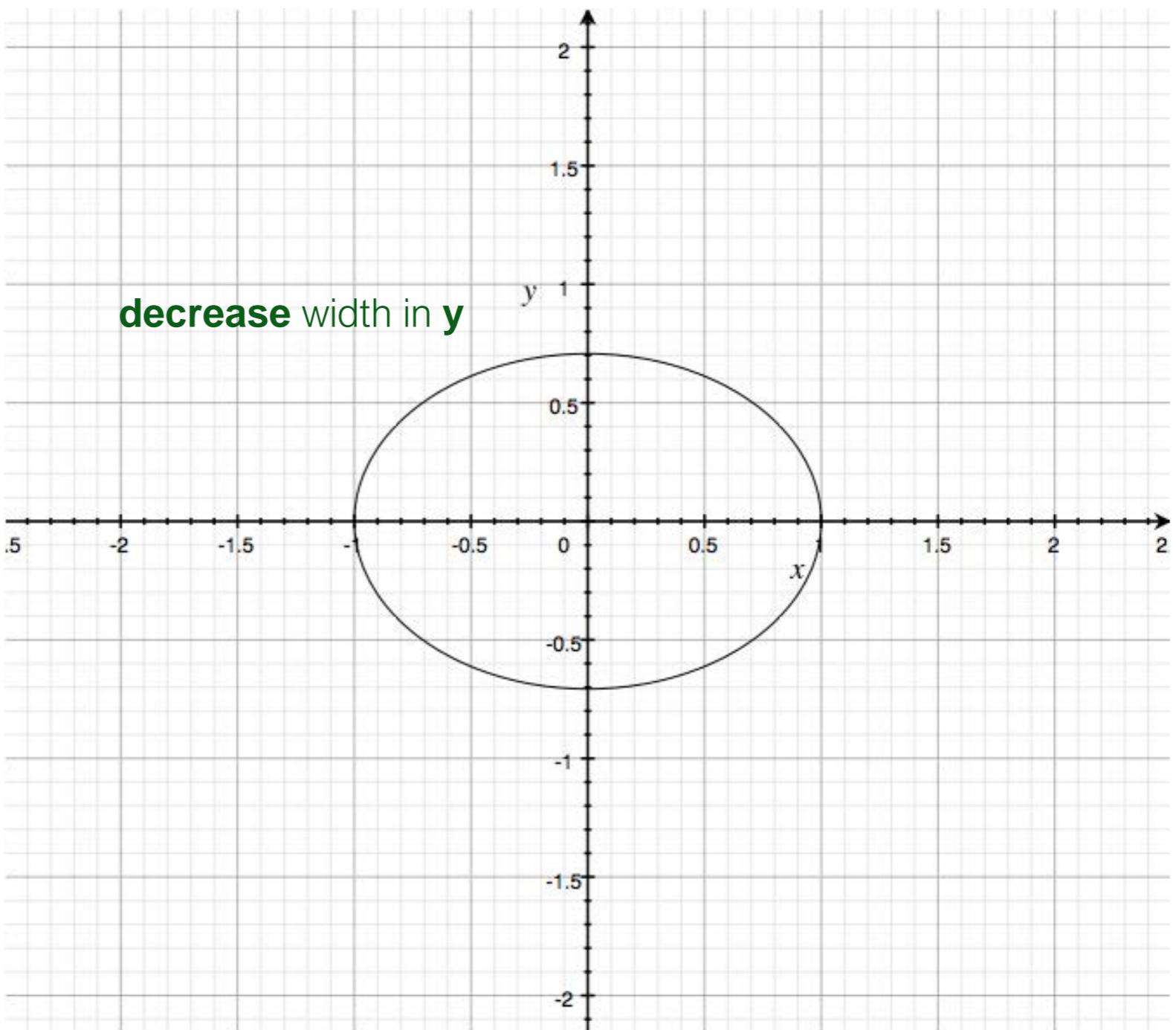
$$f(x, y) = \begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

and slice at 1

*What happens if you **increase** coefficient on y?*

$$f(x, y) = \begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

and slice at 1



$$f(x, y) = x^2 + y^2$$

can be written in matrix form like this...

$$f(x, y) = \begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

*What's the shape?*

*What are the eigenvectors?*

*What are the eigenvalues?*

$$f(x, y) = x^2 + y^2$$

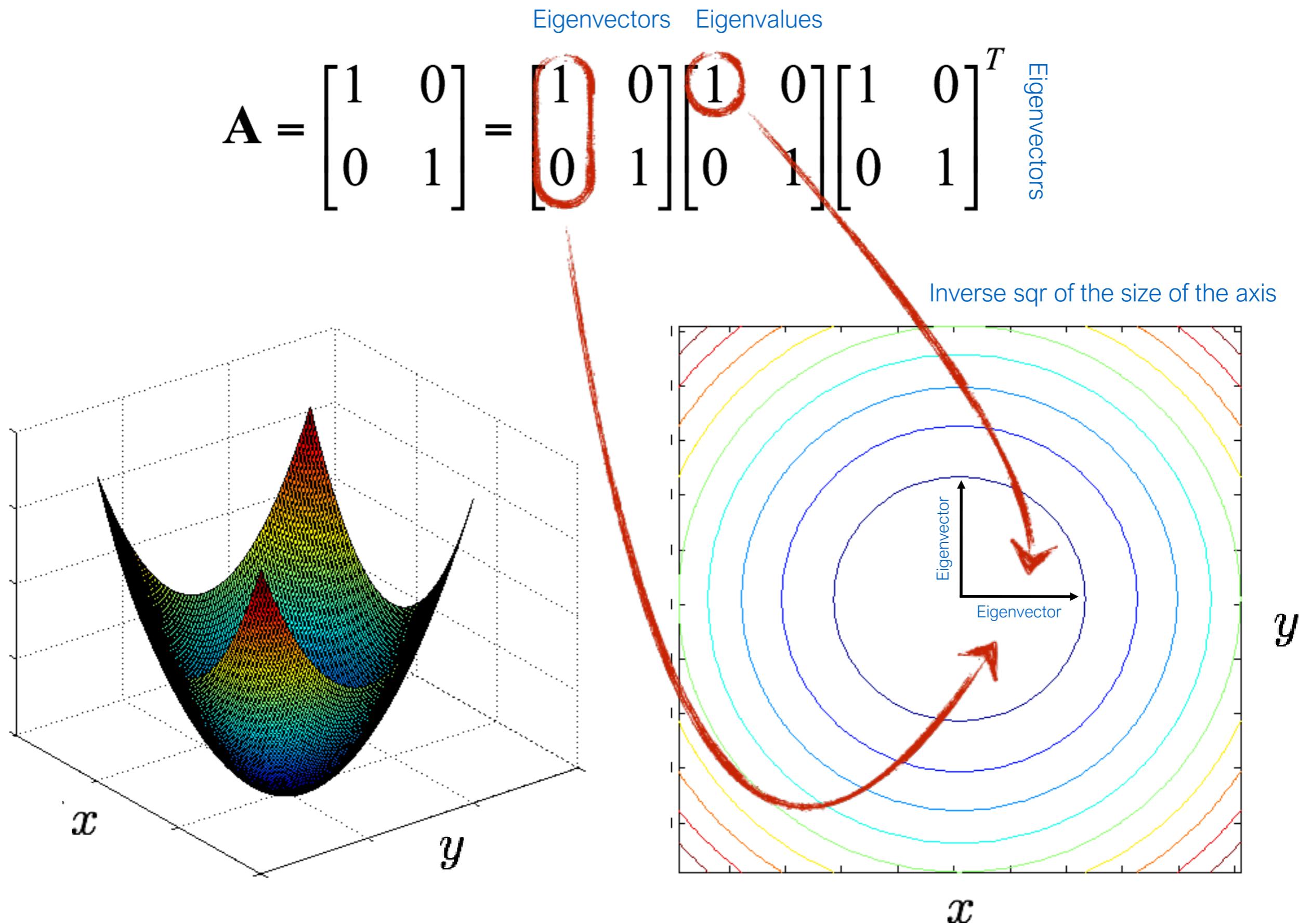
can be written in matrix form like this...

$$f(x, y) = \begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

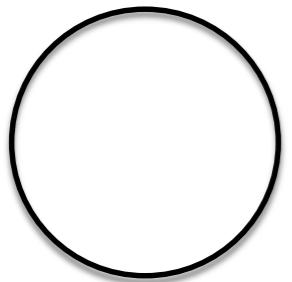
### Result of Singular Value Decomposition (SVD)

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \text{eigenvectors} \\ \text{axis of the 'ellipse slice'} \end{bmatrix} \begin{bmatrix} \text{eigenvalues along diagonal} \\ \text{Inverse sqrt of length of the quadratic along the axis} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}^T$$

The equation shows the decomposition of the identity matrix  $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$  into three components. The first component is labeled "eigenvectors" and "axis of the 'ellipse slice'". It is represented by a 2x2 matrix with columns  $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$  and  $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ , both of which are circled in red. The second component is labeled "eigenvalues along diagonal" and "Inverse sqrt of length of the quadratic along the axis". It is represented by a 2x2 diagonal matrix with entries 1 and 1, both of which are circled in red. The third component is the transpose of the second matrix, represented by  $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}^T$ .

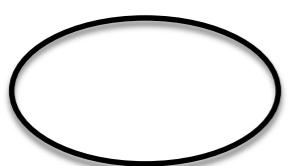


Recall:



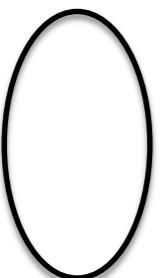
$$f(x, y) = \begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

you can smash this bowl in the **y** direction



$$f(x, y) = \begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

you can smash this bowl in the **x** direction

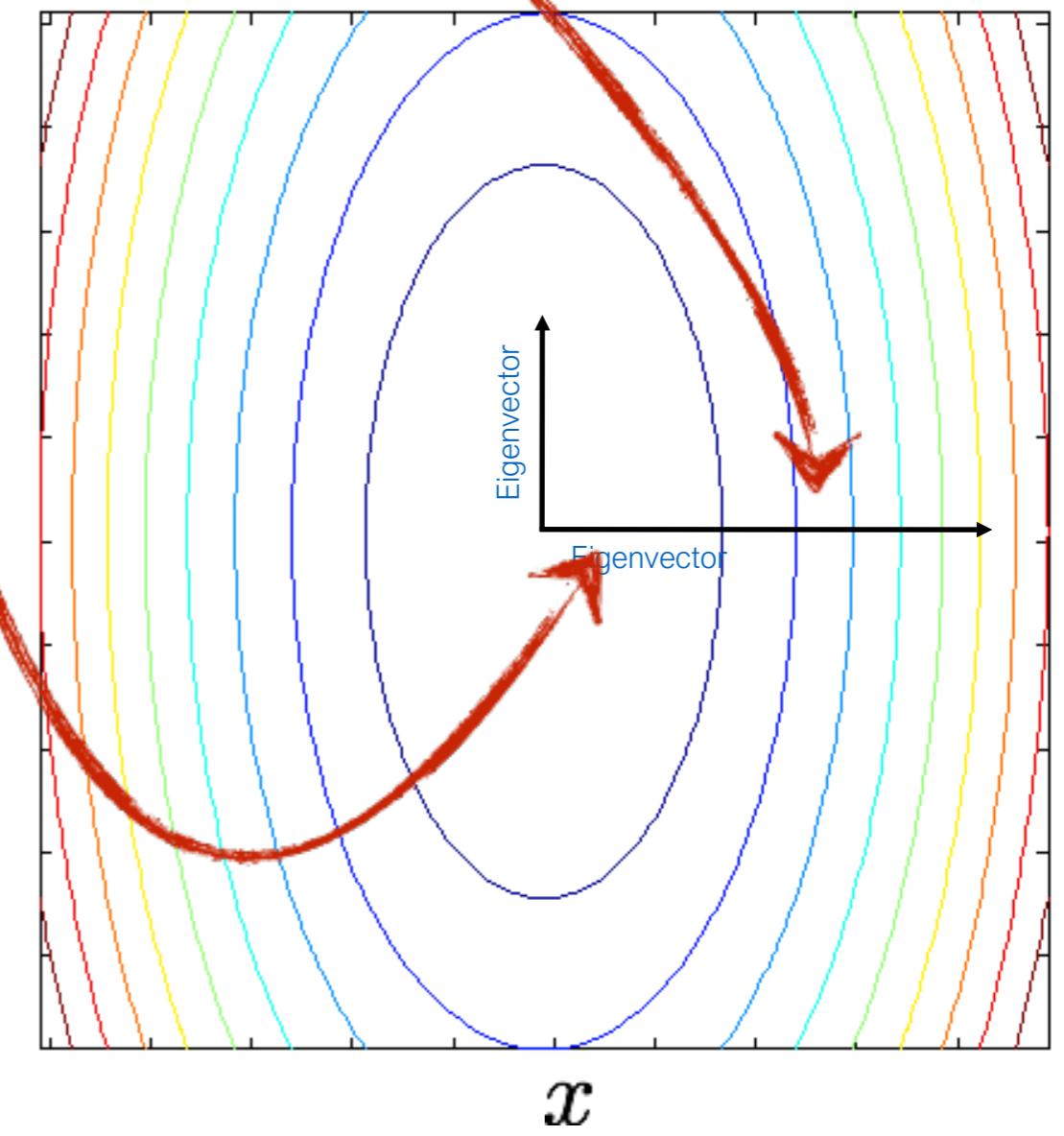
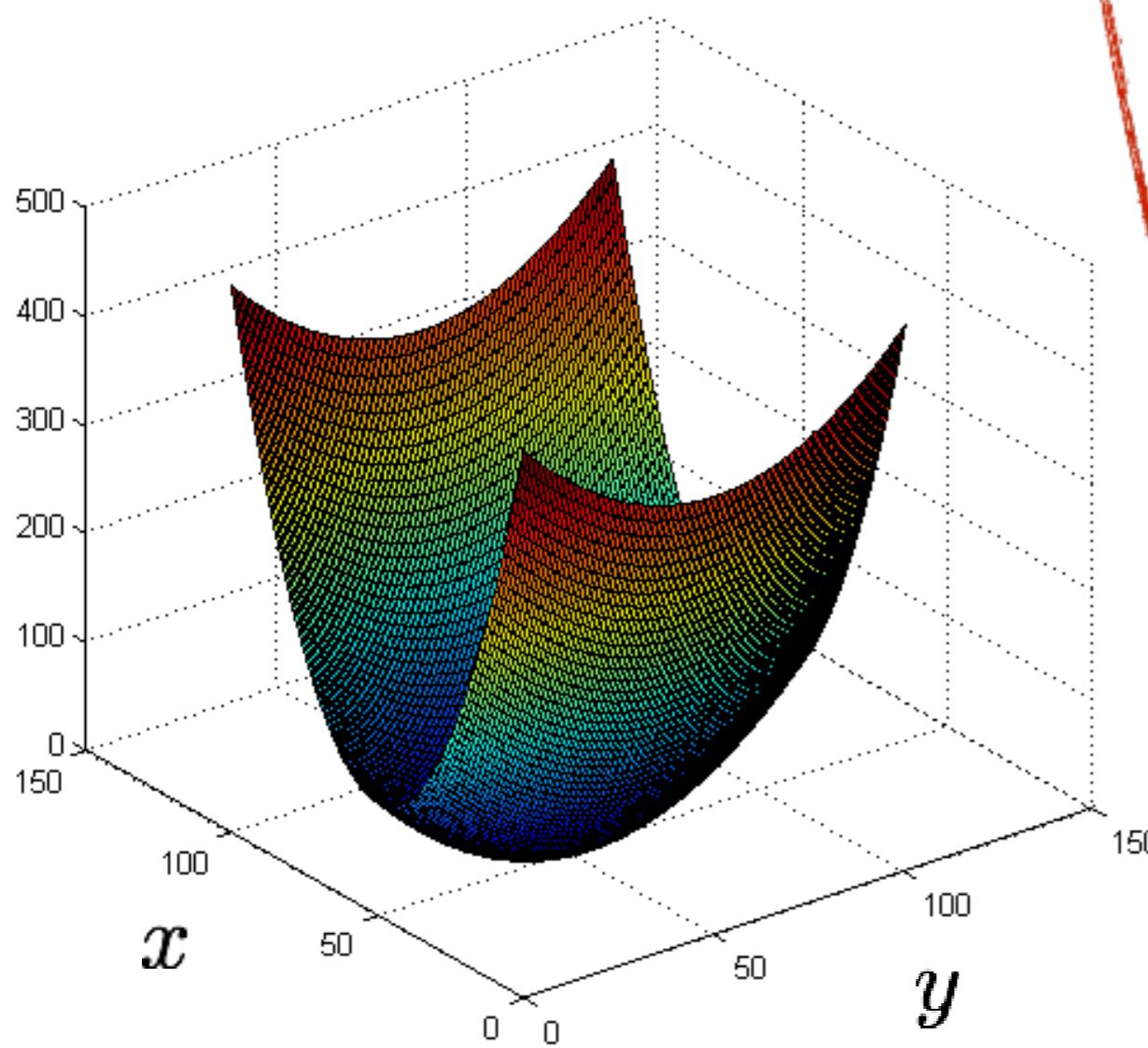


$$f(x, y) = \begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} 4 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$A = \begin{bmatrix} 4 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 4 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}^T$$

Eigenvalues  
Eigenvectors  
Eigenvectors

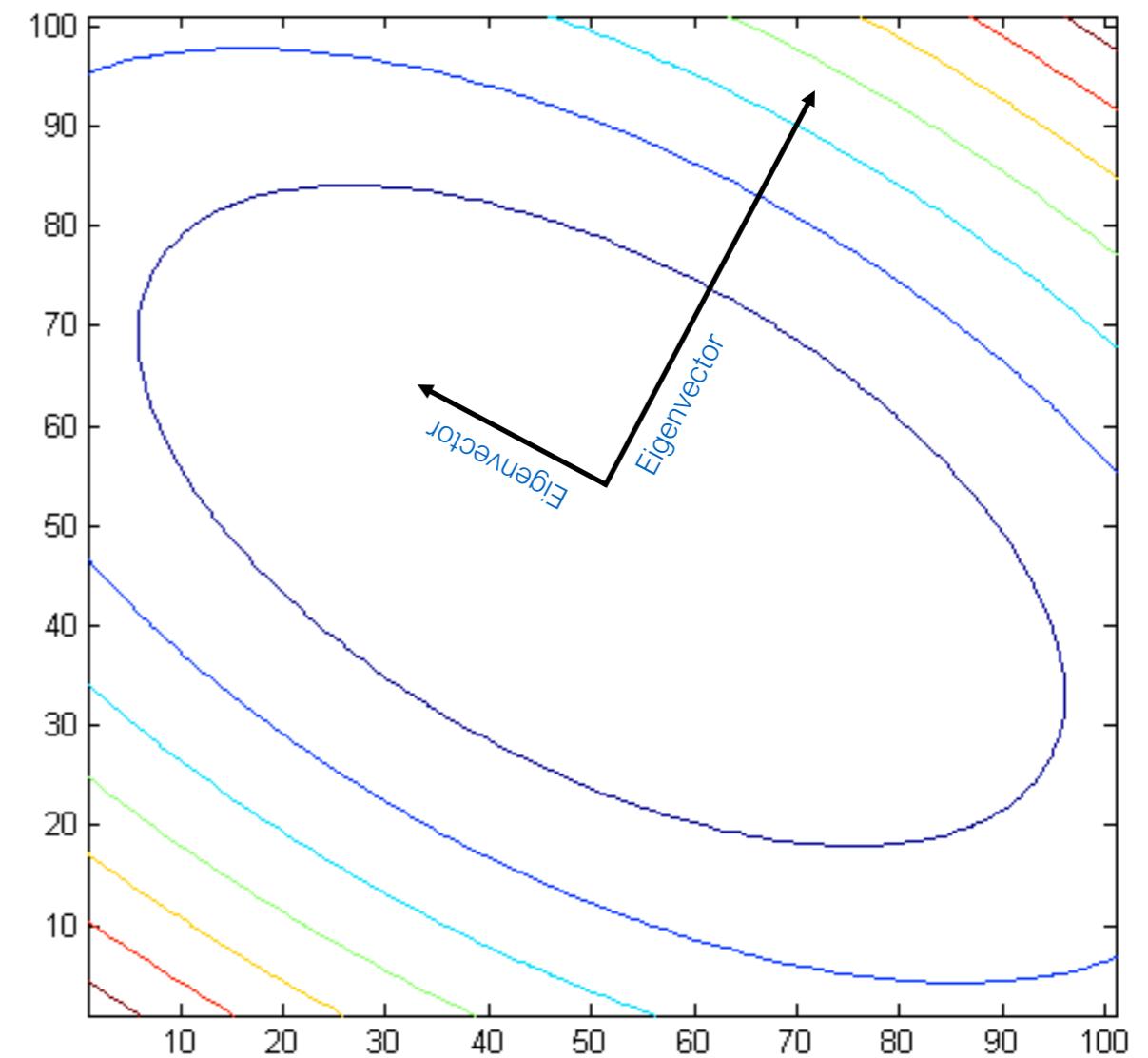
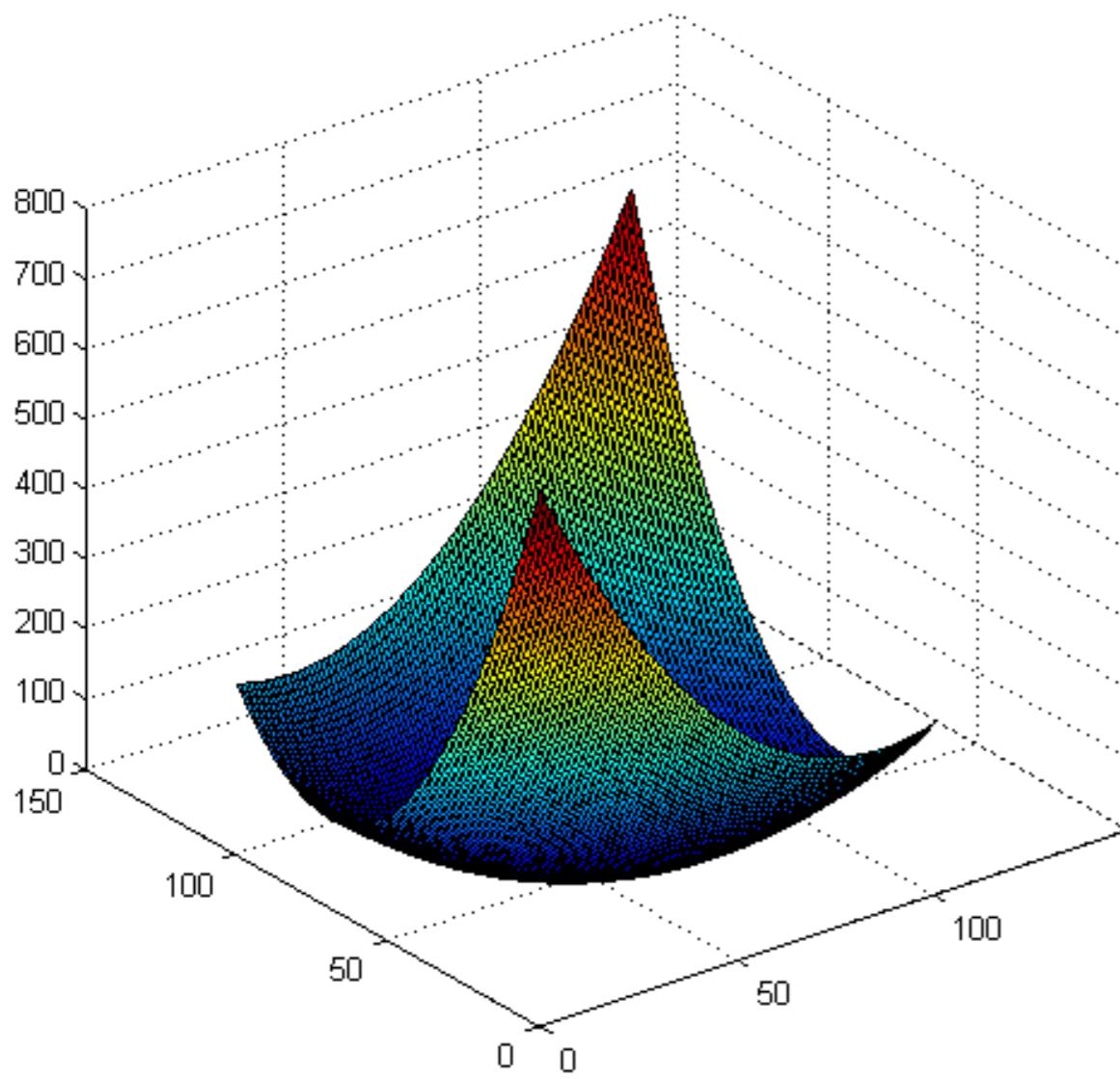
Inverse sqrt of length of axis



$$A = \begin{bmatrix} 3.25 & 1.30 \\ 1.30 & 1.75 \end{bmatrix} = \begin{bmatrix} 0.50 & -0.87 \\ -0.87 & -0.50 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix} \begin{bmatrix} 0.50 & -0.87 \\ -0.87 & -0.50 \end{bmatrix}^T$$

Eigenvalues  
Eigenvalues

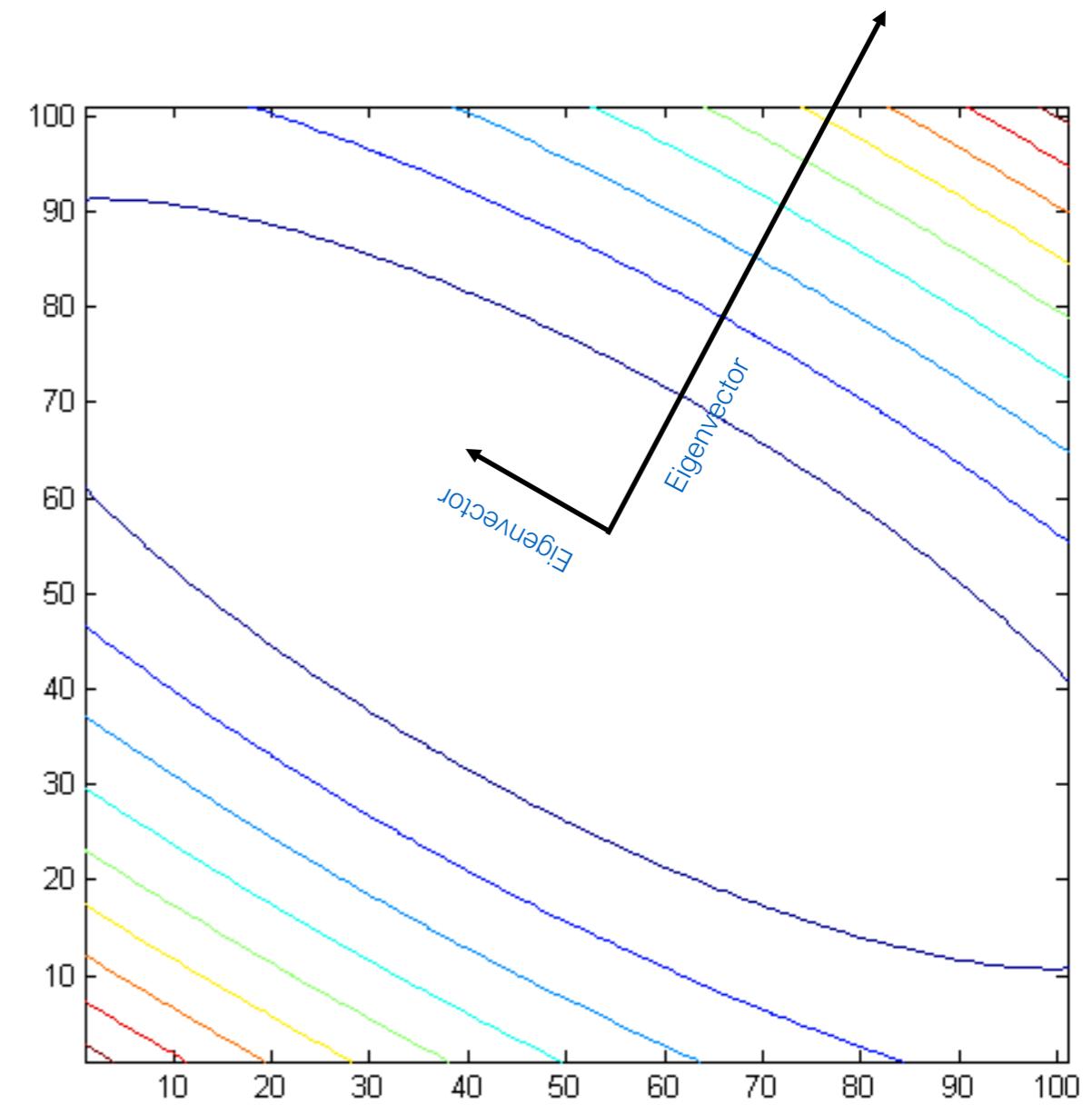
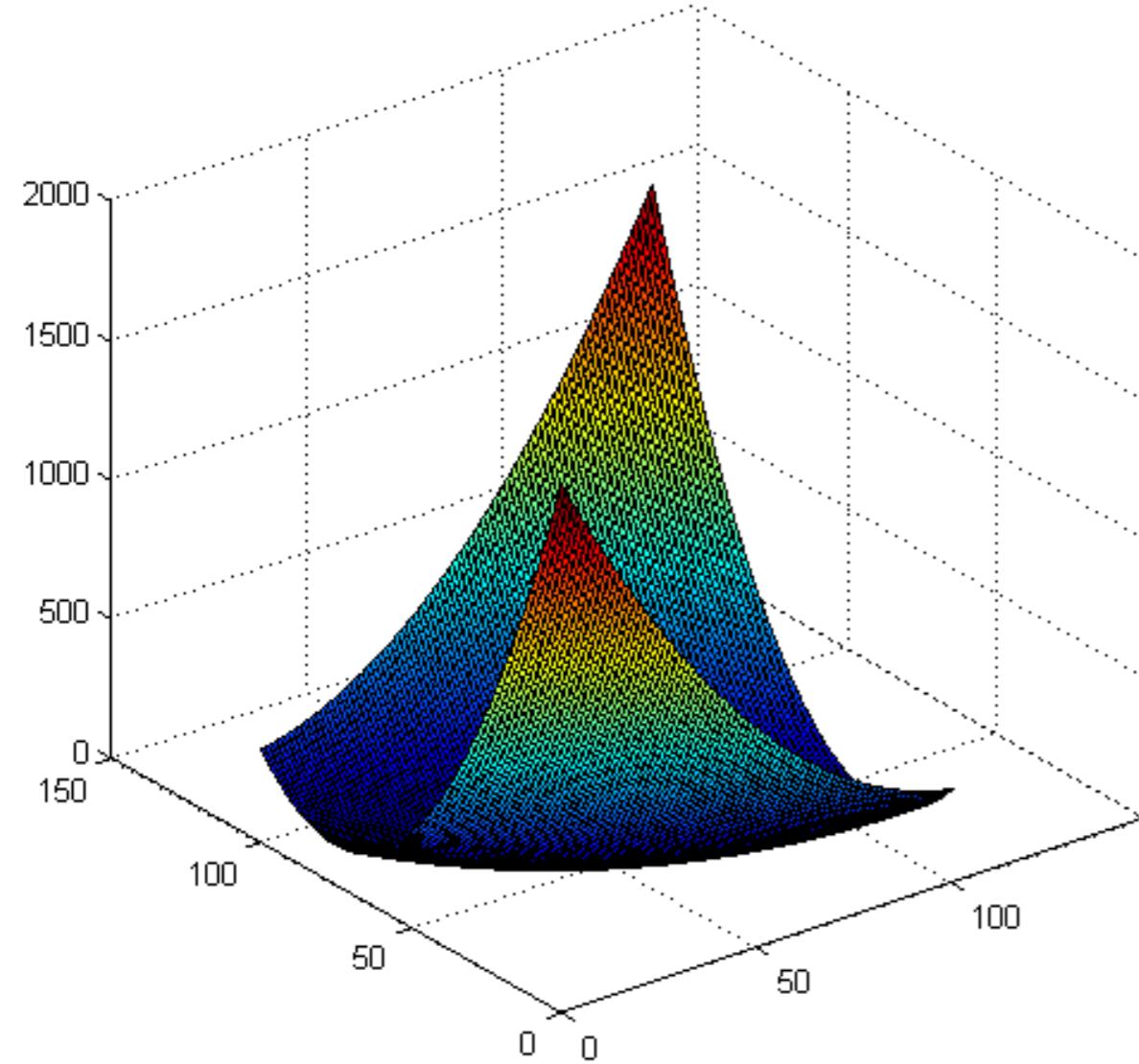
Eigenvectors  
Eigenvectors



$$A = \begin{bmatrix} 7.75 & 3.90 \\ 3.90 & 3.25 \end{bmatrix} = \begin{bmatrix} 0.50 & -0.87 \\ -0.87 & -0.50 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 10 \end{bmatrix} \begin{bmatrix} 0.50 & -0.87 \\ -0.87 & -0.50 \end{bmatrix}^T$$

Eigenvalues  
Eigenvalues

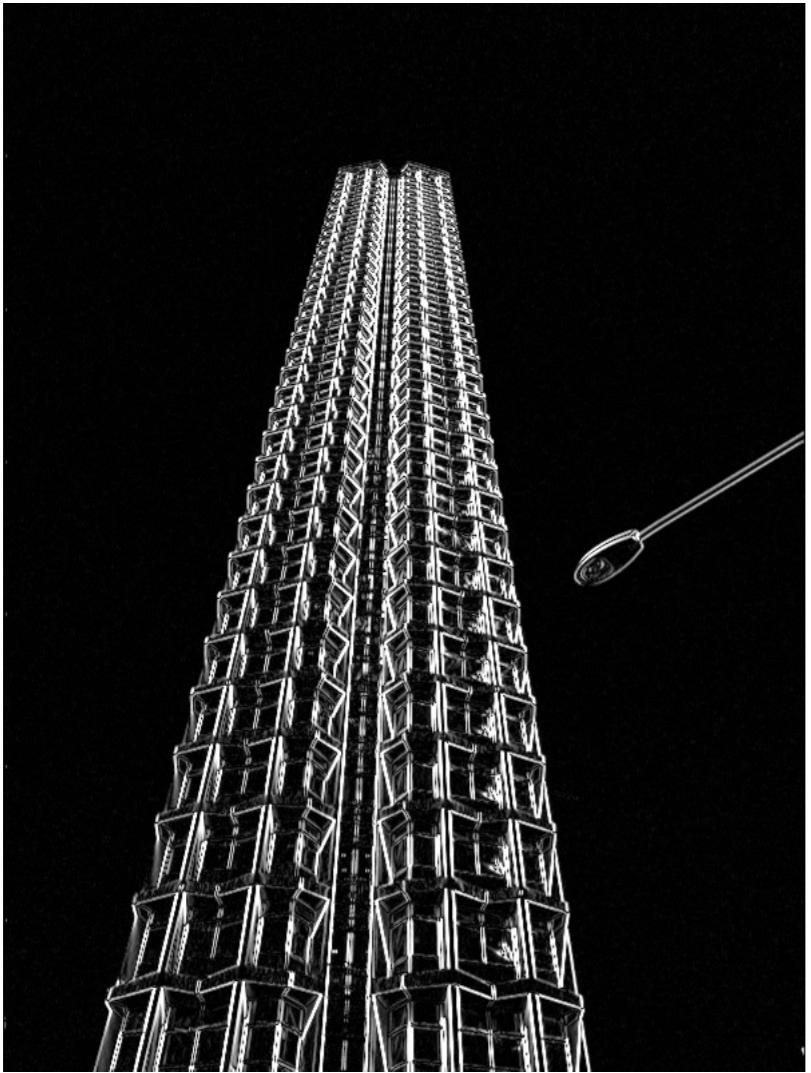
Eigenvectors  
Eigenvectors



# Reminder II:

# Image derivatives

# Horizontal derivative



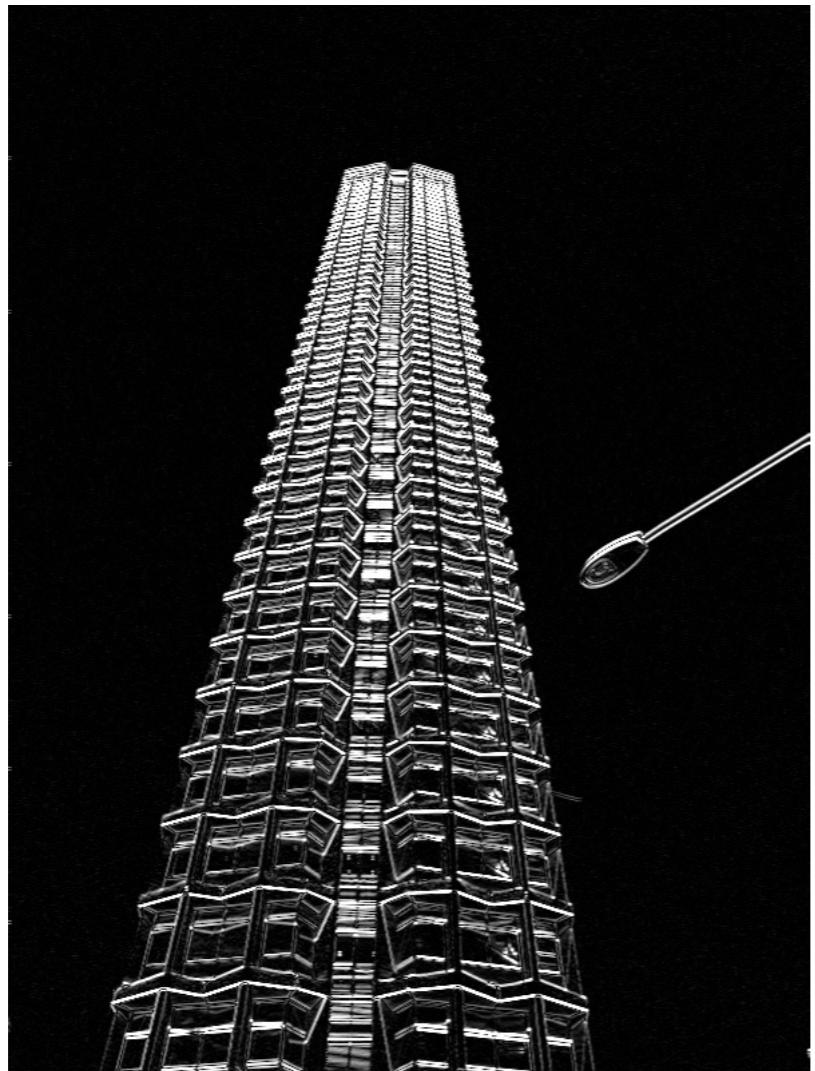
horizontal derivative

$$= \begin{bmatrix} 1 & 0 & -1 \end{bmatrix} *$$



original

# Vertical derivative



Vertical derivative

=

$$\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$$

\*

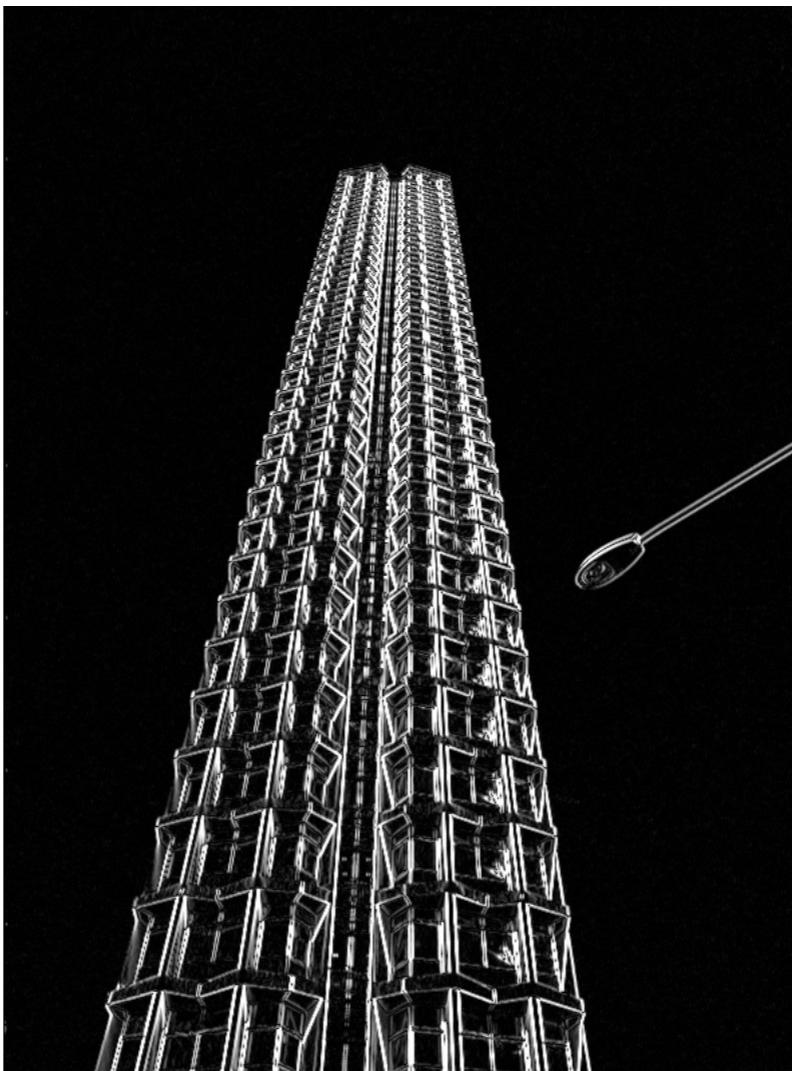


original

Better derivative filters?



original



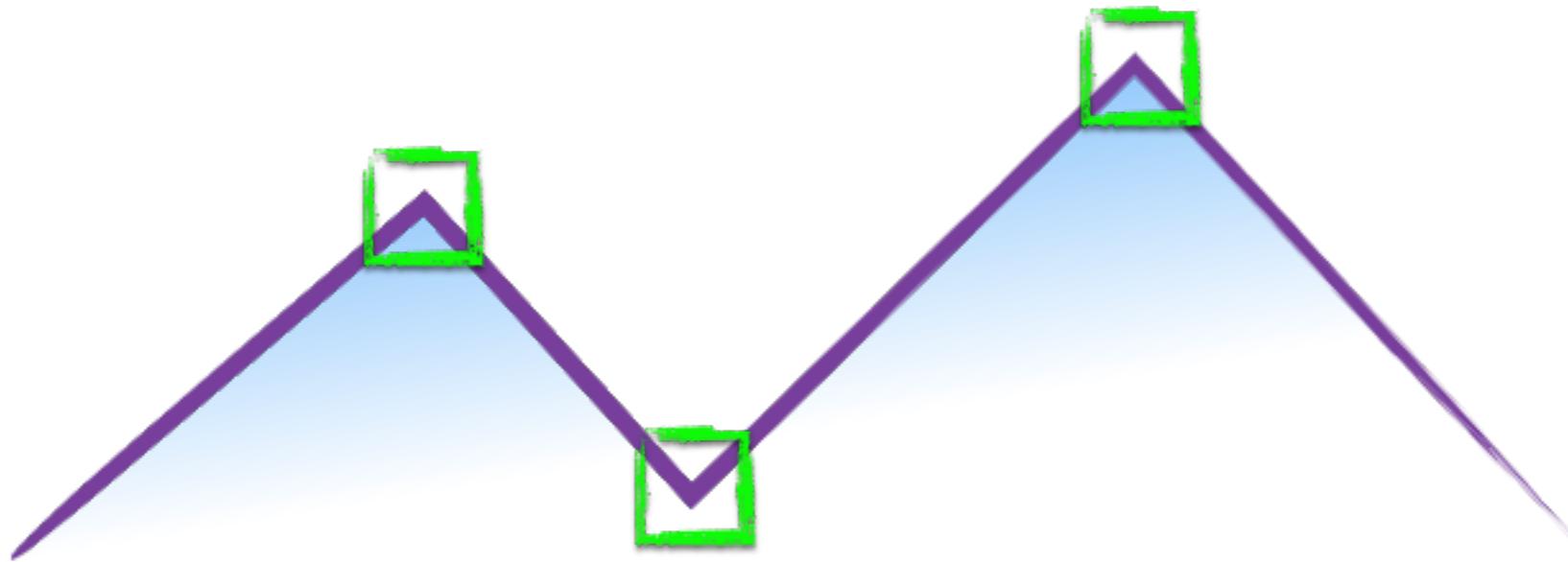
horizontal derivative



vertical derivative

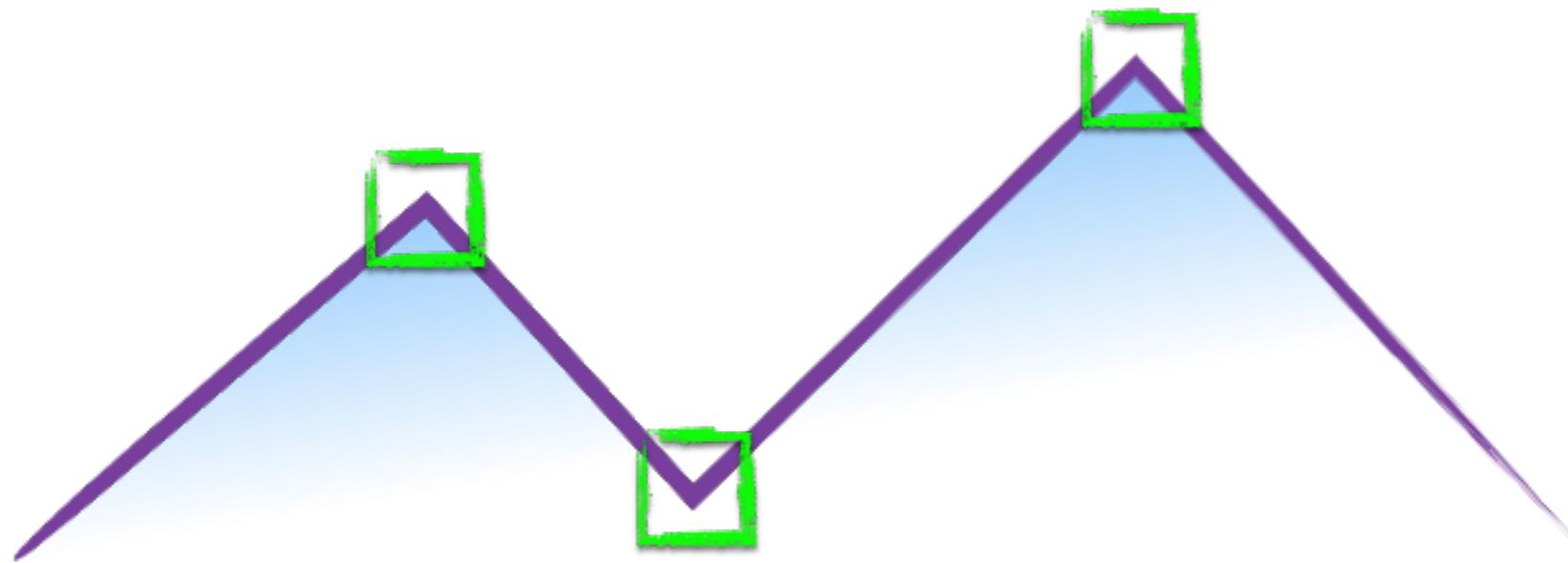
# Harris corner detector

# How do you find a corner?



# How do you find a corner?

[Moravec 1980]

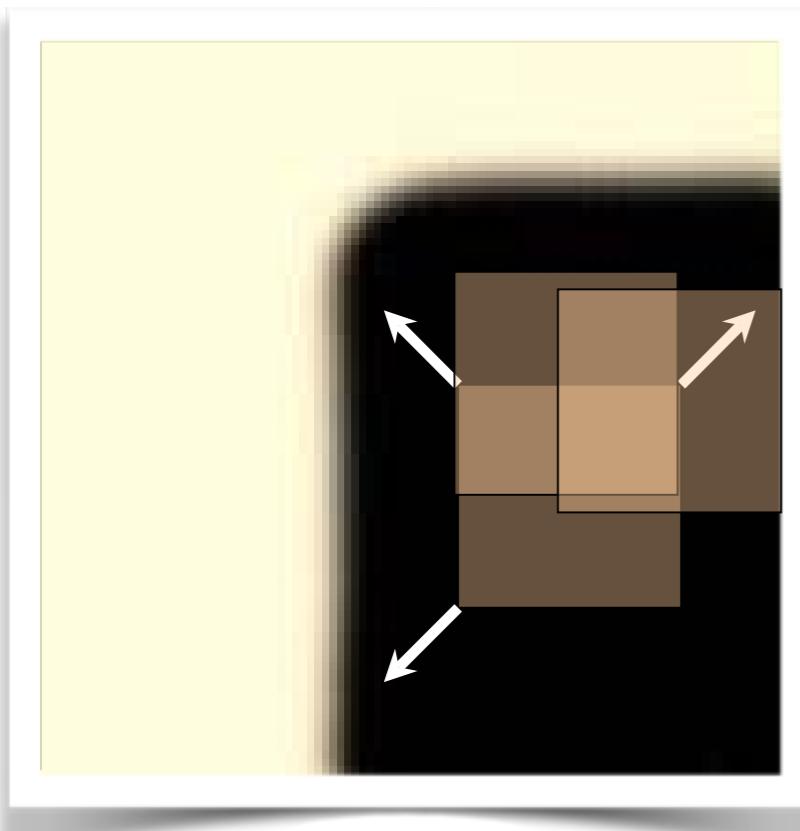


Easily recognized by looking through a small window

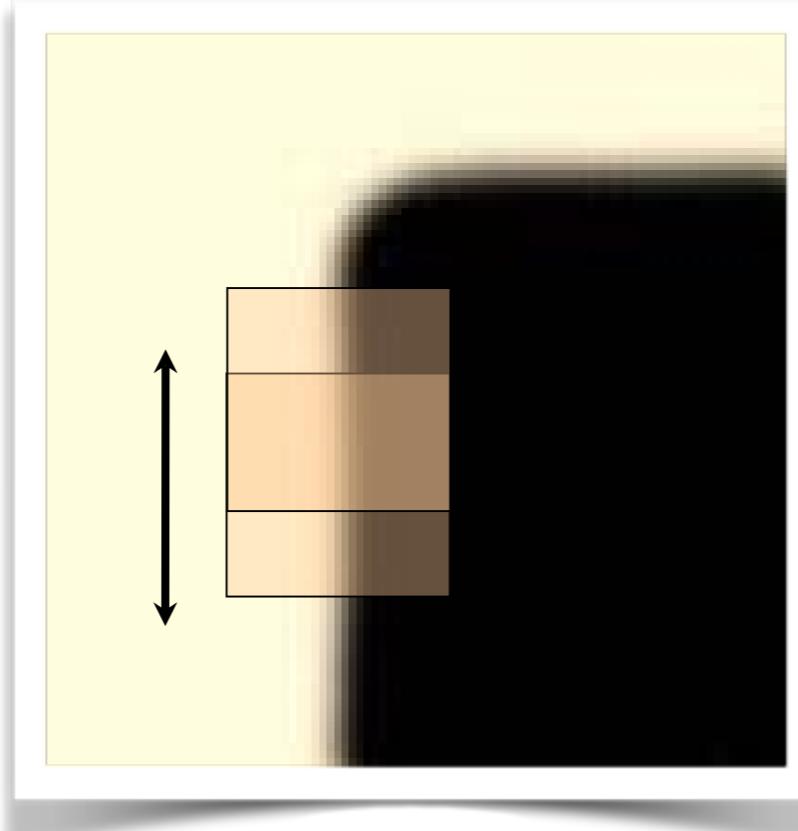
Shifting the window should give large change in intensity

Easily recognized by looking through a small window

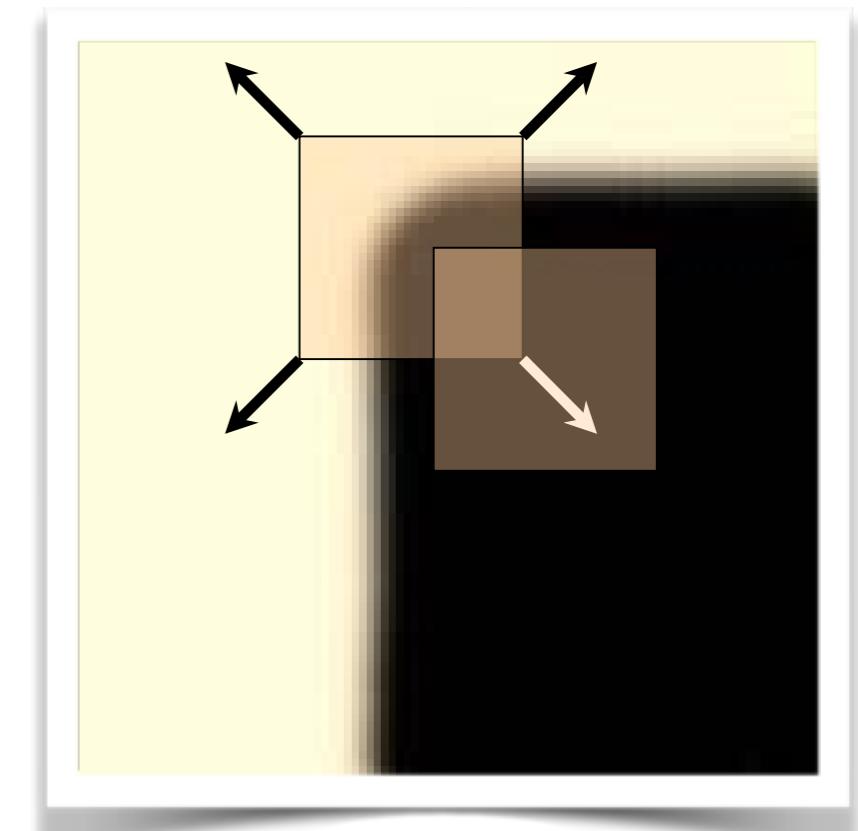
Shifting the window should give large change in intensity



“flat” region:  
no change in all  
directions



“edge”:  
no change along the edge  
direction



“corner”:  
significant change in all  
directions

Design a program to detect corners  
(hint: use image gradients)

# Finding corners (a.k.a. PCA)

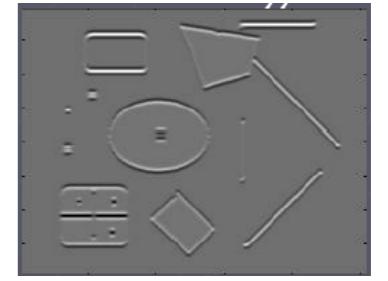
$$I_x = \frac{\partial I}{\partial x}$$

$$I_y = \frac{\partial I}{\partial y}$$

1. Compute image gradients over small region



2. Subtract mean from each image gradient



3. Compute the covariance matrix

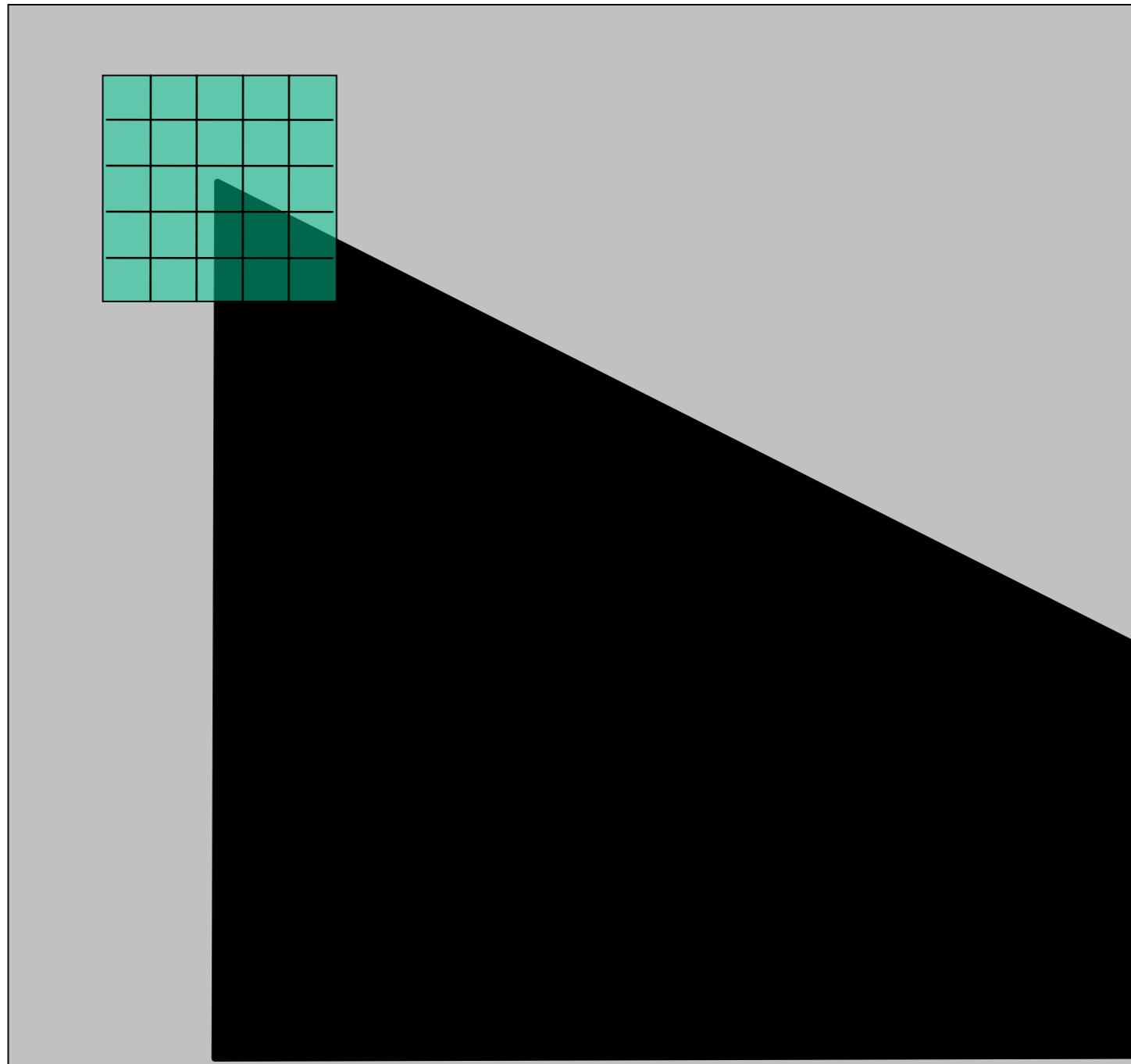
$$\begin{bmatrix} \sum_{p \in P} I_x I_x & \sum_{p \in P} I_x I_y \\ \sum_{p \in P} I_y I_x & \sum_{p \in P} I_y I_y \end{bmatrix}$$

4. Compute eigenvectors and eigenvalues

5. Use threshold on eigenvalues to detect corners

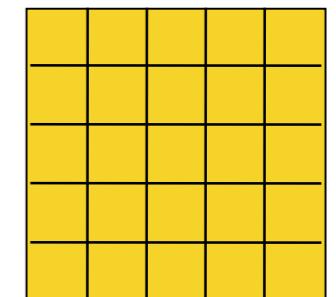
1. Compute image gradients over a small region  
(not just a single pixel)

# 1. Compute image gradients over a small region (not just a single pixel)



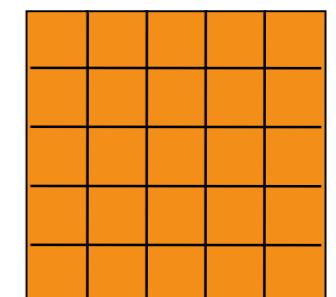
array of x gradients

$$I_x = \frac{\partial I}{\partial x}$$



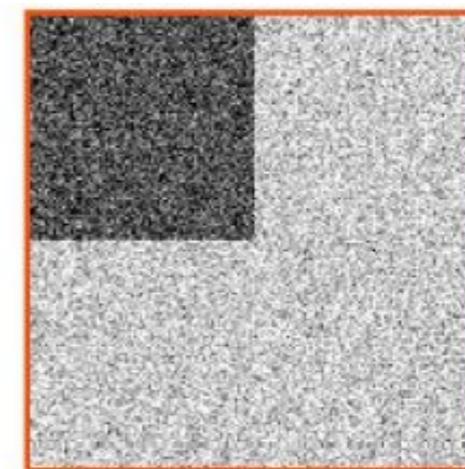
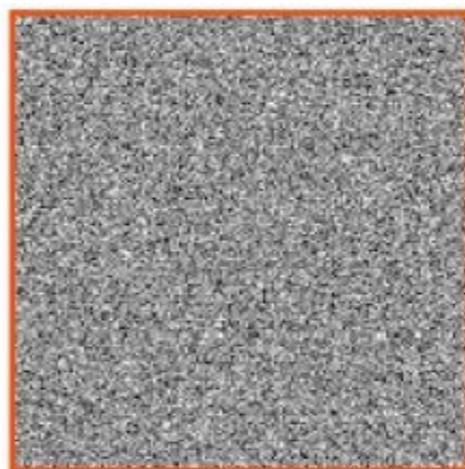
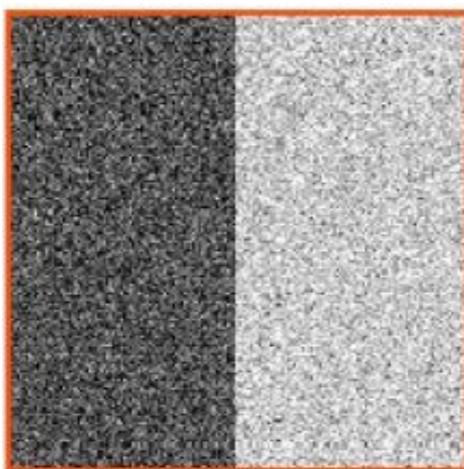
array of y gradients

$$I_y = \frac{\partial I}{\partial y}$$

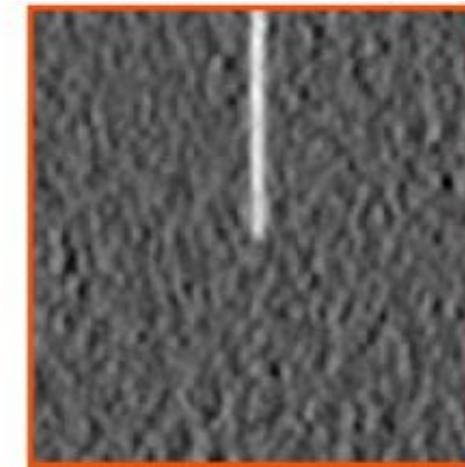
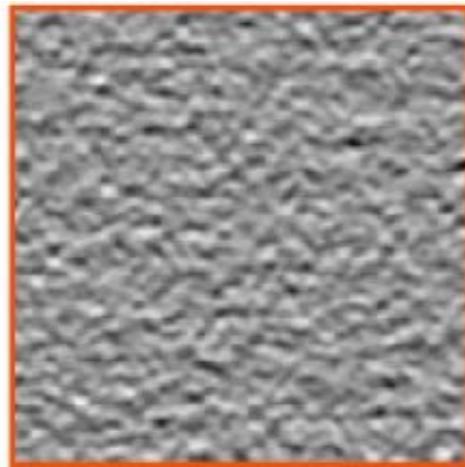
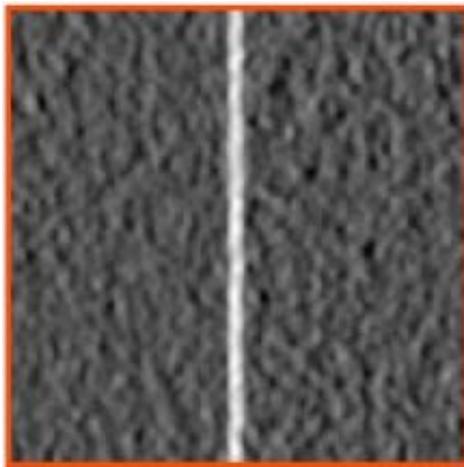


# visualization of gradients

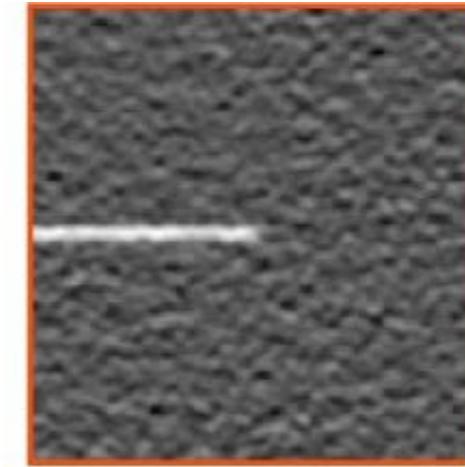
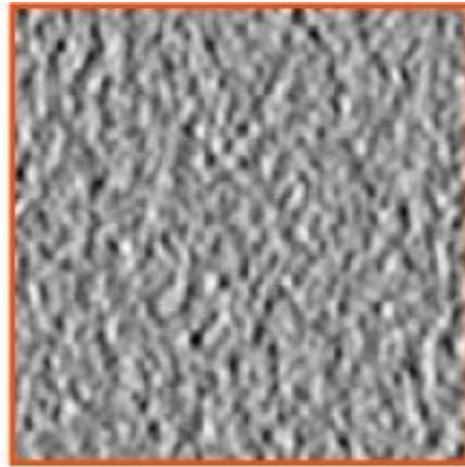
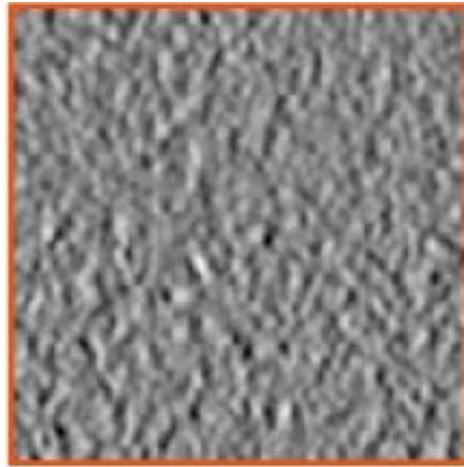
image

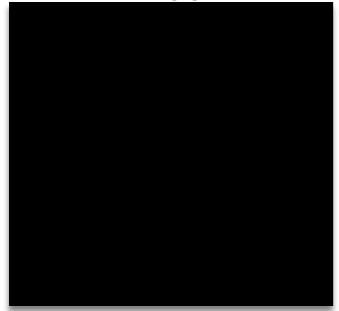
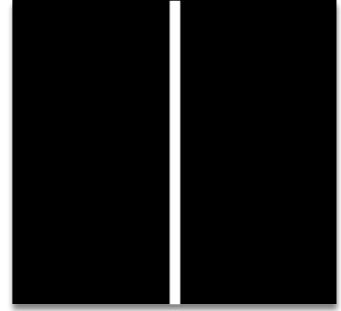
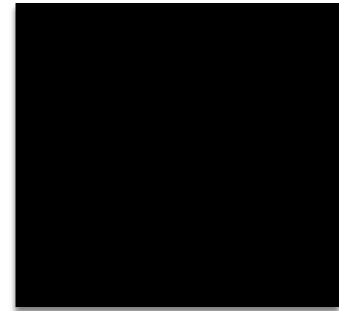
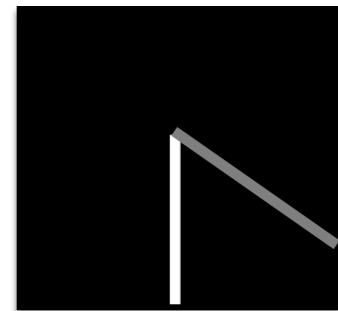
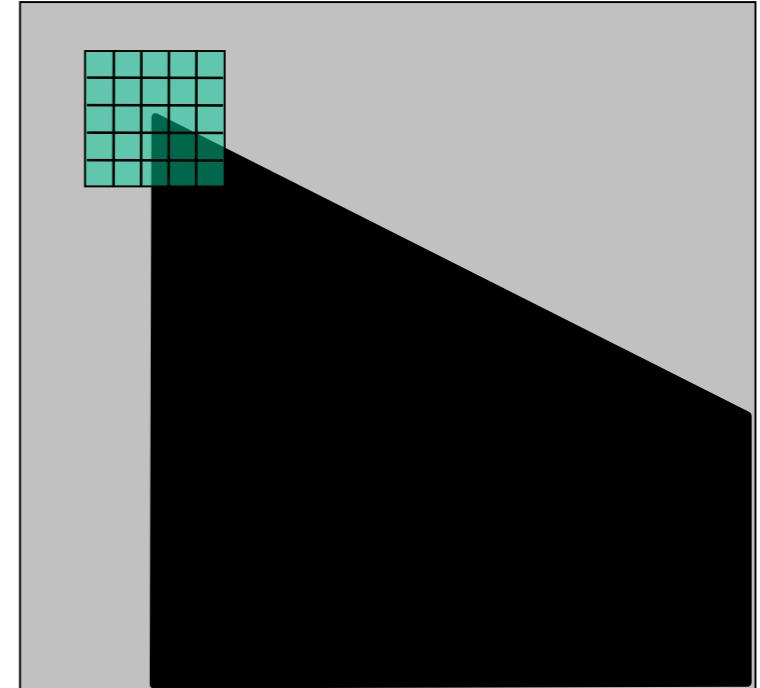
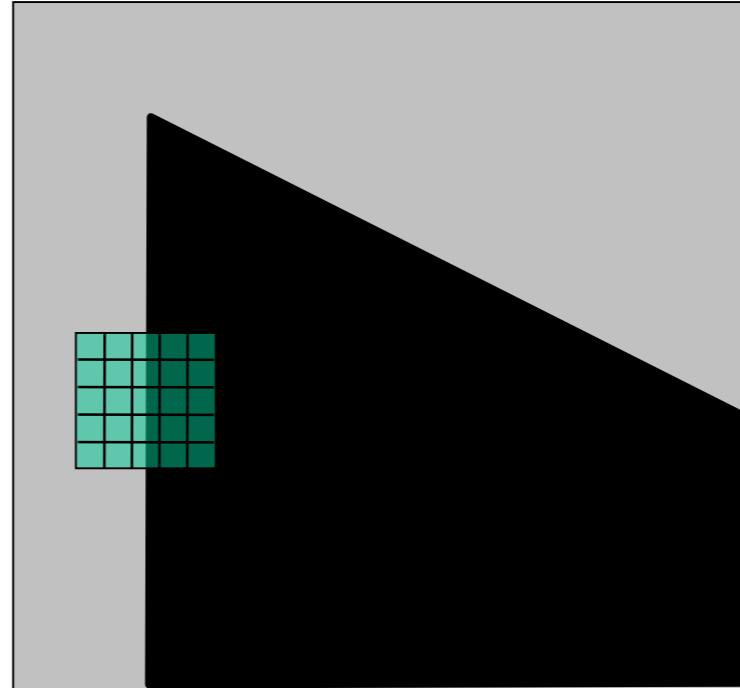
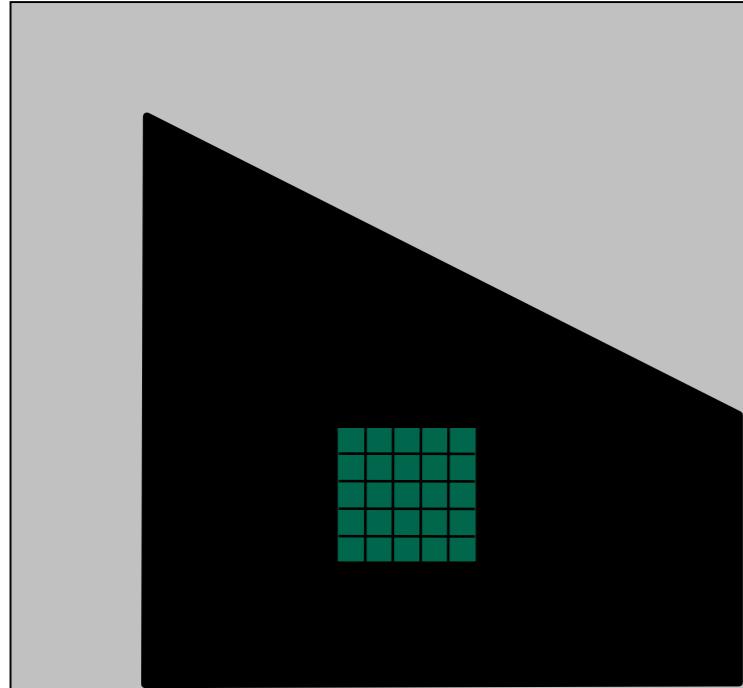
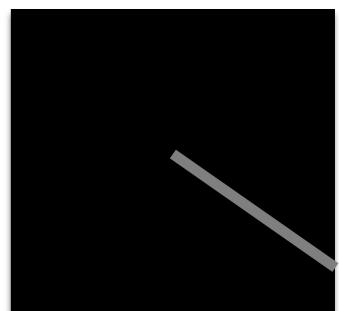


X derivative

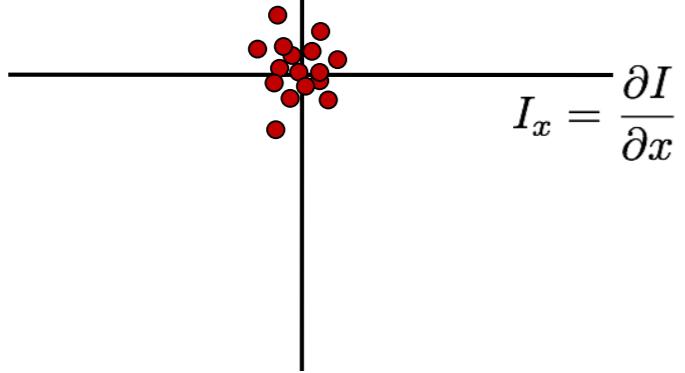


Y derivative

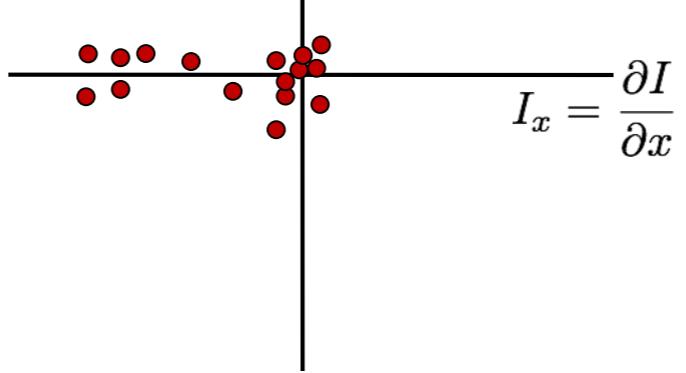


$I_x$  $I_y$  $I_x$  $I_y$  $I_x$  $I_y$ 

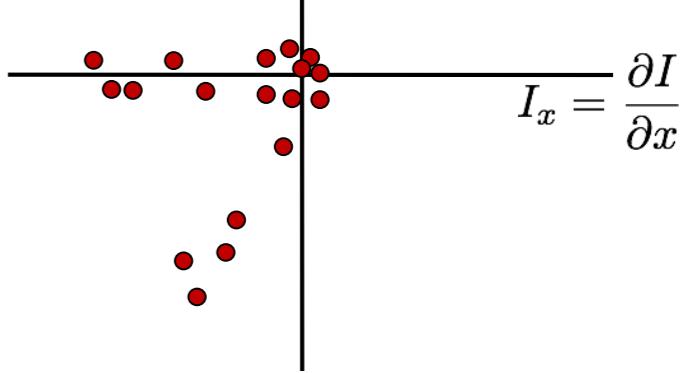
$$I_y = \frac{\partial I}{\partial y}$$



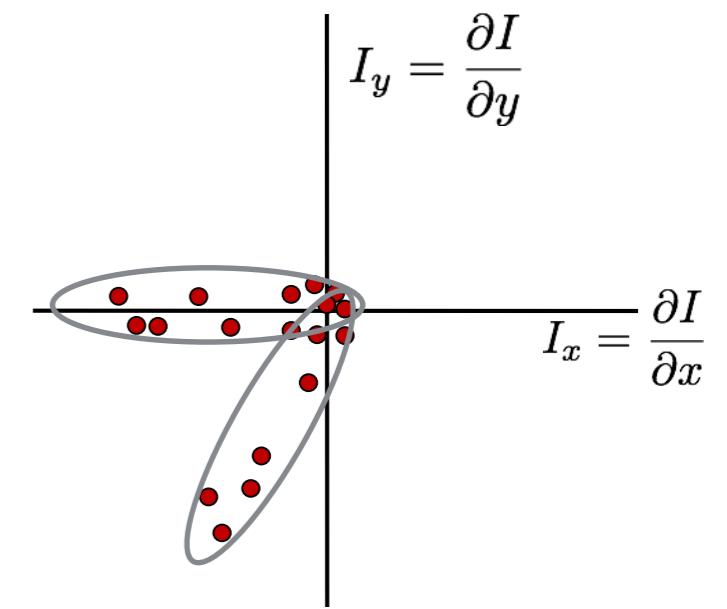
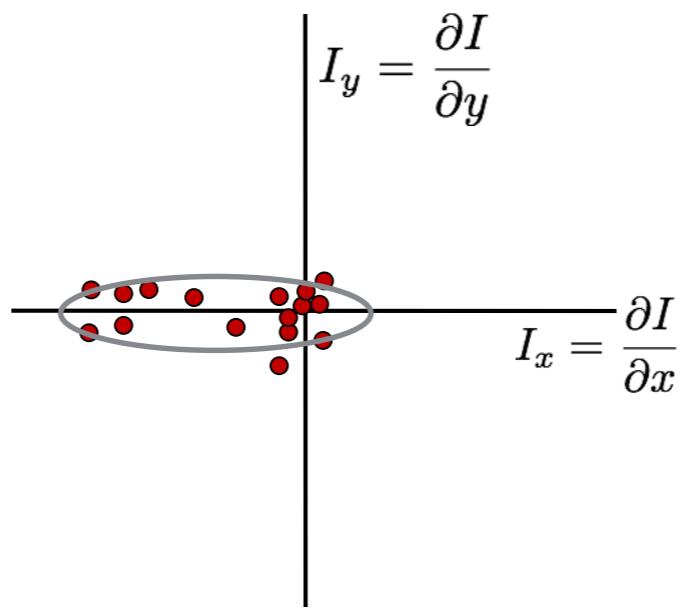
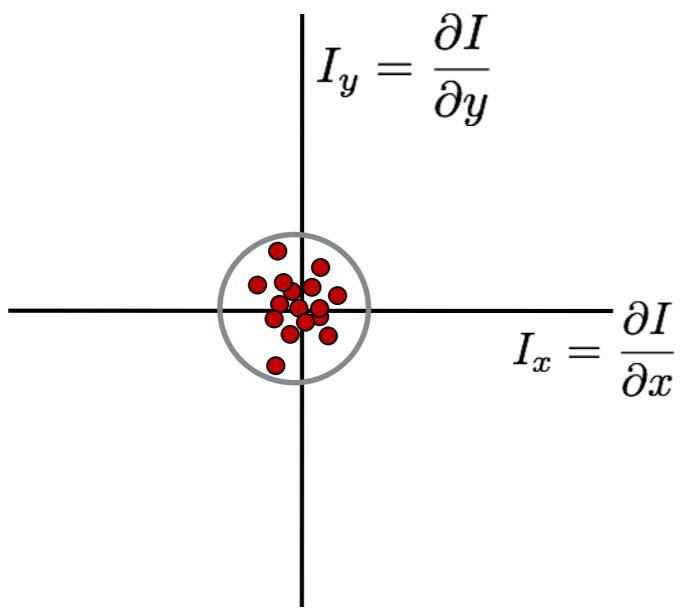
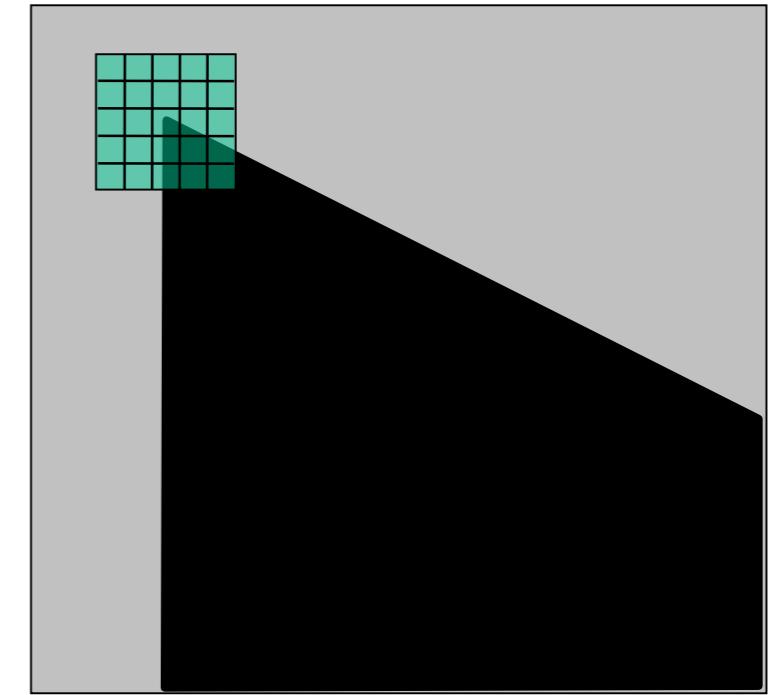
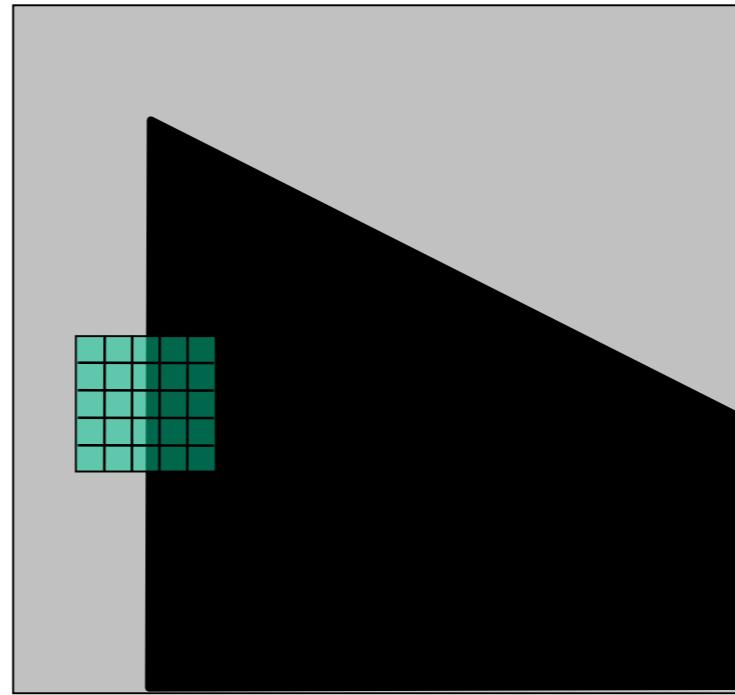
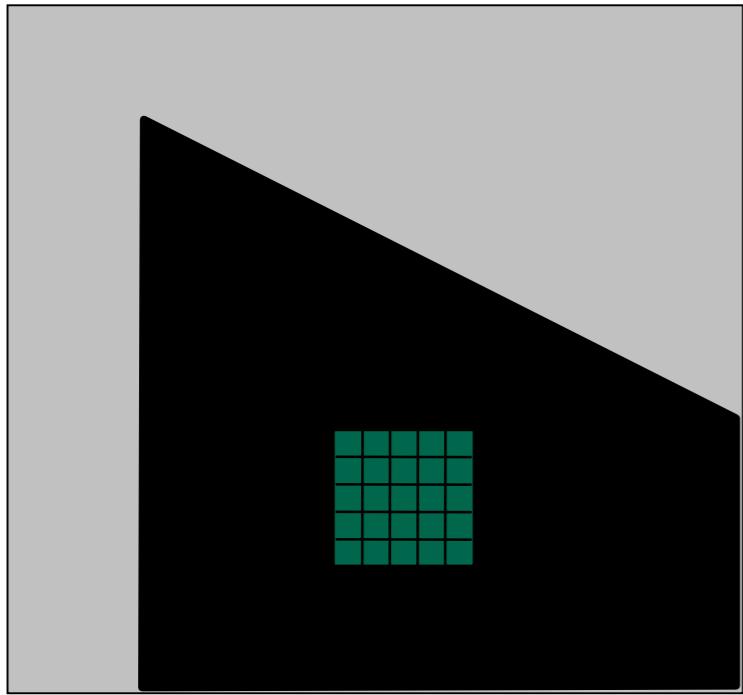
$$I_y = \frac{\partial I}{\partial y}$$



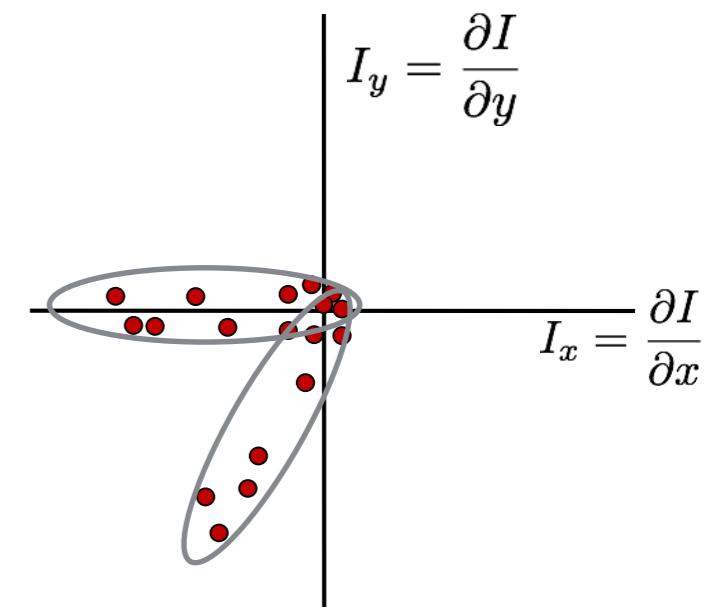
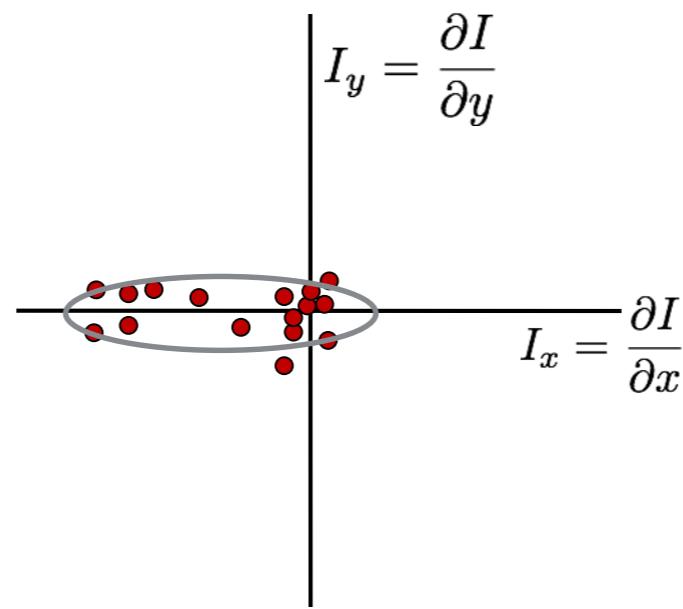
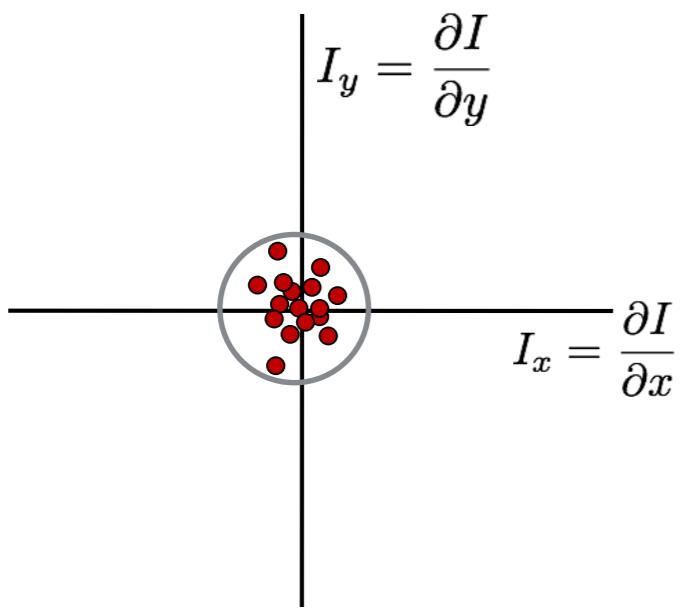
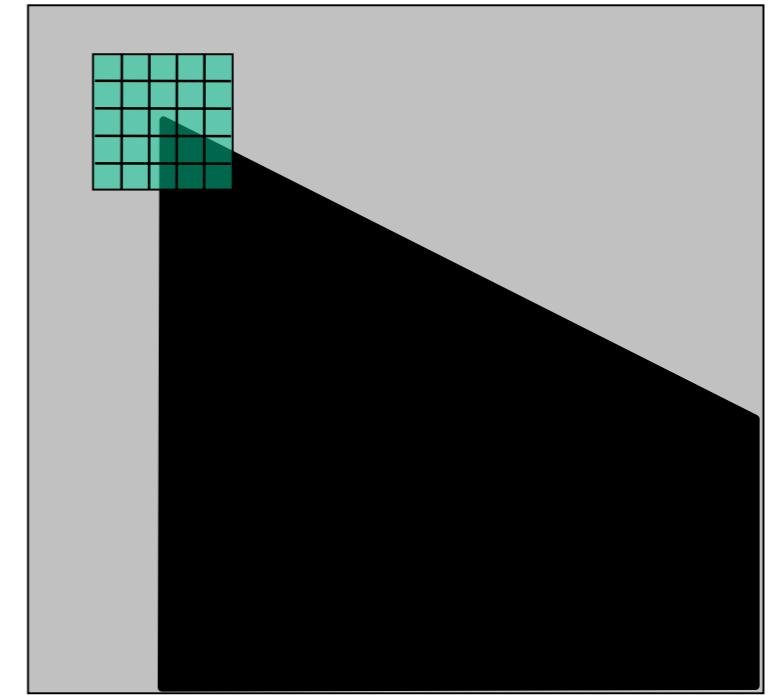
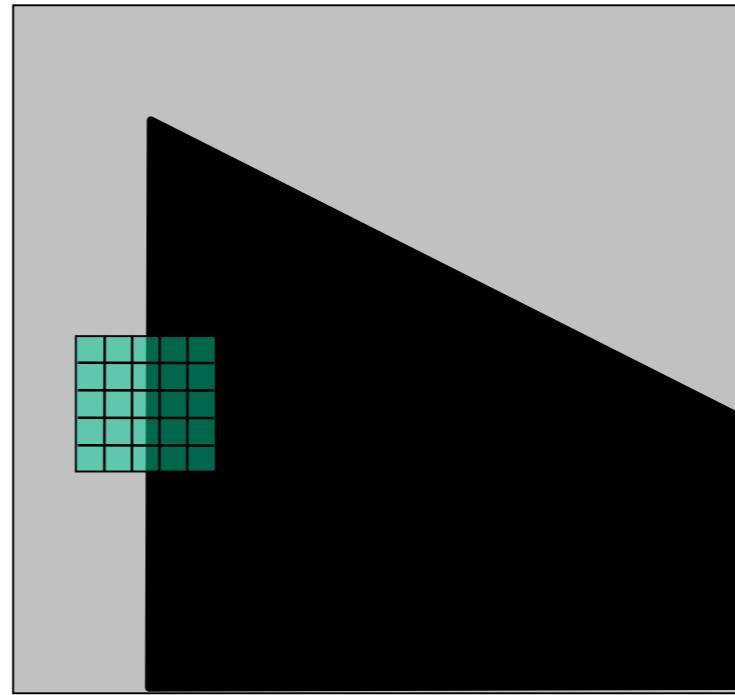
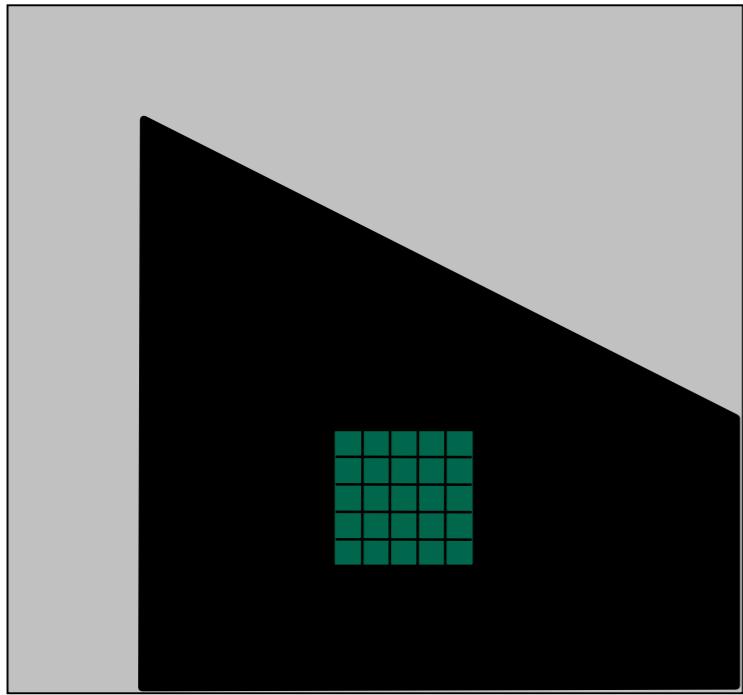
$$I_y = \frac{\partial I}{\partial y}$$



*What does the distribution tell you about the region?*



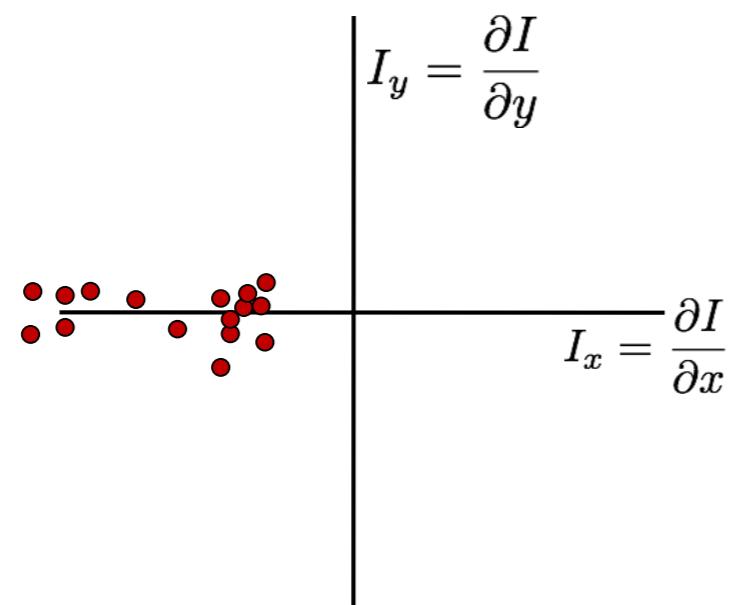
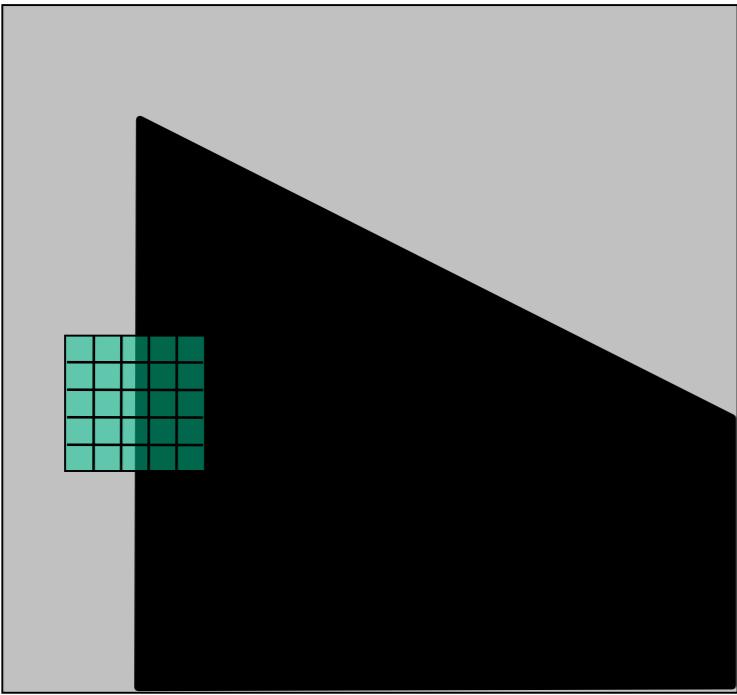
distribution reveals edge orientation and magnitude



*How do you quantify orientation and magnitude?*

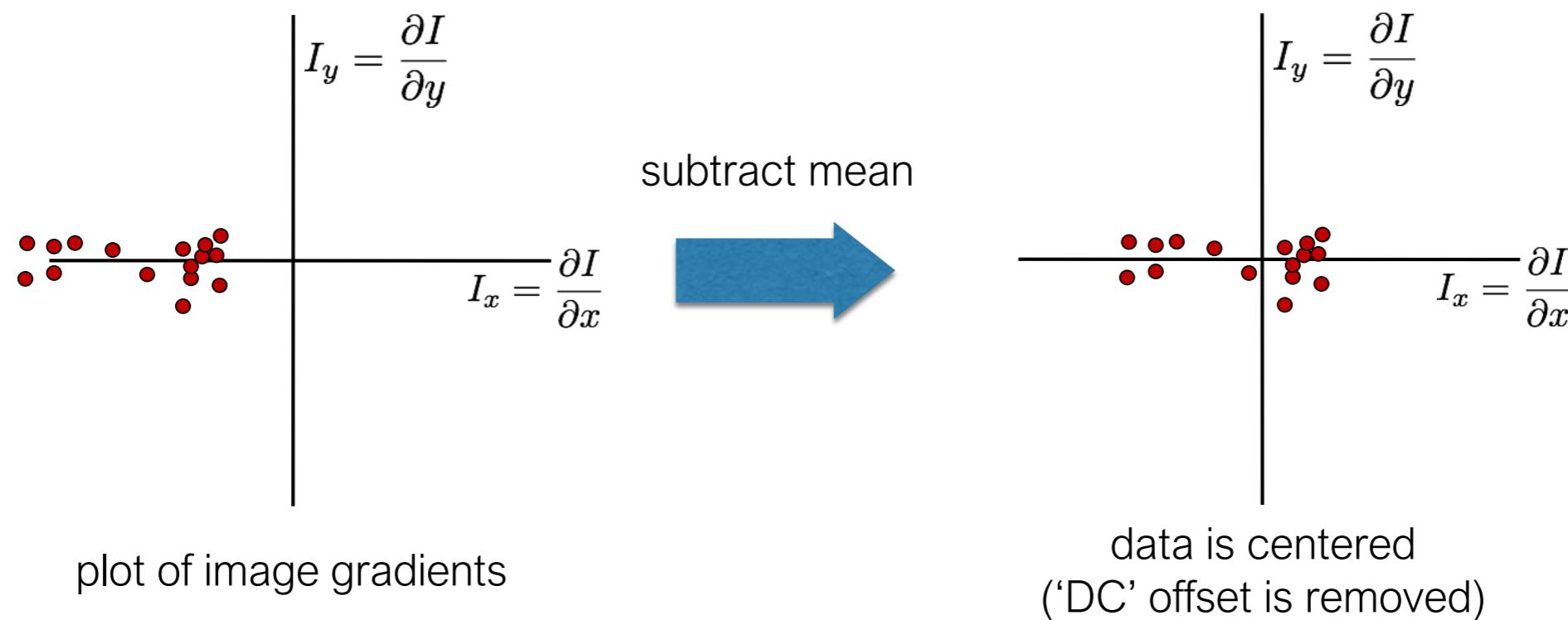
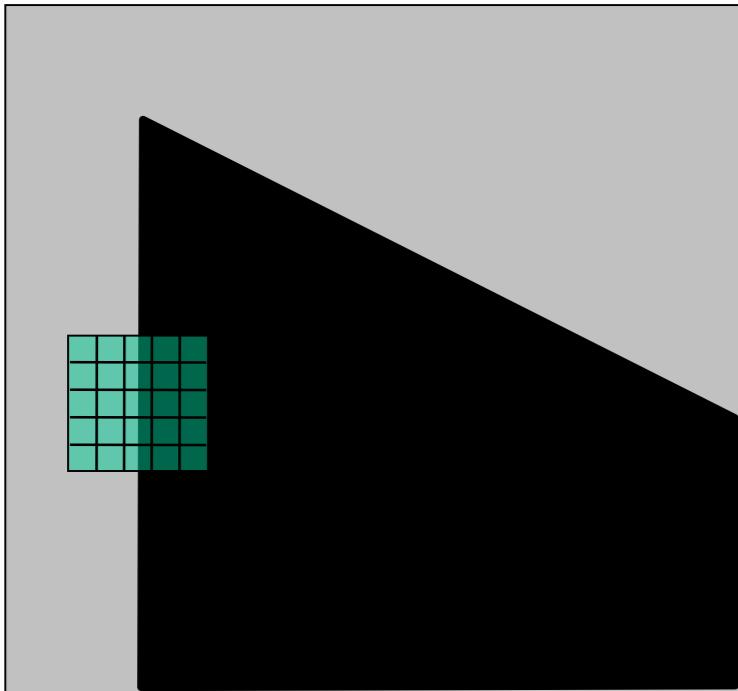
2. Subtract the mean from each image gradient

## 2. Subtract the mean from each image gradient



plot of image gradients

## 2. Subtract the mean from each image gradient



3. Compute the covariance matrix

### 3. Compute the covariance matrix

$$\begin{bmatrix} \sum_{p \in P} I_x I_x & \sum_{p \in P} I_x I_y \\ \sum_{p \in P} I_y I_x & \sum_{p \in P} I_y I_y \end{bmatrix}$$

$$\sum_{p \in P} I_x I_y = \text{sum}\left( \begin{array}{|c|c|c|c|} \hline & & & \\ \hline \end{array} \right) \cdot \ast \cdot \begin{array}{|c|c|c|c|} \hline & & & \\ \hline \end{array} )$$

array of x gradients    array of y gradients

$$I_x = \frac{\partial I}{\partial x}$$
$$I_y = \frac{\partial I}{\partial y}$$

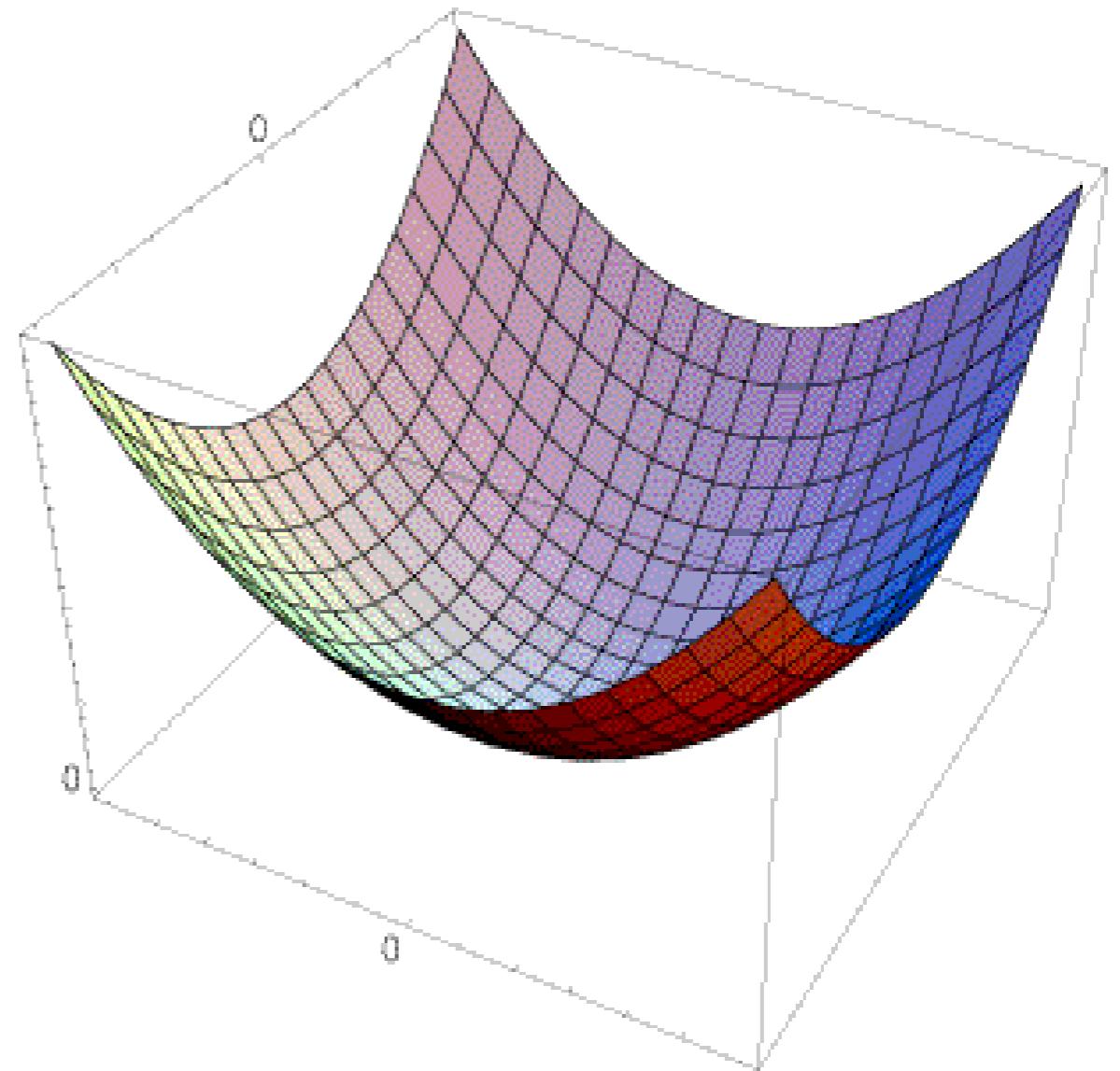
*Where does this covariance matrix come from?*

# Visualization of a quadratic

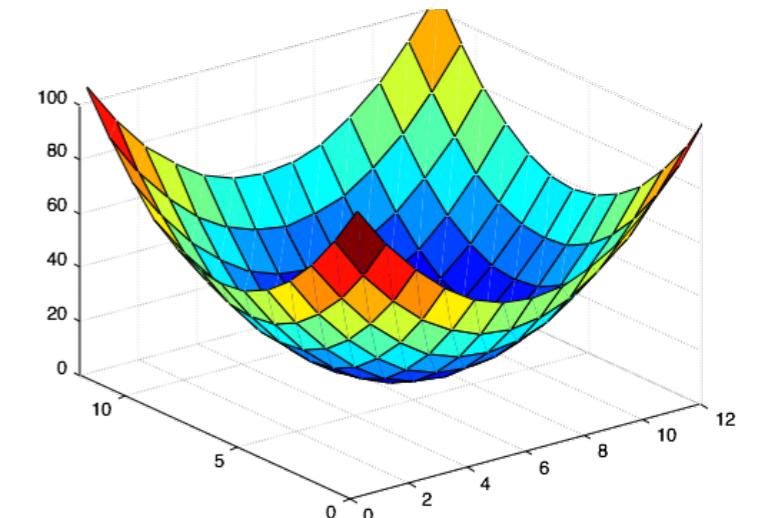
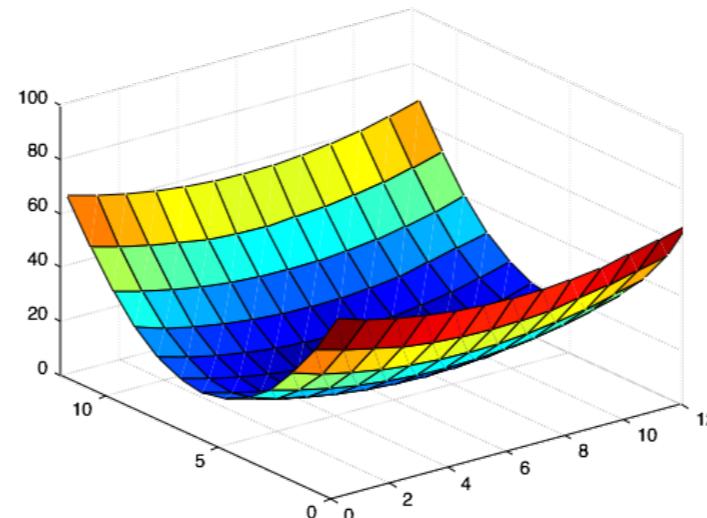
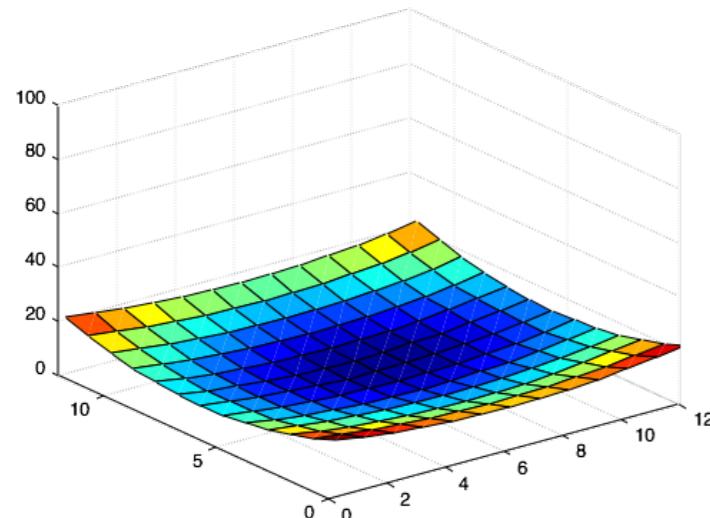
The surface  $E(u,v)$  is locally approximated by a quadratic form

$$E(u,v) \approx [u \ v] M \begin{bmatrix} u \\ v \end{bmatrix}$$

$$M = \sum \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

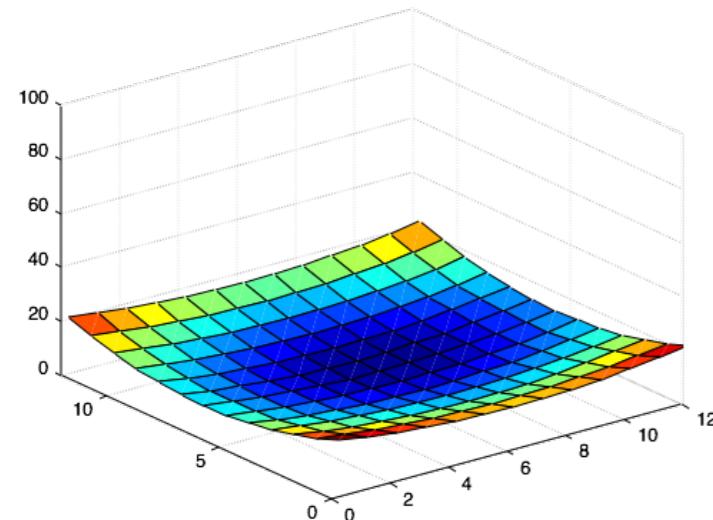


*Which error surface indicates a good image feature?*

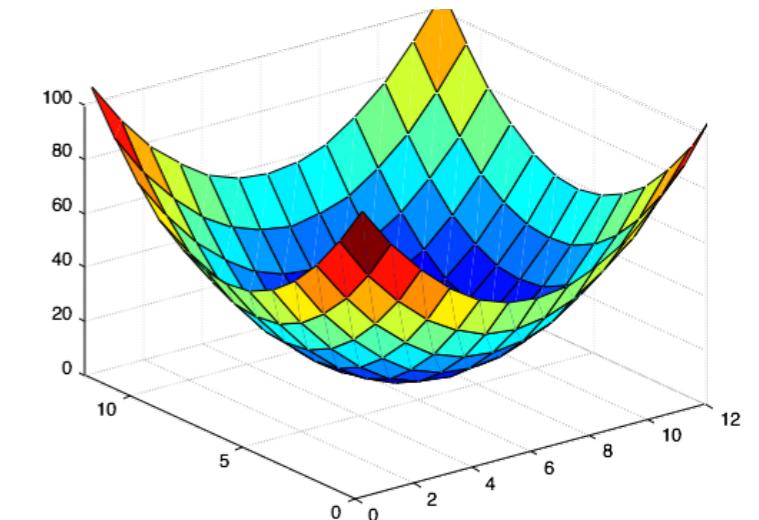
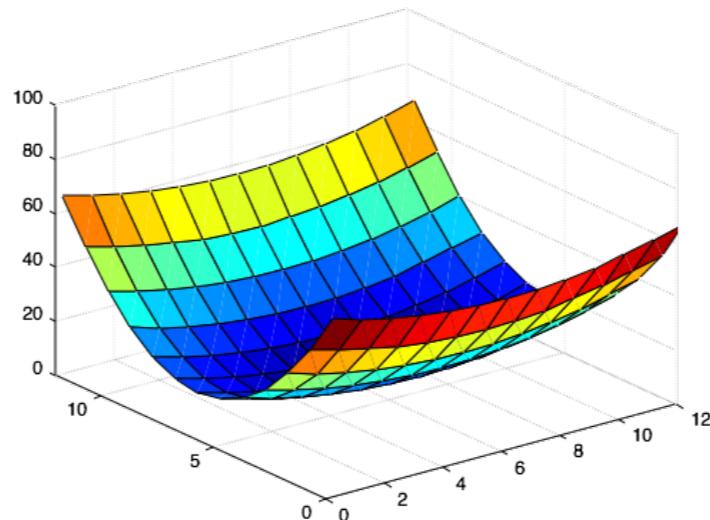


*What kind of image patch do these surfaces represent?*

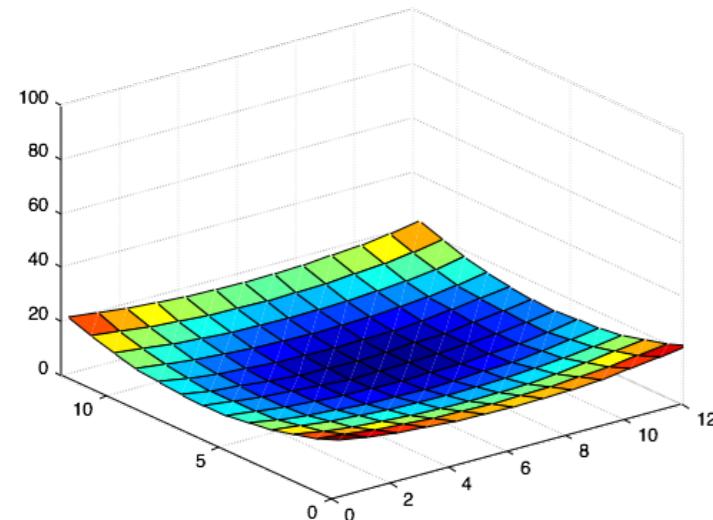
*Which error surface indicates a good image feature?*



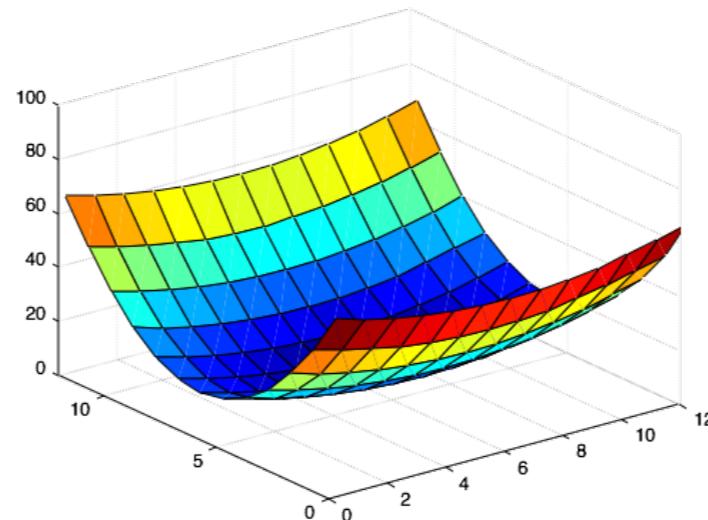
flat



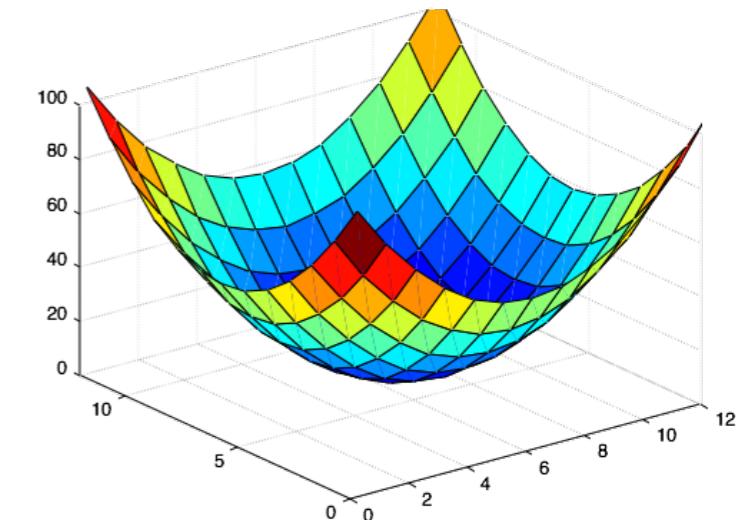
## *Which error surface indicates a good image feature?*



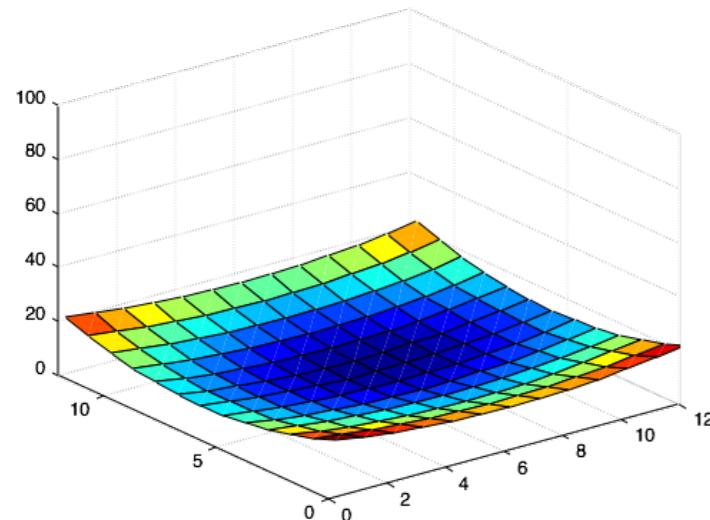
flat



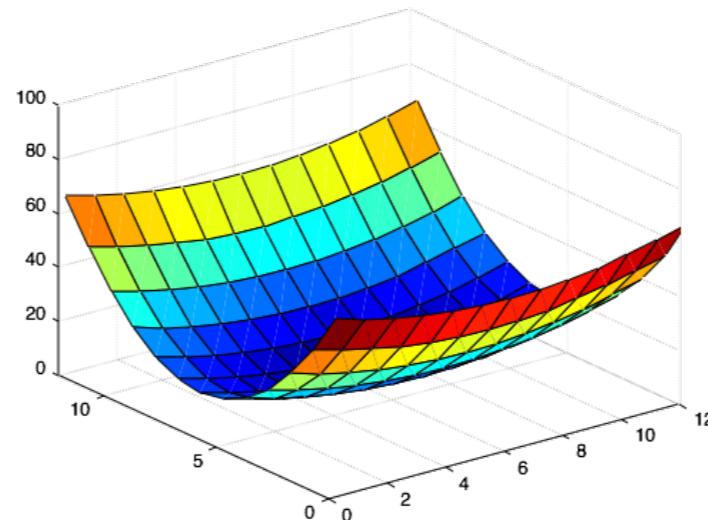
edge  
'line'



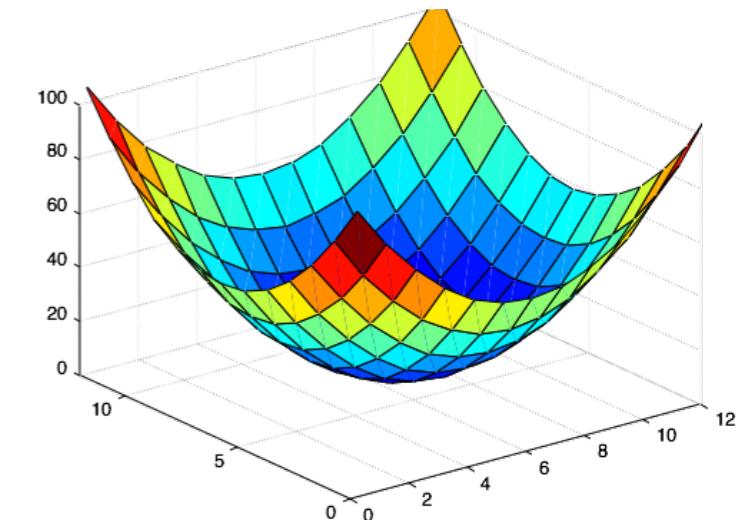
## *Which error surface indicates a good image feature?*



flat

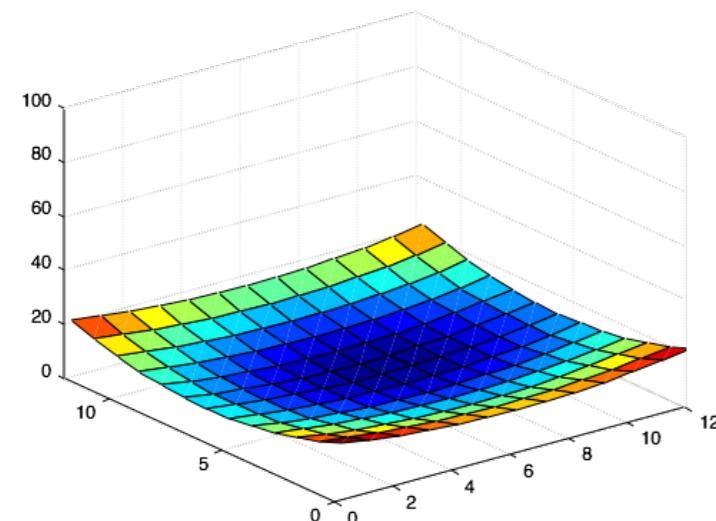


edge  
'line'

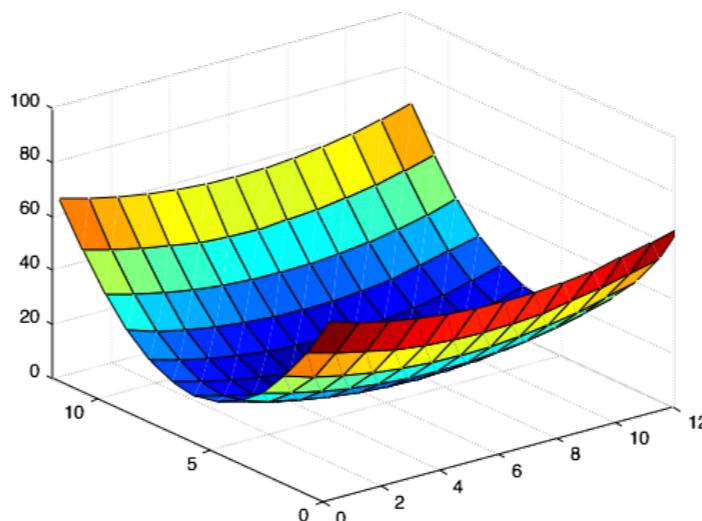


corner  
'dot'

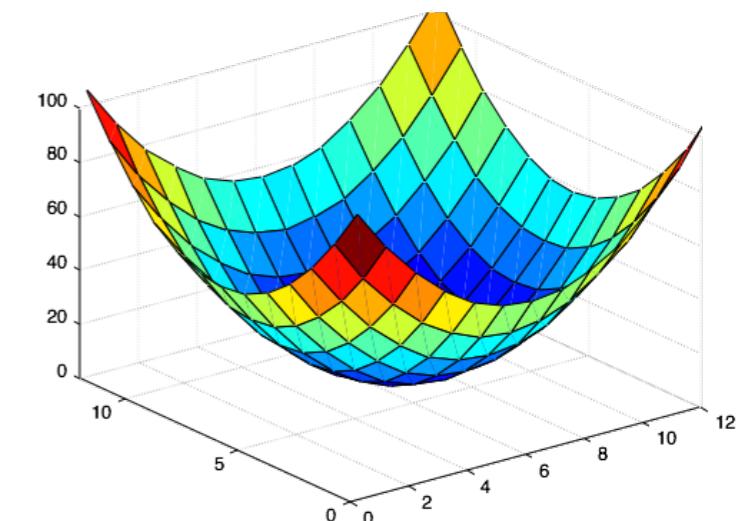
# *Which error surface indicates a good image feature?*



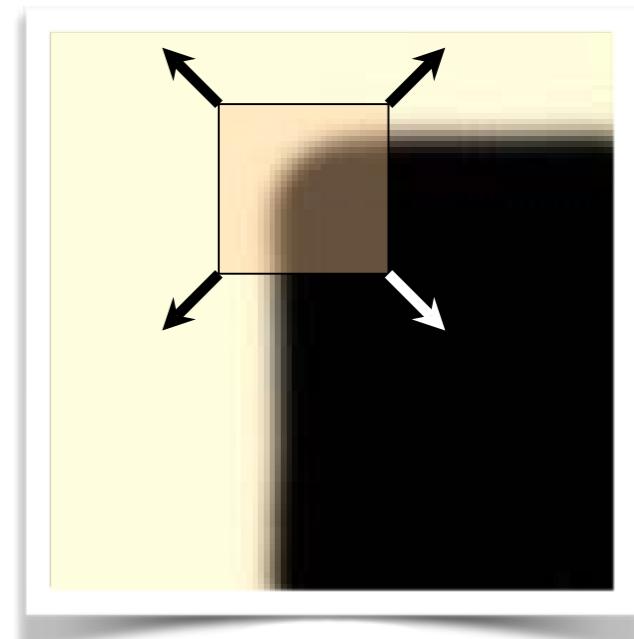
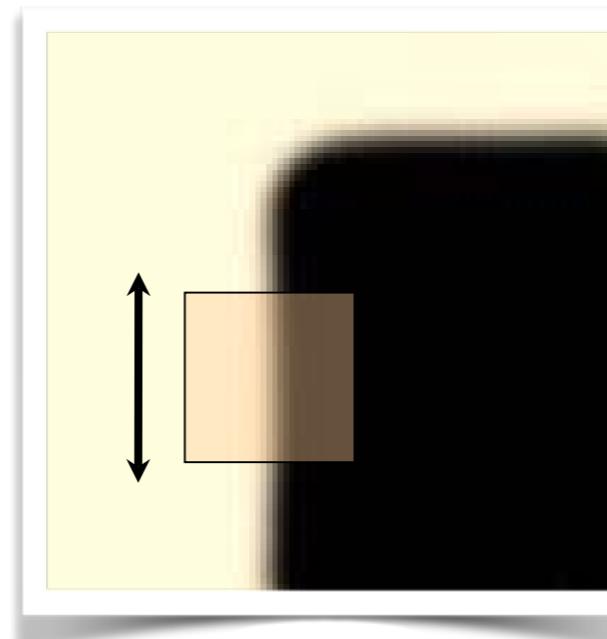
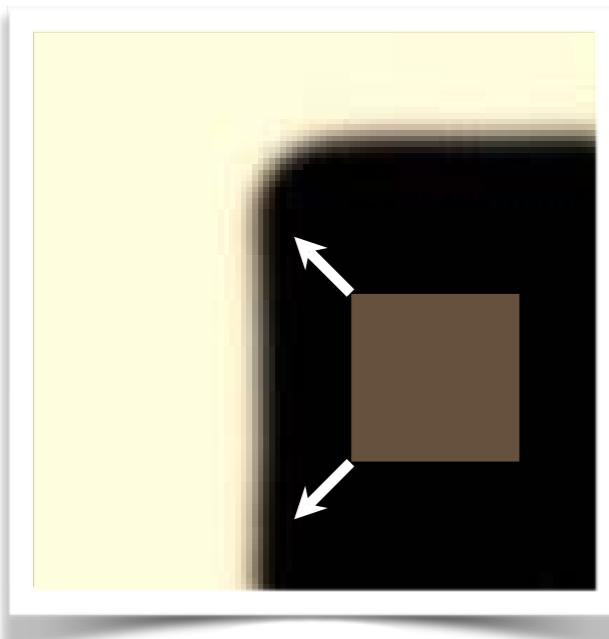
flat



edge  
'line'



corner  
'dot'



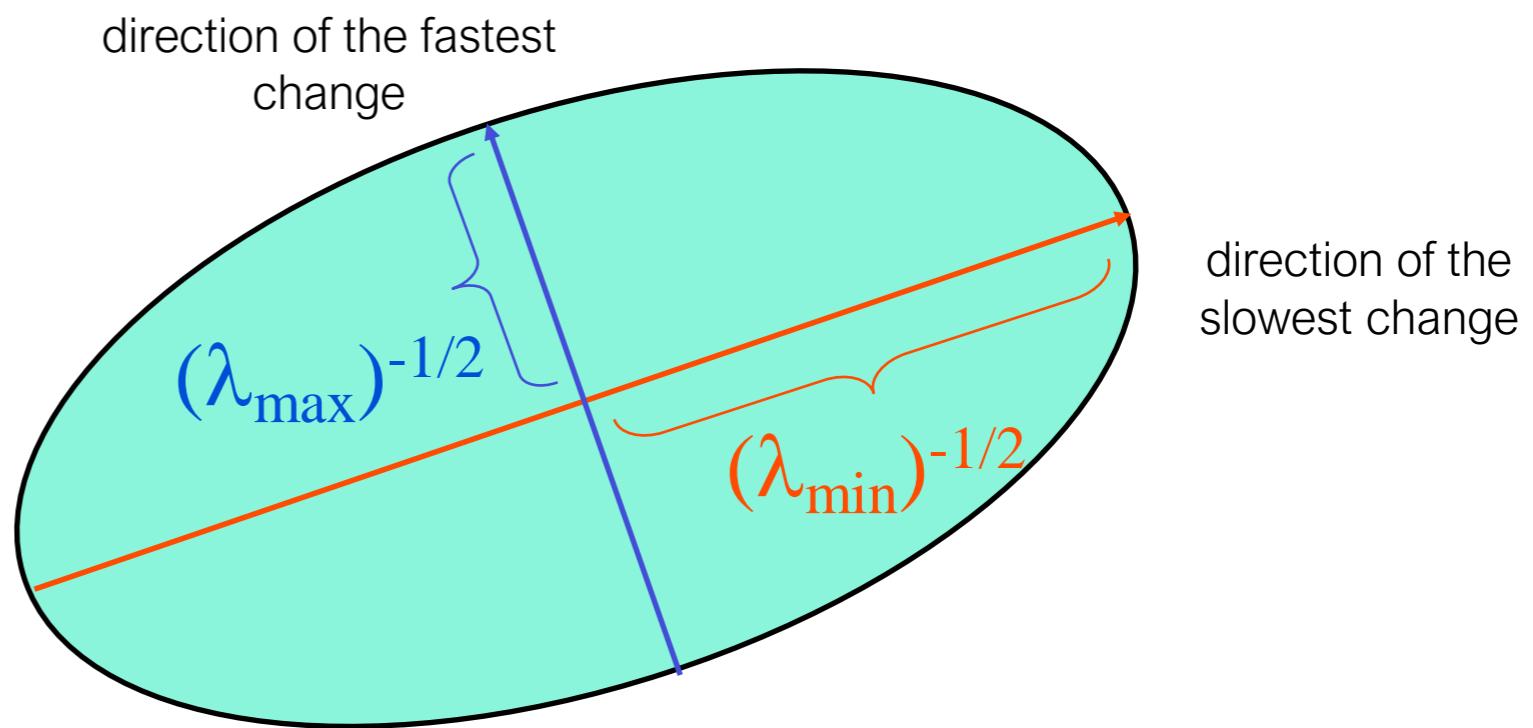
# Visualization as an ellipse

Since  $M$  is symmetric, we have  $M = R^{-1} \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} R$

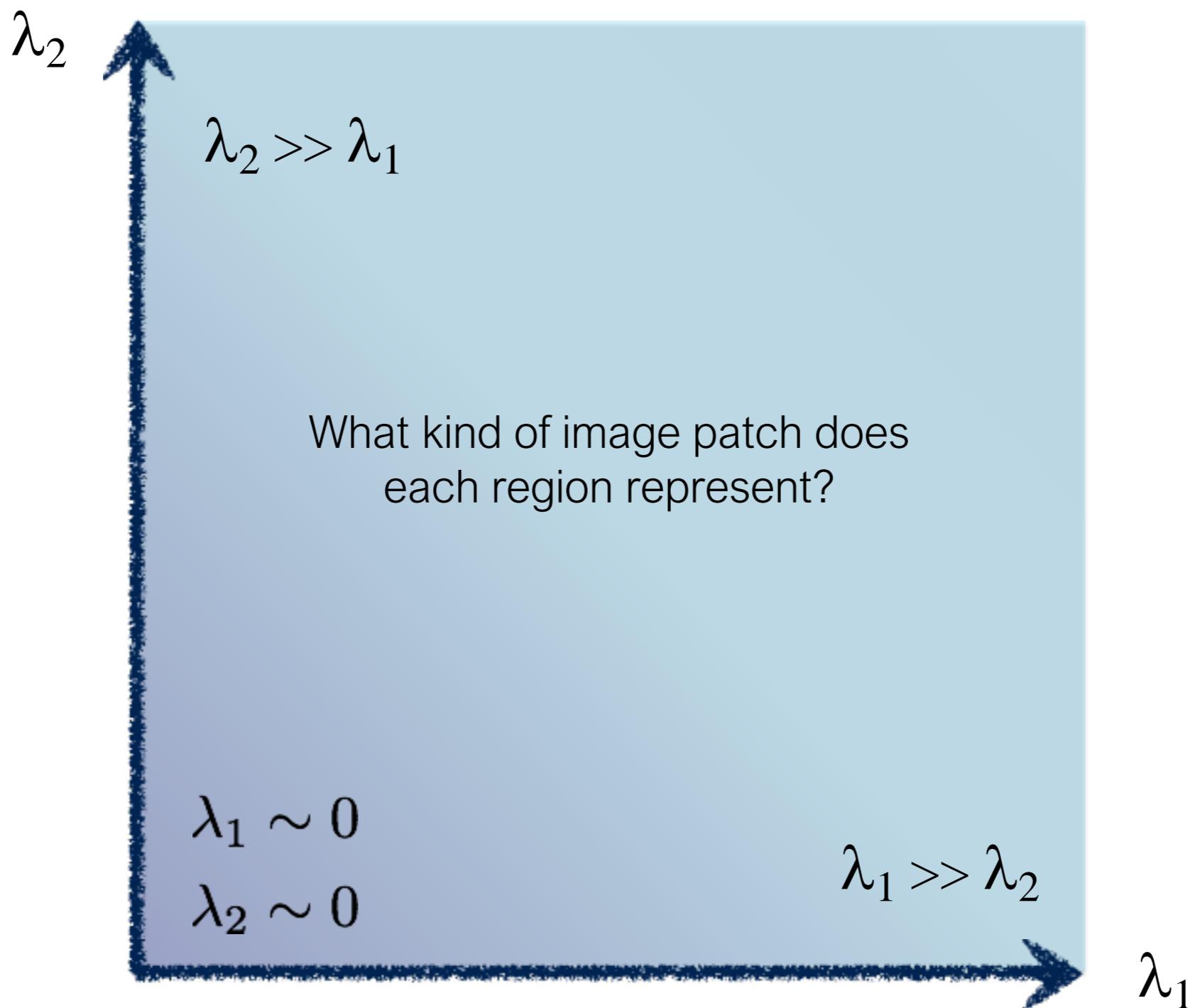
We can visualize  $M$  as an ellipse with axis lengths determined by the eigenvalues and orientation determined by  $R$

Ellipse equation:

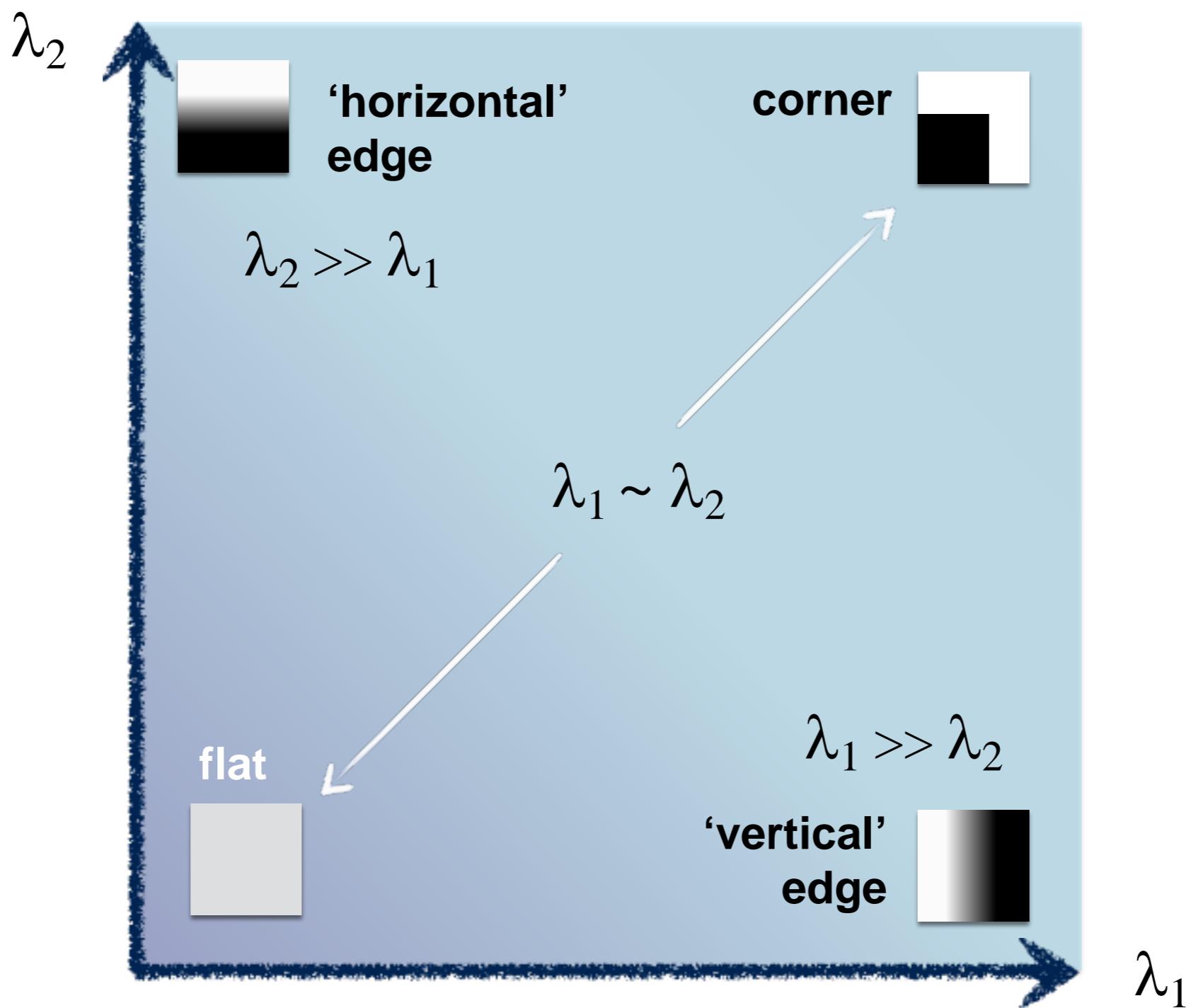
$$[u \ v] M \begin{bmatrix} u \\ v \end{bmatrix} = \text{const}$$



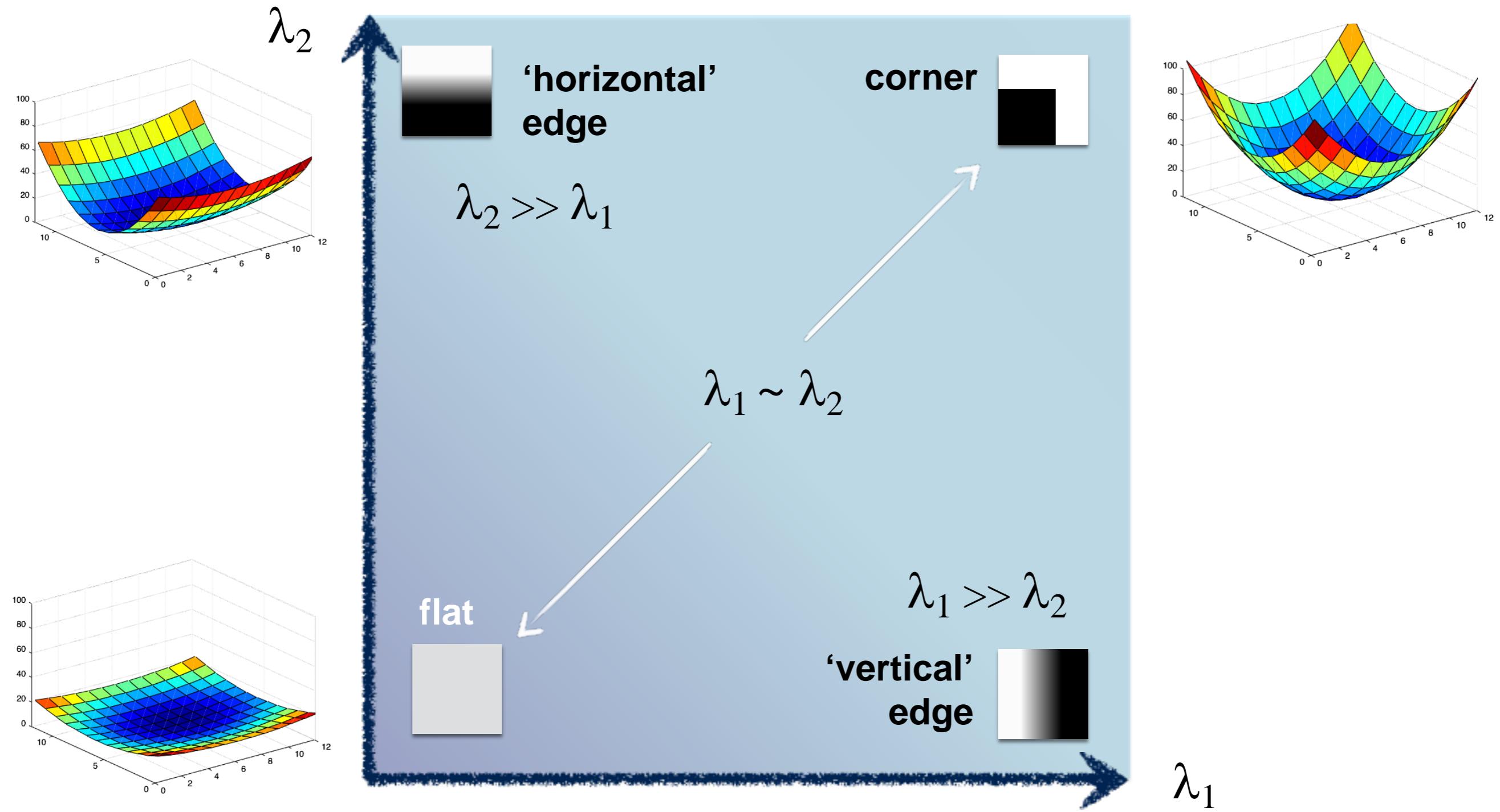
# interpreting eigenvalues



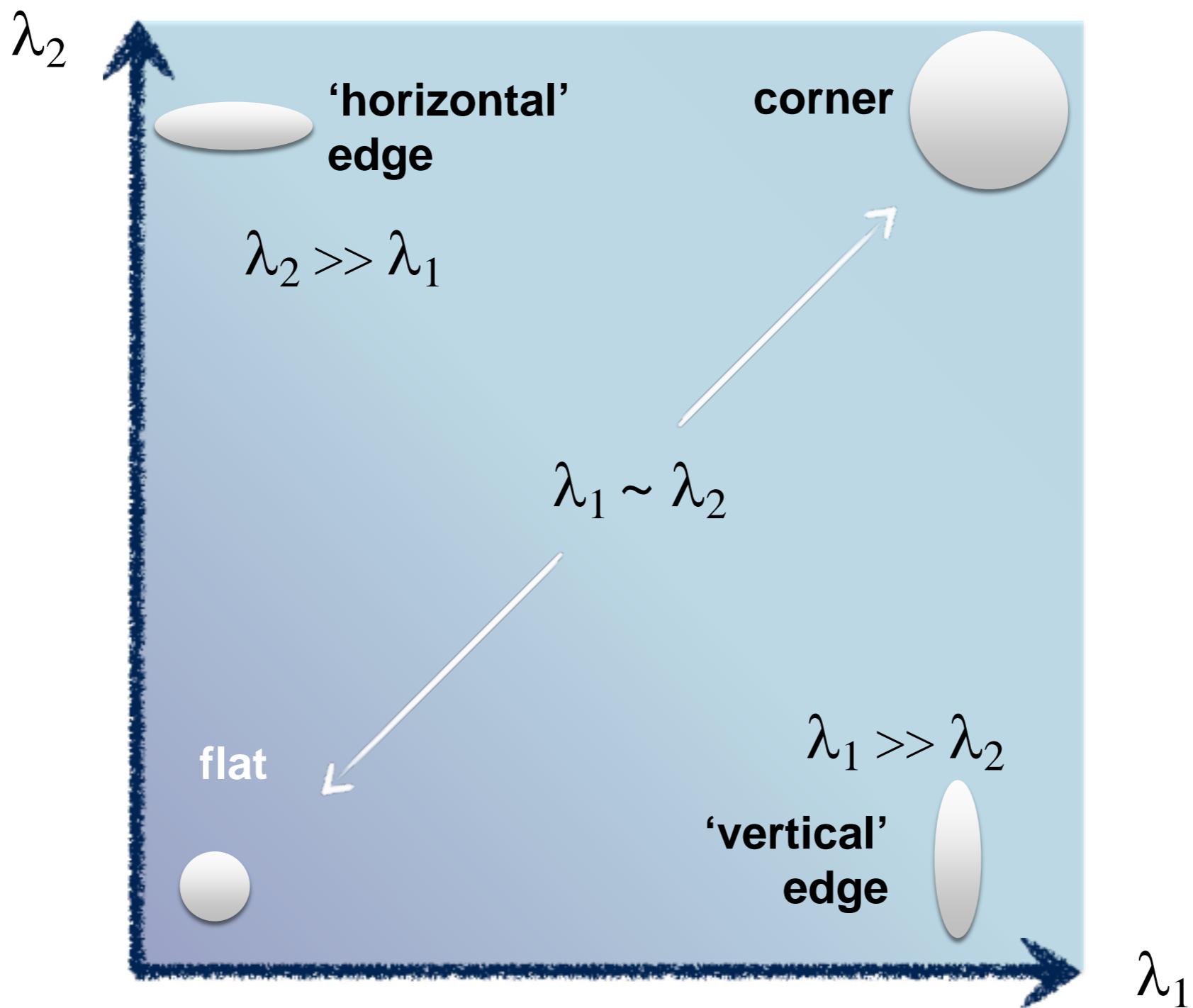
# interpreting eigenvalues



# interpreting eigenvalues

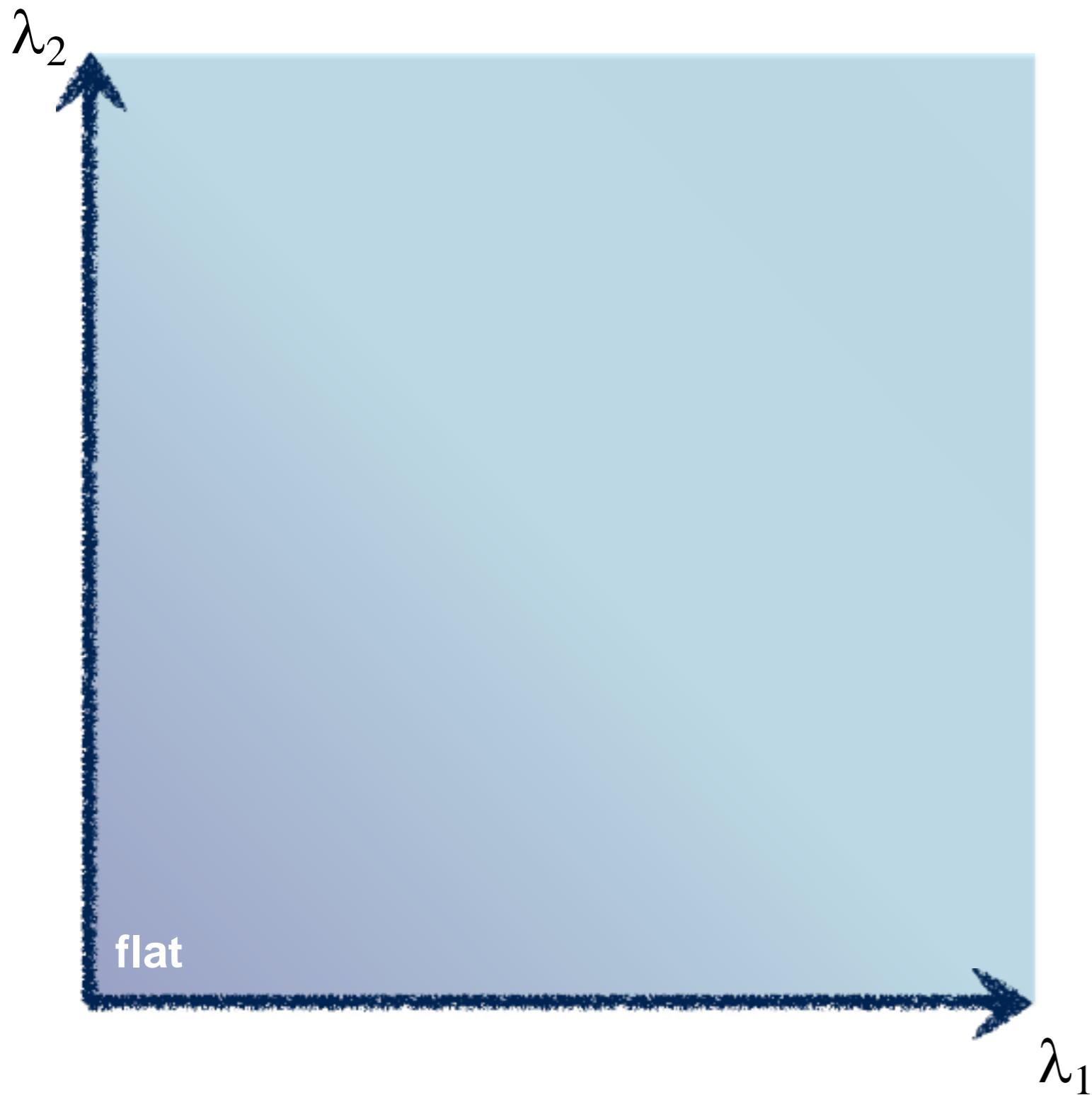


# interpreting eigenvalues



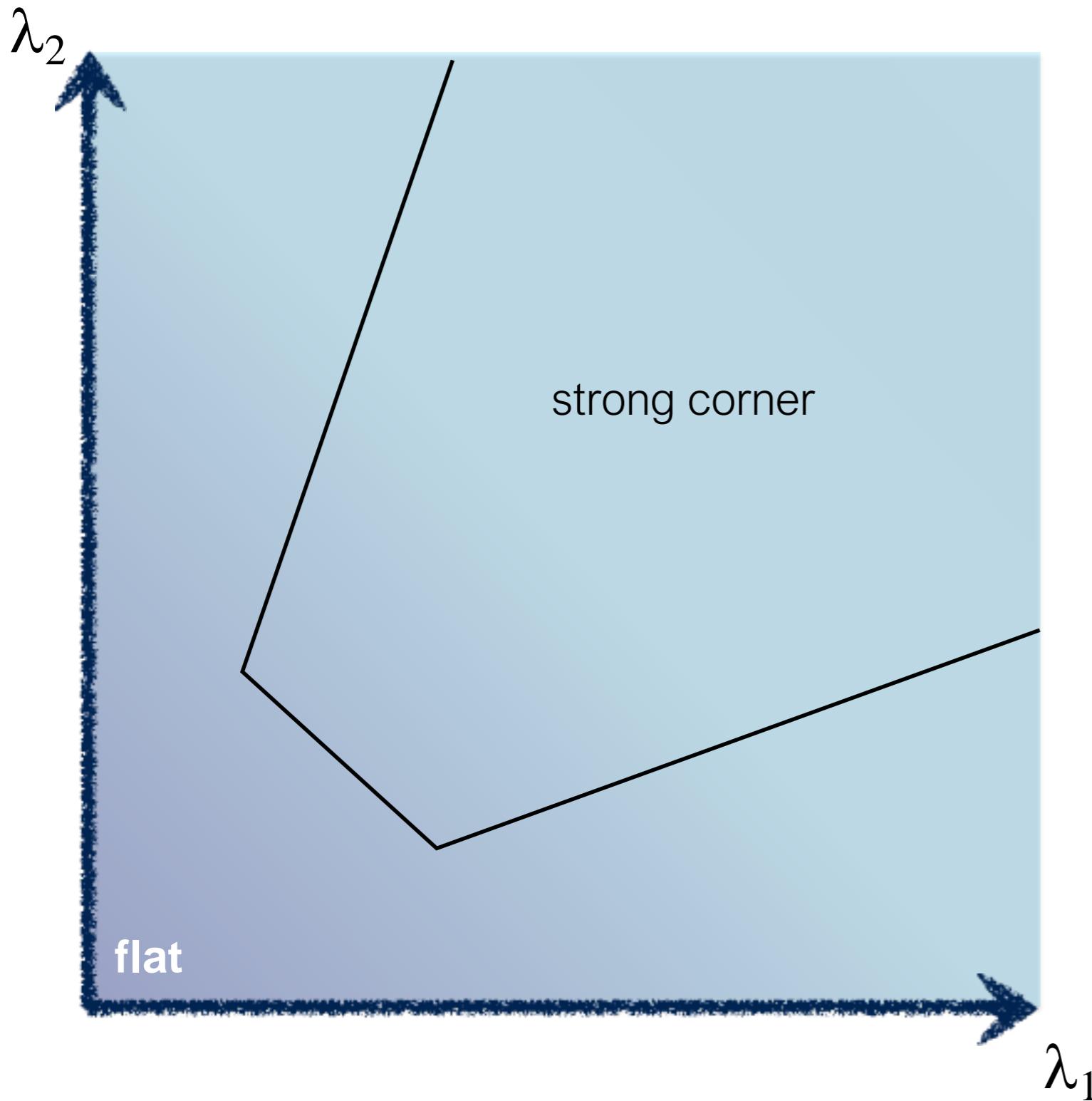
5. Use threshold on eigenvalues to detect corners

## 5. Use threshold on eigenvalues to detect corners



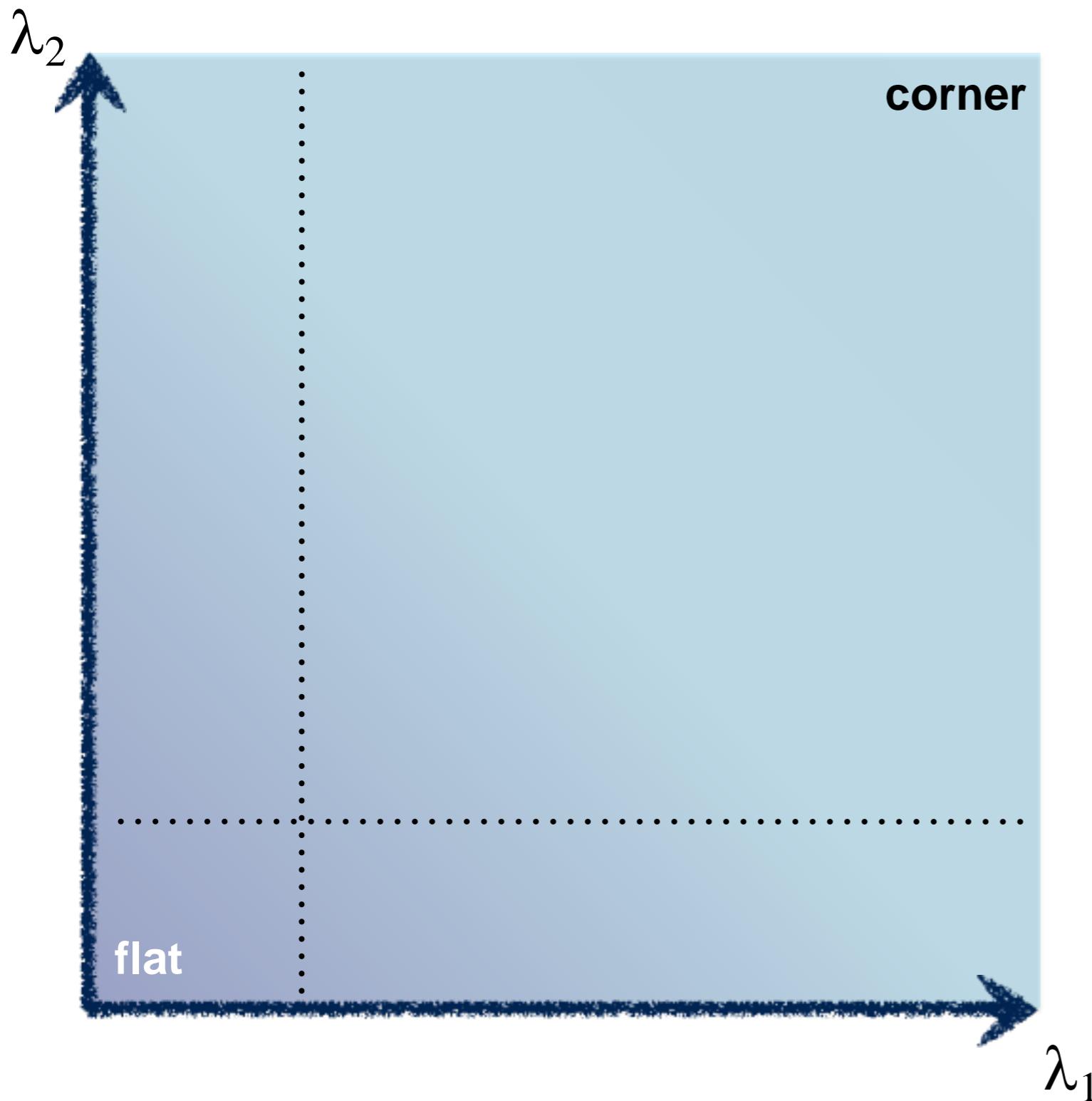
Think of a function to score ‘cornerness’

## 5. Use threshold on eigenvalues to detect corners



Think of a function to score ‘cornerness’

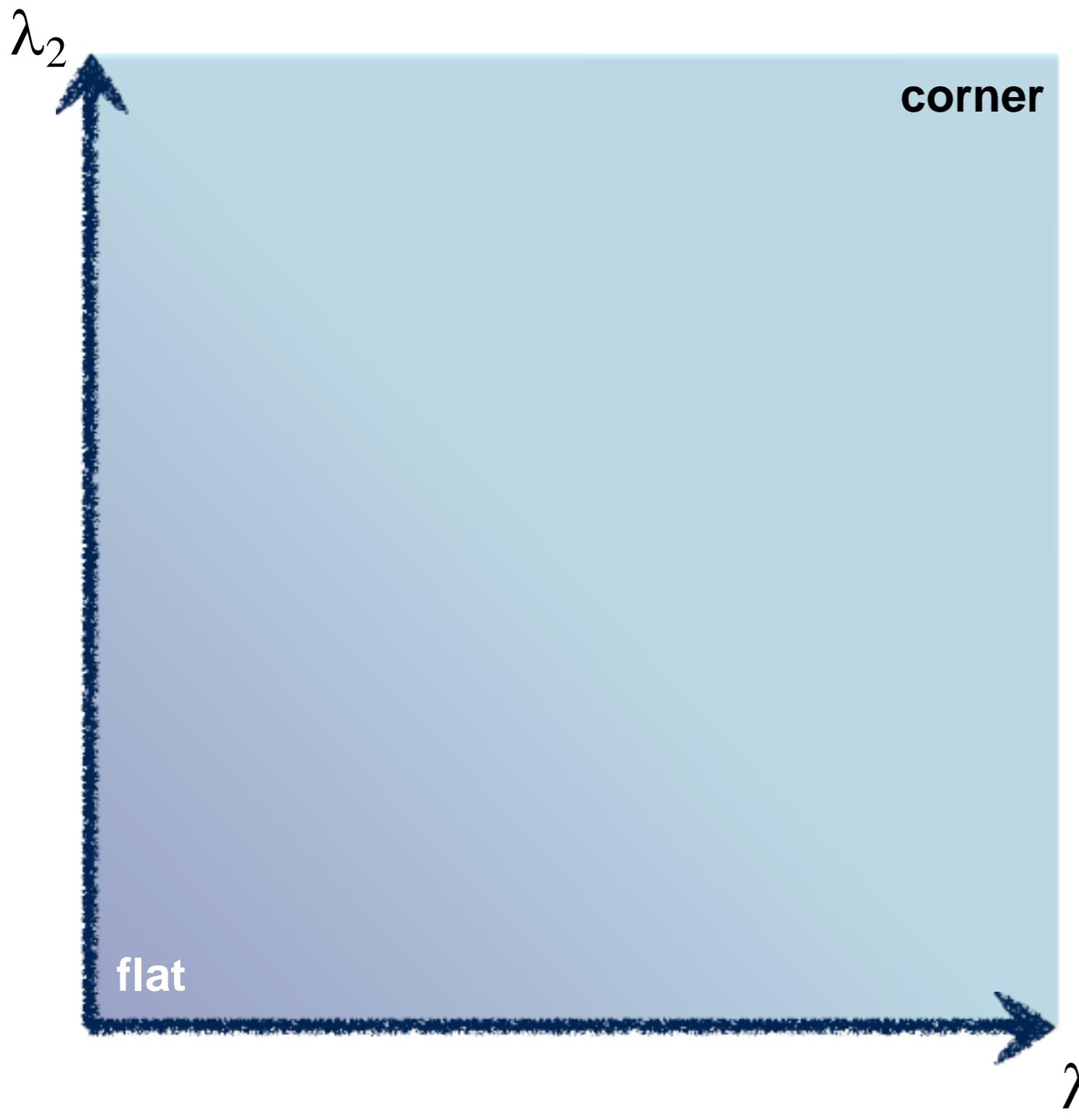
## 5. Use threshold on eigenvalues to detect corners $\wedge$ (a function of )



Use the smallest eigenvalue as the response function

$$R = \min(\lambda_1, \lambda_2)$$

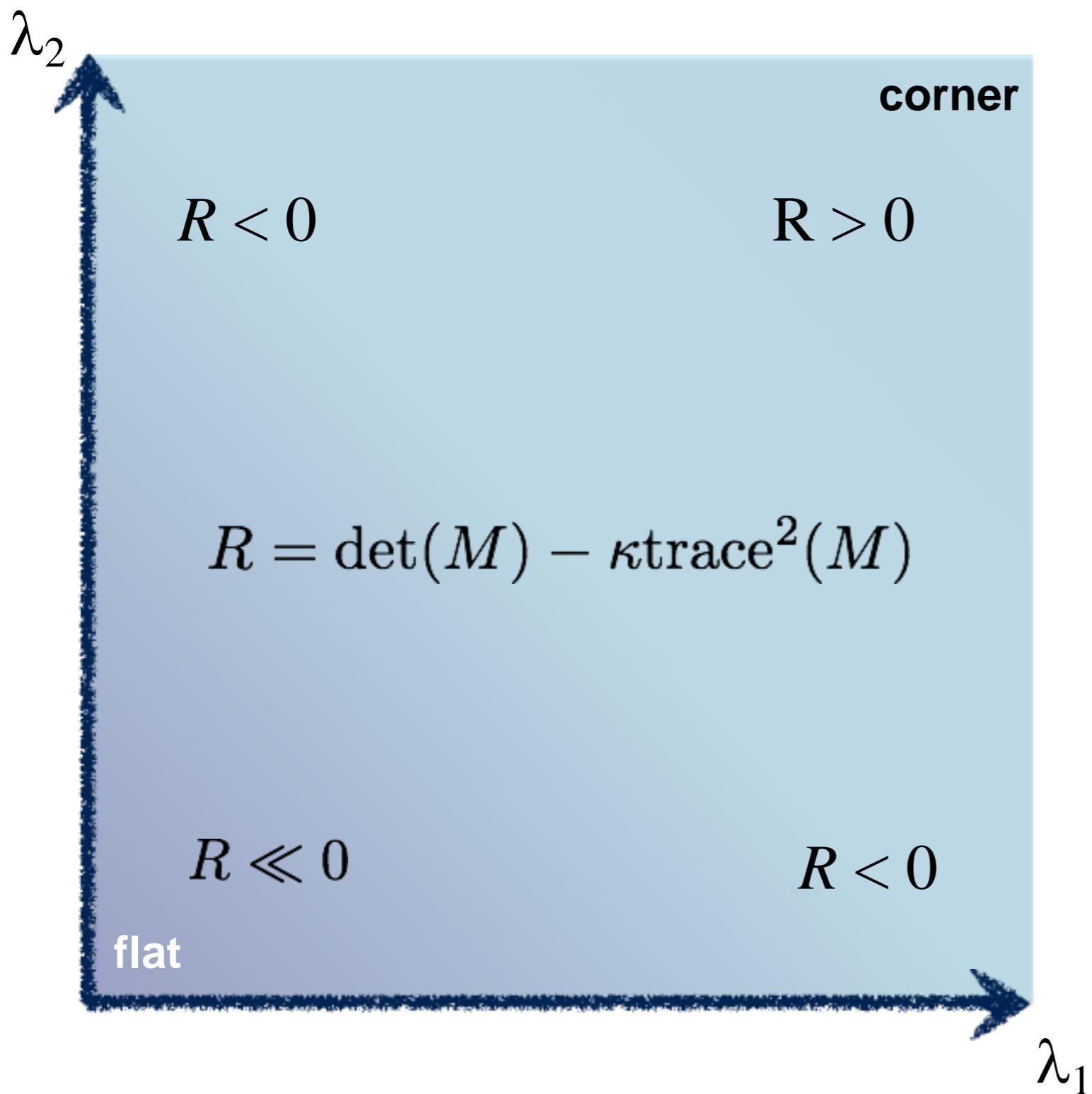
## 5. Use threshold on eigenvalues to detect corners $\wedge$ (a function of )



$$R = \lambda_1 \lambda_2 - \kappa(\lambda_1 + \lambda_2)^2$$

Can compute this more efficiently...

## 5. Use threshold on eigenvalues to detect corners ( $\wedge$ a function of )



$$\det M = \lambda_1 \lambda_2$$

$$\text{trace } M = \lambda_1 + \lambda_2$$

$$\det \begin{pmatrix} a & b \\ c & d \end{pmatrix} = ad - bc$$

$$\text{trace} \begin{pmatrix} a & b \\ c & d \end{pmatrix} = a + d$$

Harris & Stephens (1988)

$$R = \det(M) - \kappa \text{trace}^2(M)$$

Kanade & Tomasi (1994)

$$R = \min(\lambda_1, \lambda_2)$$

Nobel (1998)

$$R = \frac{\det(M)}{\text{trace}(M) + \epsilon}$$

# Harris Detector

C.Harris and M.Stephens. "A Combined Corner and Edge Detector."1988.

1. Compute x and y derivatives of image

$$I_x = G_\sigma^x * I \quad I_y = G_\sigma^y * I$$

2. Compute products of derivatives at every pixel

$$I_{x^2} = I_x \cdot I_x \quad I_{y^2} = I_y \cdot I_y \quad I_{xy} = I_x \cdot I_y$$

3. Compute the sums of the products of derivatives at each pixel

$$S_{x^2} = G_{\sigma'} * I_{x^2} \quad S_{y^2} = G_{\sigma'} * I_{y^2} \quad S_{xy} = G_{\sigma'} * I_{xy}$$

$G_{\sigma'} *$  =Gaussian blur

$G_\sigma^x * G_\sigma^y *$  =Gaussian derivatives

# Harris Detector

C.Harris and M.Stephens. "A Combined Corner and Edge Detector."1988.

4. Define the matrix at each pixel

$$M(x, y) = \begin{bmatrix} S_{x^2}(x, y) & S_{xy}(x, y) \\ S_{xy}(x, y) & S_{y^2}(x, y) \end{bmatrix}$$

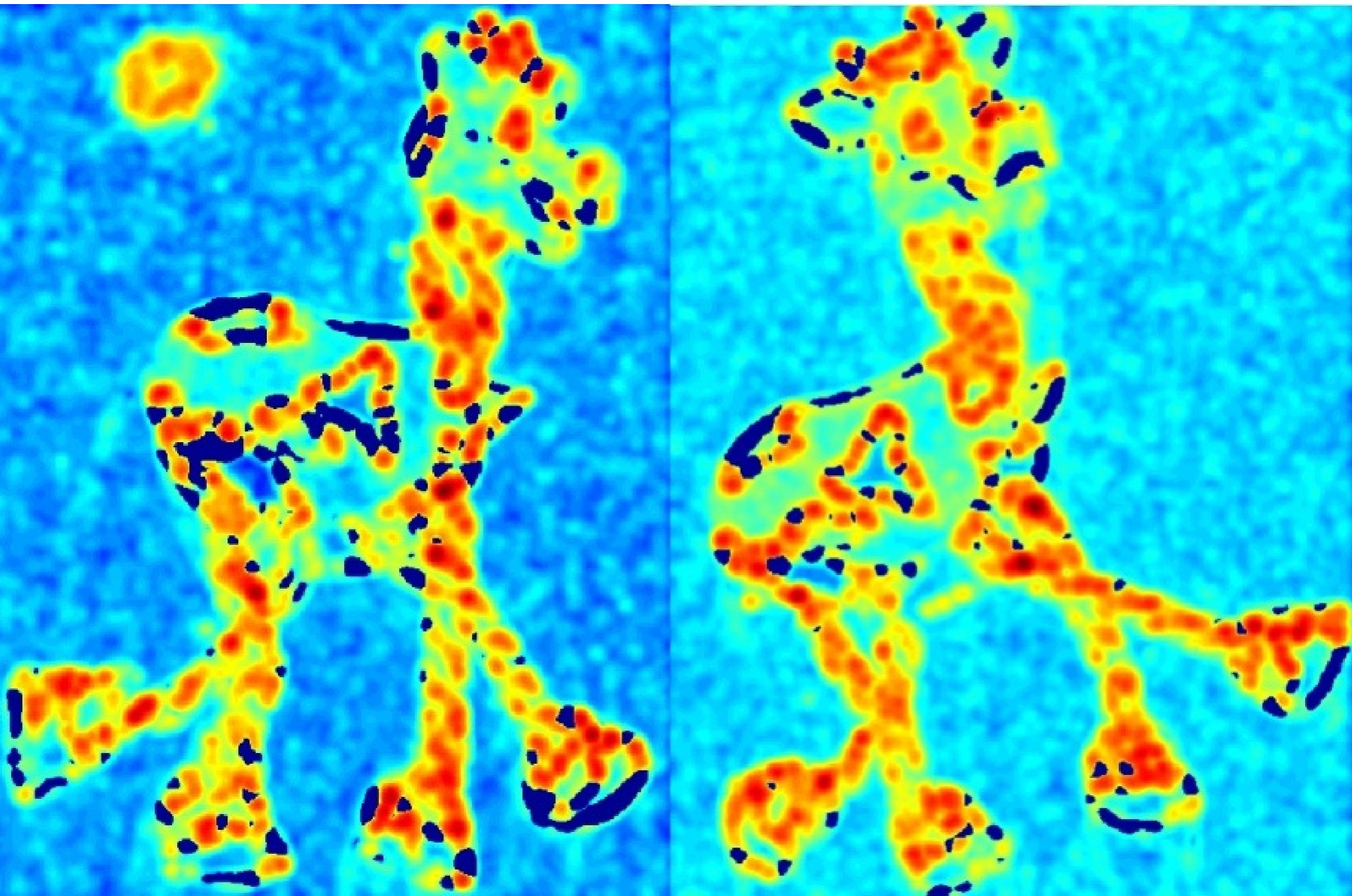
5. Compute the response of the detector at each pixel

$$R = \det M - k(\text{trace} M)^2$$

6. Threshold on value of R; compute non-max suppression.

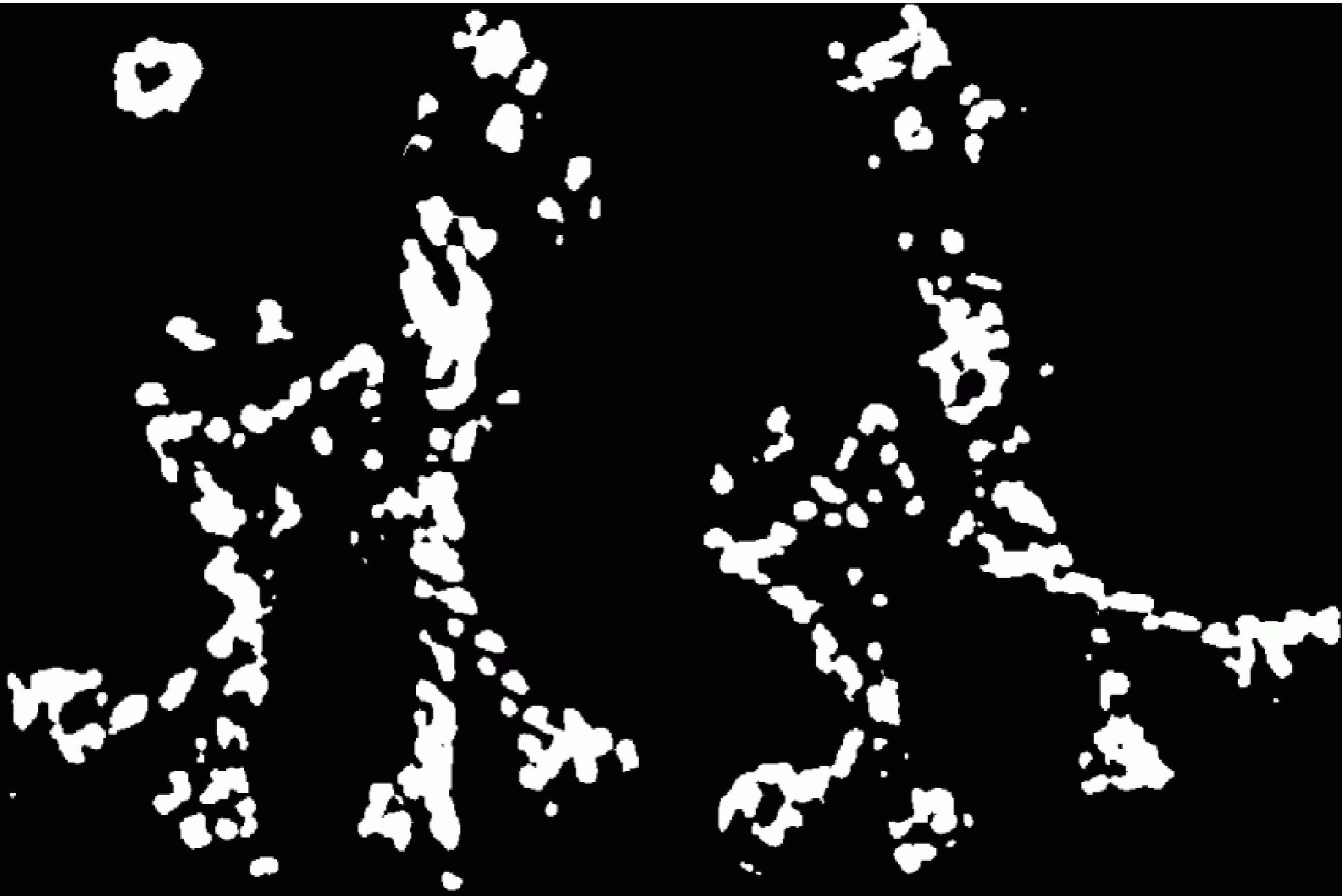


# Corner response

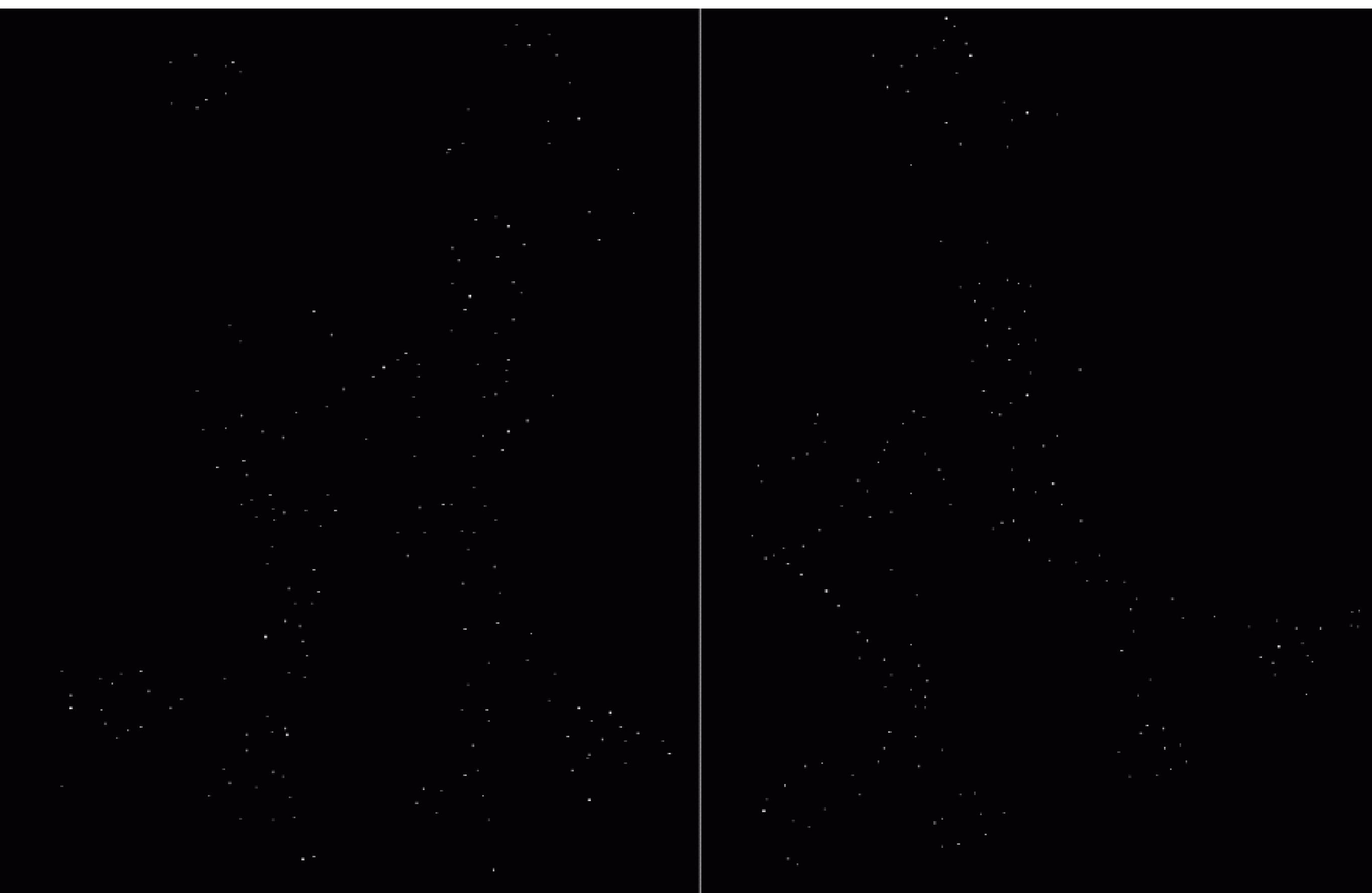




# Thresholded corner response

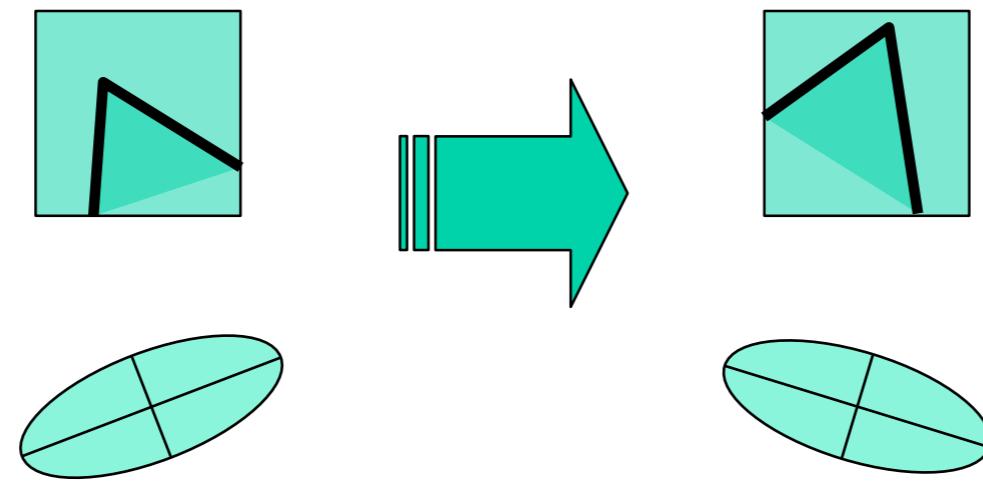


# Non-maximal suppression





# Harris corner response is invariant to rotation



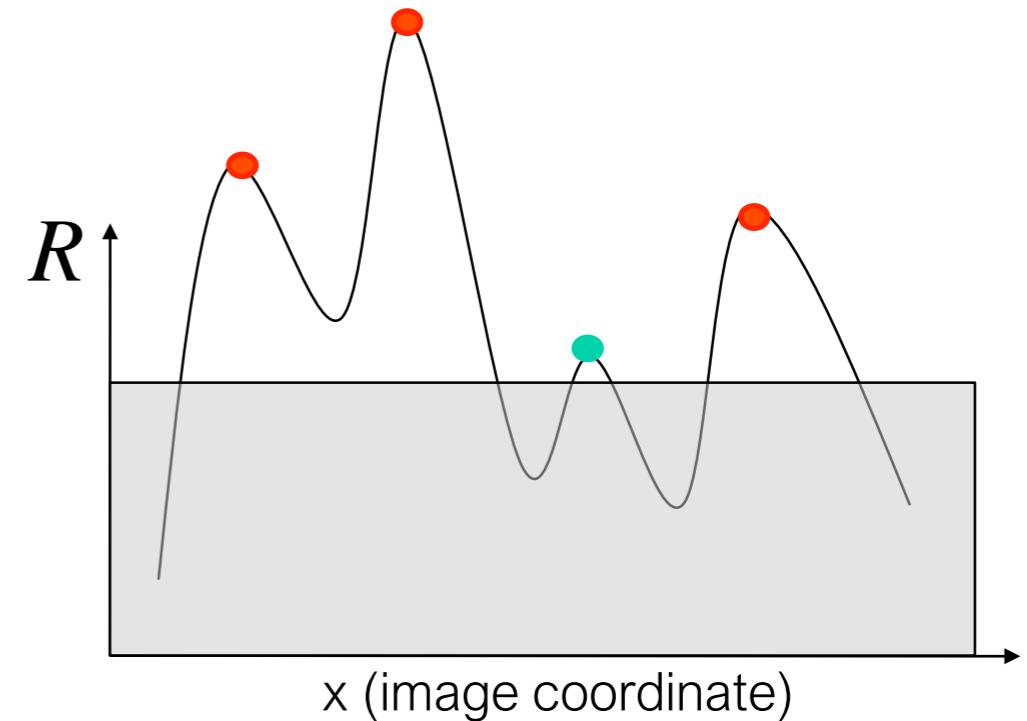
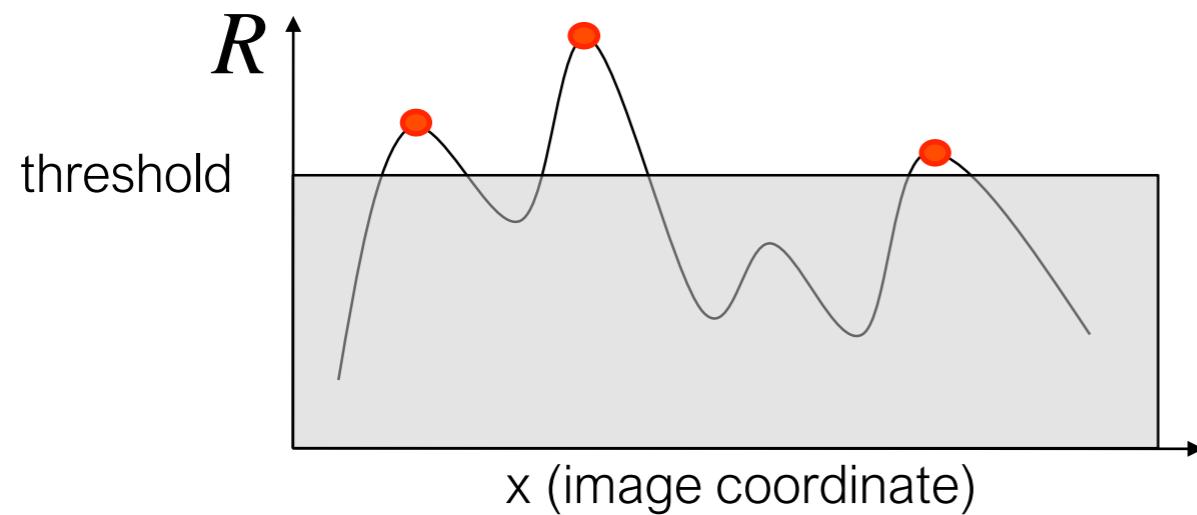
Ellipse rotates but its shape  
**(eigenvalues)** remains the same

**Corner response R is invariant to image rotation**

# Harris corner response is invariant to intensity changes

Partial invariance to *affine intensity* change

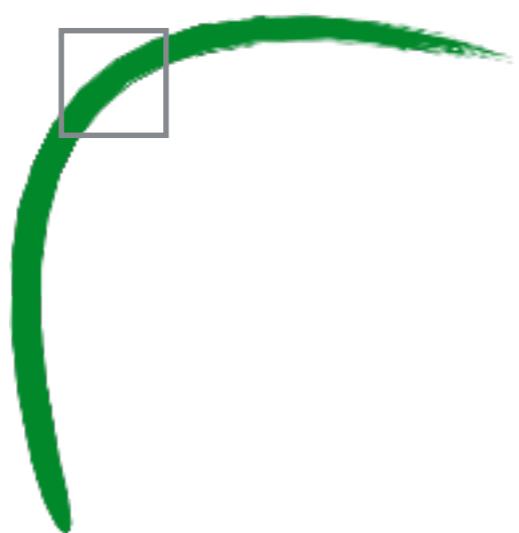
- Only derivatives are used => invariance to intensity shift  $I \rightarrow I + b$
- Intensity scale:  $I \rightarrow a I$



The Harris detector is not invariant to changes in ...

The Harris corner detector is not  
invariant to scale

edge!



corner!



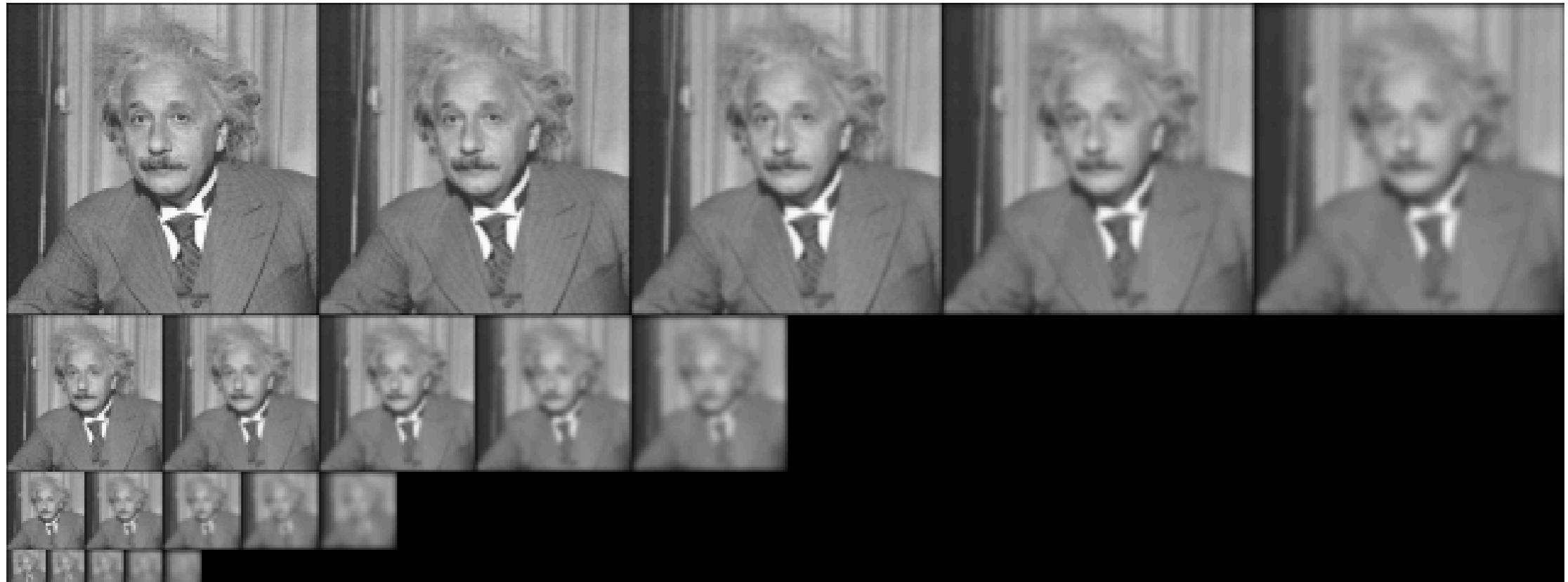
How can we make a feature detector scale-invariant?

# Laplacian feature detection

So far we have seen the detection of corners. Now let's discuss a simpler type of feature.

Laplacian.

What does this filter like?



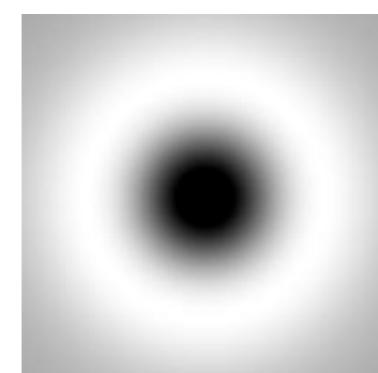
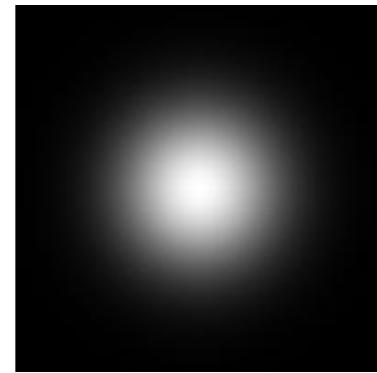
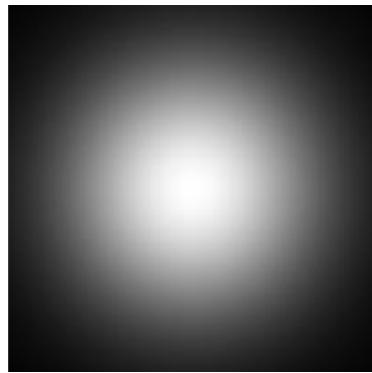
Gaussian



Laplacian

# Difference of Gaussians

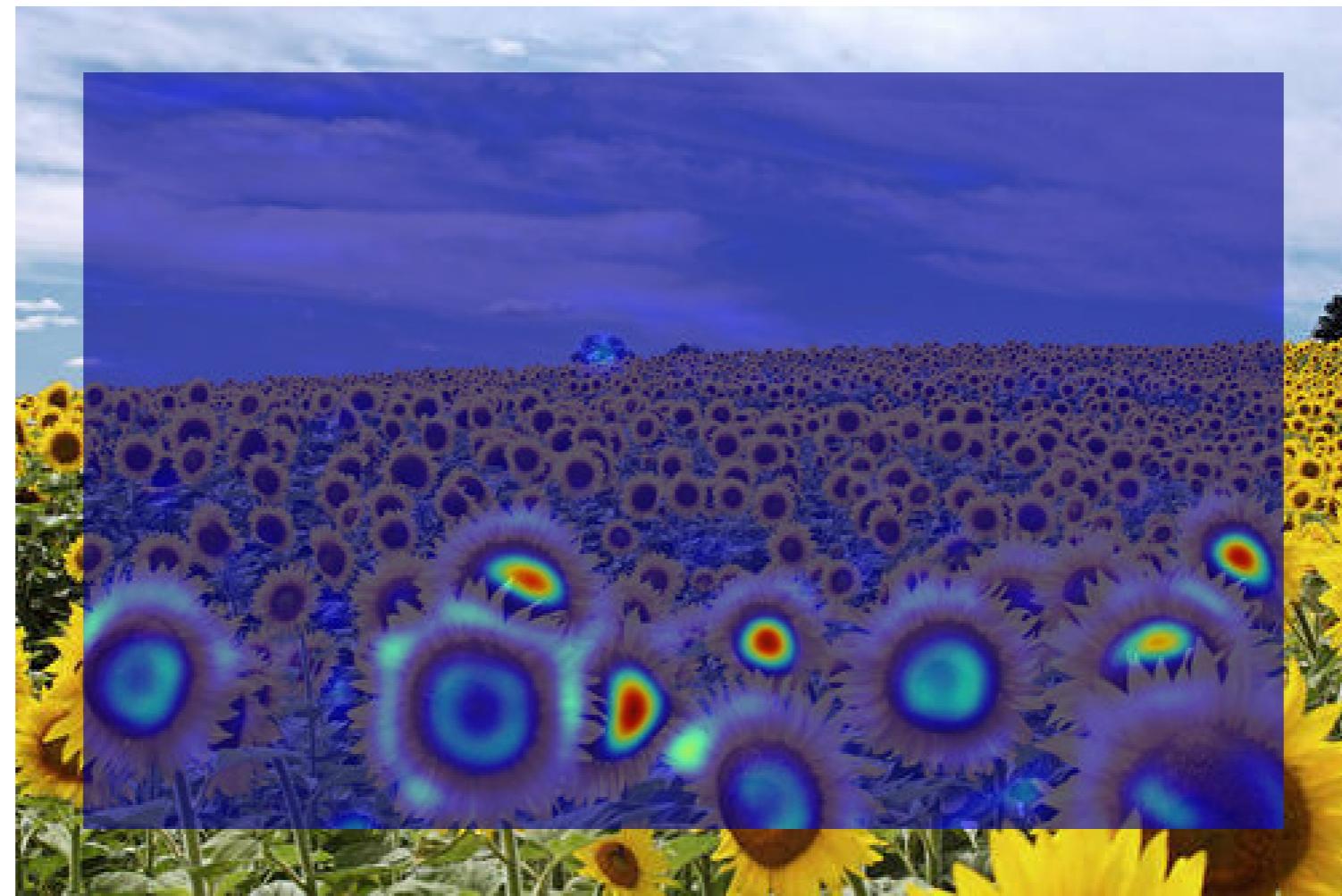
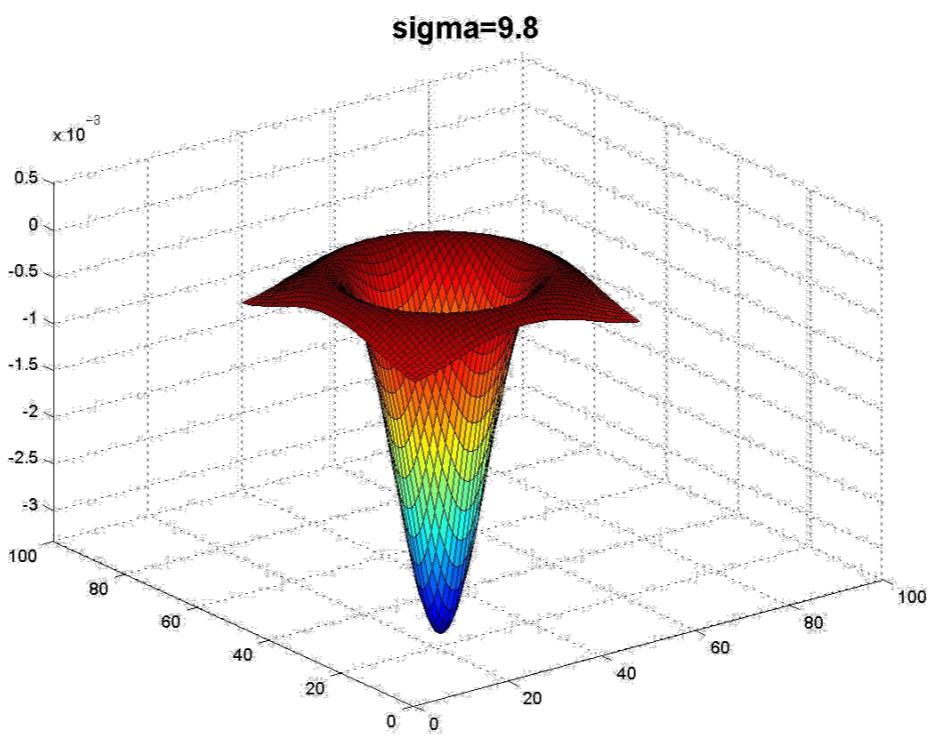
$$G_{\sigma_1} * I - G_{\sigma_2} * I = (G_{\sigma_1} - G_{\sigma_2}) * I$$



Laplacian

Q: What does this filter like?

Blobby circular image areas



# Multi-scale detection

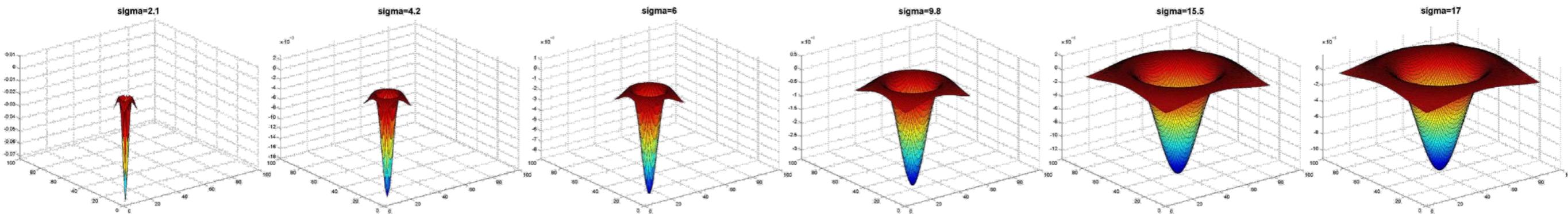
How can we make a feature detector scale-invariant?

How can we automatically select the scale?

# Multi-scale blob detection



# What happens if you apply different Laplacian filters?



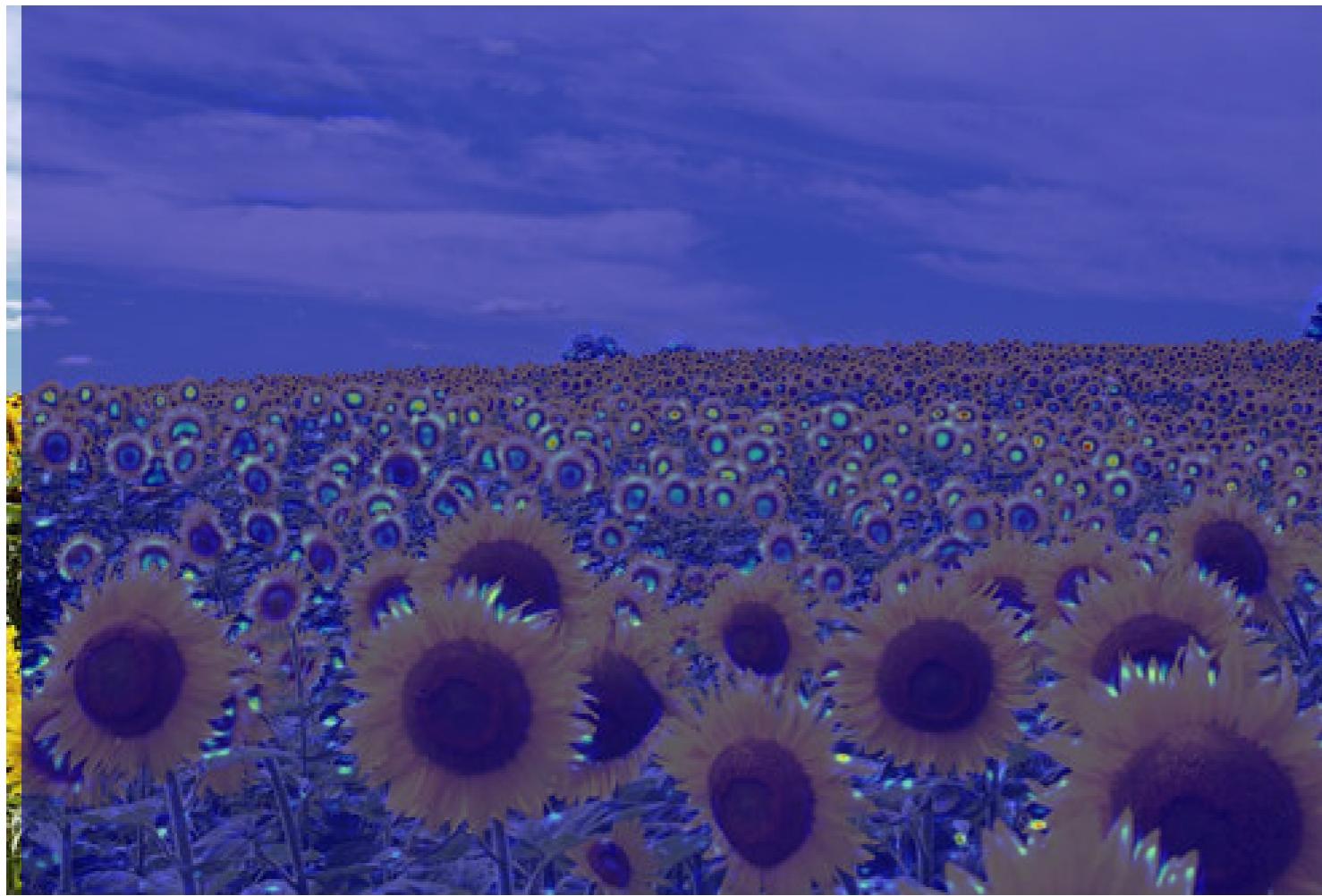
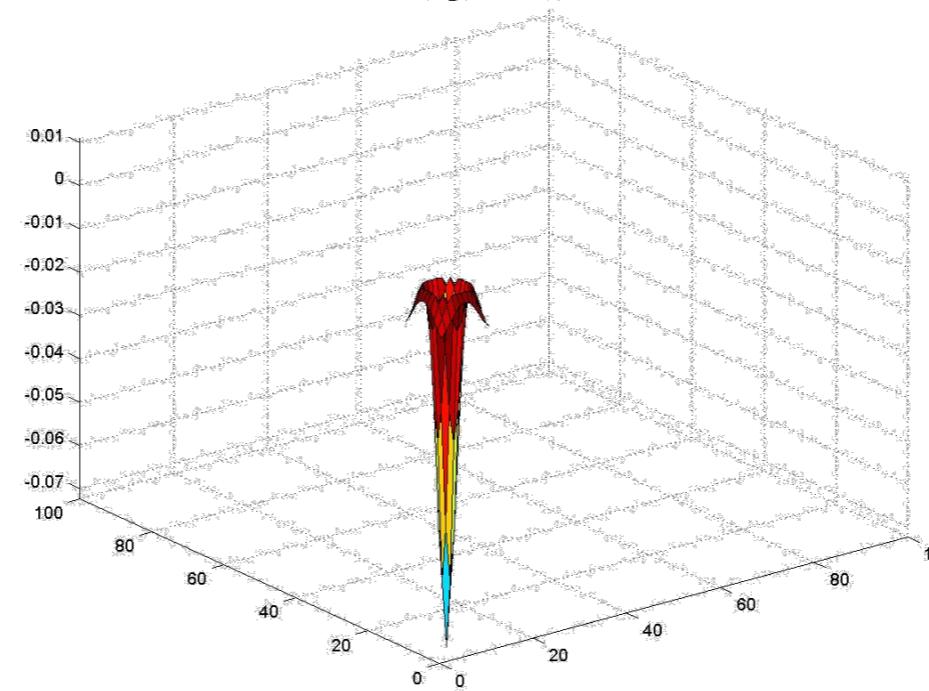
Full size



3/4 size

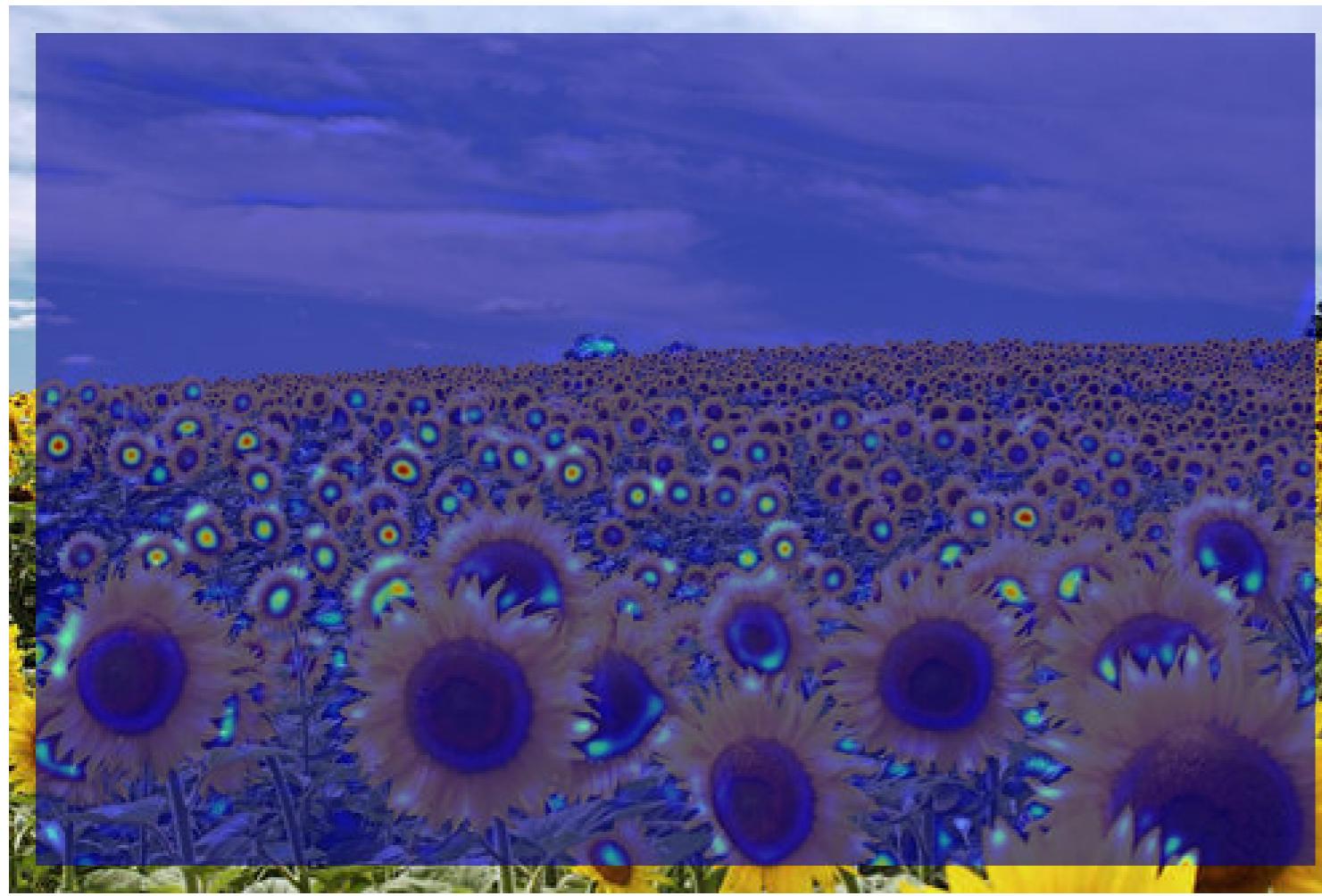
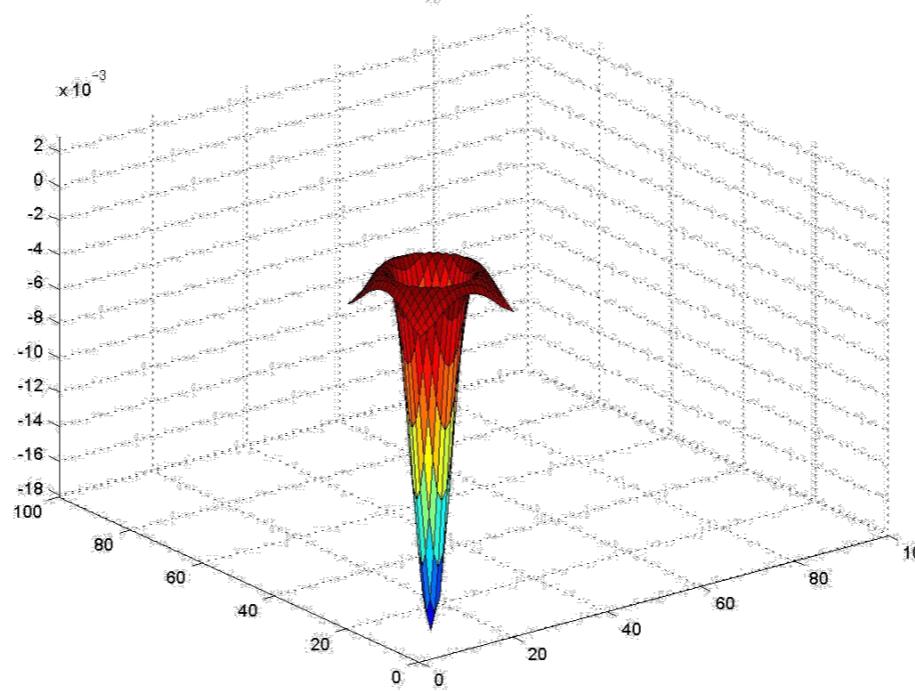


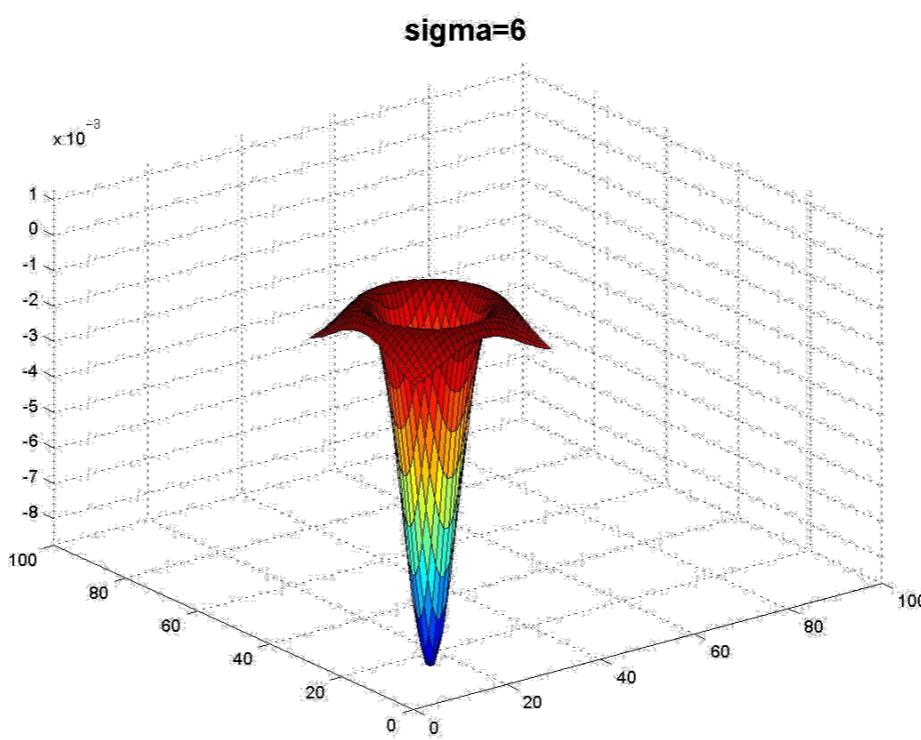
**sigma=2.1**

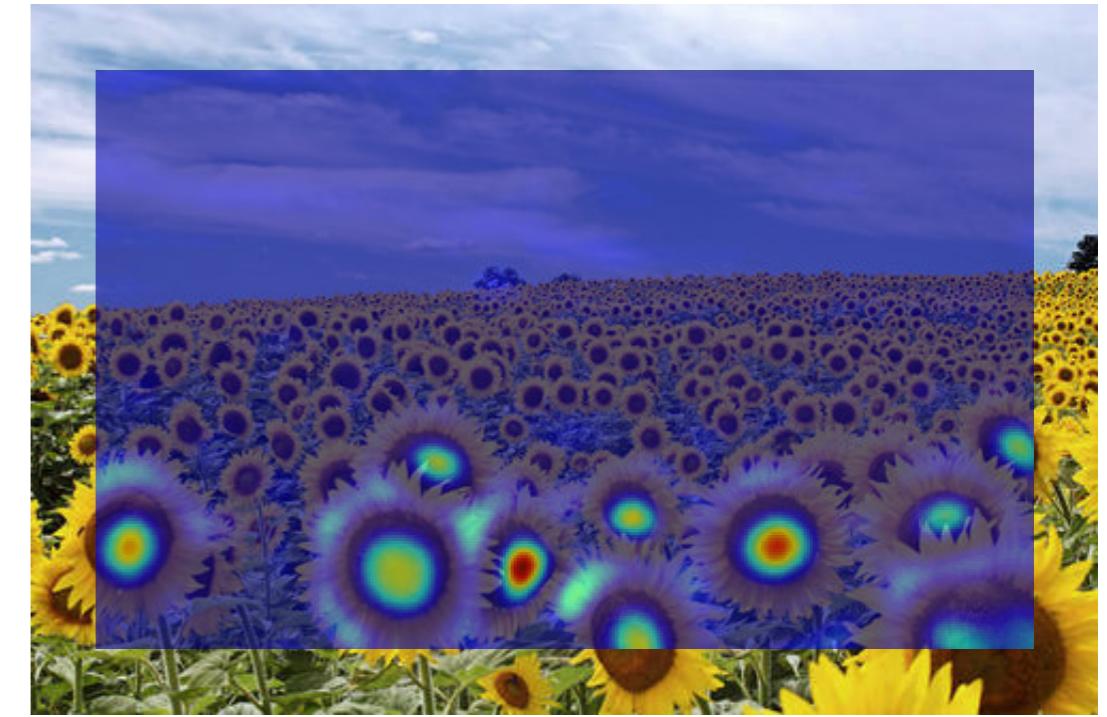
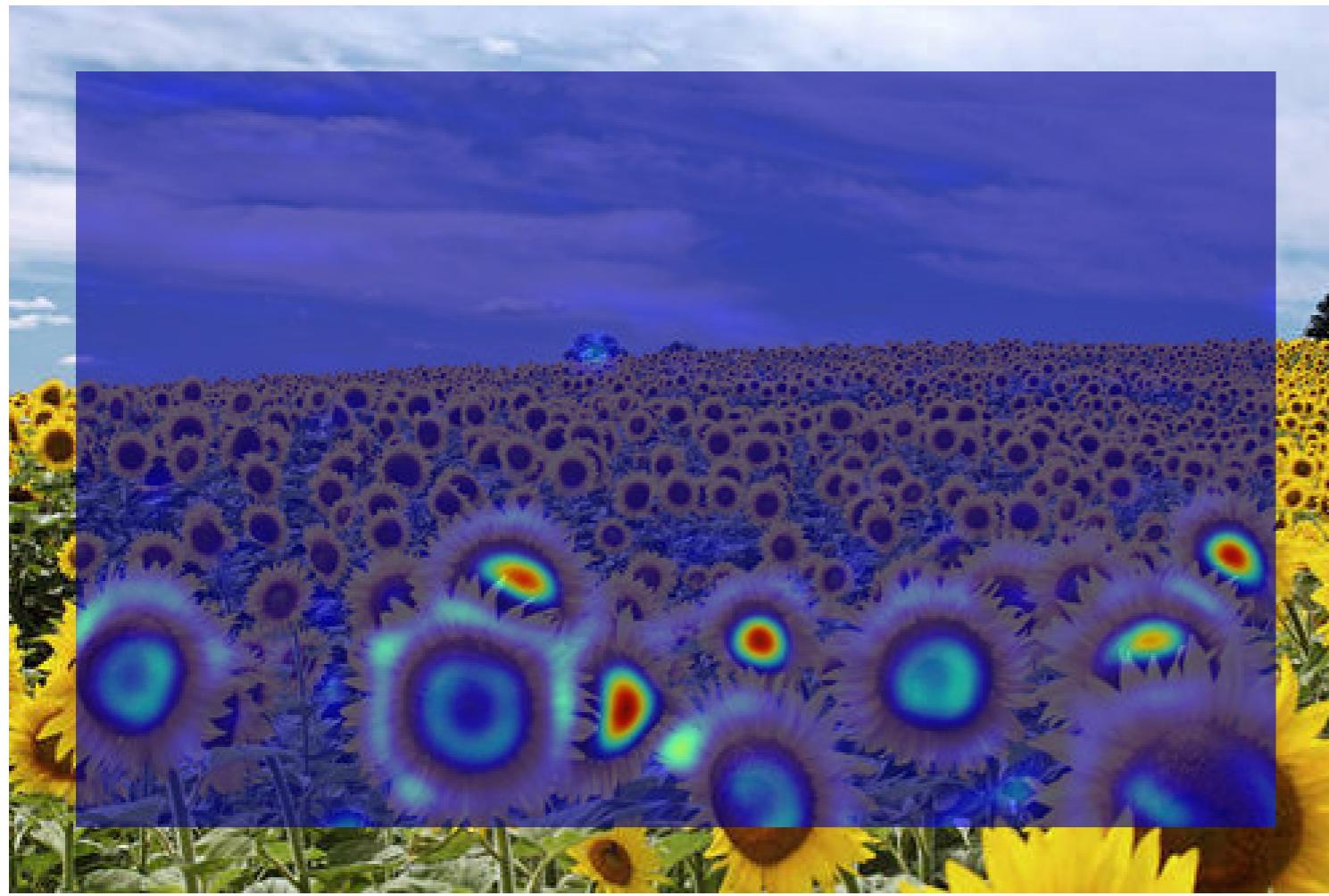
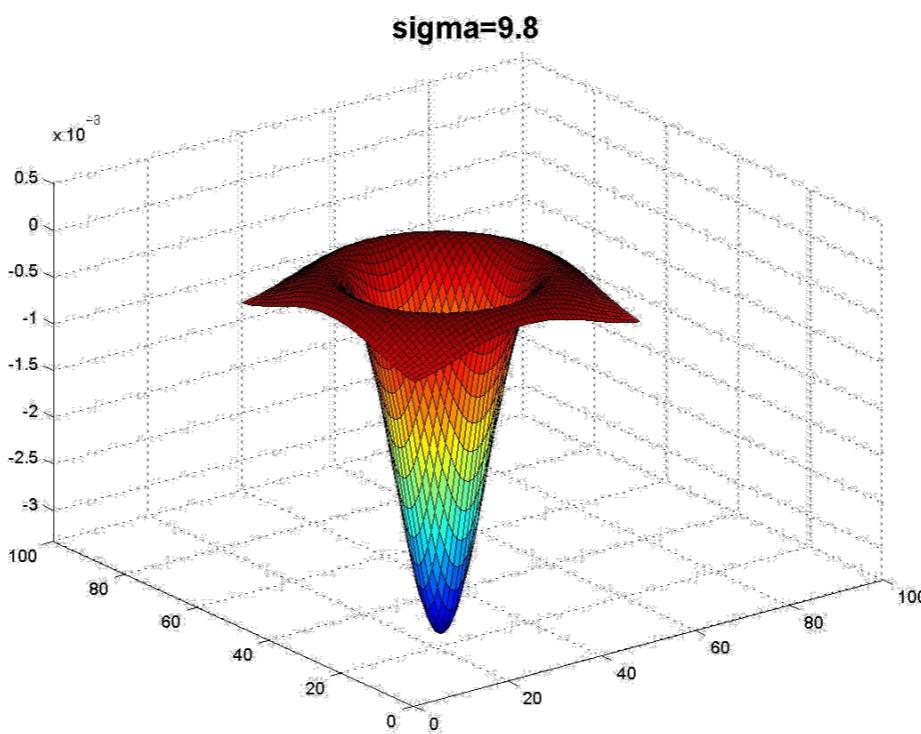


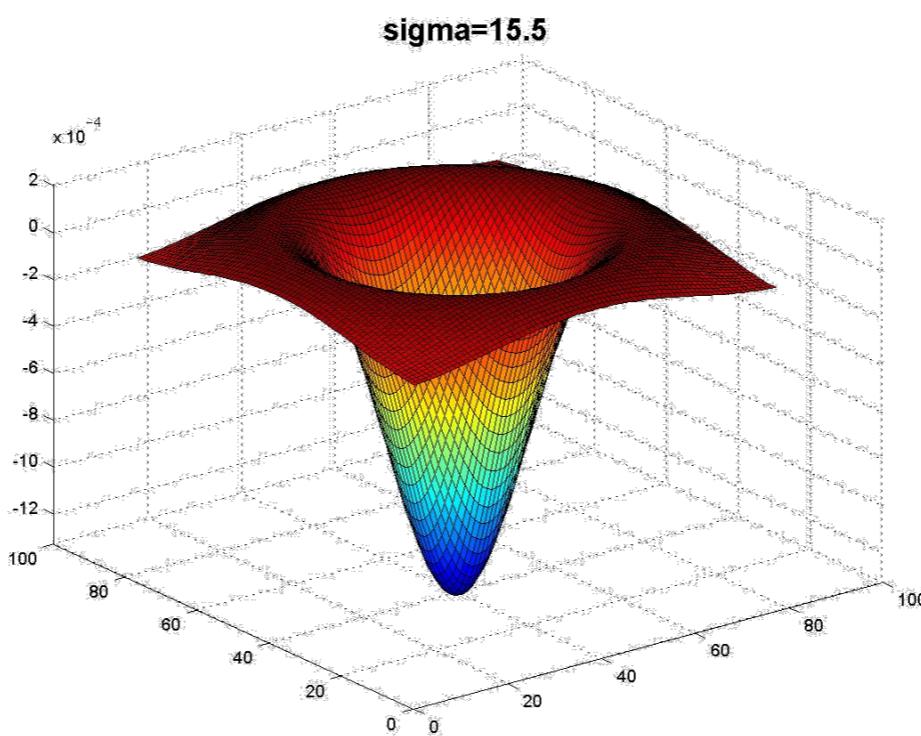
**jet** color scale  
blue: low, red: high

**sigma=4.2**

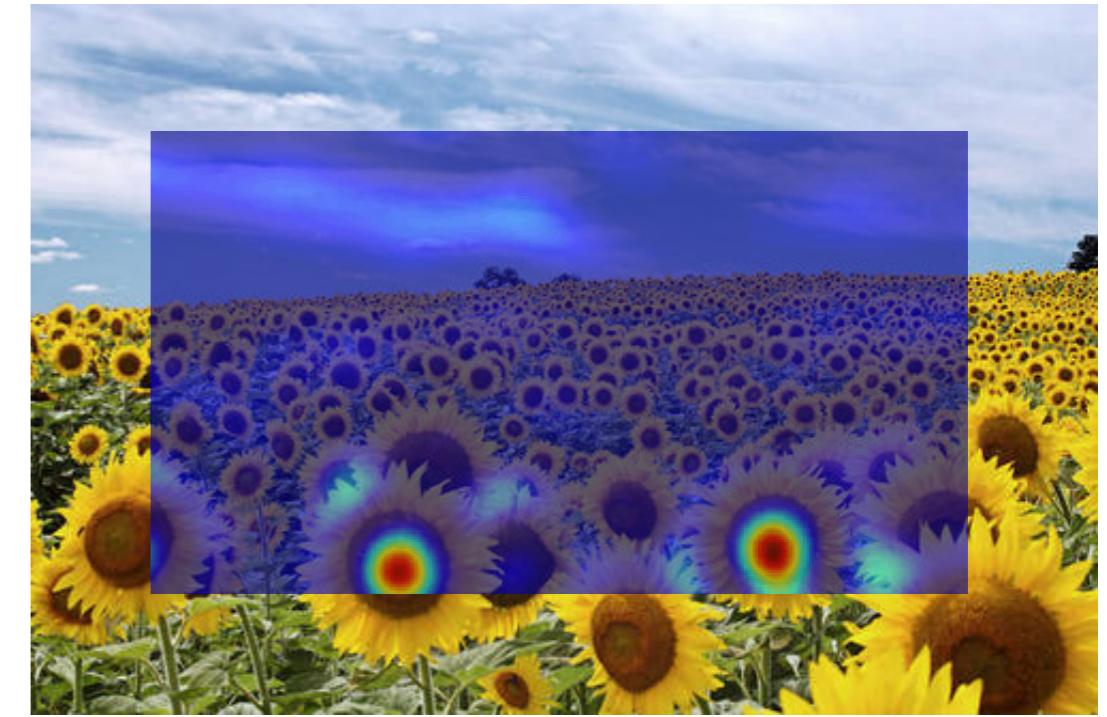
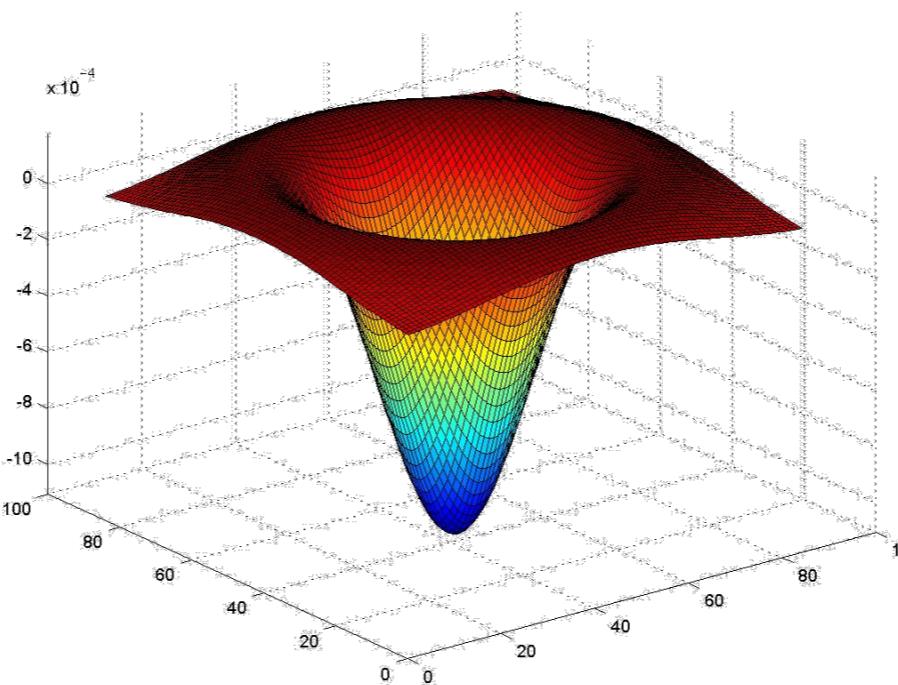








**sigma=17**



What happened when you applied different Laplacian filters?

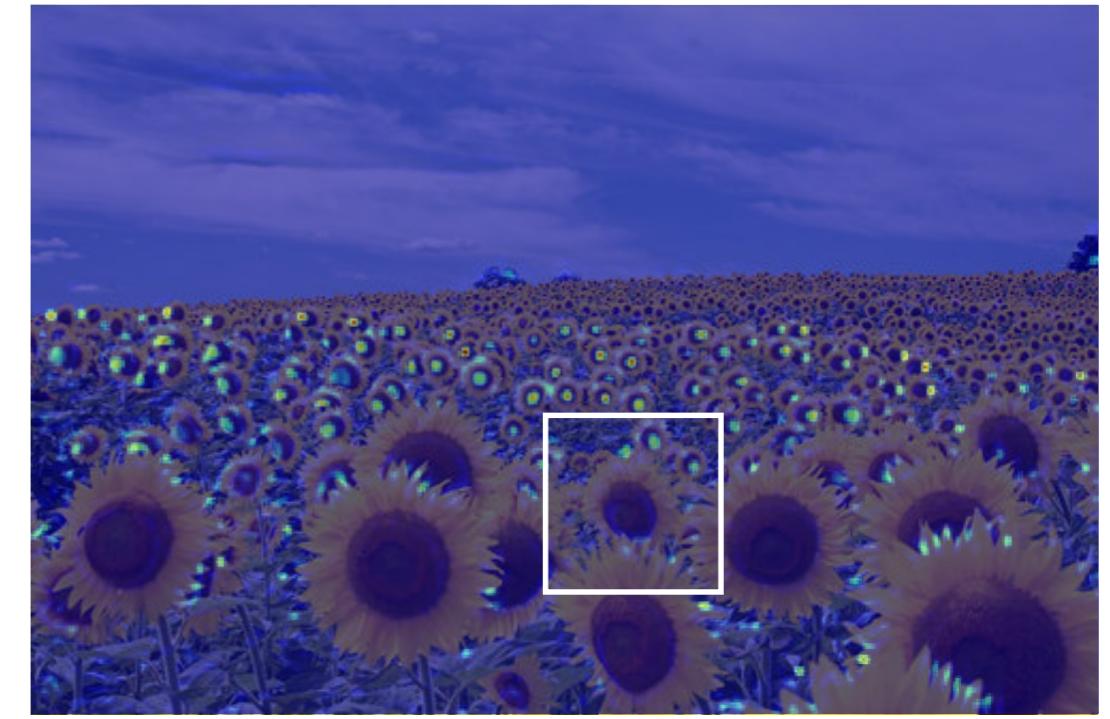
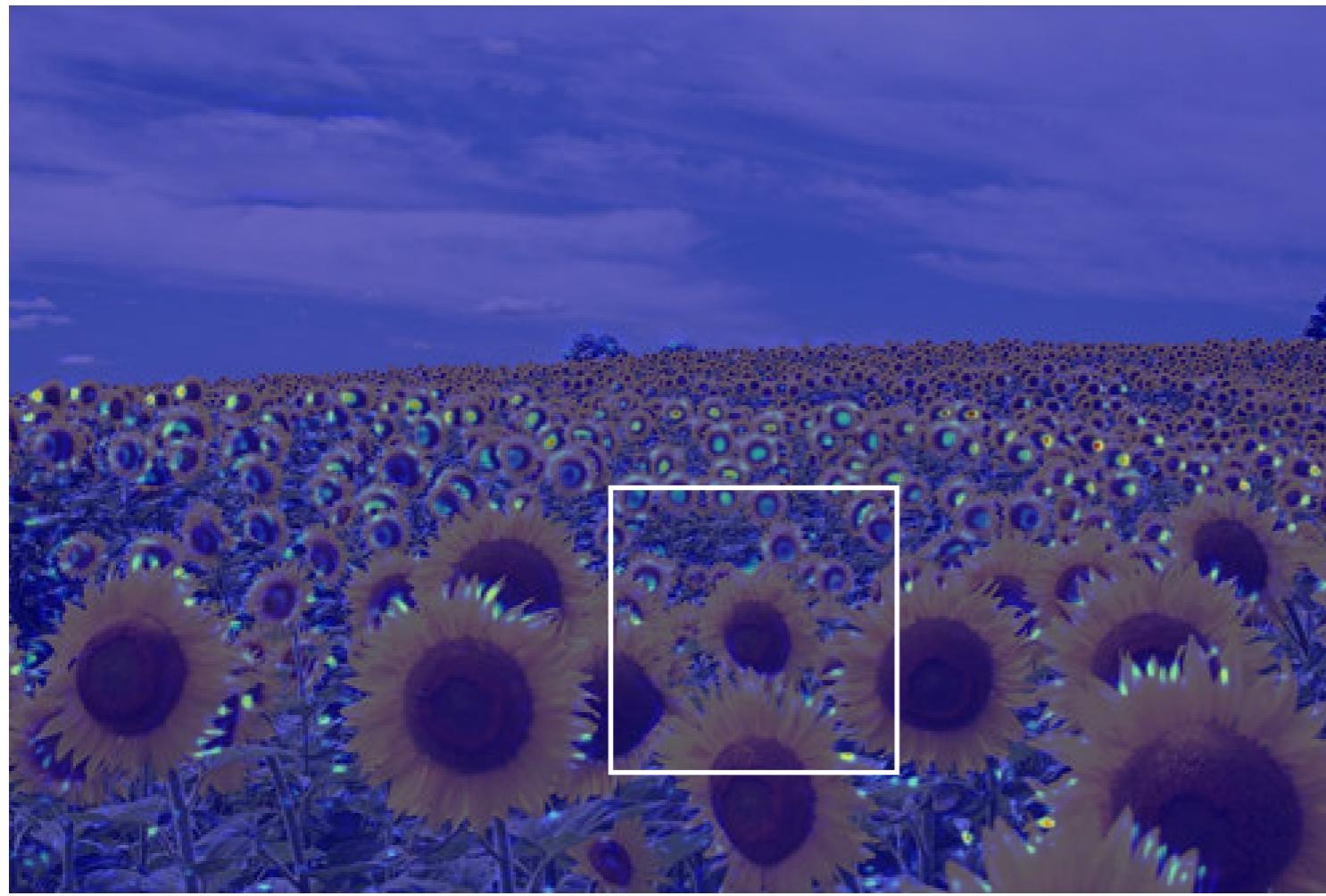
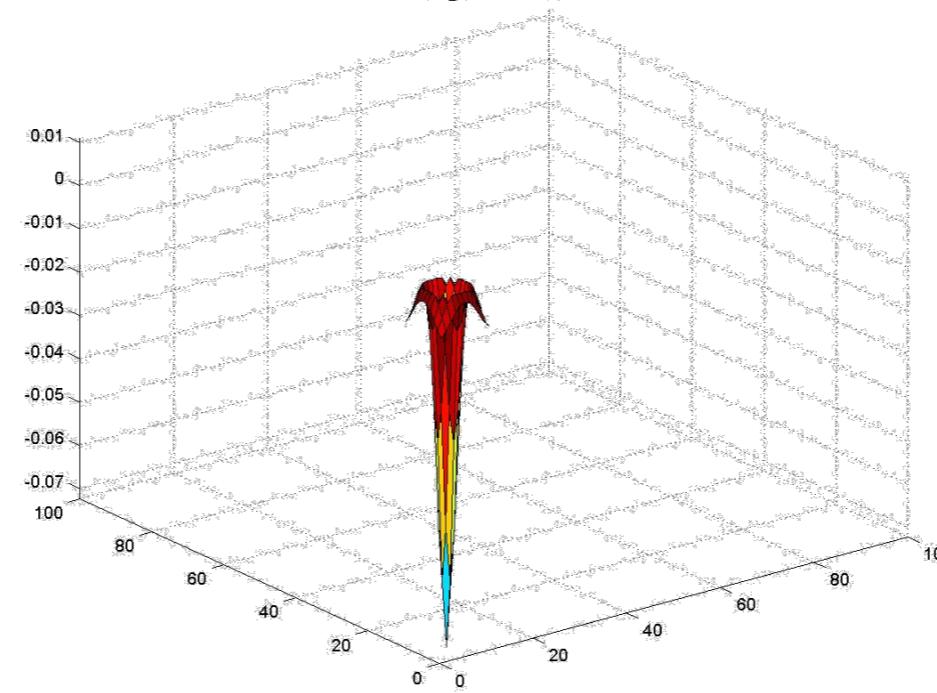
Full size



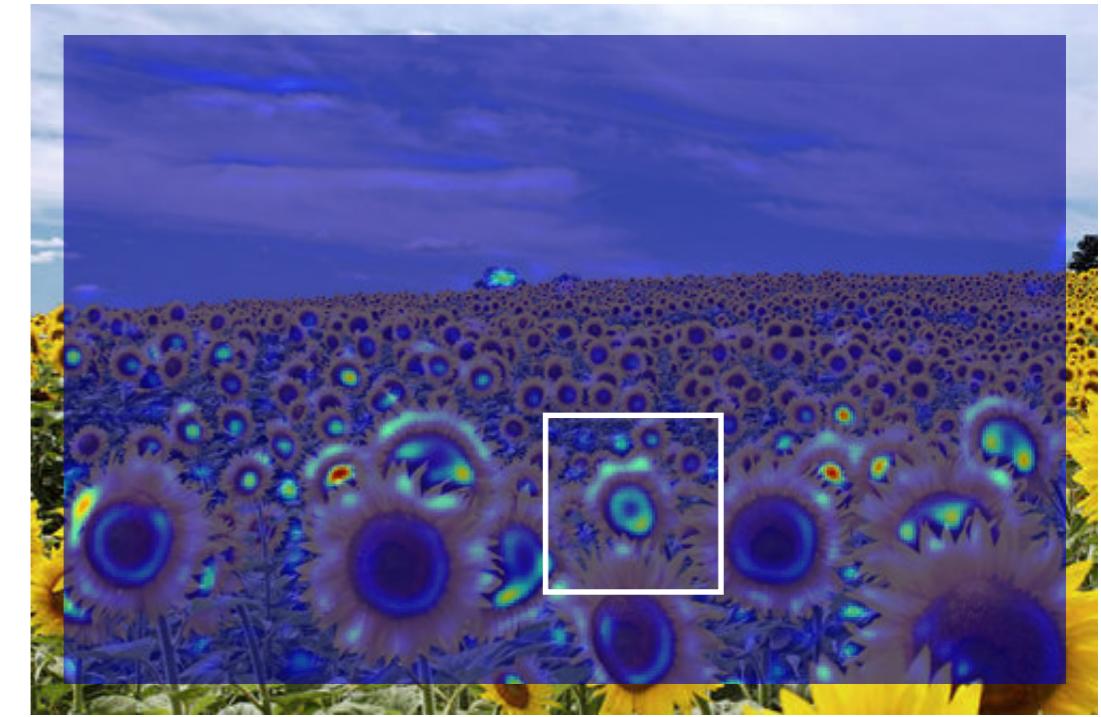
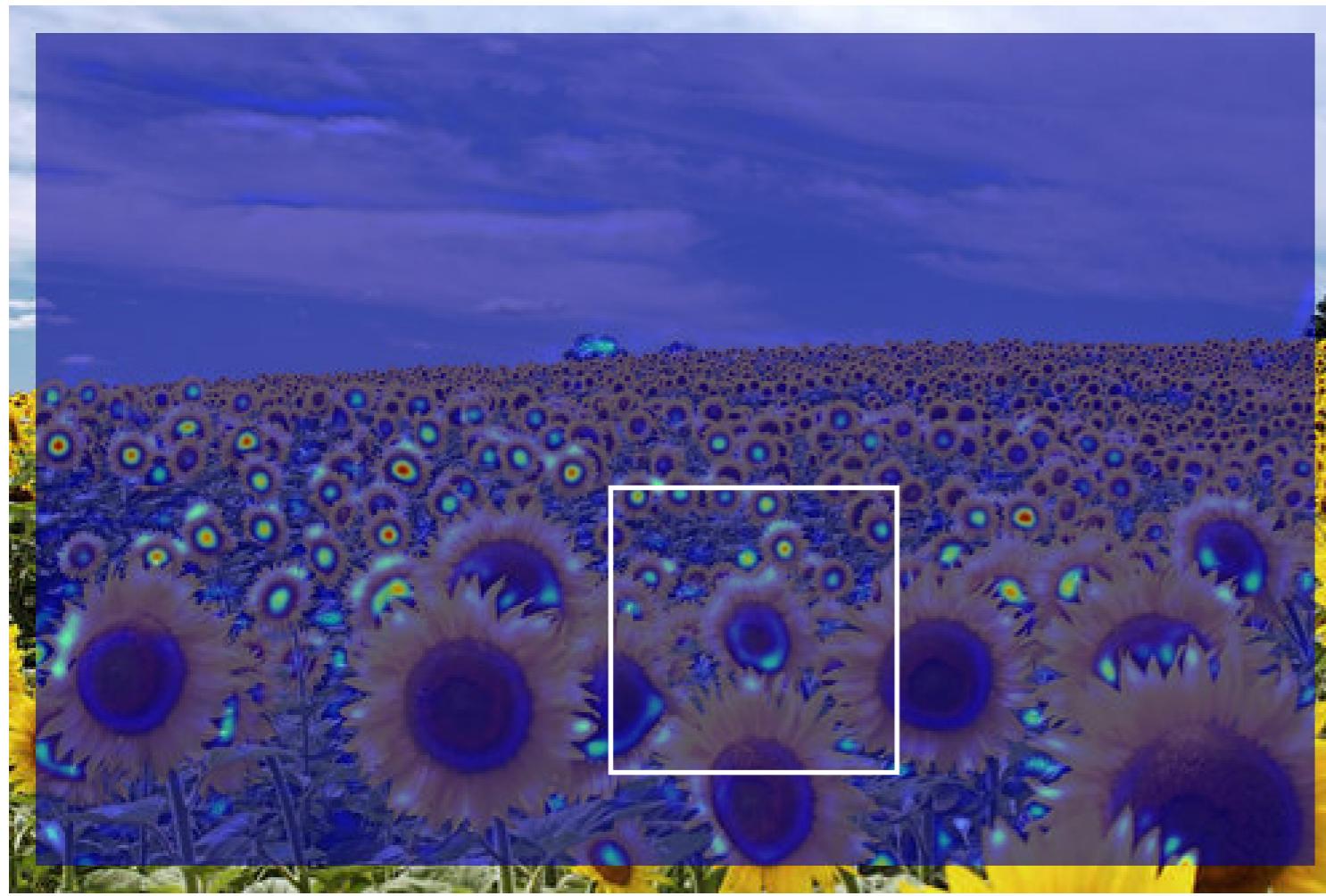
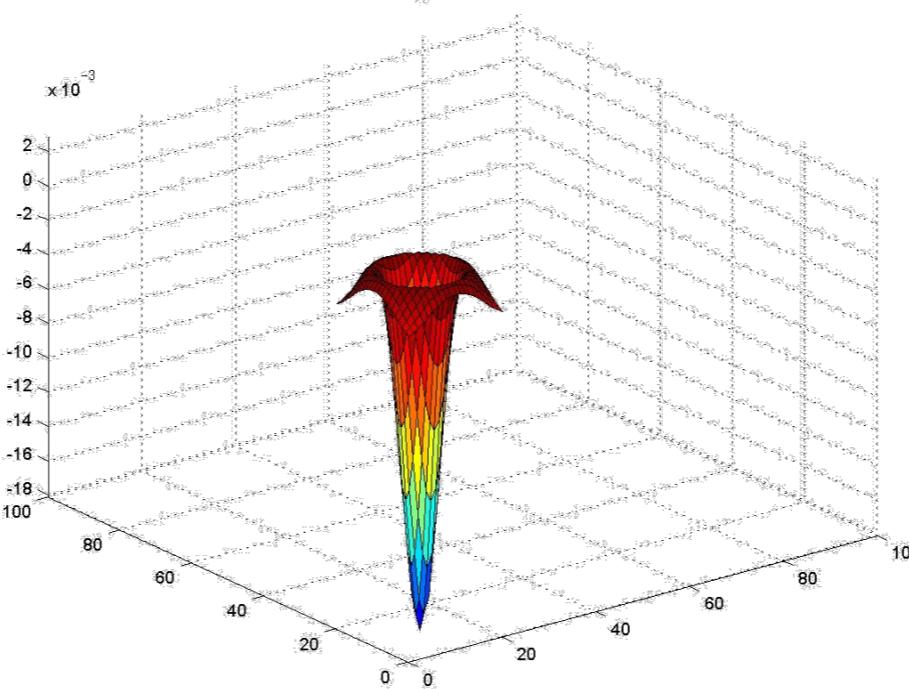
3/4 size

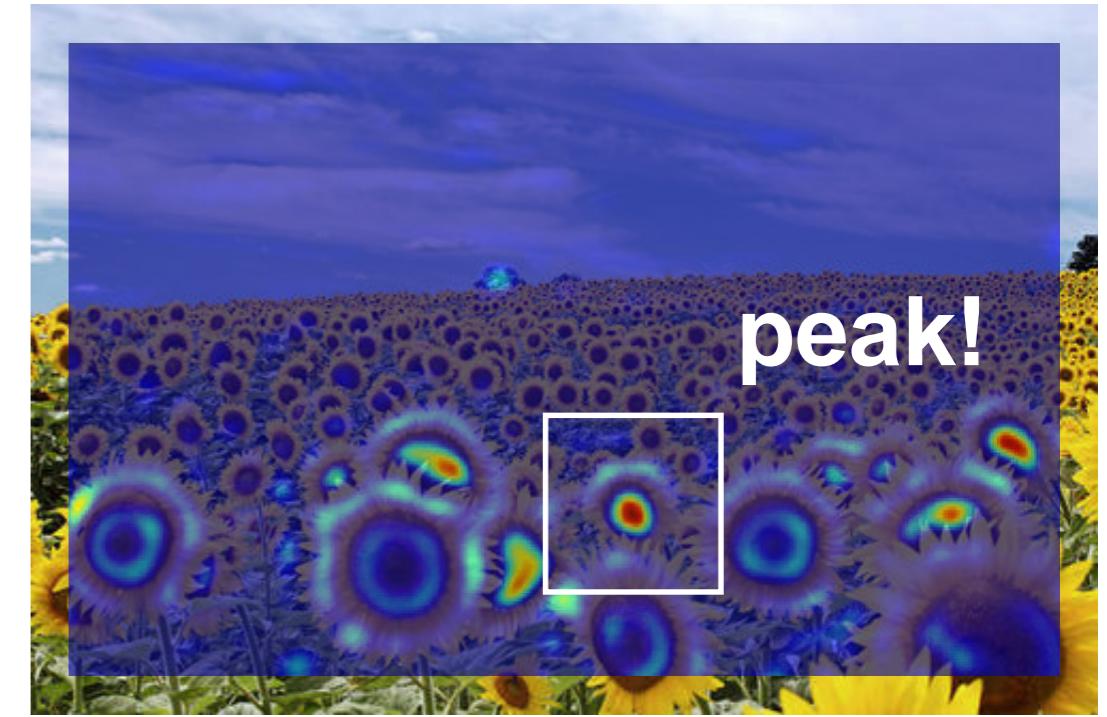
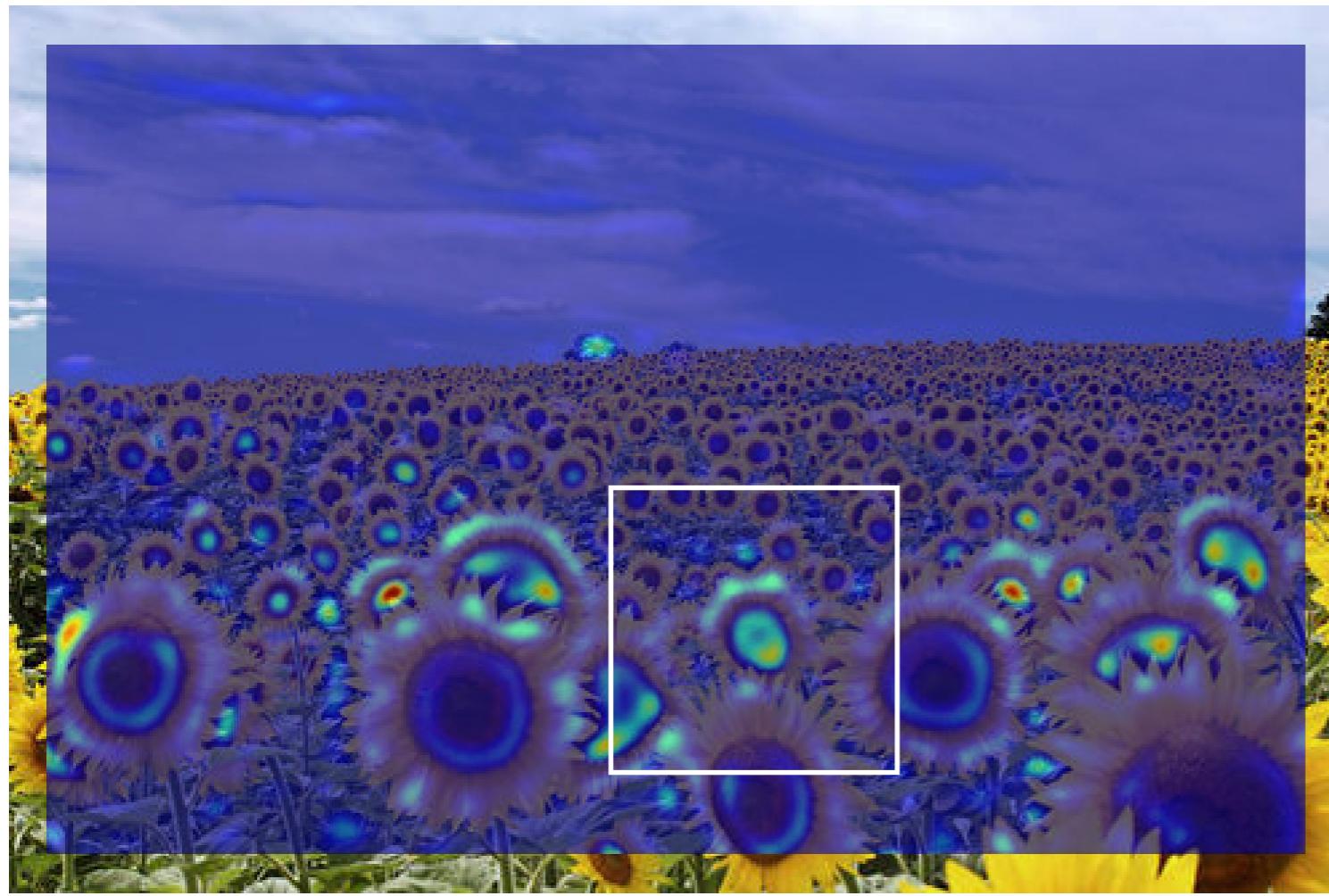
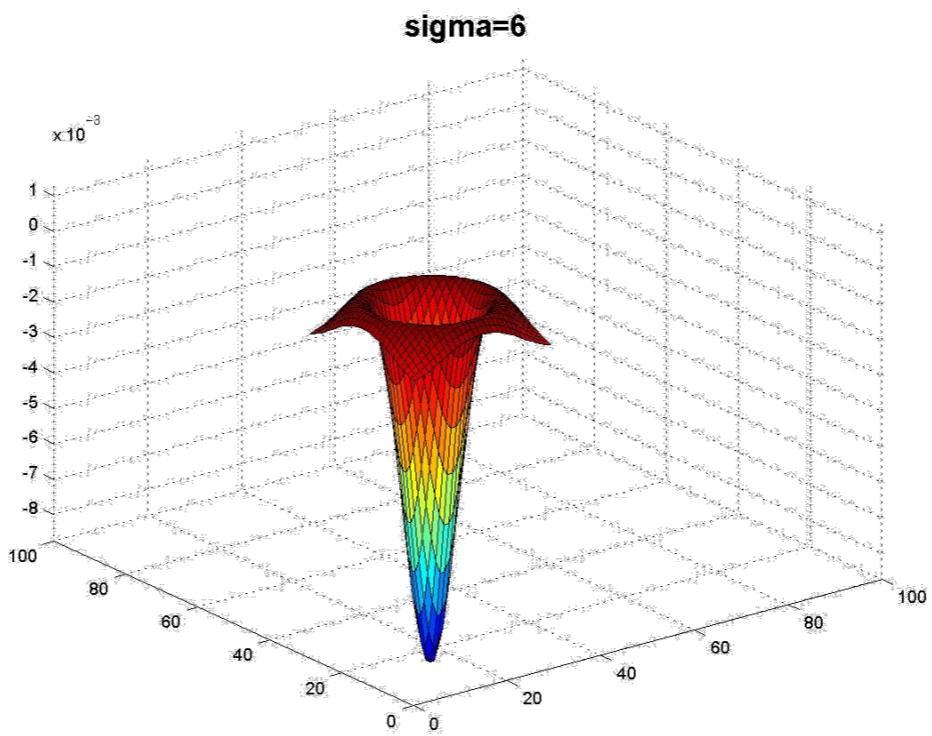


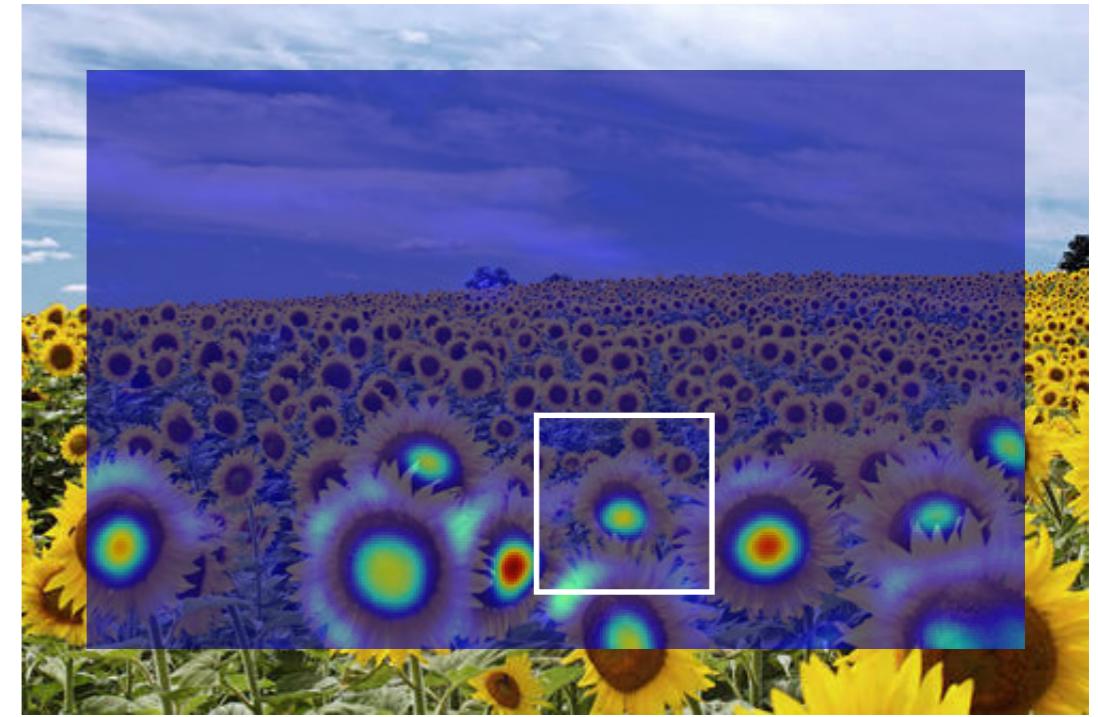
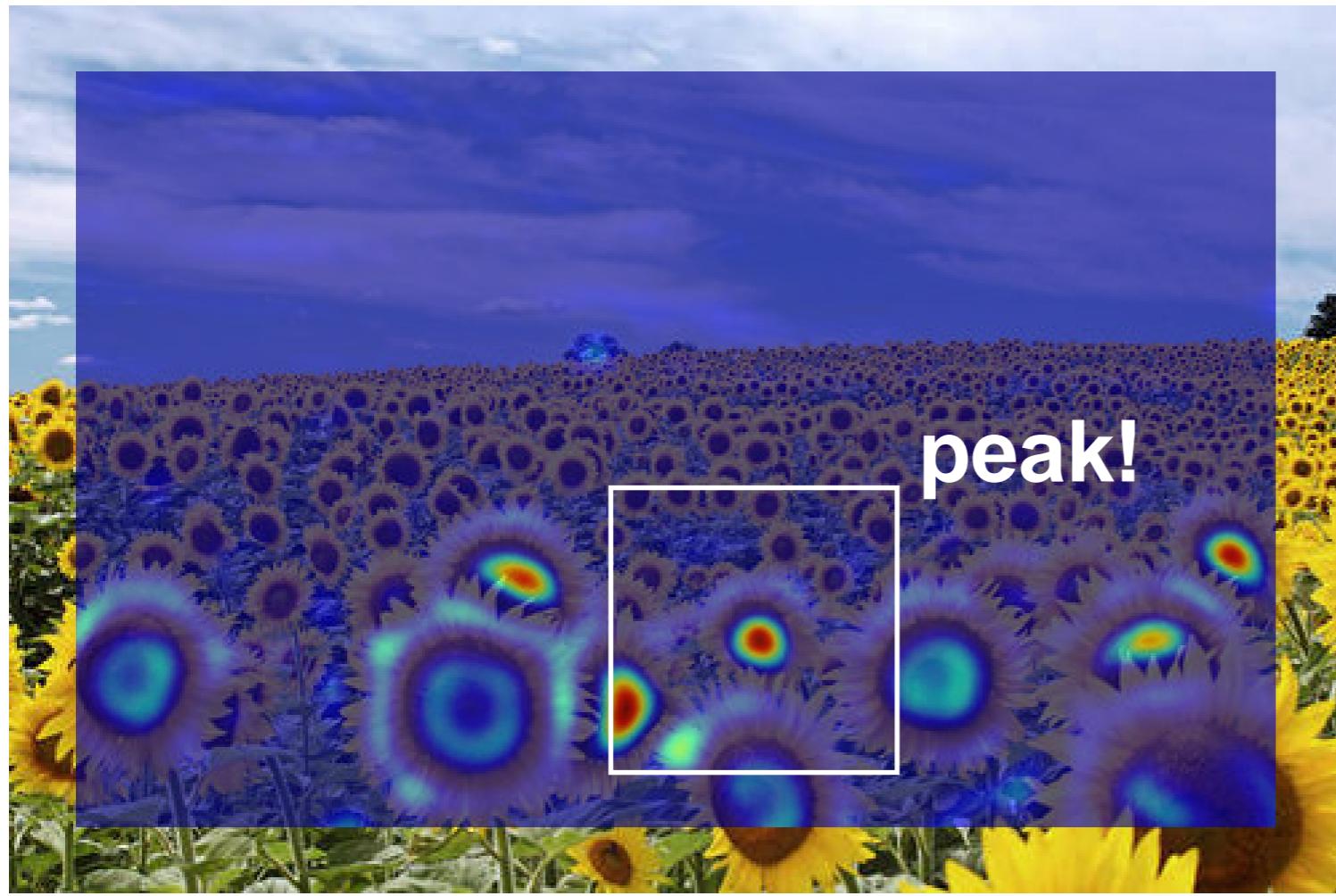
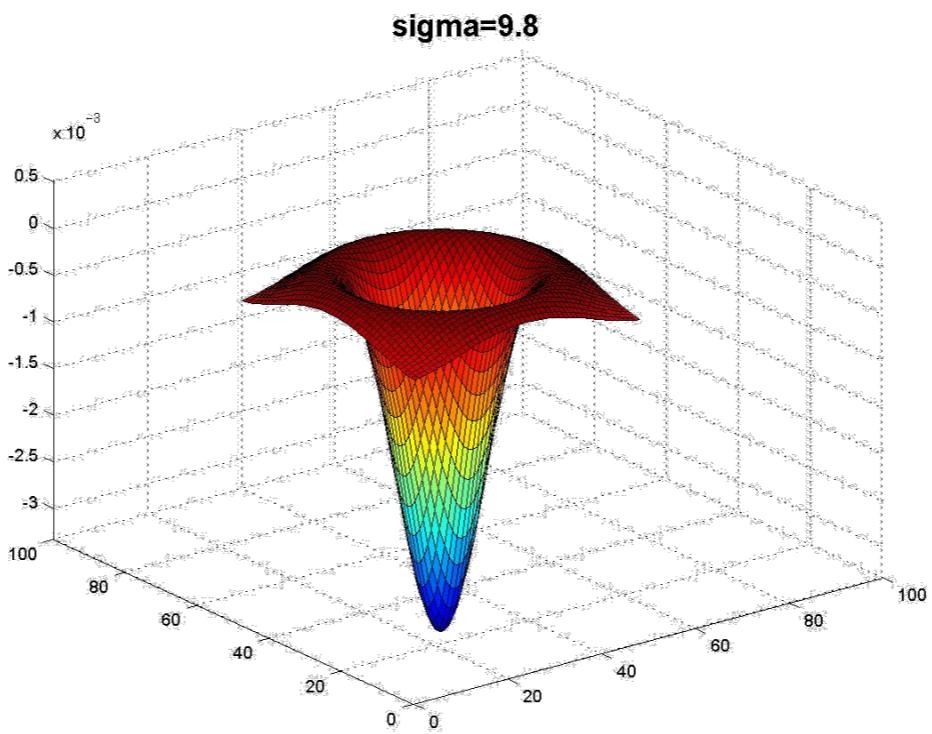
**sigma=2.1**



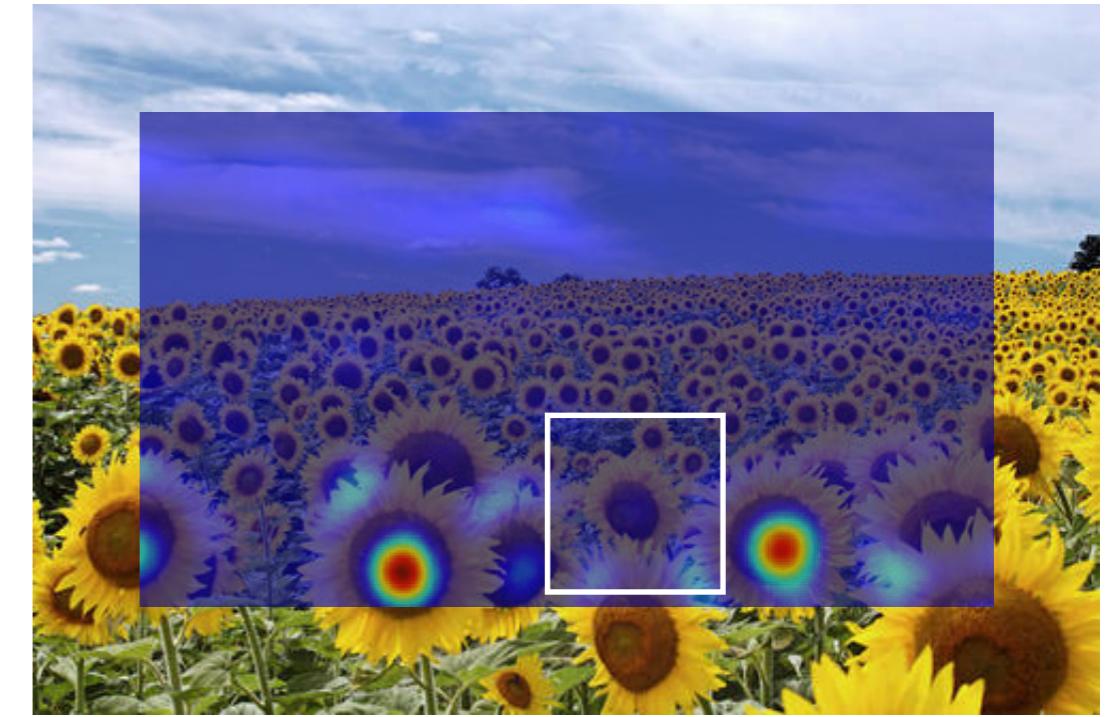
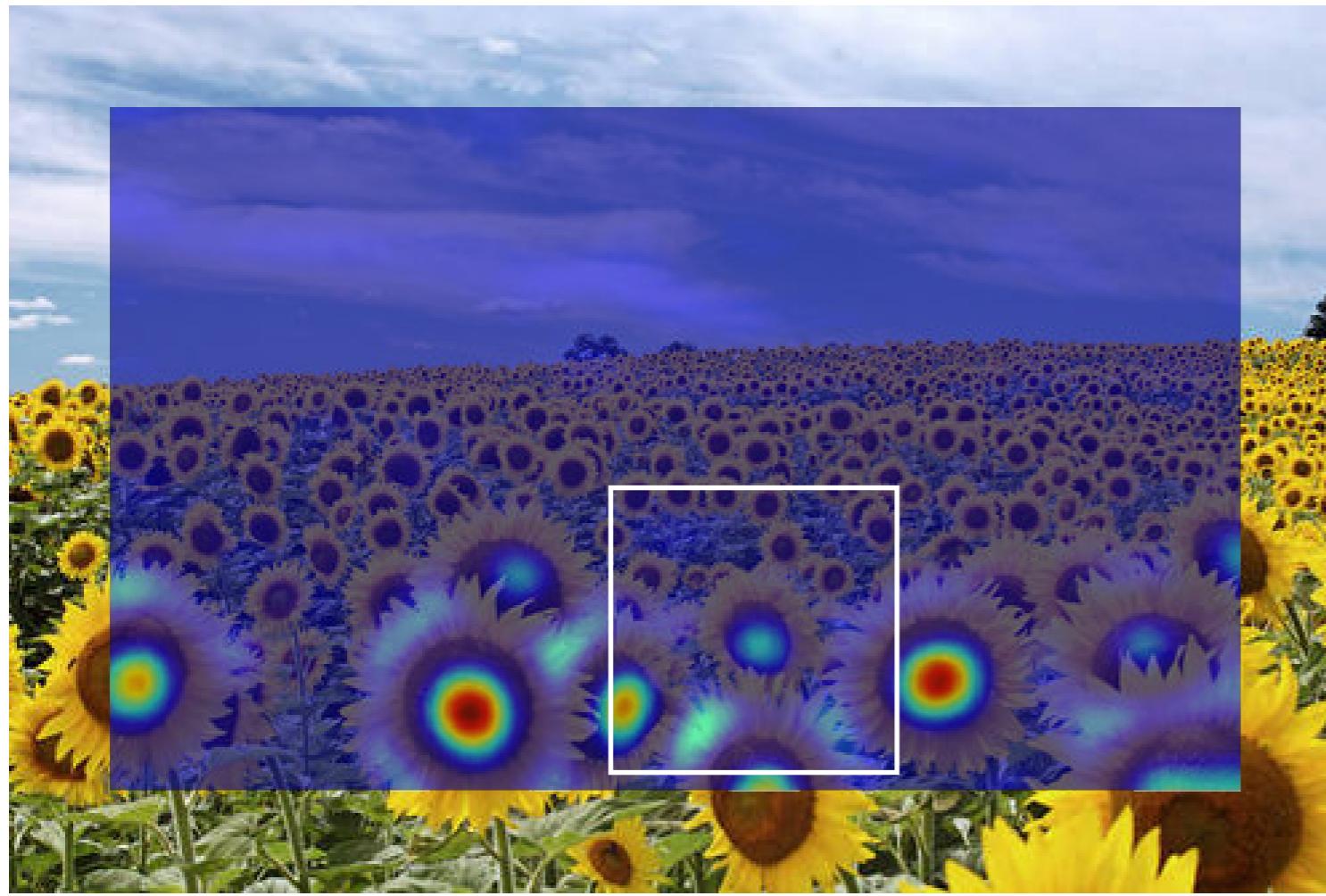
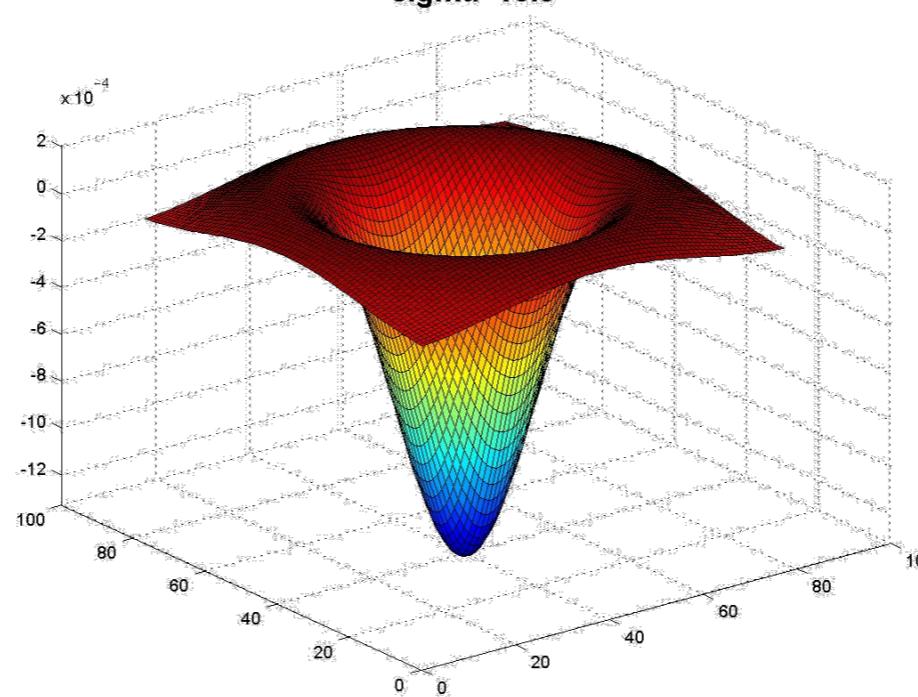
**sigma=4.2**



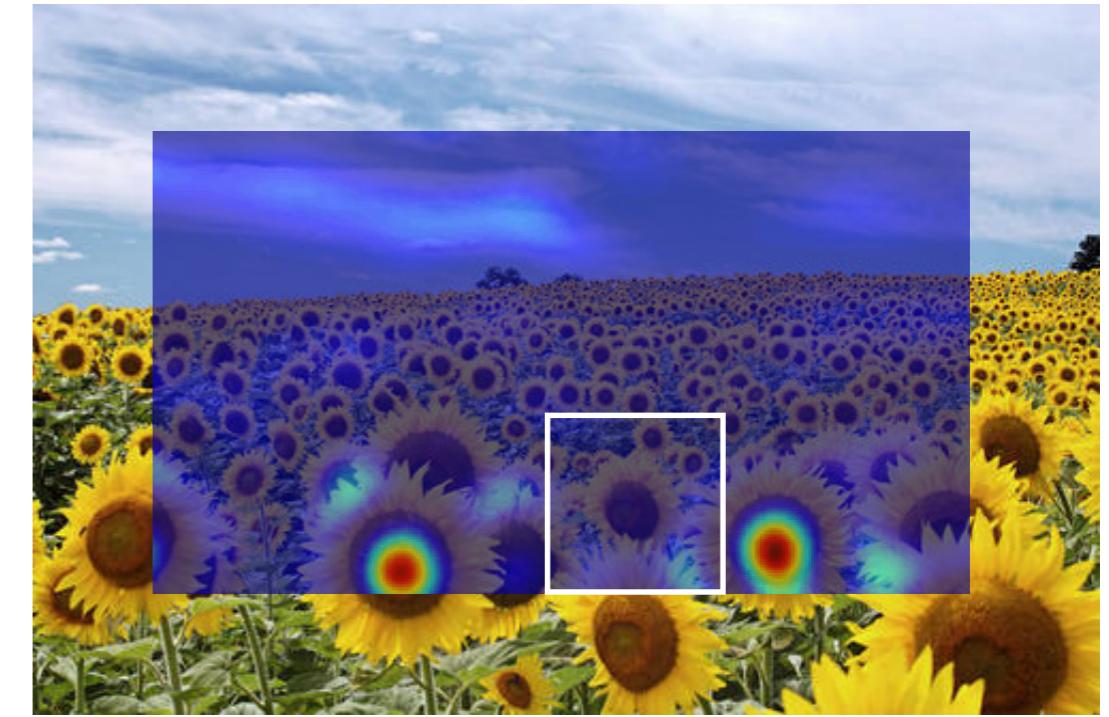
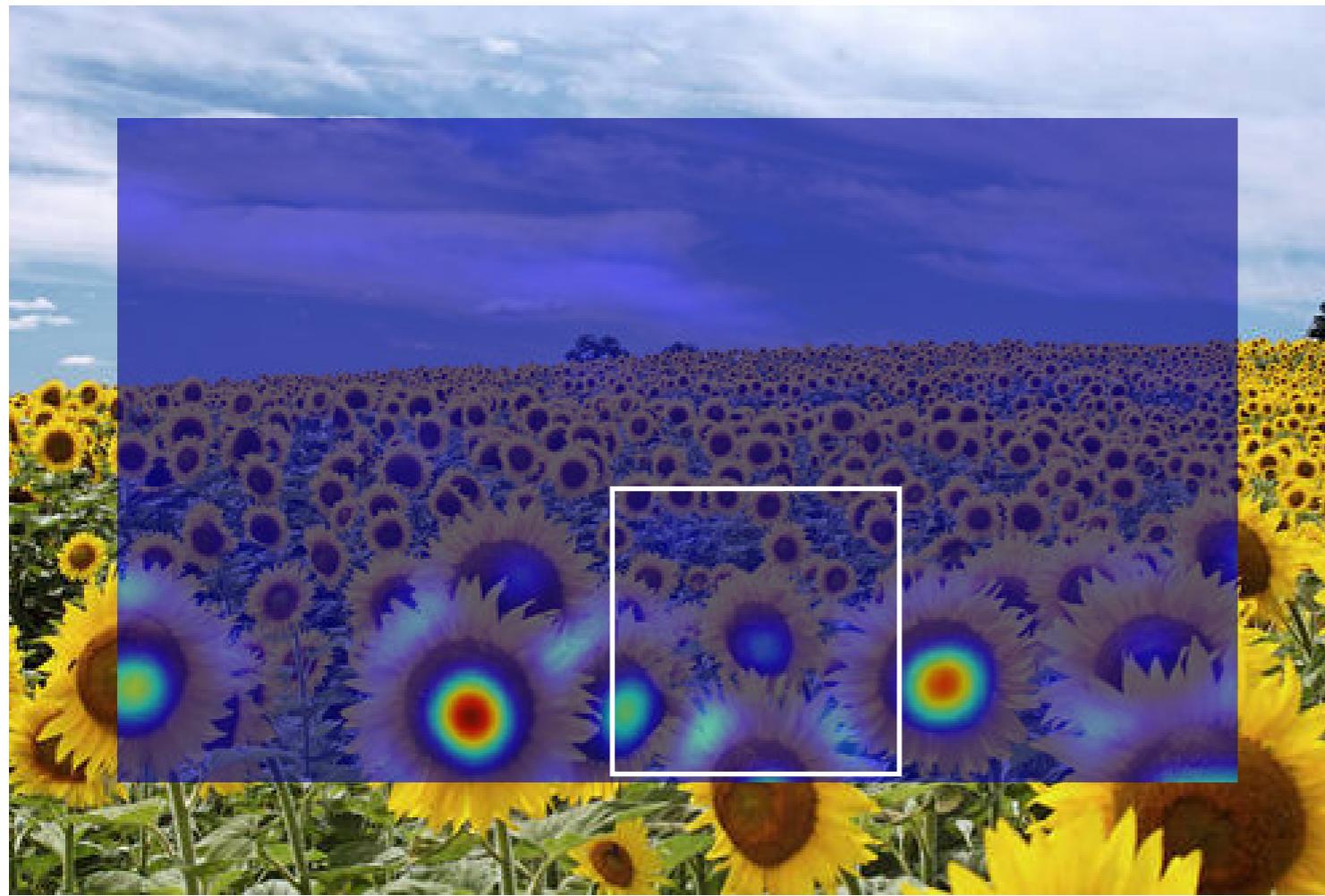
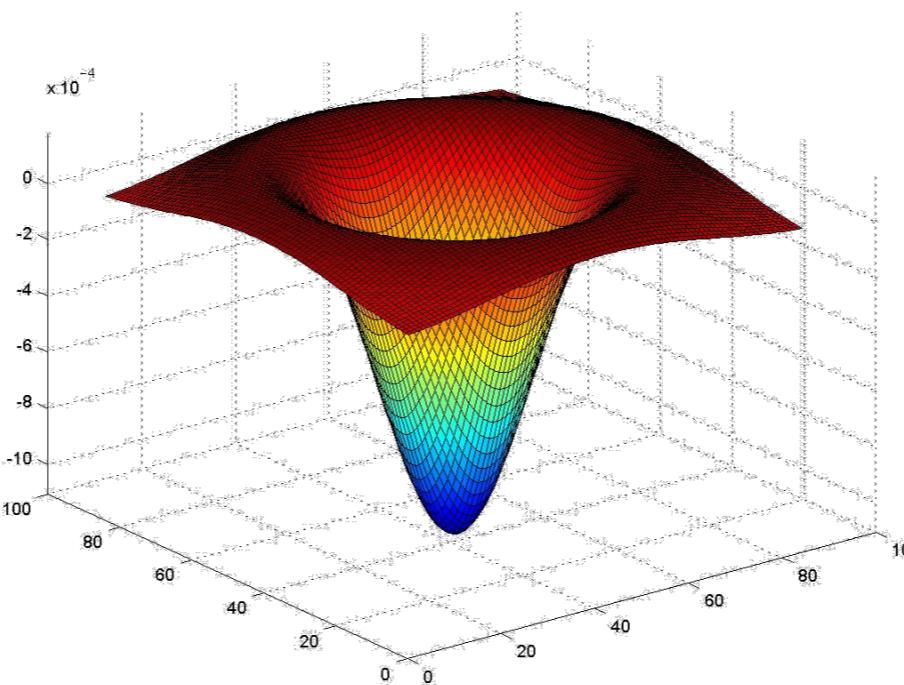




**sigma=15.5**



**sigma=17**



What happened when you applied different Laplacian filters?

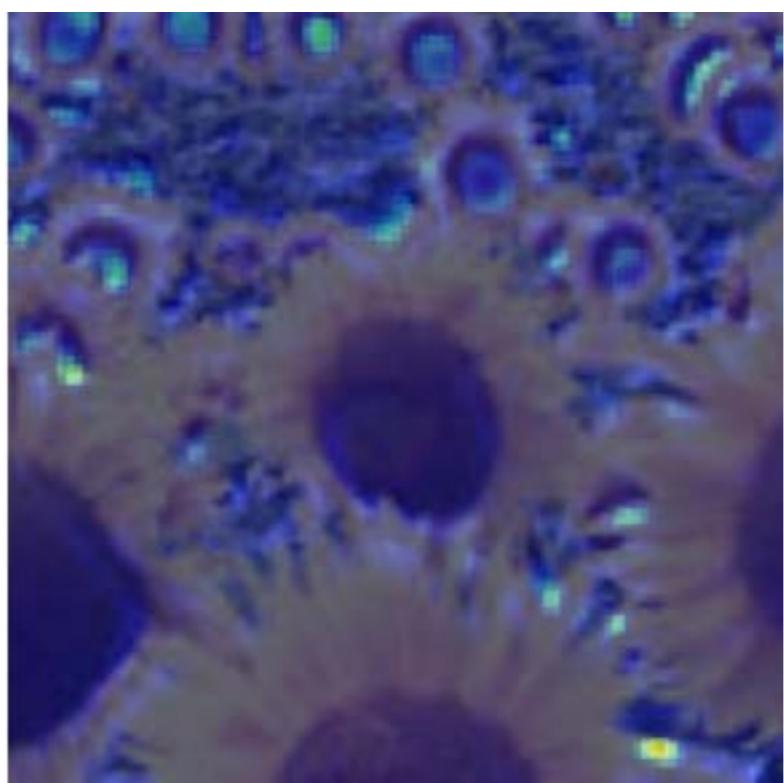
Full size



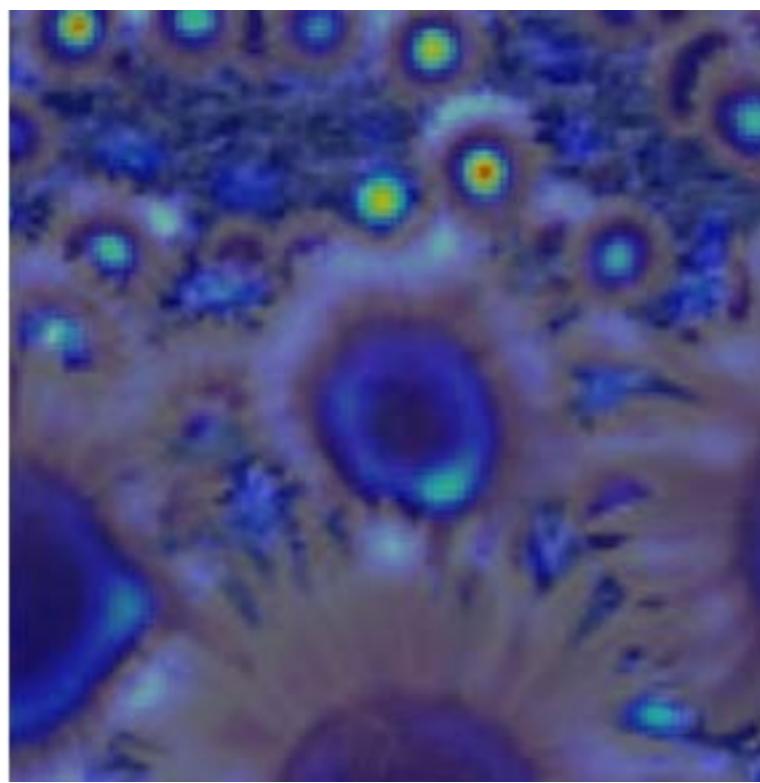
3/4 size



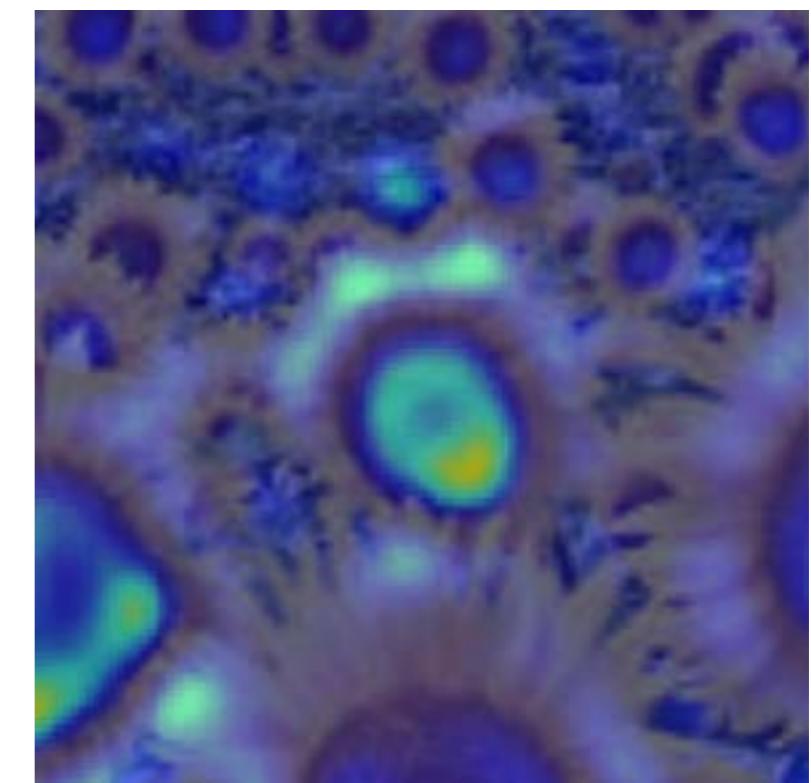
2.1



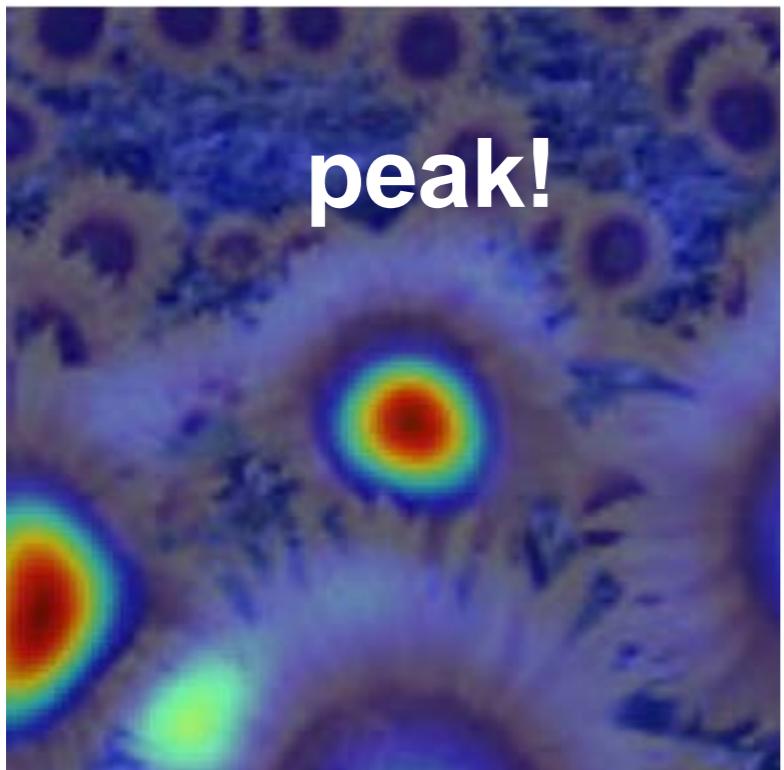
4.2



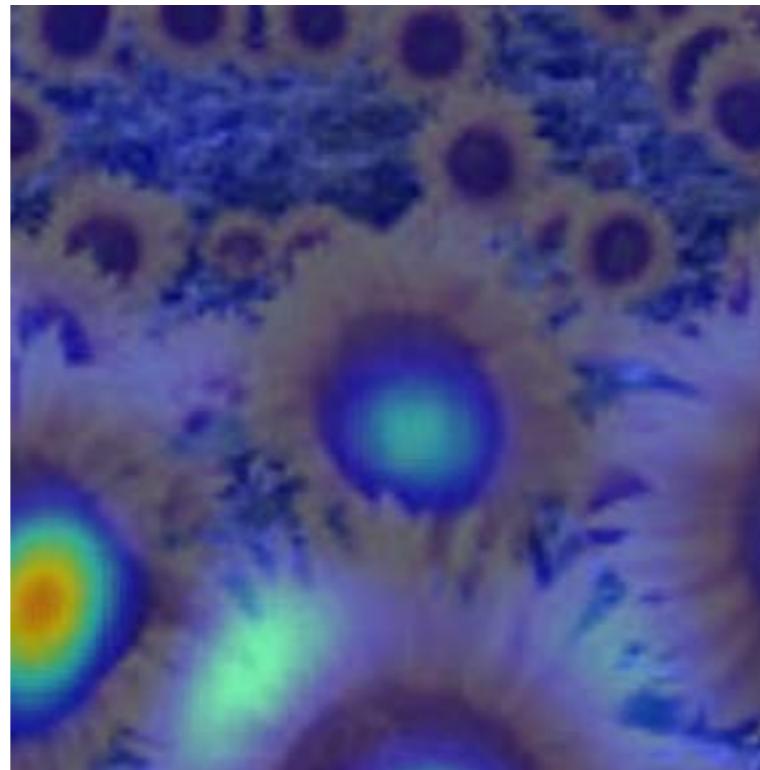
6.0



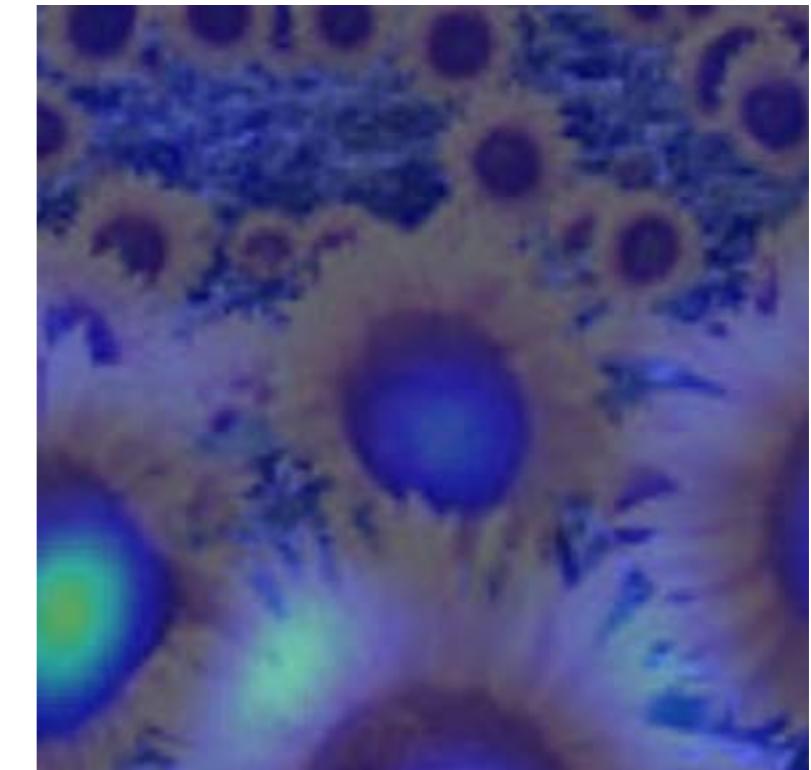
9.8



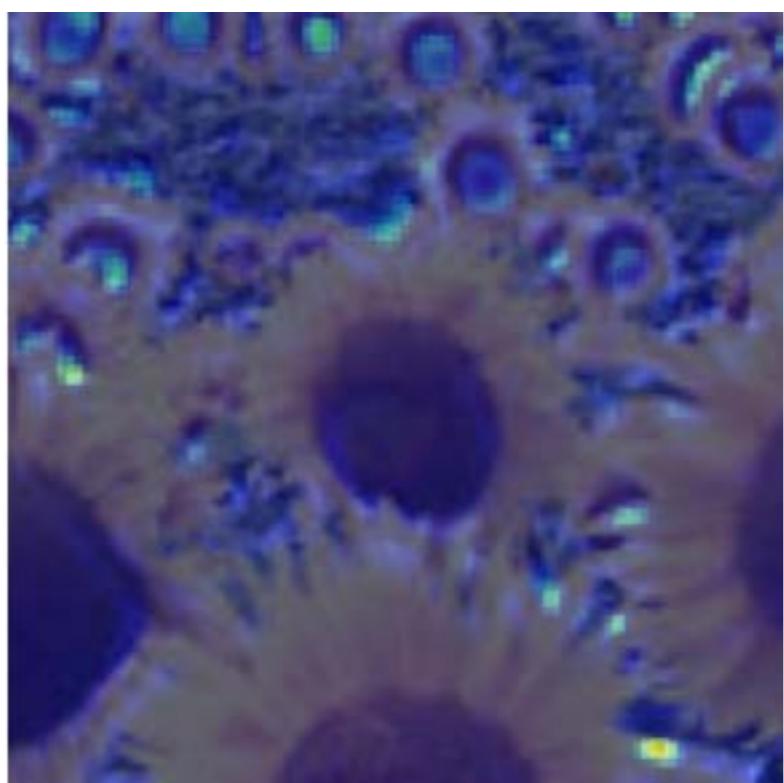
15.5



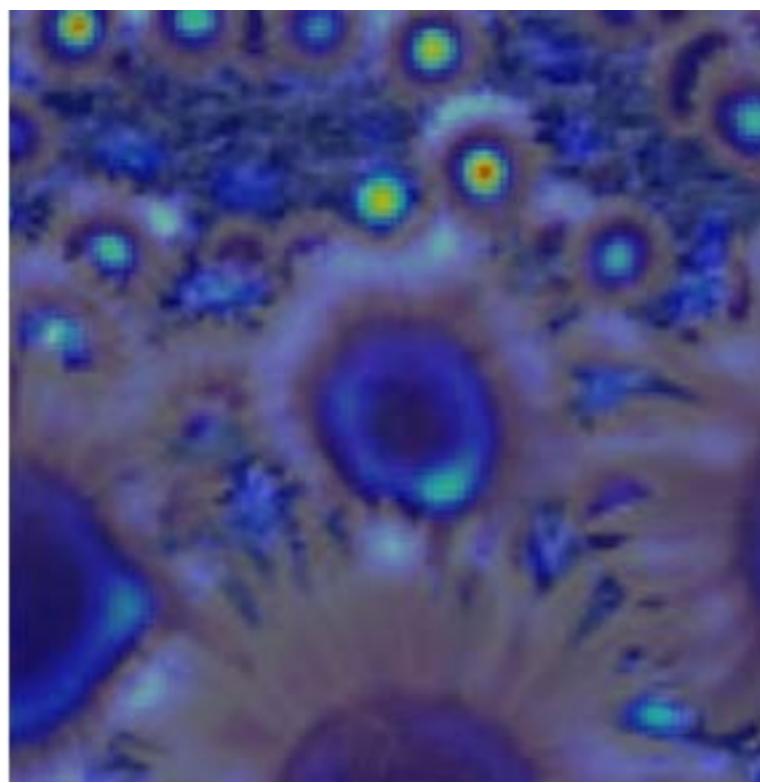
17.0



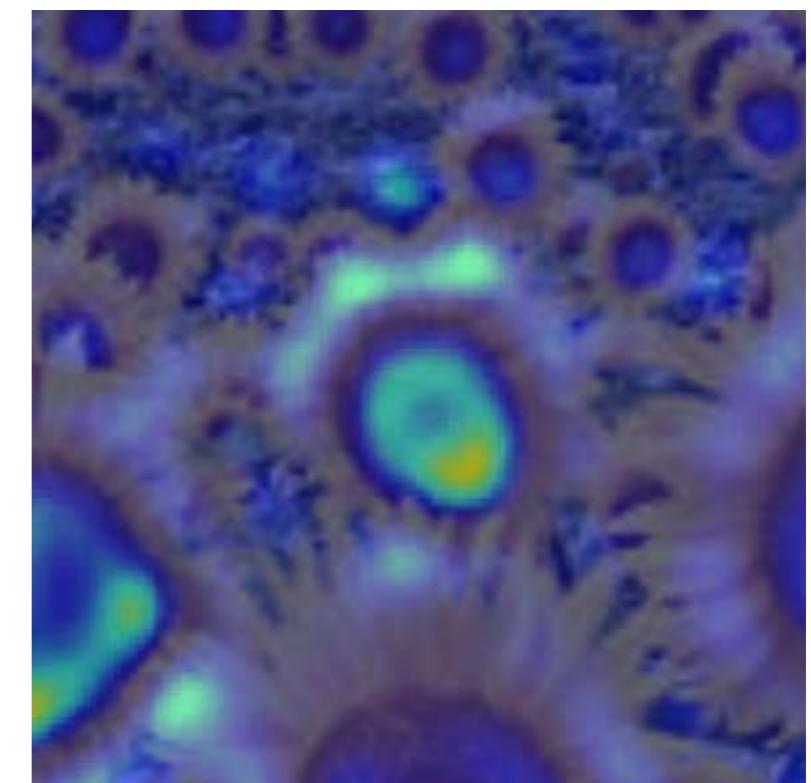
2.1



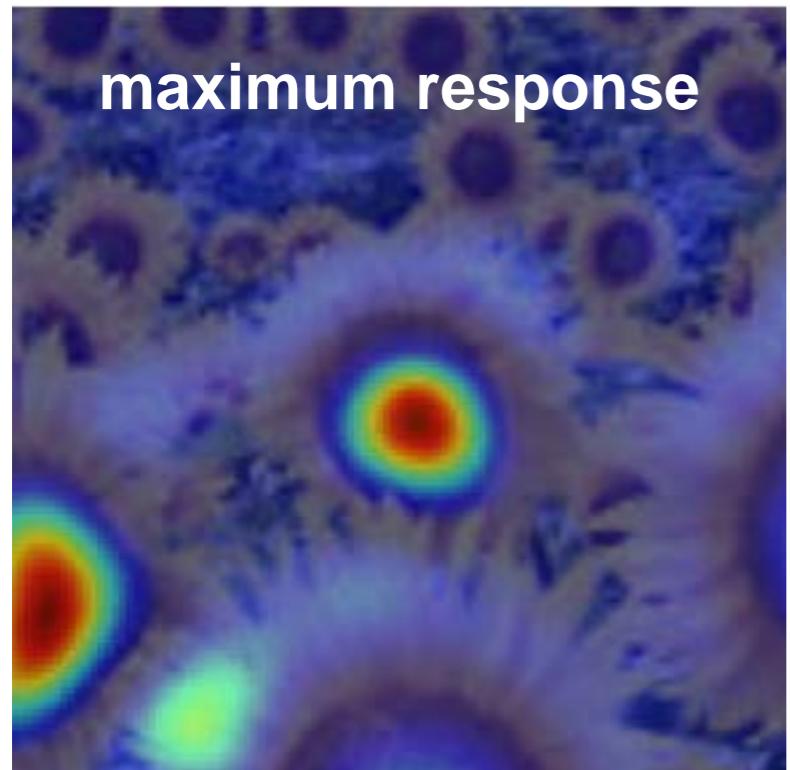
4.2



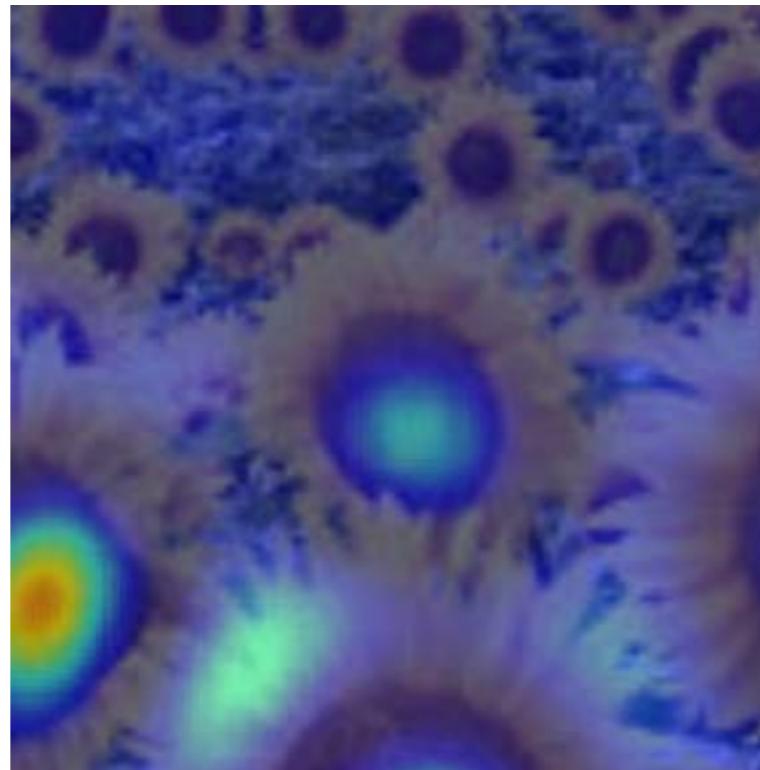
6.0



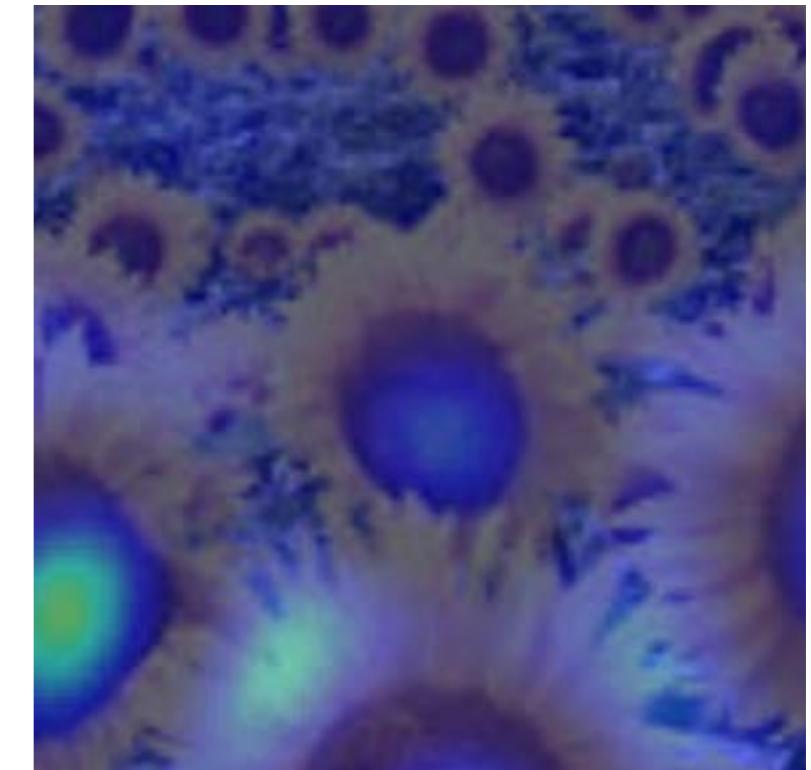
9.8



15.5



17.0



# optimal scale

2.1

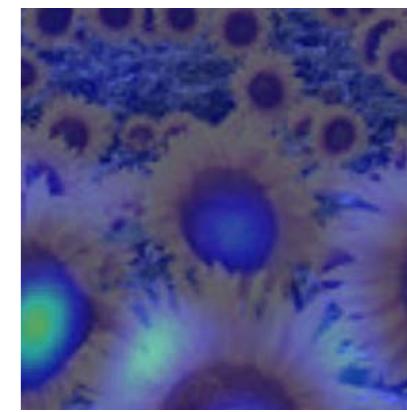
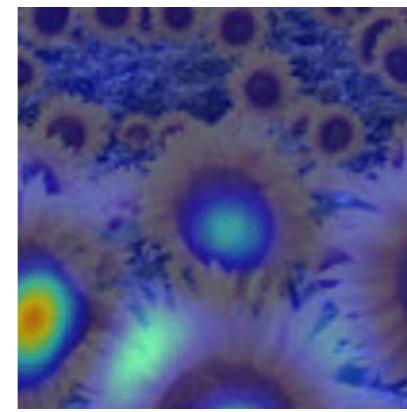
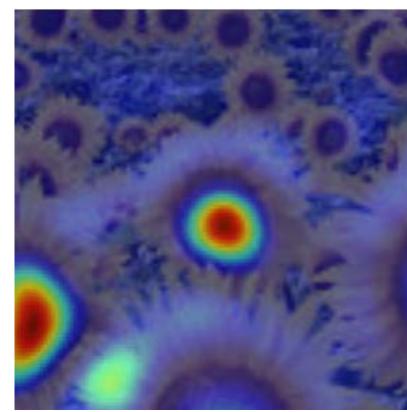
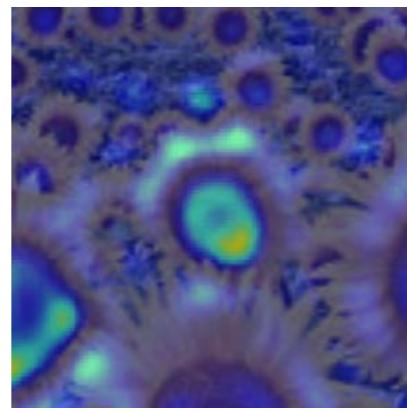
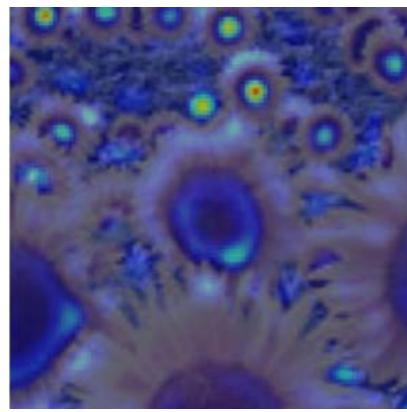
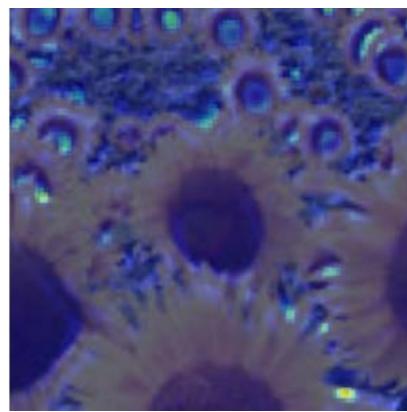
4.2

6.0

9.8

15.5

17.0



Full size image

2.1

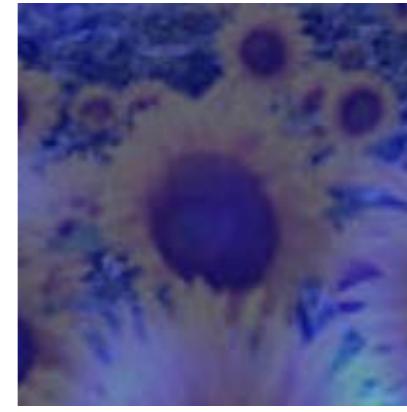
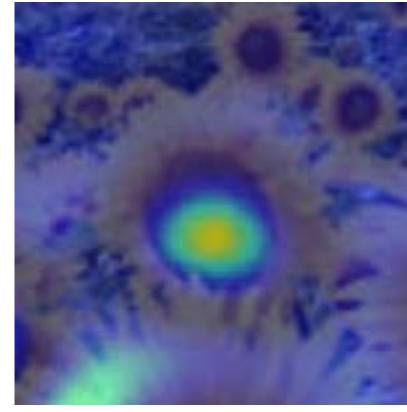
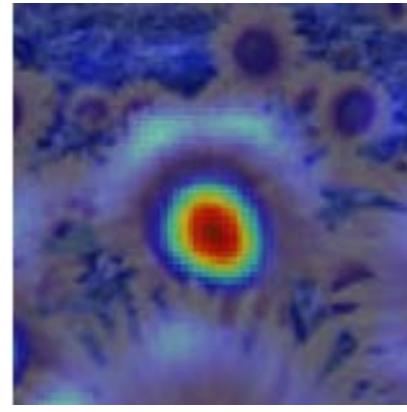
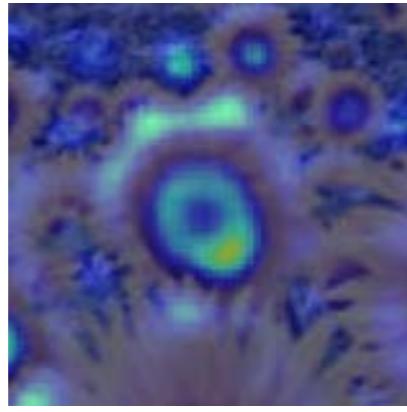
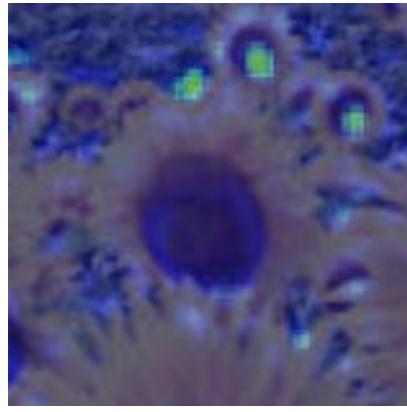
4.2

6.0

9.8

15.5

17.0



3/4 size image

# optimal scale

2.1

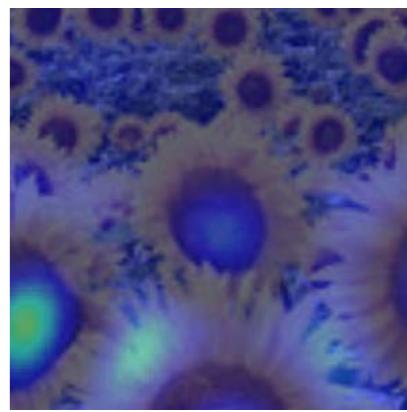
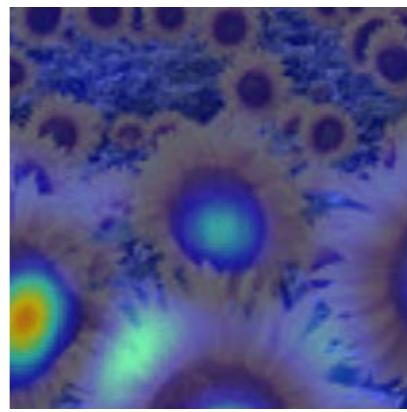
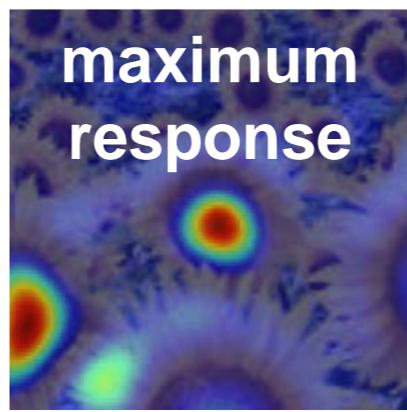
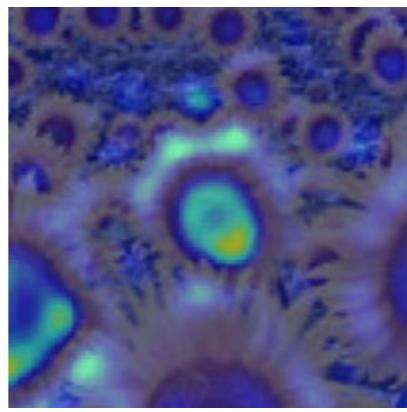
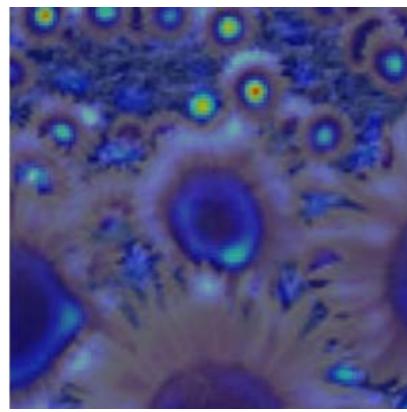
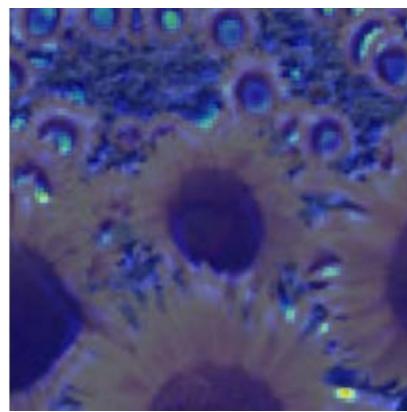
4.2

6.0

9.8

15.5

17.0



Full size image

2.1

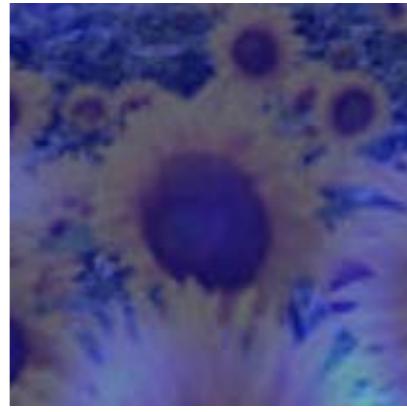
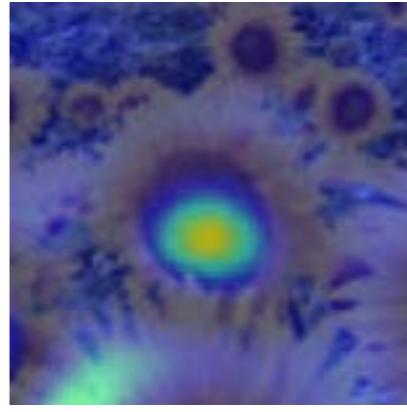
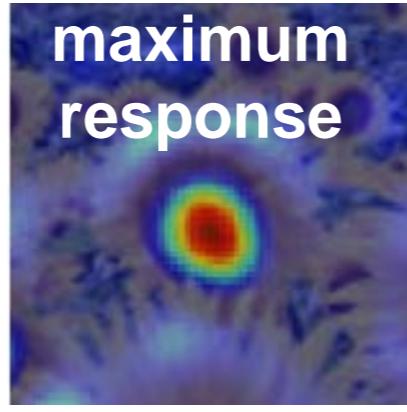
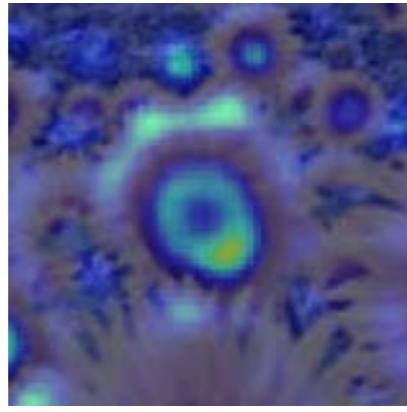
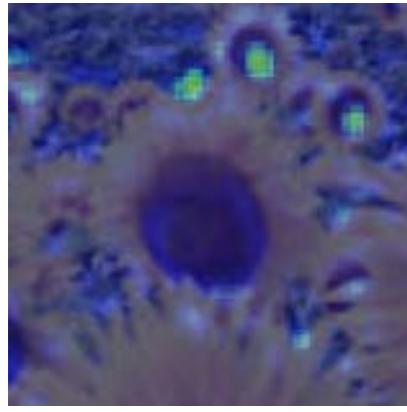
4.2

6.0

9.8

15.5

17.0



3/4 size image

Intuitively...

Find local maxima in both **position** and **scale**

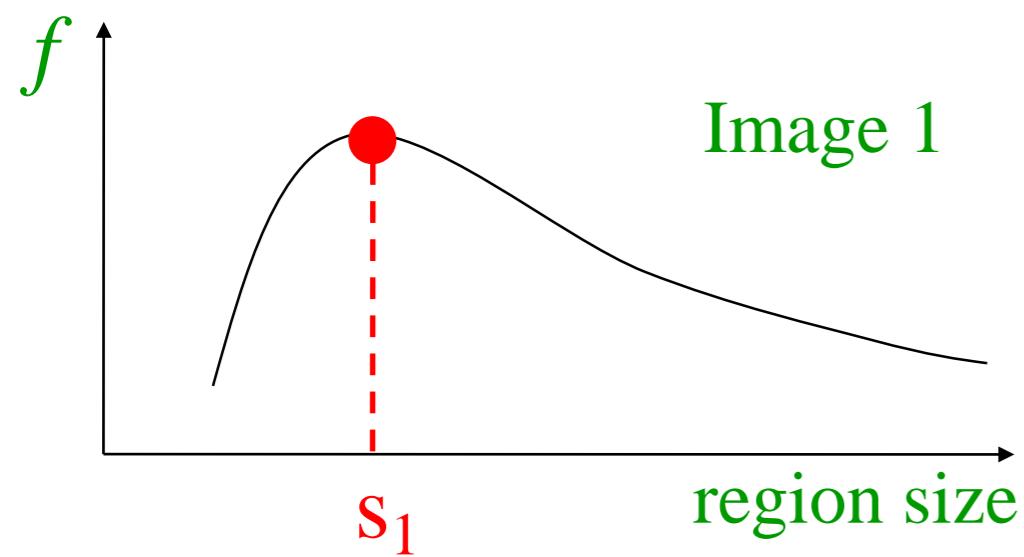
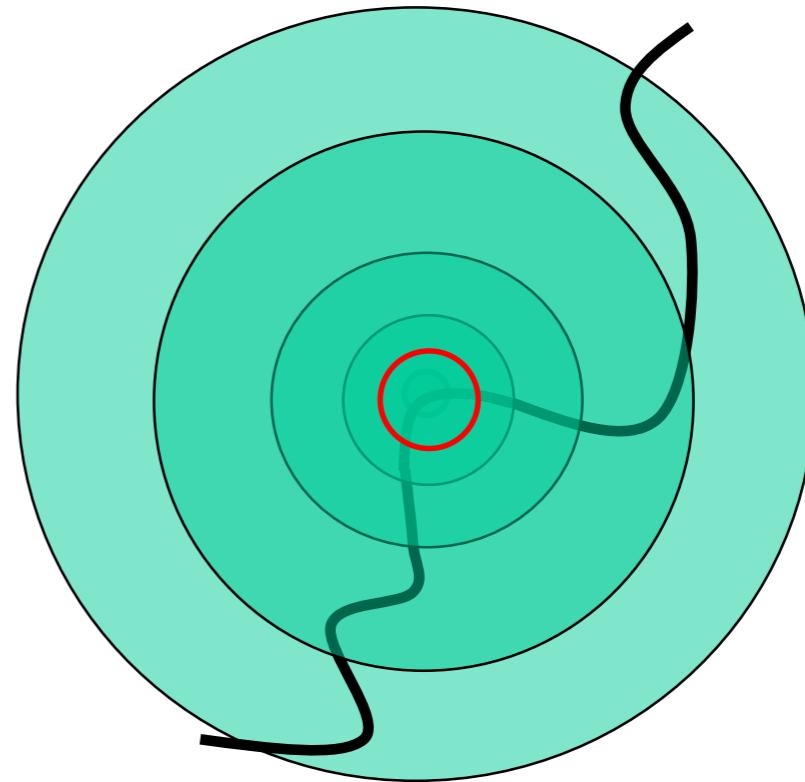
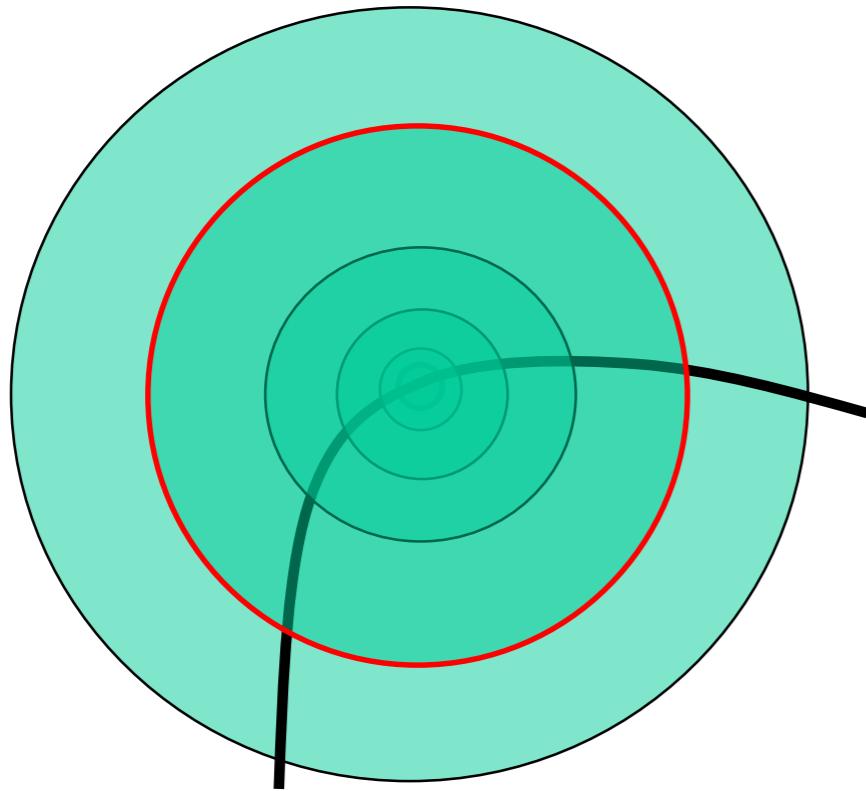


Image 1

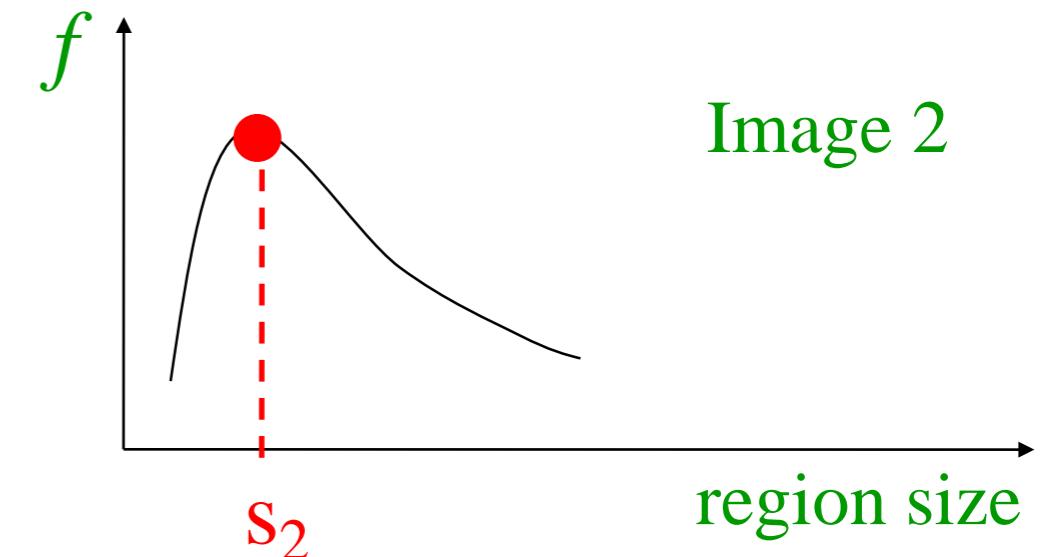
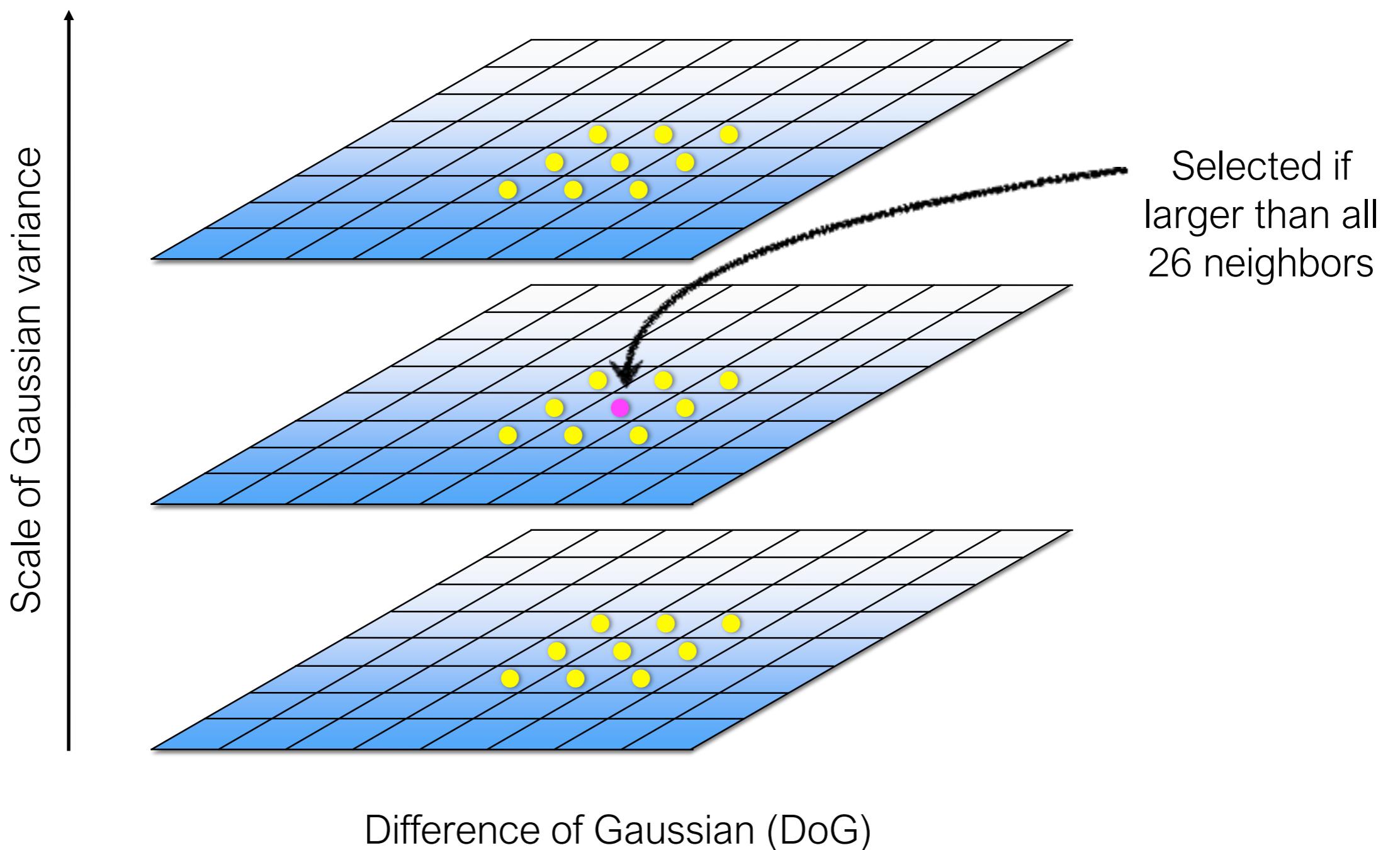


Image 2

How would you implement  
scale selection?

# Scale-space extrema



# implementation

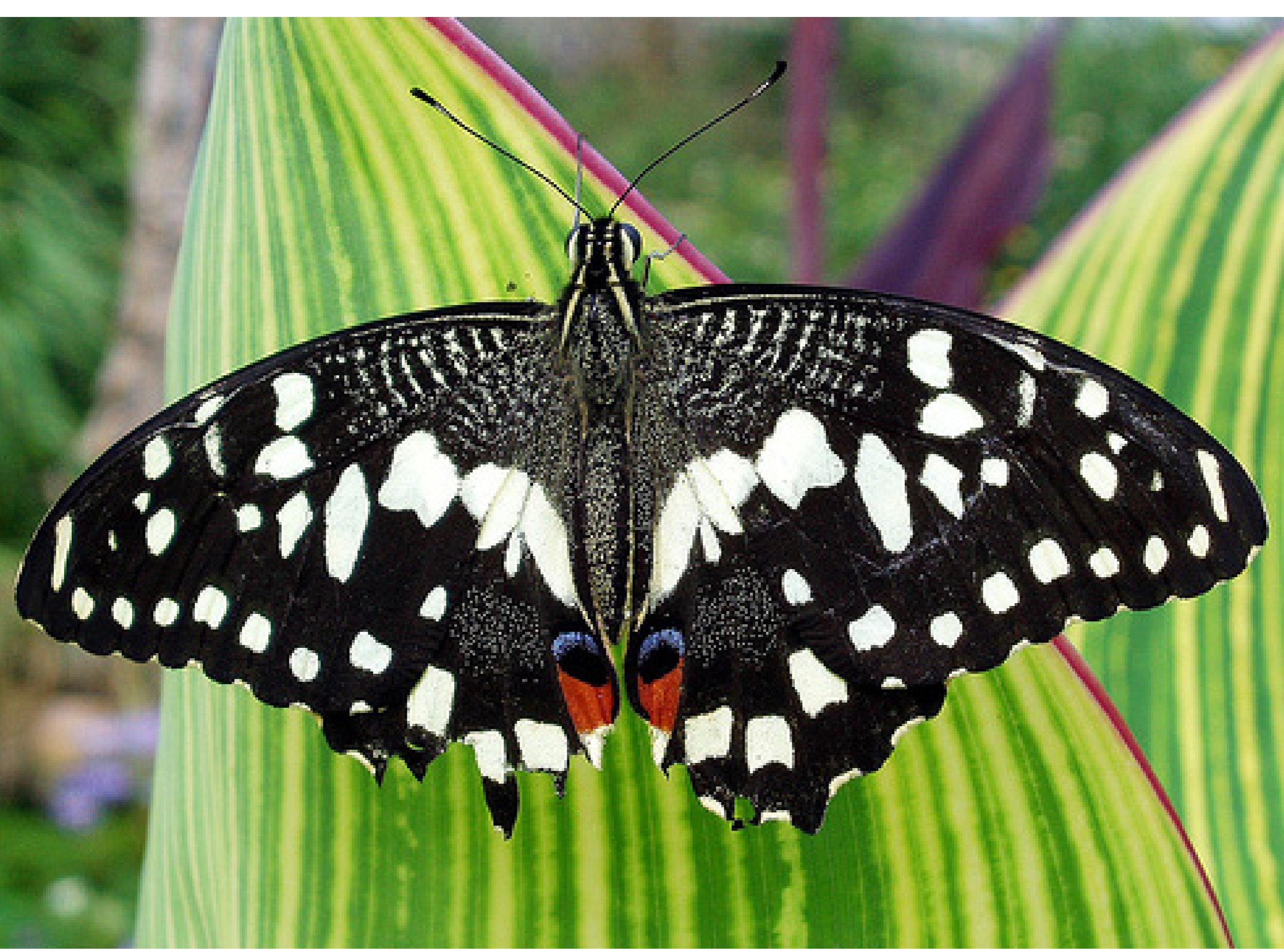
For each level of the Gaussian pyramid

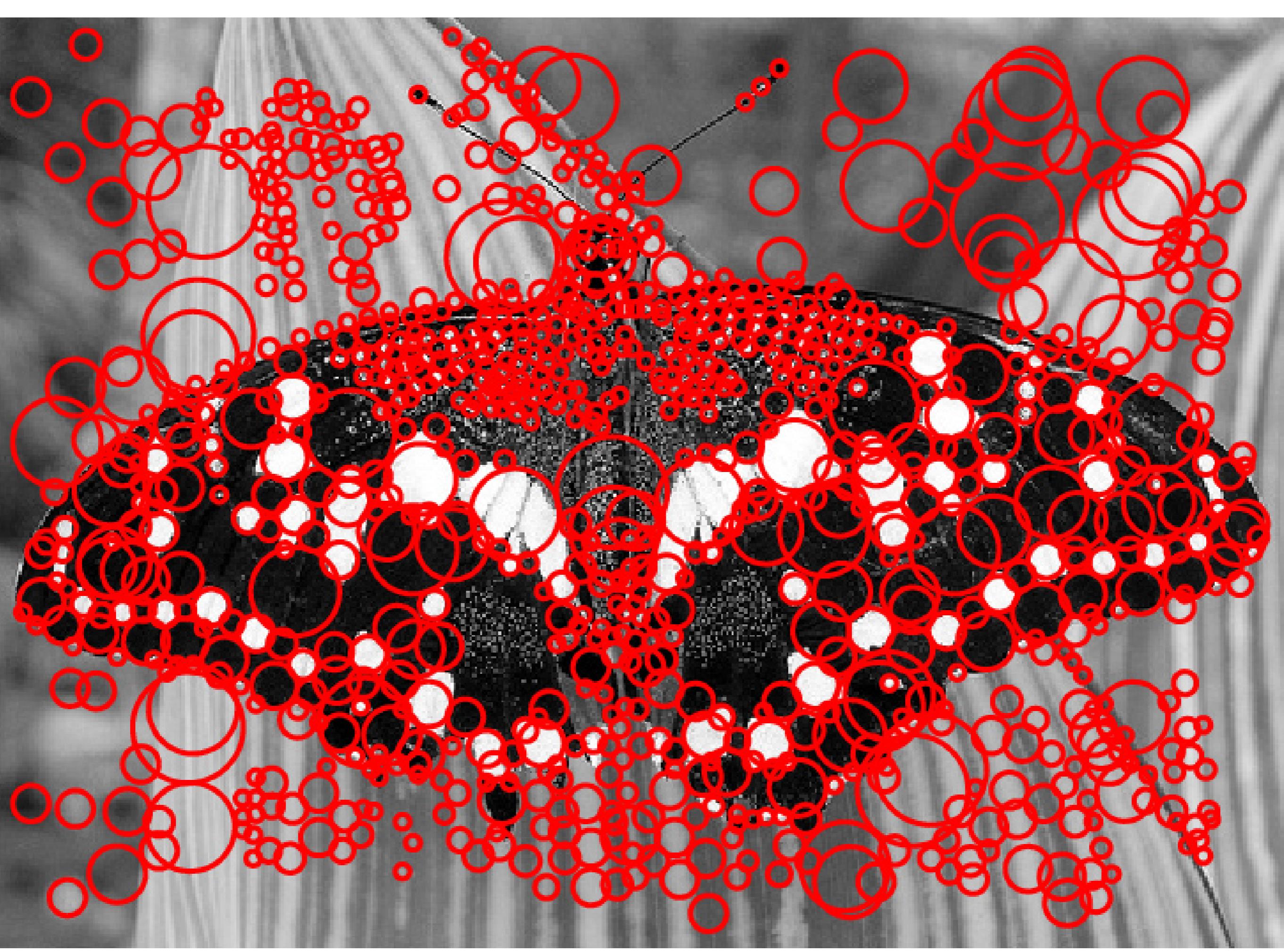
compute feature response (e.g. Harris, Laplacian)

For each level of the Gaussian pyramid

if local maximum and cross-scale

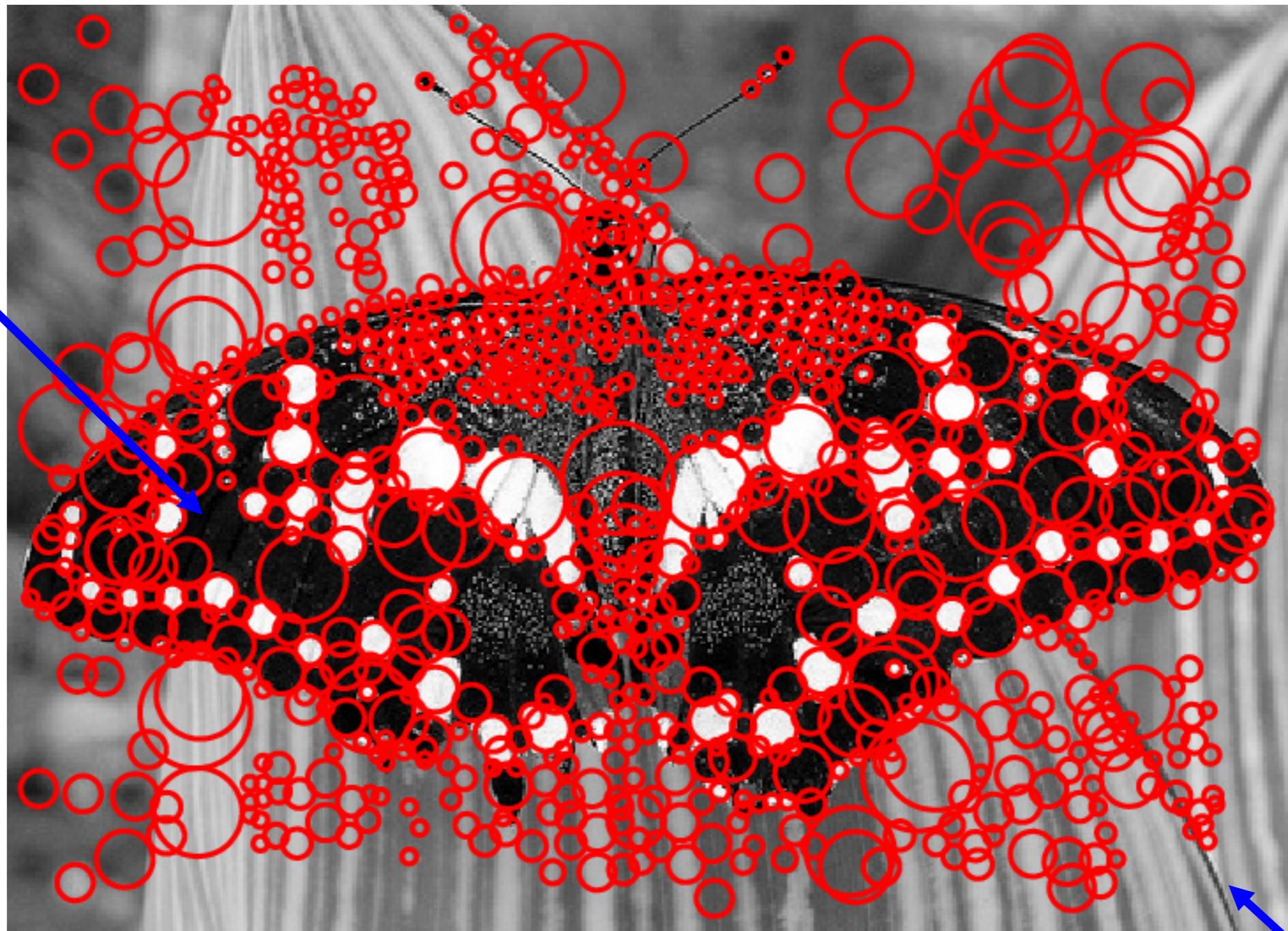
**save** scale and location of feature  $(x, y, s)$





# What do we detect here?

Blobby  
features



Line features

# Avoiding edge features?

Laplacian scale-space features fire on blobby features, but also on 1D lines. These are less desired as a feature as their location can easily shift in different images

# Avoiding edge features?

Test eigen values of local hessian

$$H = \begin{bmatrix} D_{xx} & D_{xy} \\ D_{yx} & D_{yy} \end{bmatrix}$$

D- difference of Gaussian image

$$R = \frac{TR(H)^2}{\text{Det}(H)} = \frac{(\lambda_{min} + \lambda_{max})^2}{\lambda_{min}\lambda_{max}}$$

Minimized when  
 $\lambda_{min} = \lambda_{max}$

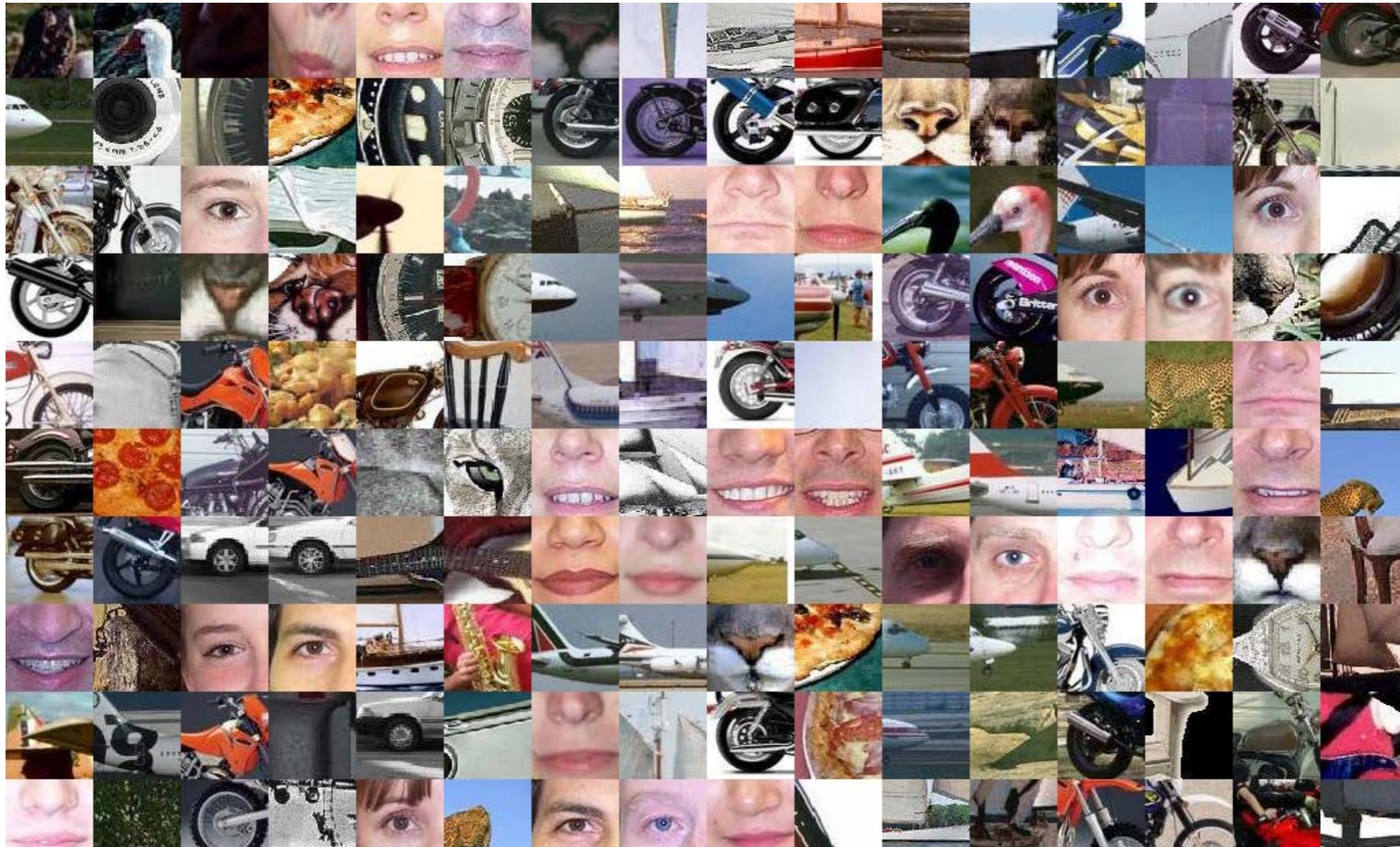
Edges  $\lambda_{min} \rightarrow 0, R \rightarrow \infty$

Discard features with  $R > \theta_r$ .

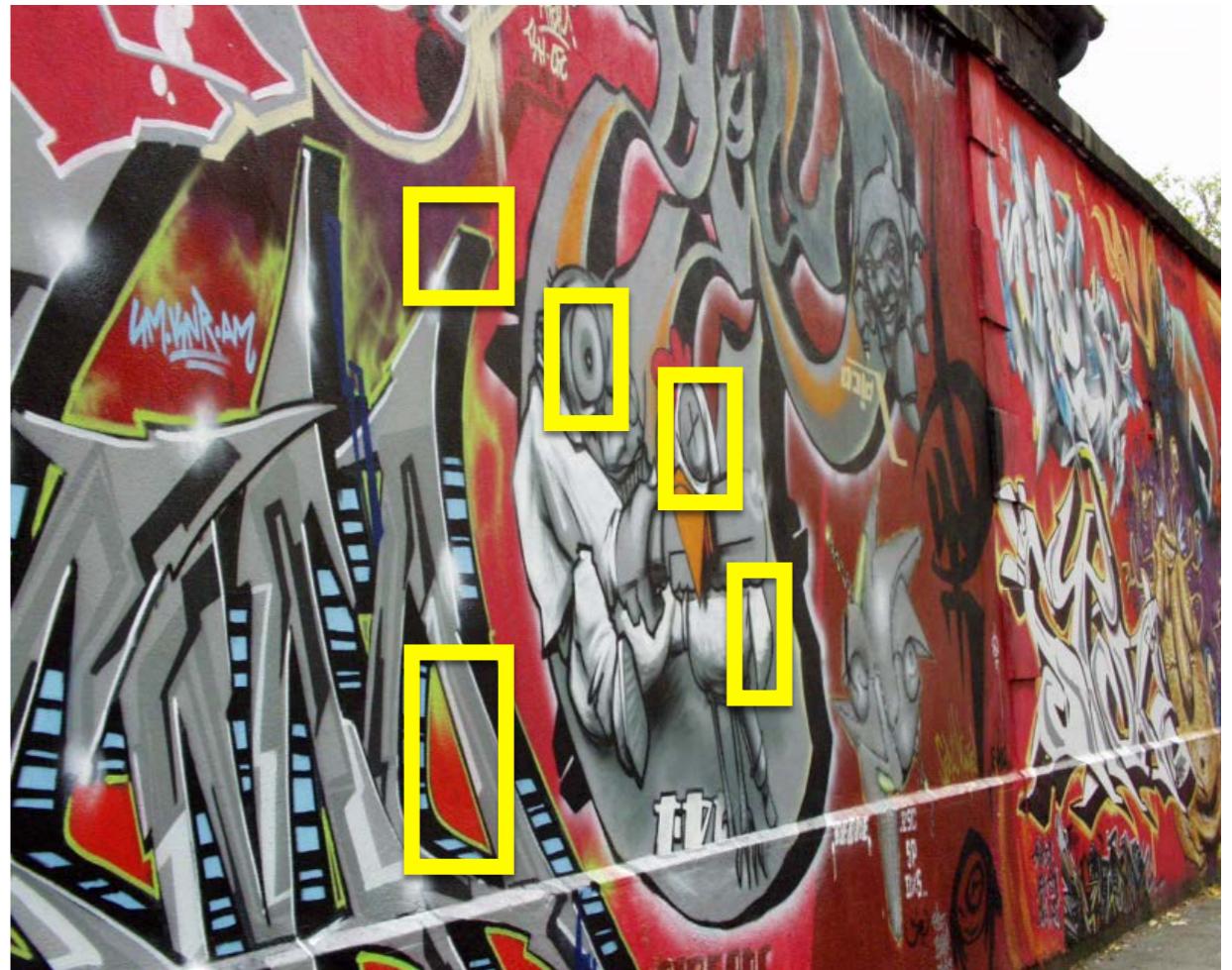
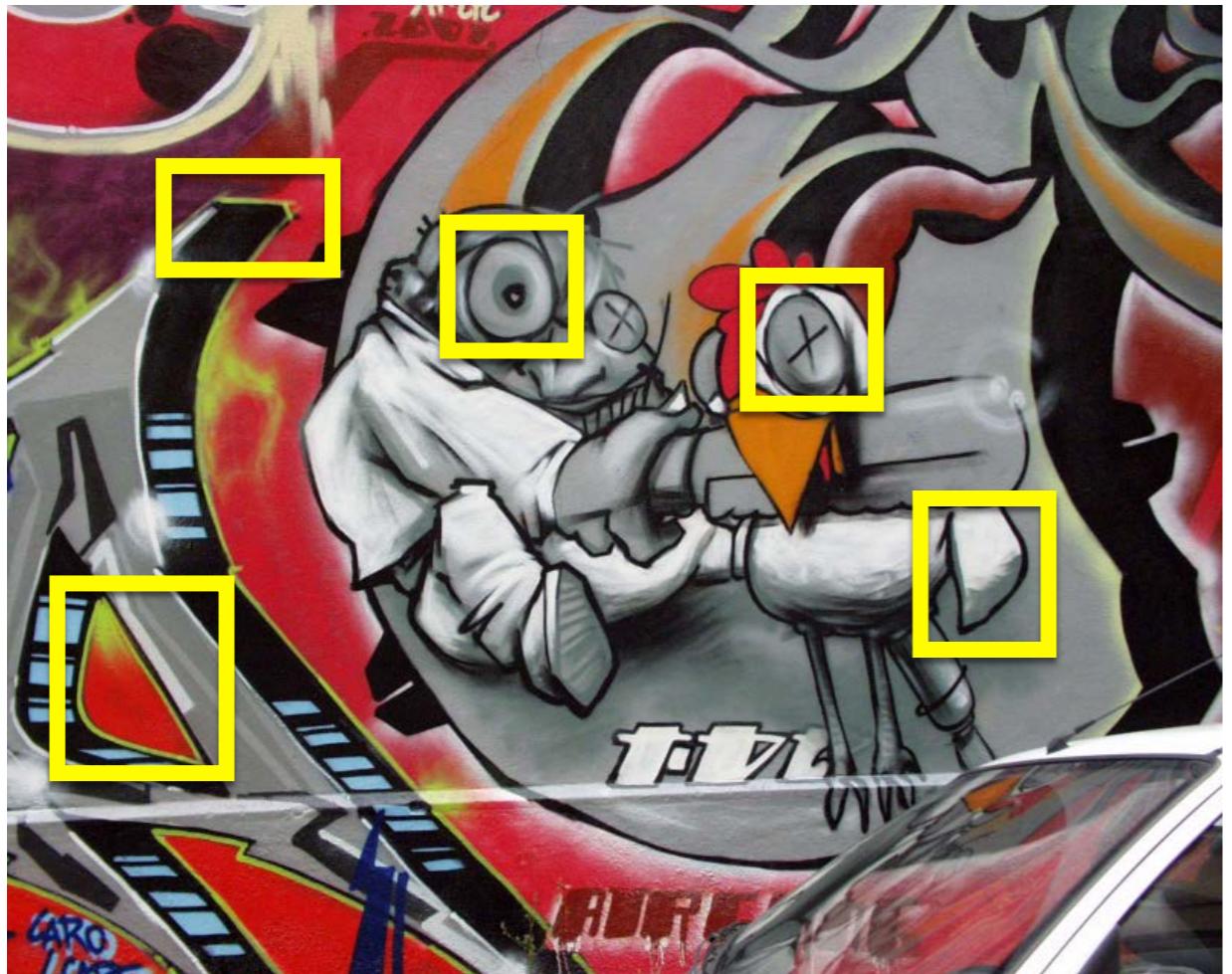
$$D_{xx} = (G_x * G_x) * D, \quad D_{xy} = (G_x * G_y) * D, \quad D_{yy} = (G_y * G_y) * D$$

e.g.  $(G_x * G_x) = [-1 \ 2 \ -1]$  (better use smooth filters )

# Feature descriptors



Why do we need feature  
descriptors?



If we know where the good features are,  
how do we match them?

# Object instance recognition



Schmid and Mohr 1997



Sivic and Zisserman, 2003

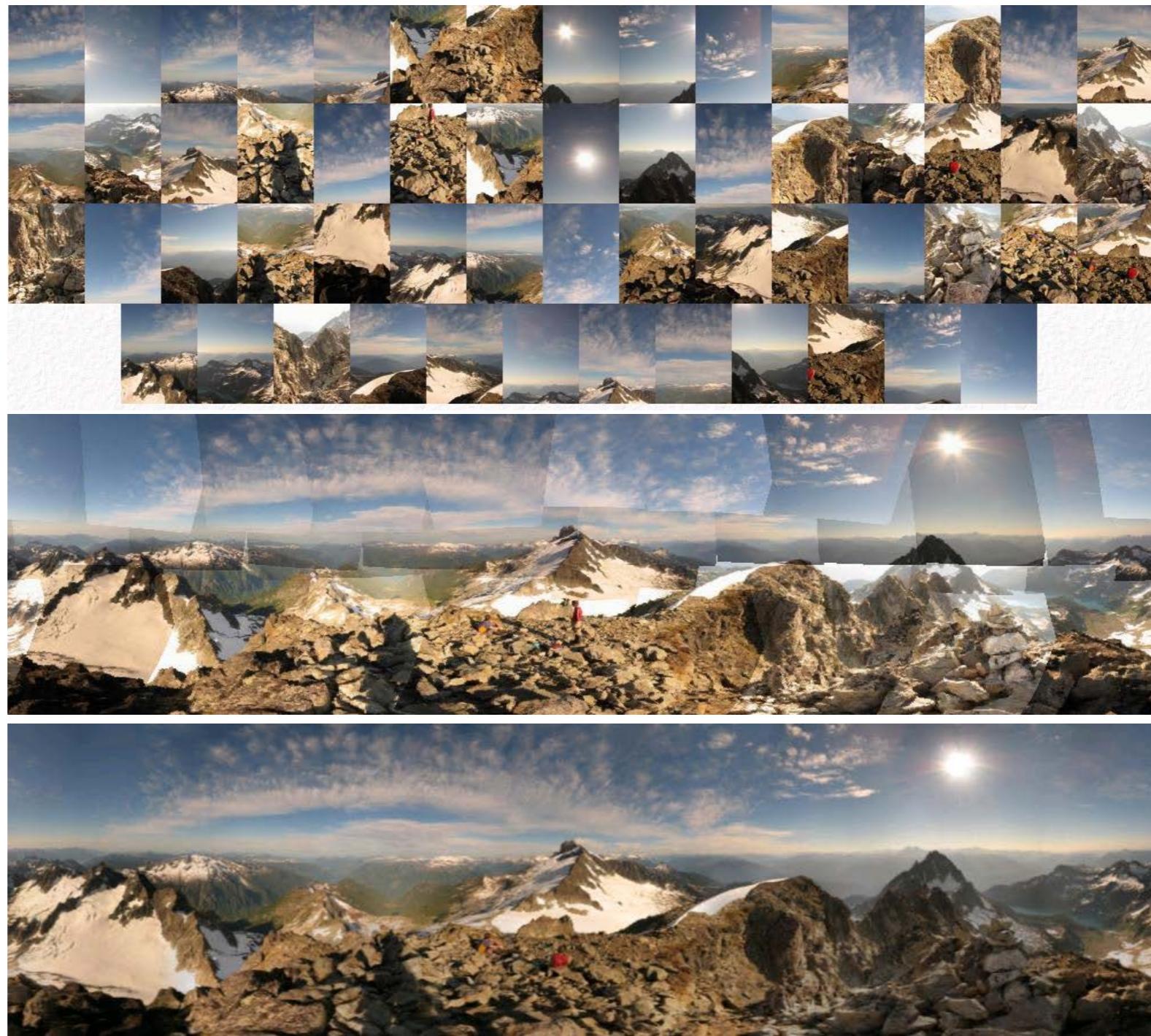


Rothganger et al. 2003



Lowe 2002

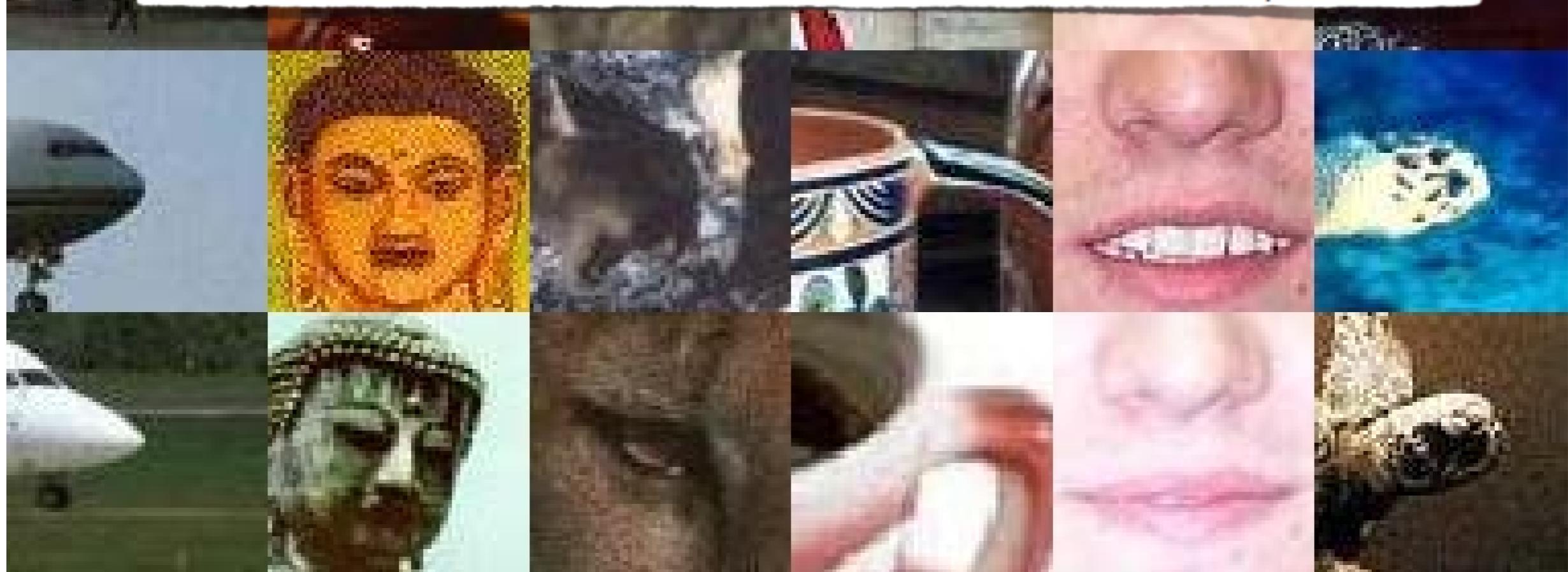
# Image mosaicing





## How do we describe an image patch?

Patches with similar content should have similar descriptors.

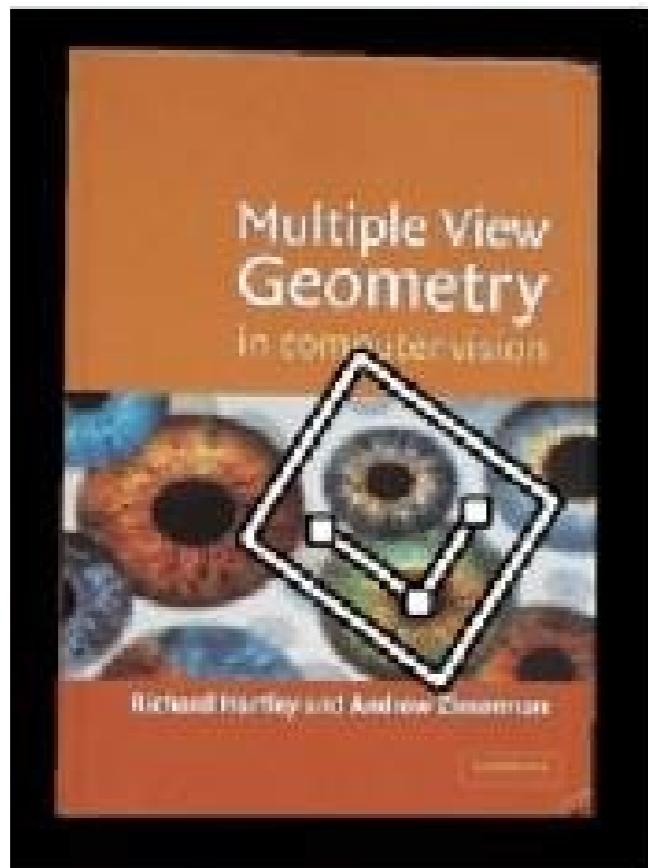


# Designing feature descriptors

# Photometric transformations



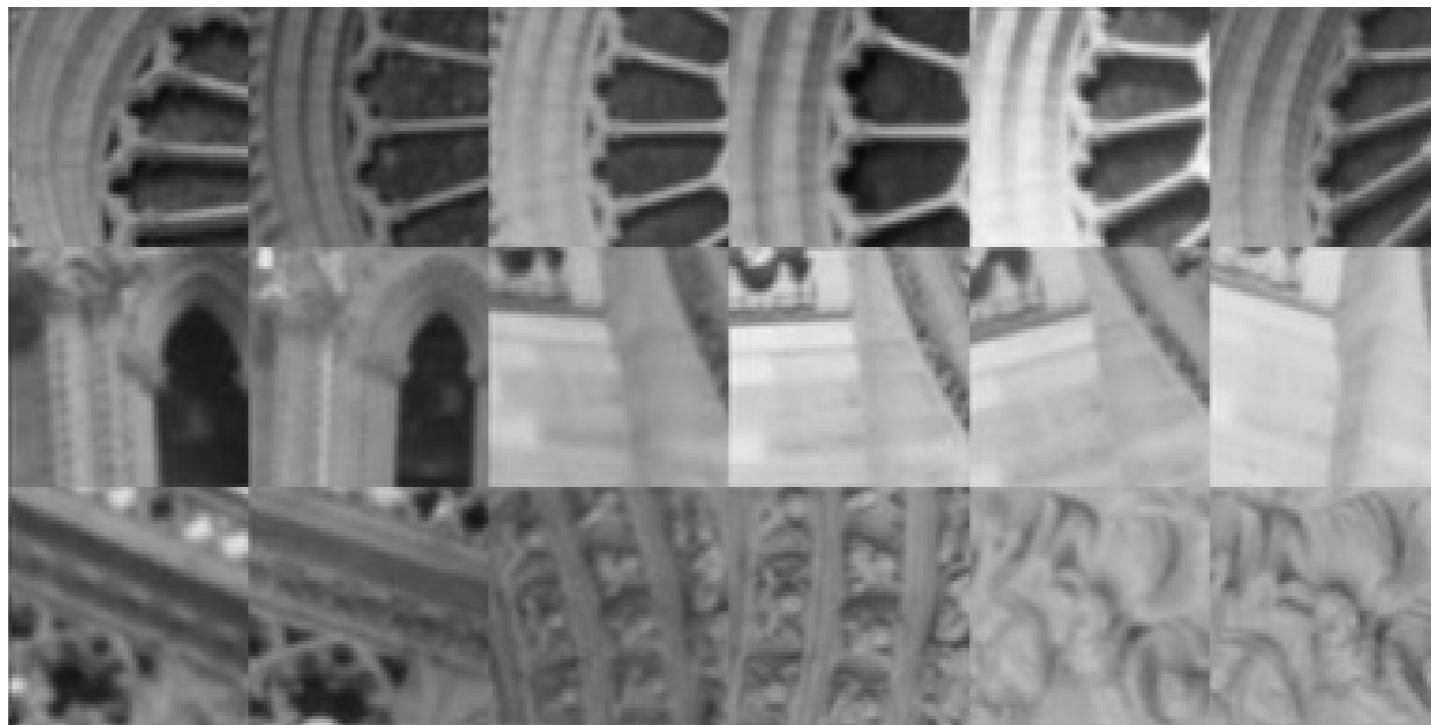
# Geometric transformations



objects will appear at different scales,  
translation and rotation



*What is the best descriptor for an image feature?*



# Image patch

Just use the pixel values of the patch



Perfectly fine if geometry and appearance is unchanged  
(a.k.a. template matching)

# Tiny Images



Just down-sample it!  
Simple, fast, robust to small affine transforms.



# Image patch

Just use the pixel values of the patch



Perfectly fine if geometry and appearance is unchanged  
(a.k.a. template matching)

*What are the problems?*

# Image patch

Just use the pixel values of the patch



Perfectly fine if geometry and appearance is unchanged  
(a.k.a. template matching)

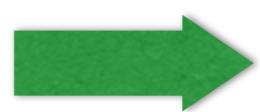
*What are the problems?*

*How can you be less sensitive to absolute intensity values?*

# Image gradients

Use pixel differences

1	2	3
4	5	6
7	8	9



$$( \quad - \quad + \quad + \quad - \quad - \quad + \quad )$$

vector of x derivatives

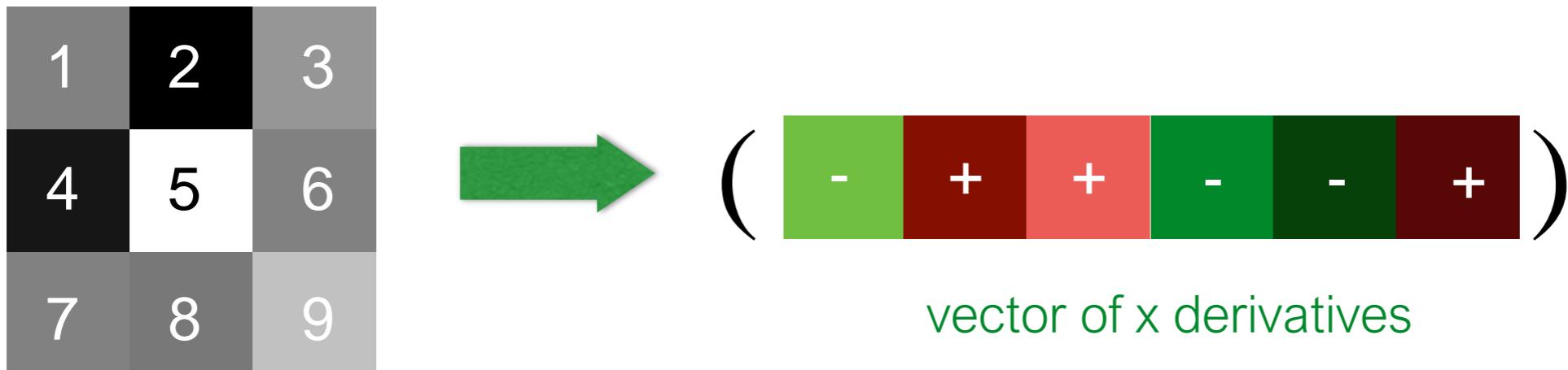
'binary descriptor'

Feature is invariant to absolute intensity values

*What are the problems?*

# Image gradients

Use pixel differences



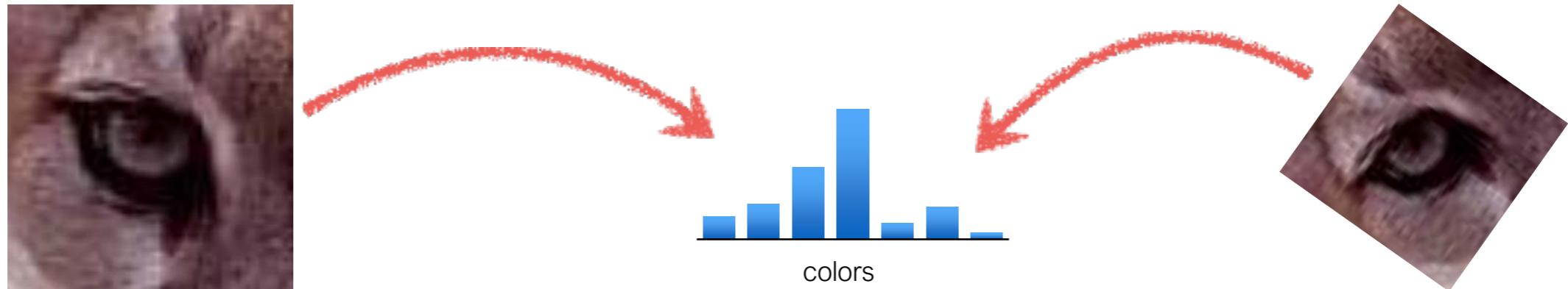
Feature is invariant to absolute intensity values

*What are the problems?*

*How can you be less sensitive to deformations?*

# Color histogram

Count the colors in the image using a histogram

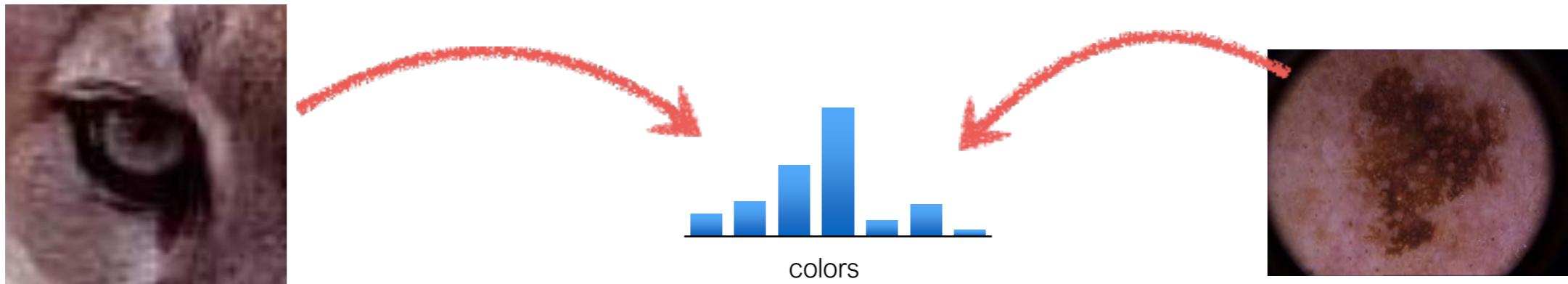


Invariant to changes in scale and rotation

*What are the problems?*

# Color histogram

Count the colors in the image using a histogram

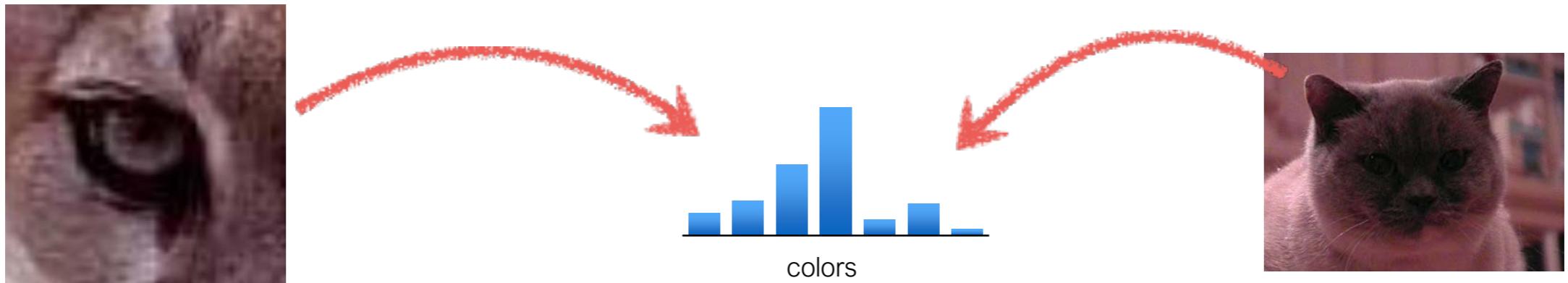


Invariant to changes in scale and rotation

*What are the problems?*

# Color histogram

Count the colors in the image using a histogram



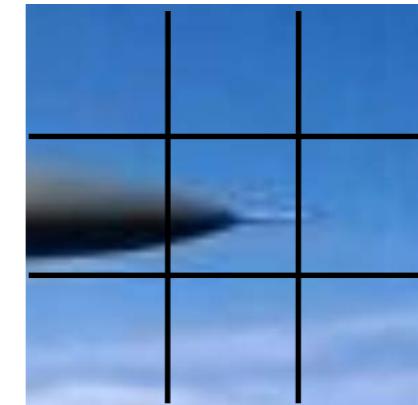
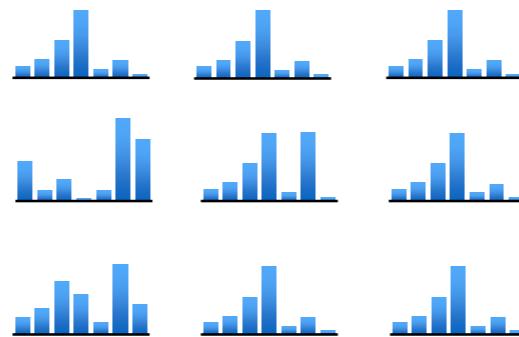
Invariant to changes in scale and rotation

*What are the problems?*

*How can you be more sensitive to spatial layout?*

# Spatial histograms

Compute histograms over spatial ‘cells’

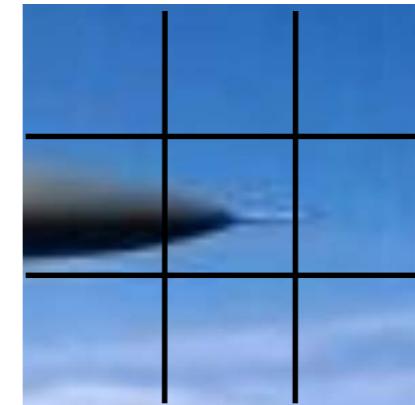
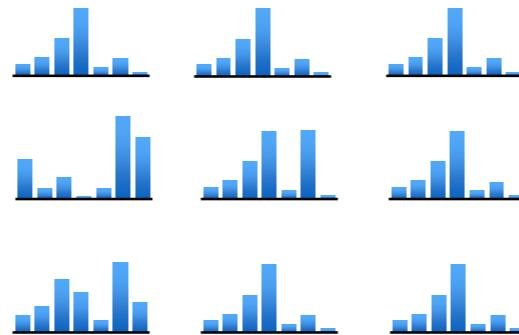


Retains rough spatial layout  
Some invariance to deformations

*What are the problems?*

# Spatial histograms

Compute histograms over spatial ‘cells’



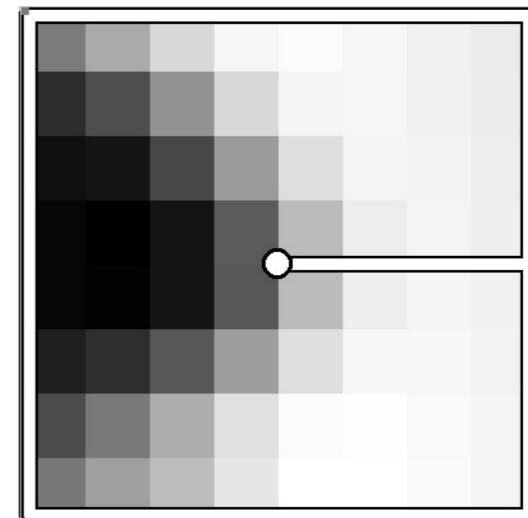
Retains rough spatial layout  
Some invariance to deformations

*What are the problems?*

*How can you be completely invariant to rotation?*

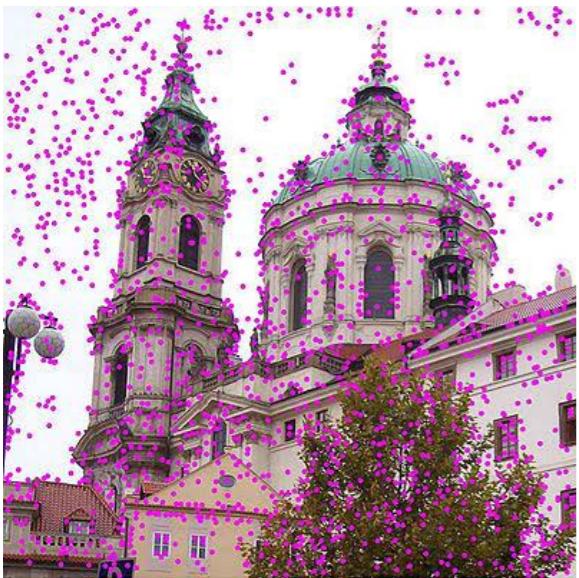
# Orientation normalization

Use the dominant image gradient direction to normalize the orientation of the patch



save the orientation angle  $\theta$  along with  $(x, y, s)$

# SIFT descriptor



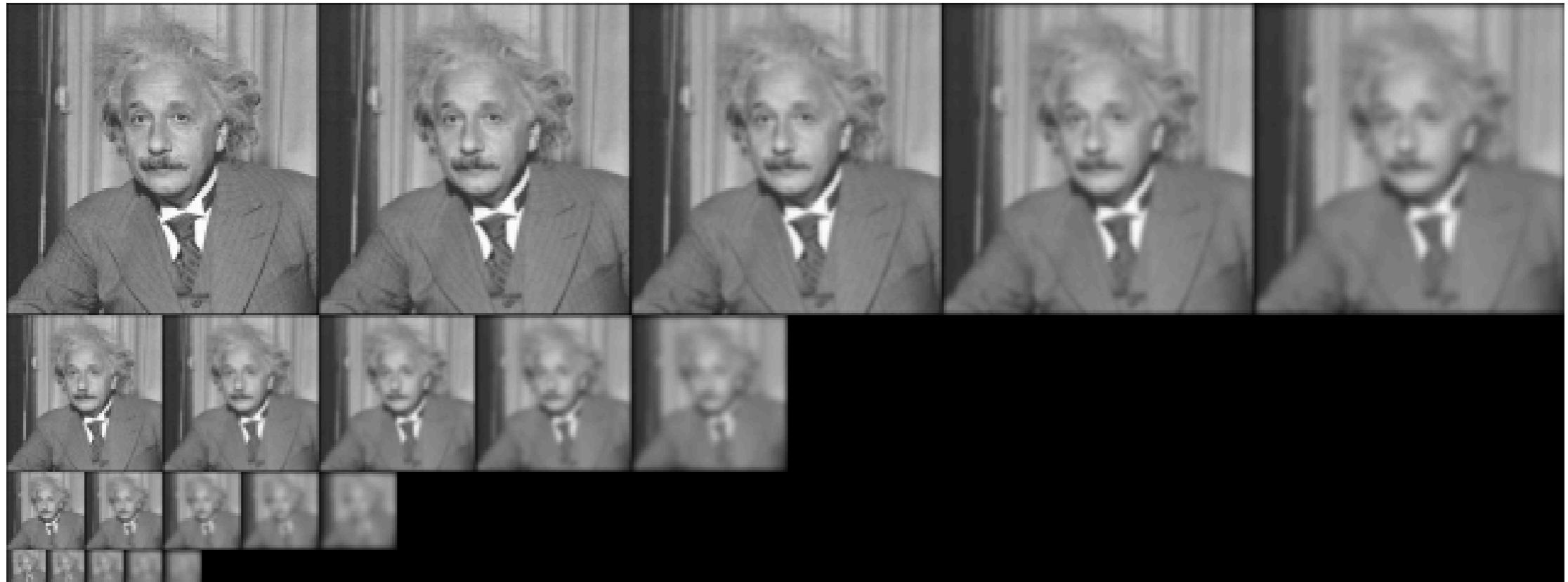
# SIFT

(Scale Invariant Feature Transform)

SIFT describes both a **detector** and **descriptor**

1. Multi-scale extrema detection
2. Keypoint localization
3. Orientation assignment
4. Keypoint descriptor

Discussed  
earlier

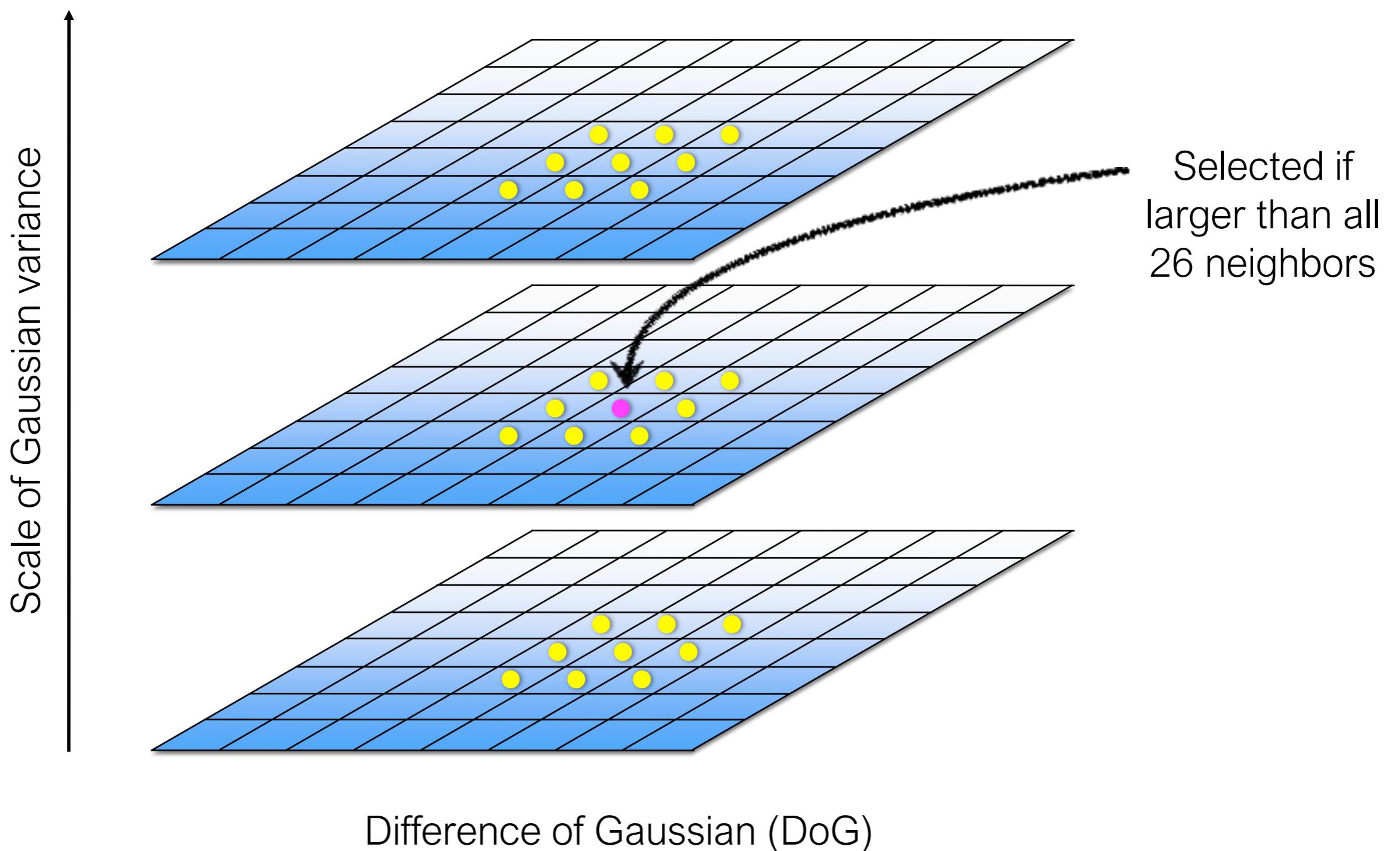


Gaussian



Laplacian

# Scale-space extrema



# 3. Orientation assignment

For a keypoint,  $\mathbf{L}$  is the **Gaussian-smoothed** image with the closest scale,

$$m(x, y) = \sqrt{(L(x + 1, y) - L(x - 1, y))^2 + (L(x, y + 1) - L(x, y - 1))^2}$$

x-derivative   y-derivative

$$\theta(x, y) = \tan^{-1}((L(x, y + 1) - L(x, y - 1)) / (L(x + 1, y) - L(x - 1, y)))$$

Detection process returns

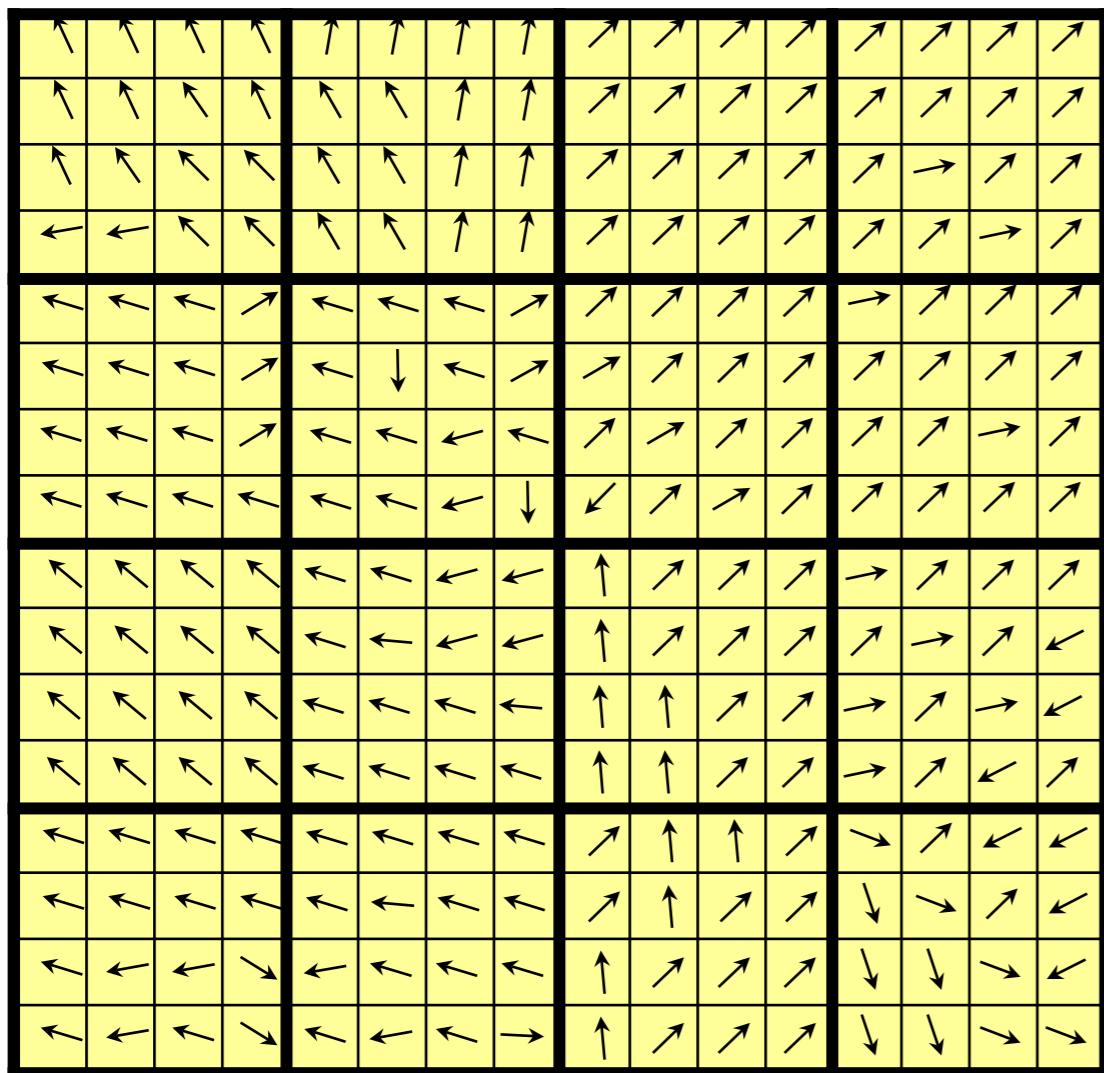
$$\{x, y, \sigma, \theta\}$$

location    scale    orientation

# 4. Keypoint descriptor

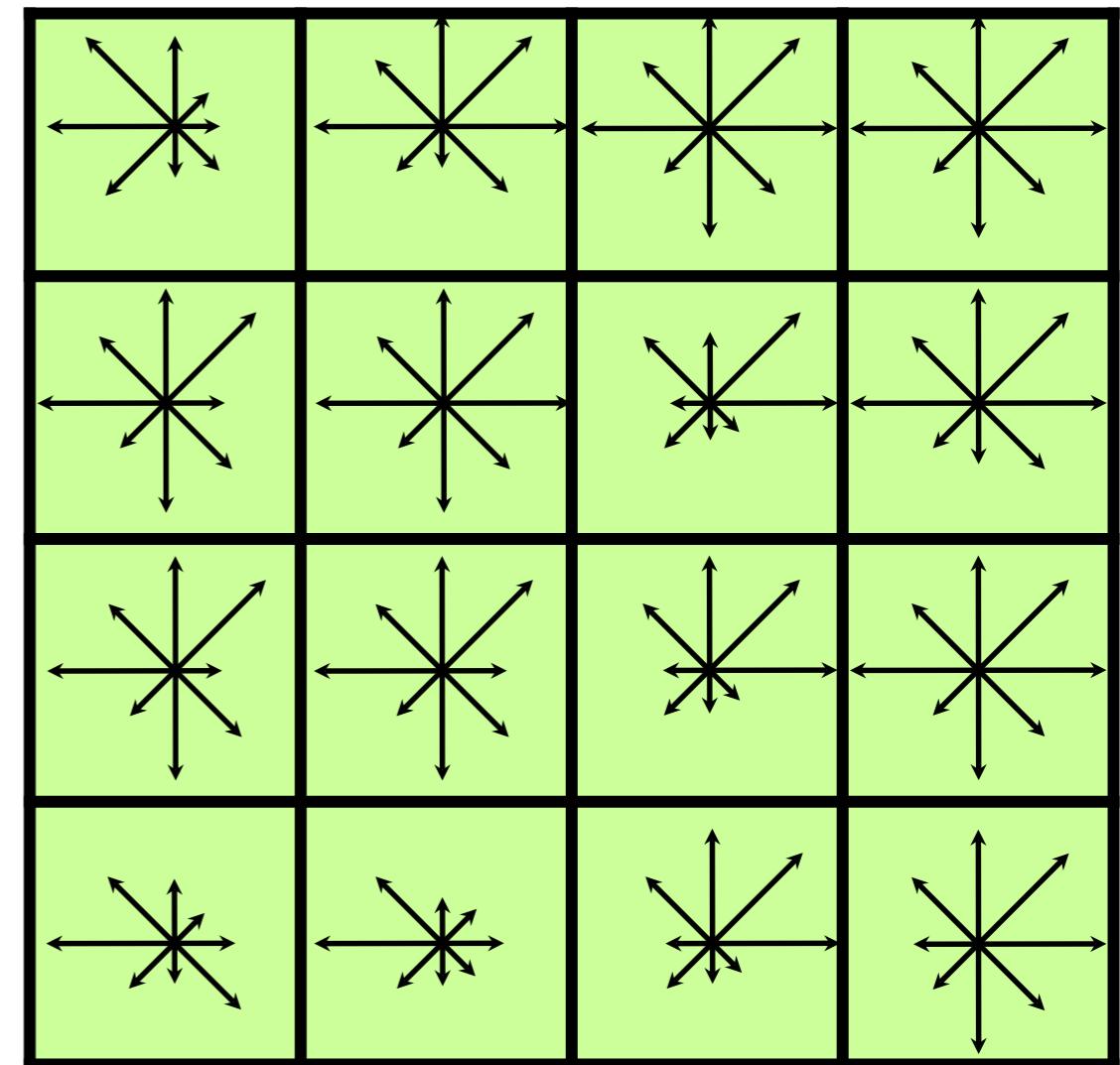
Image Gradients

( $4 \times 4$  pixel per cell,  $4 \times 4$  cells)



SIFT descriptor

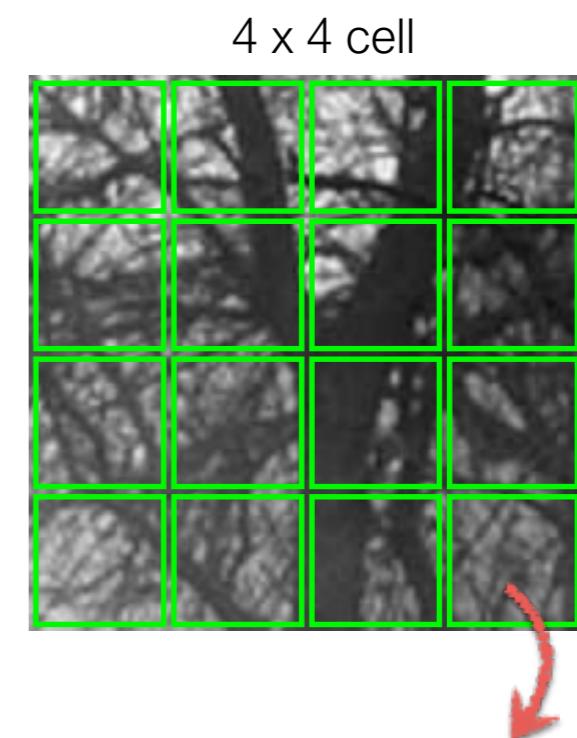
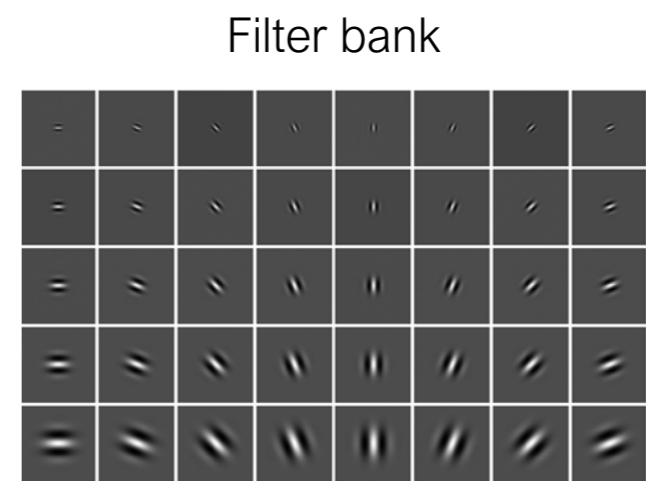
( $16$  cells  $\times$   $8$  directions =  $128$  dims)



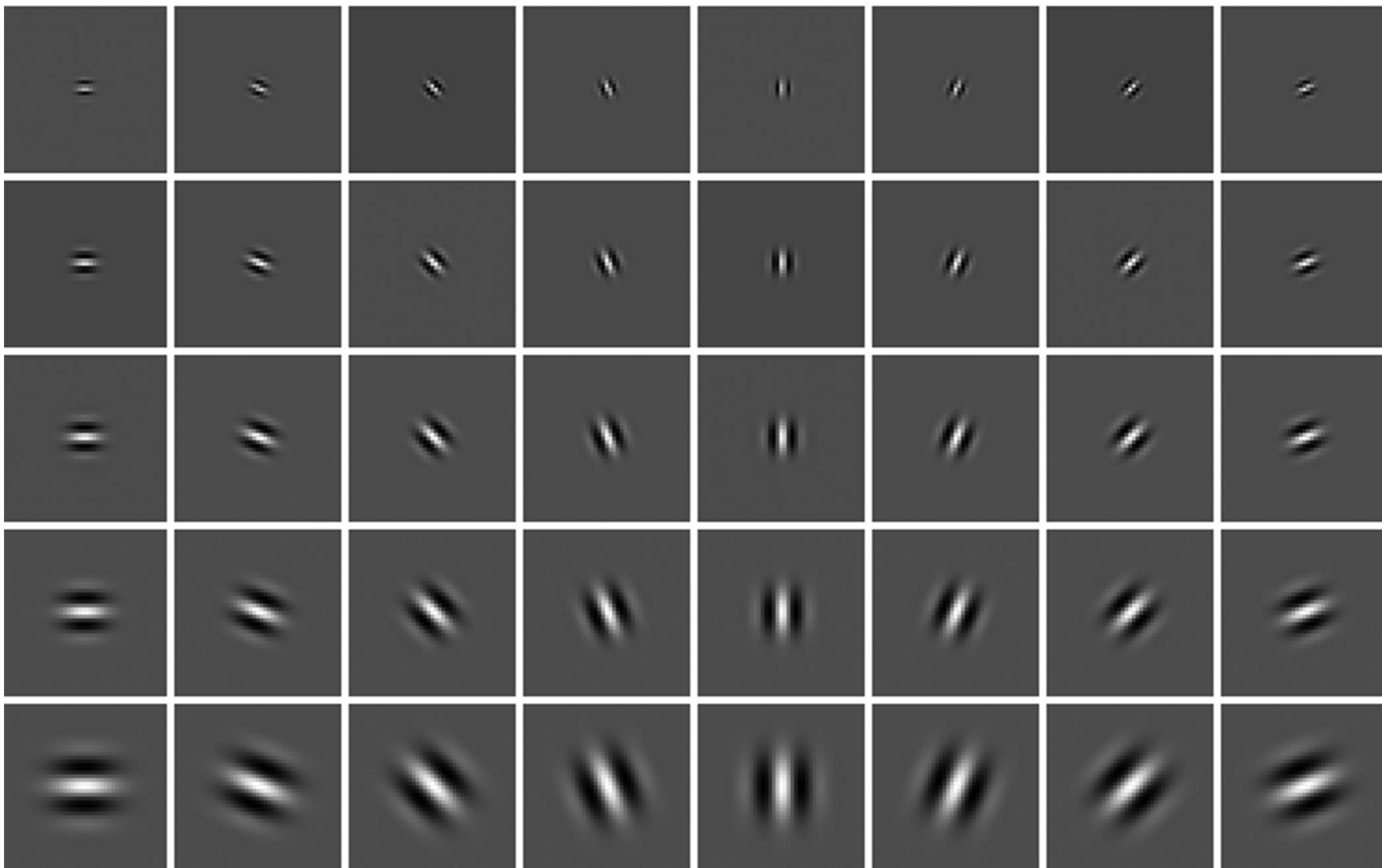
# GIST descriptor

# GIST

1. Compute filter responses (filter bank of Gabor filters)
2. Divide image patch into  $4 \times 4$  cells
3. Compute filter response averages for each cell
4. Size of descriptor is  $4 \times 4 \times N$ , where  $N$  is the size of the filter bank

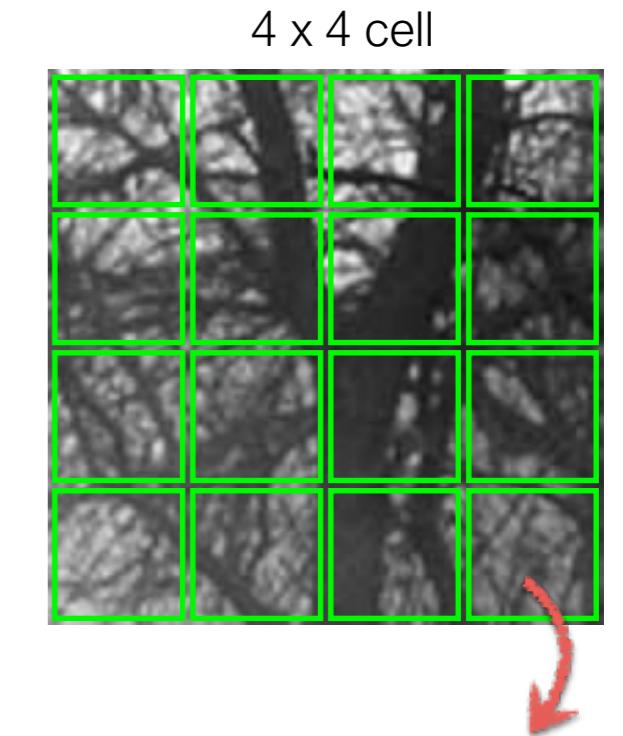
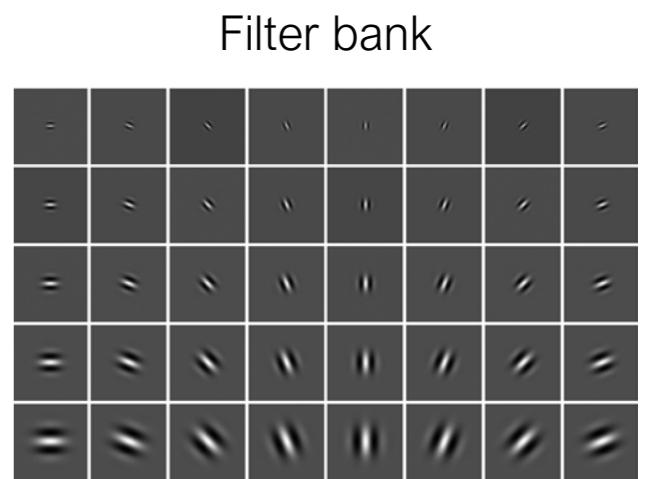


# Directional edge detectors



# GIST

1. Compute filter responses (filter bank of Gabor filters)
2. Divide image patch into  $4 \times 4$  cells
3. Compute filter response averages for each cell
4. Size of descriptor is  $4 \times 4 \times N$ , where  $N$  is the size of the filter bank

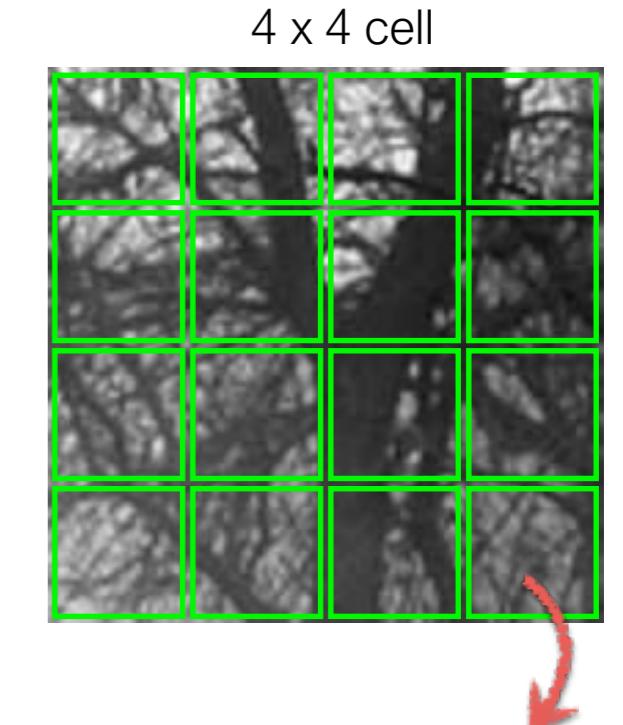
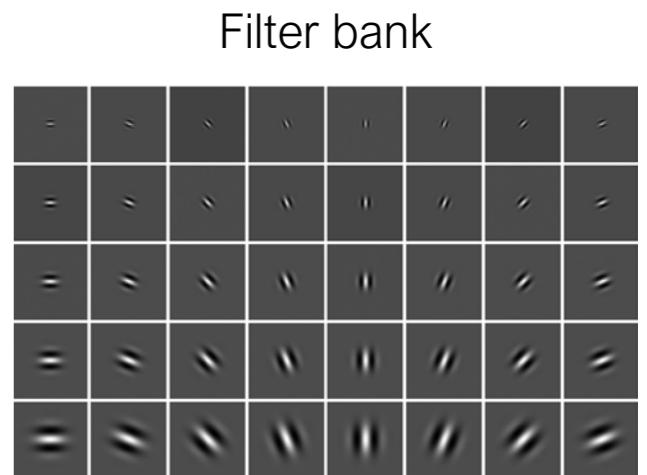


*What is the GIST descriptor encoding?*



# GIST

1. Compute filter responses (filter bank of Gabor filters)
2. Divide image patch into  $4 \times 4$  cells
3. Compute filter response averages for each cell
4. Size of descriptor is  $4 \times 4 \times N$ , where  $N$  is the size of the filter bank



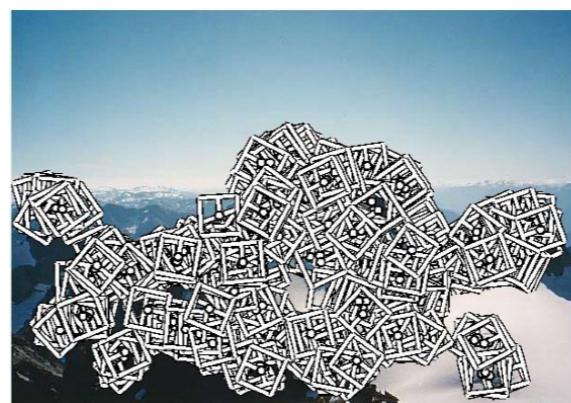
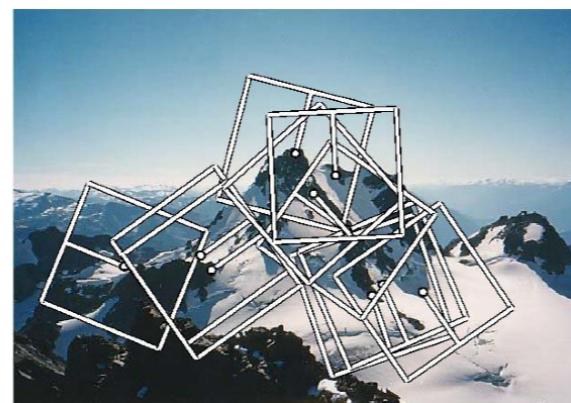
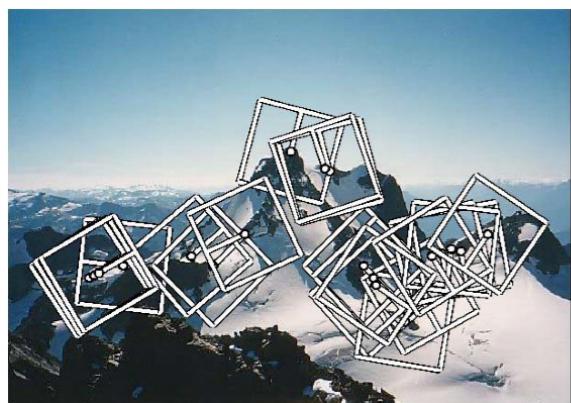
*What is the GIST descriptor encoding?*

Rough spatial distribution of image gradients

# MOPS descriptor

# Multi-Scale Oriented Patches (MOPS)

Multi-Image Matching using Multi-Scale Oriented Patches. M. Brown, R. Szeliski and S. Winder.  
International Conference on Computer Vision and Pattern Recognition (CVPR2005). pages 510-517



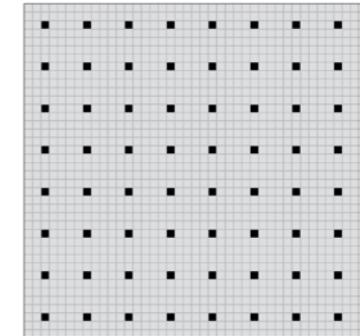
# Multi-Scale Oriented Patches (MOPS)

Multi-Image Matching using Multi-Scale Oriented Patches. M. Brown, R. Szeliski and S. Winder.  
International Conference on Computer Vision and Pattern Recognition (CVPR2005). pages 510-517

Given a feature  $(x, y, s, \theta)$

Get  $40 \times 40$  image patch, subsample  
every 5th pixel

(*what's the purpose of this step?*)



Subtract the mean, divide by standard  
deviation

(*what's the purpose of this step?*)

Haar Wavelet Transform

(*what's the purpose of this step?*)

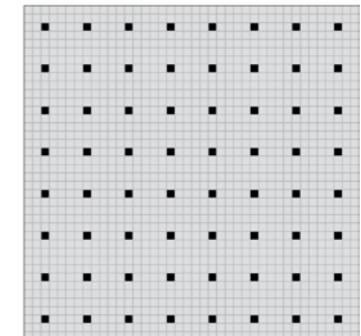
# Multi-Scale Oriented Patches (MOPS)

Multi-Image Matching using Multi-Scale Oriented Patches. M. Brown, R. Szeliski and S. Winder.  
International Conference on Computer Vision and Pattern Recognition (CVPR2005). pages 510-517

Given a feature  $(x, y, s, \theta)$

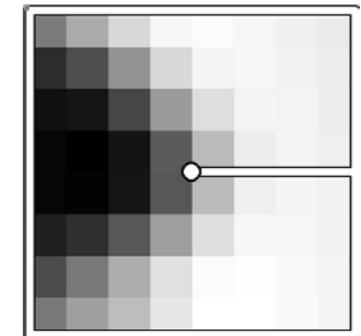
Get  $40 \times 40$  image patch, subsample  
every 5th pixel

(low frequency filtering, absorbs localization errors)



Subtract the mean, divide by standard  
deviation

(*what's the purpose of this step?*)



Haar Wavelet Transform

(*what's the purpose of this step?*)

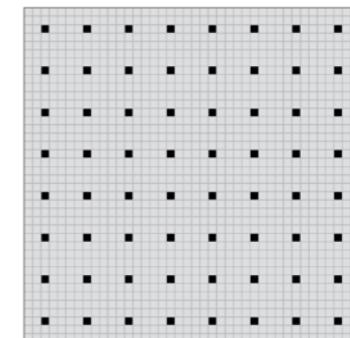
# Multi-Scale Oriented Patches (MOPS)

Multi-Image Matching using Multi-Scale Oriented Patches. M. Brown, R. Szeliski and S. Winder.  
International Conference on Computer Vision and Pattern Recognition (CVPR2005). pages 510-517

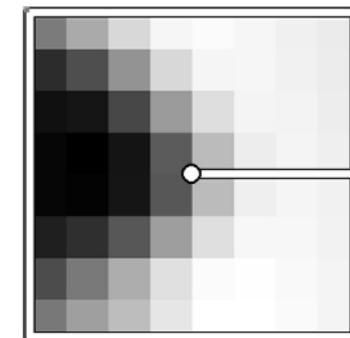
Given a feature  $(x, y, s, \theta)$

Get  $40 \times 40$  image patch, subsample  
every 5th pixel

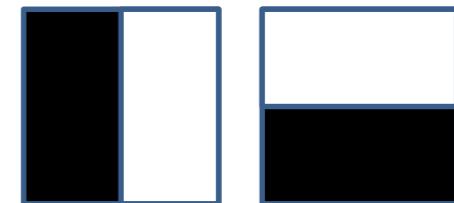
(low frequency filtering, absorbs localization errors)



Subtract the mean, divide by standard  
deviation  
(removes bias and gain)



Haar Wavelet Transform  
*(what's the purpose of this step?)*



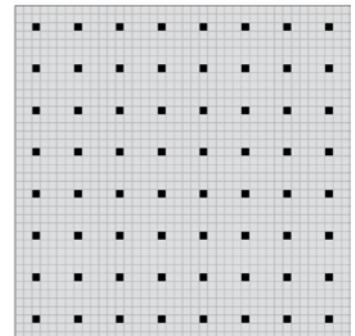
# Multi-Scale Oriented Patches (MOPS)

Multi-Image Matching using Multi-Scale Oriented Patches. M. Brown, R. Szeliski and S. Winder.  
International Conference on Computer Vision and Pattern Recognition (CVPR2005). pages 510-517

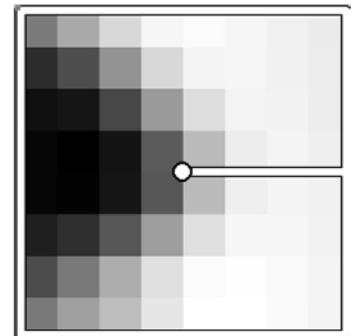
Given a feature  $(x, y, s, \theta)$

Get  $40 \times 40$  image patch, subsample  
every 5th pixel

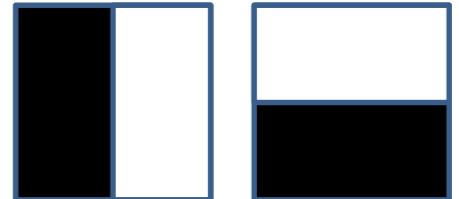
(low frequency filtering, absorbs localization errors)



Subtract the mean, divide by standard  
deviation  
(removes bias and gain)



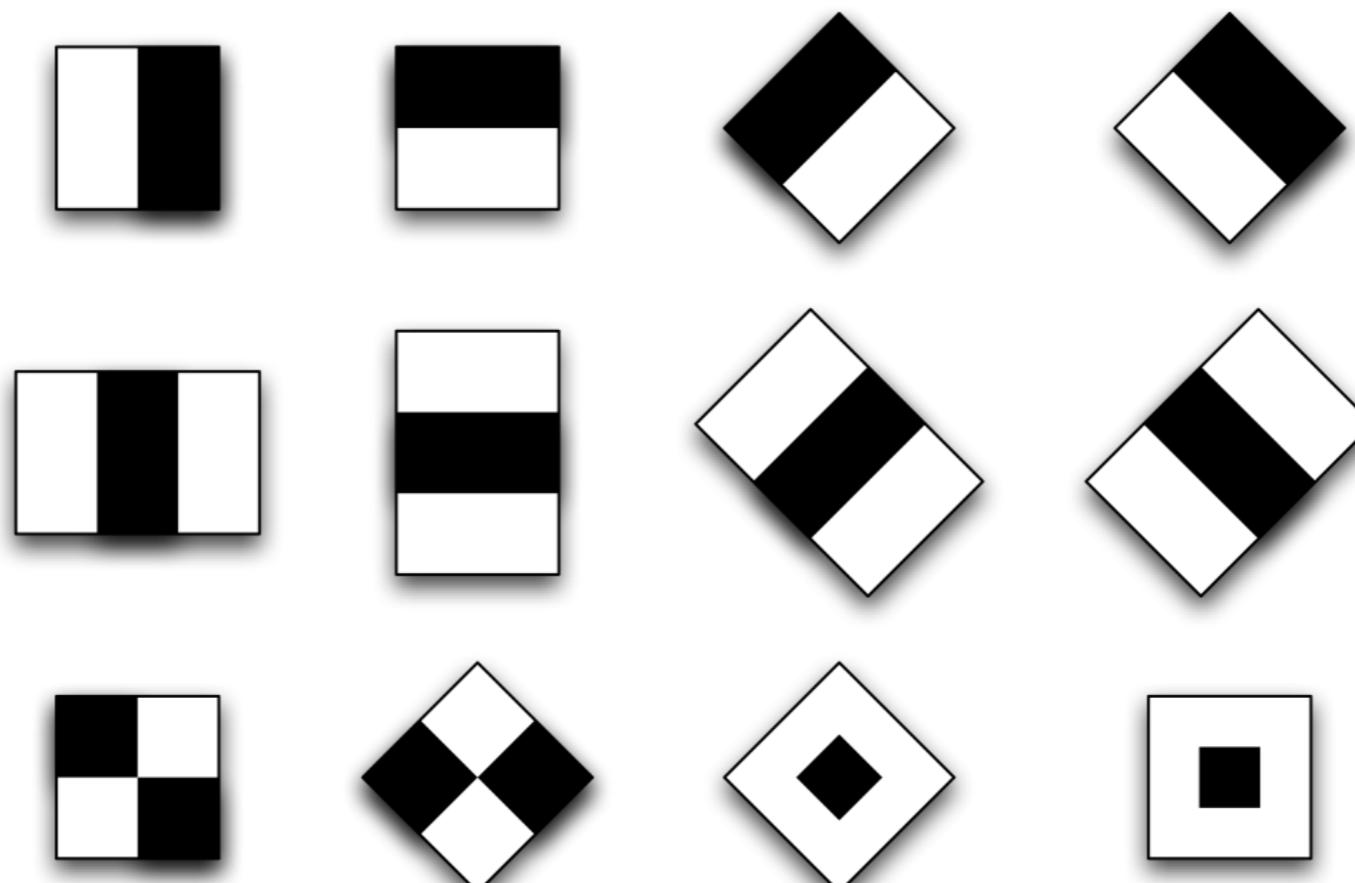
Haar Wavelet Transform  
(low frequency projection)



# Haar Wavelets

(actually, Haar-like features)

Use responses of a bank of filters as a descriptor



Can compute Haar wavelet responses **efficiently** (in constant time) with integral images

# BRIEF descriptor

Binary Robust Independent  
Elementary Features

# BRIEF Binary Robust Independent Elementary Features

Michael Calonder, Vincent Lepetit, Christoph Strecha, and Pascal Fua. ECCV2010

Select a set of  $n=256$  pairs  $(x, y)$  in patch  $I$ .

Store a binary descriptor

$$\rho_n(I, x_n, y_n) = \begin{cases} 1 & \text{if } I(x_n) < I(y_n) \\ 0 & \text{otherwise} \end{cases}$$

Invariant to affine intensity transformation?

$$\rho_n(I, x_n, y_n) = \rho_n(aI + b, x_n, y_n)$$

Invariant to scale?

Apply at characteristic scale of the interest point

Invariant to rotation?

Nope

Robustness to noise?

Smooth image before testing < relations

# BRIEF Binary Robust Independent Elementary Features

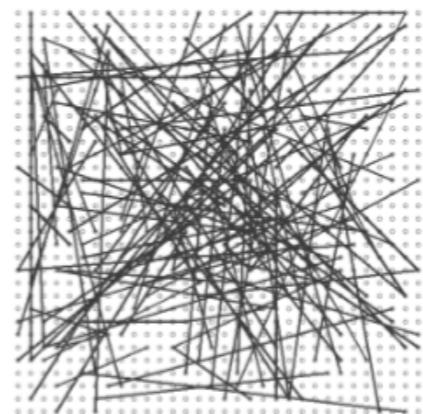
Michael Calonder, Vincent Lepetit, Christoph Strecha, and Pascal Fua. ECCV2010

Select a set of  $n=256$  pairs  $(x, y)$  in patch  $I$ .

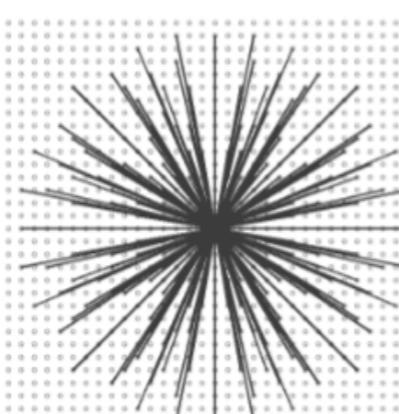
Store a binary descriptor

$$\rho_n(I, x_n, y_n) = \begin{cases} 1 & \text{if } I(x_n) < I(y_n) \\ 0 & \text{otherwise} \end{cases}$$

Which pairs to select?



Random



Polar grid

# References

Basic reading:

- Szeliski textbook, Sections 4.1.