

Segmentation



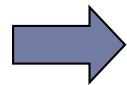
Slide credit: Ioannis Gkioulekas ,Kris Kitani, Fredo Durand, James Hays, Srinivasa Narasimhan Derek Hoiem, Lihi Zelnik

Image segmentation

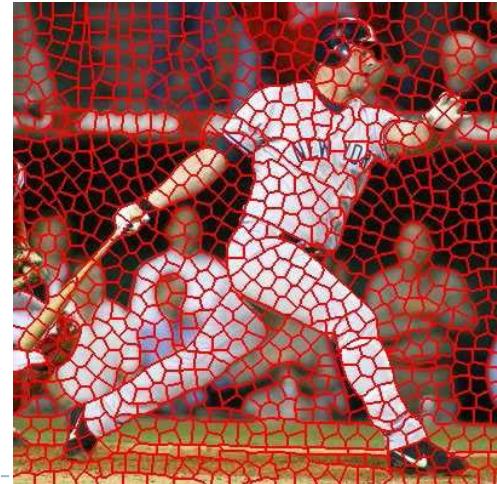
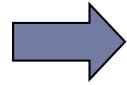
Group pixels into meaningful or perceptually similar regions



Segmentation for efficiency: “superpixels”



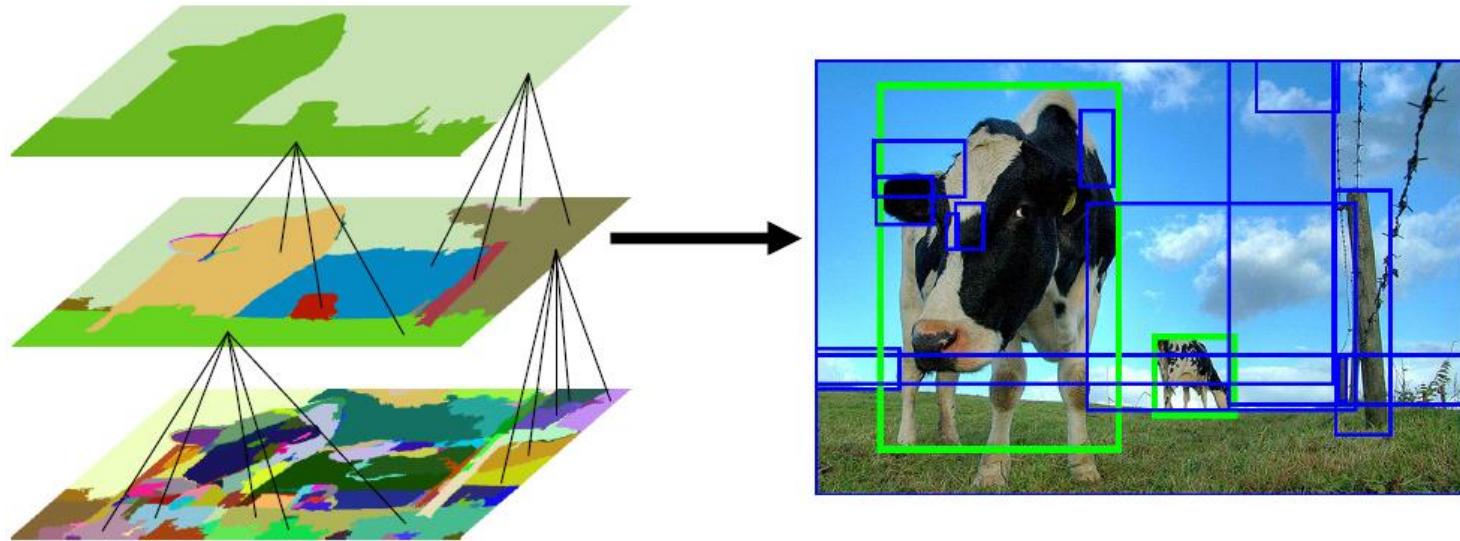
[Felzenszwalb and Huttenlocher 2004]



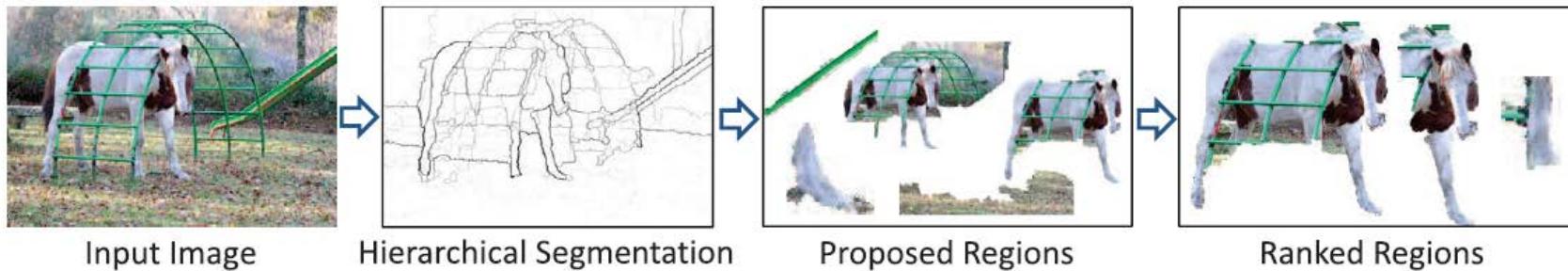
[Hoiem et al. 2005, Mori 2005]



Segmentation for object proposals



“Selective Search” [Sande, Uijlings et al. ICCV 2011, IJCV 2013]



[Endres & Hoiem ECCV 2010, IJCV 2014]

Segmentation for image editing



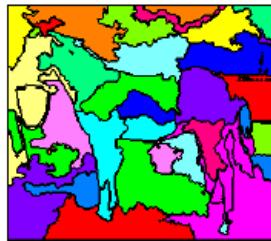
Major processes for segmentation

- ▶ Bottom-up: group tokens with similar features
- ▶ Top-down: group tokens that likely belong to the same object

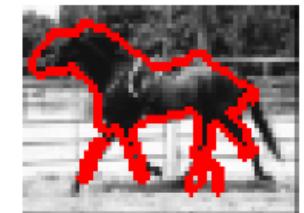
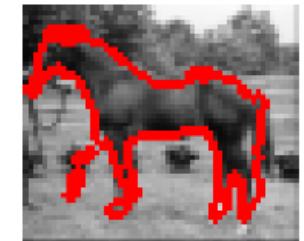
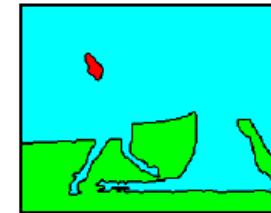
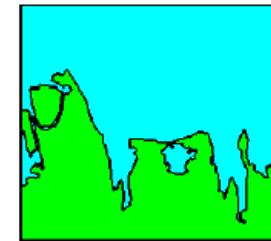
Input



Bottom-up



Top-down



(a)

(b)

(c)



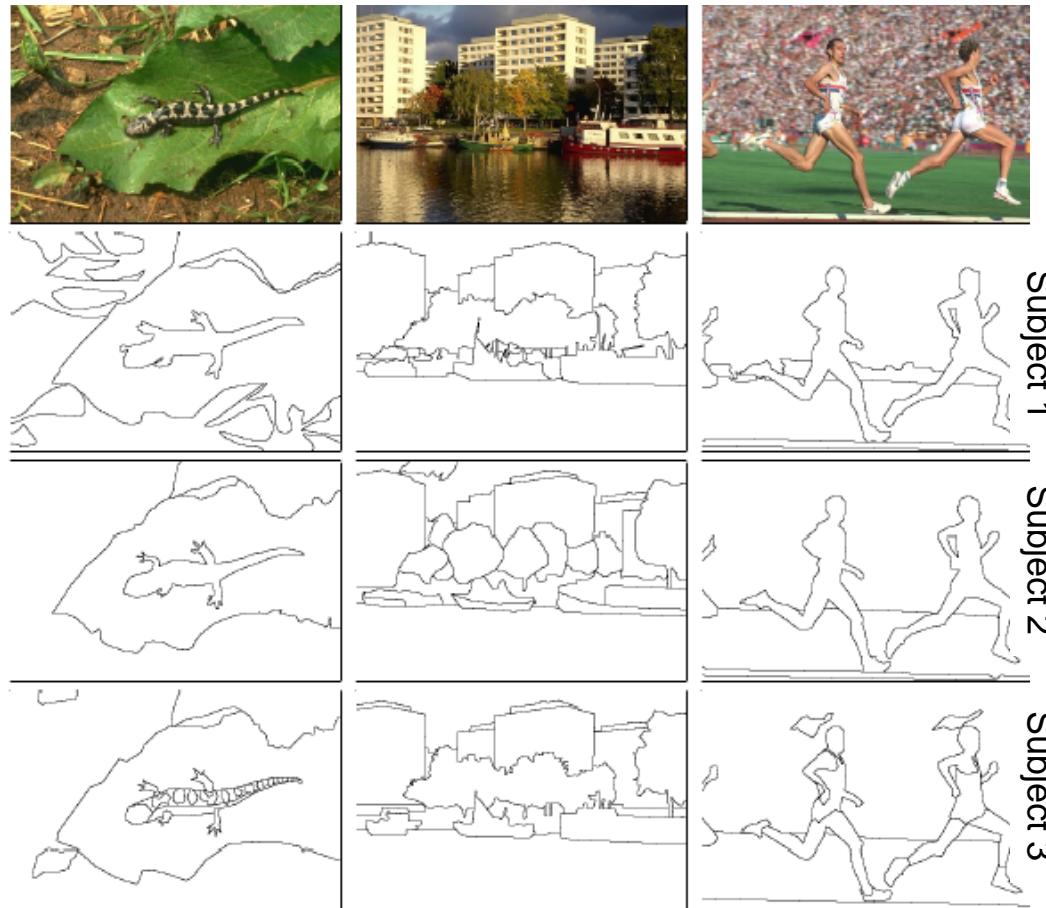


What is a “good” segmentation??



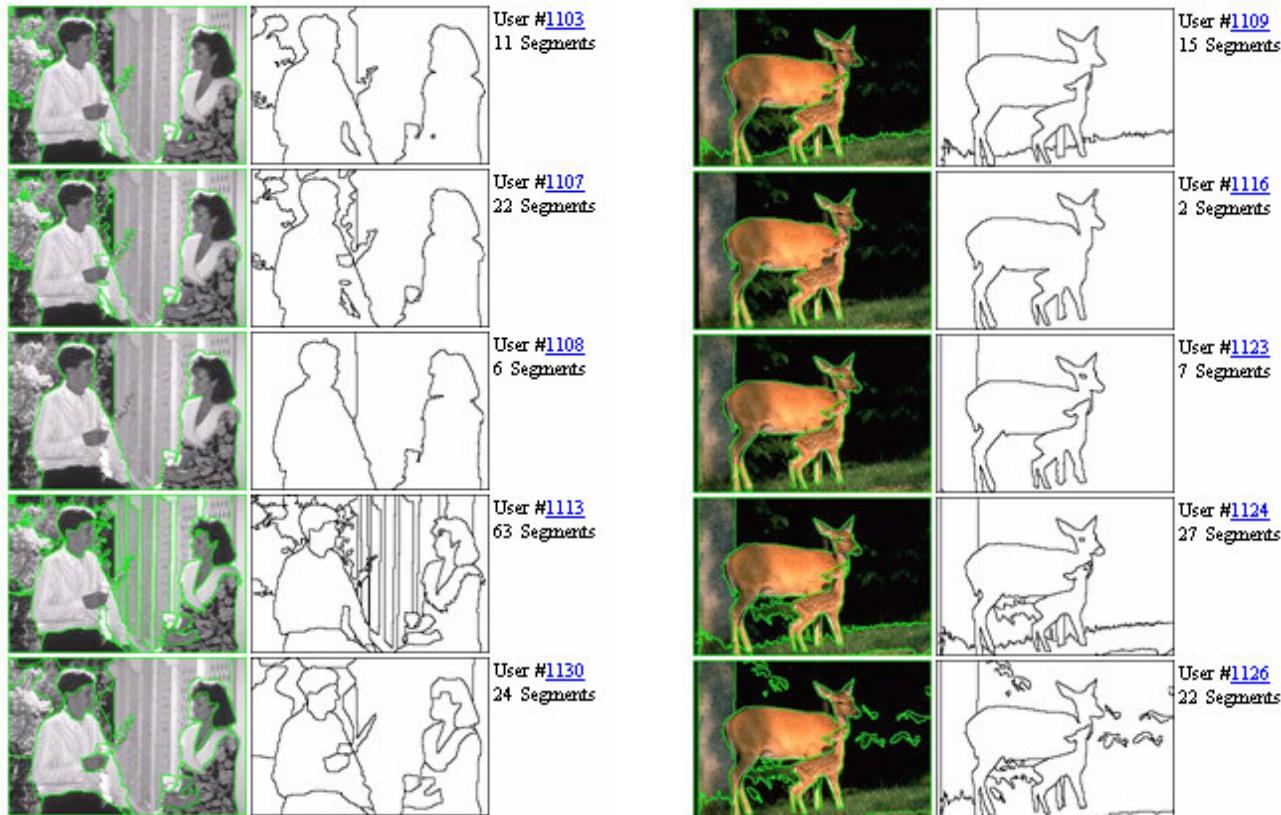
First idea: Compare to human segmentation or to “ground truth”

No objective definition of segmentation!



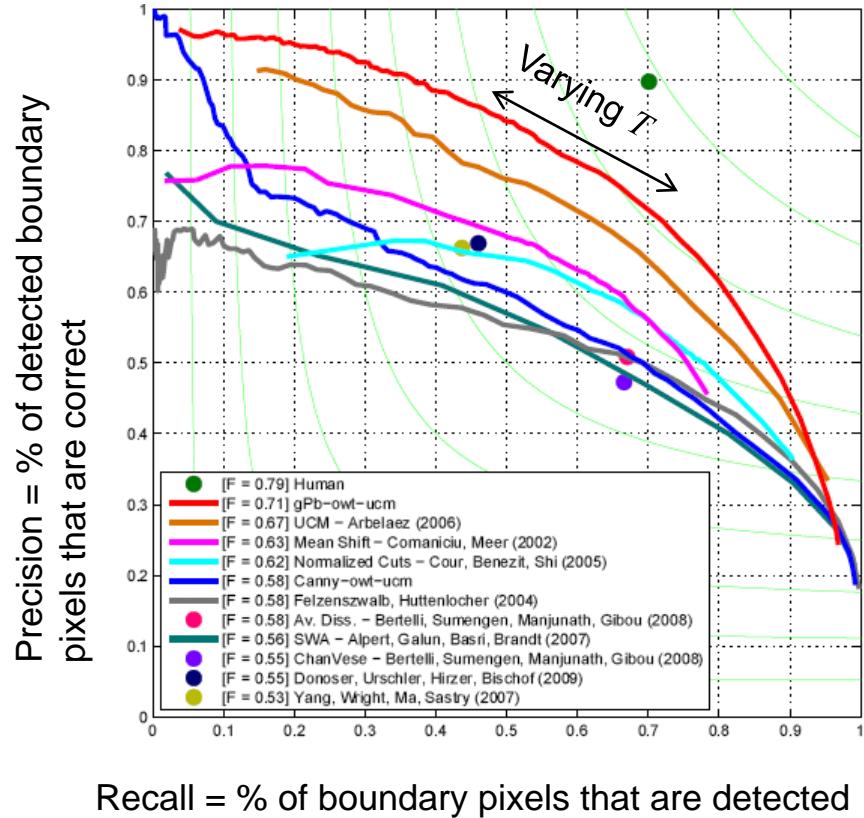
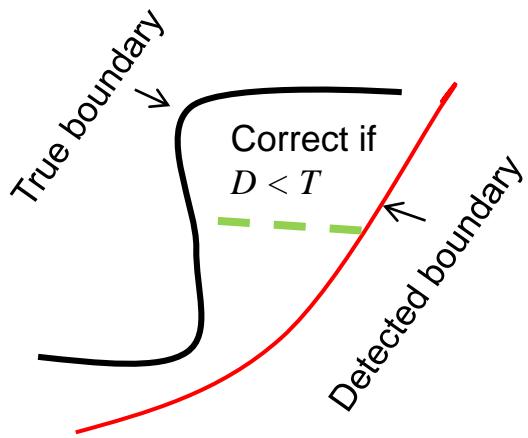
- ▶ <http://www.eecs.berkeley.edu/Research/Projects/CS/vision/grouping/resources.html>

No objective definition of segmentation!



- ▶ <http://www.eecs.berkeley.edu/Research/Projects/CS/vision/bags/BSDS300/html/dataset/images/color/317080.html>

Evaluation: Boundary agreement



Evaluation: Region overlap with ground truth



Ground Truth



Segment #1

.825

$$OS(S, G) = \frac{|S \cap G|}{|S \cup G|}$$

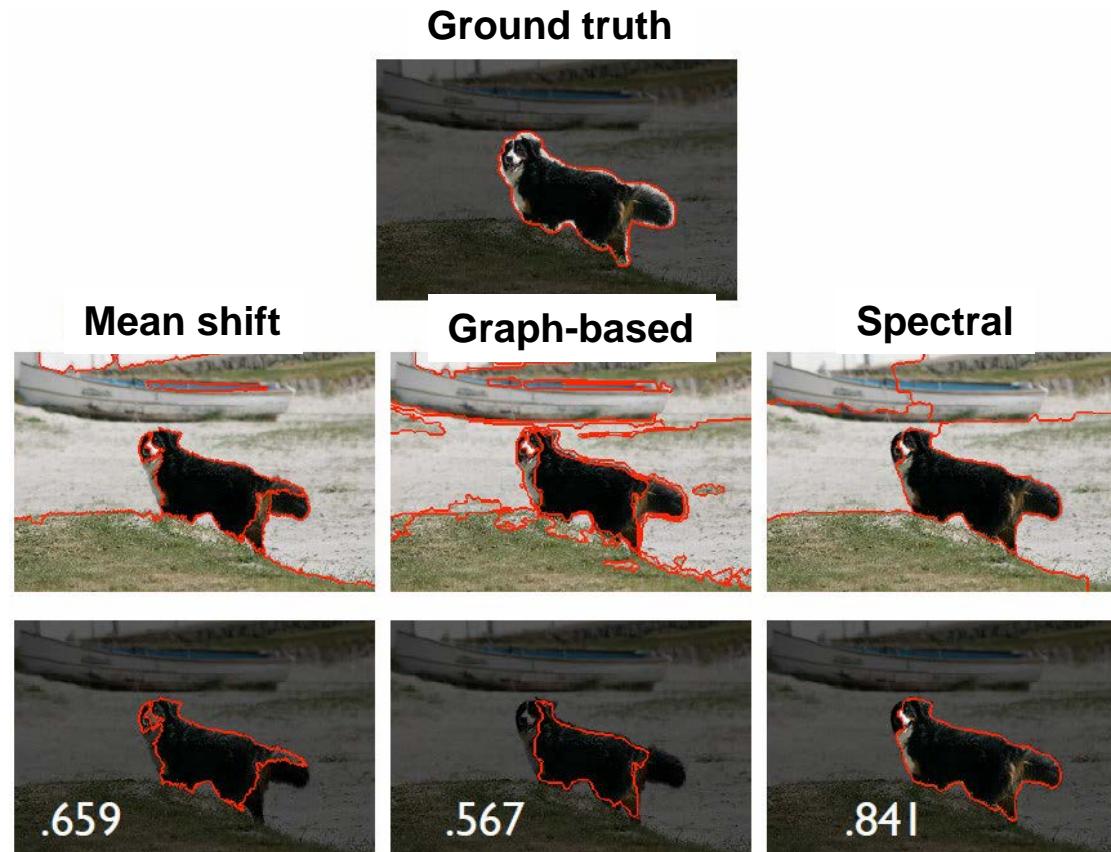


Segment #2

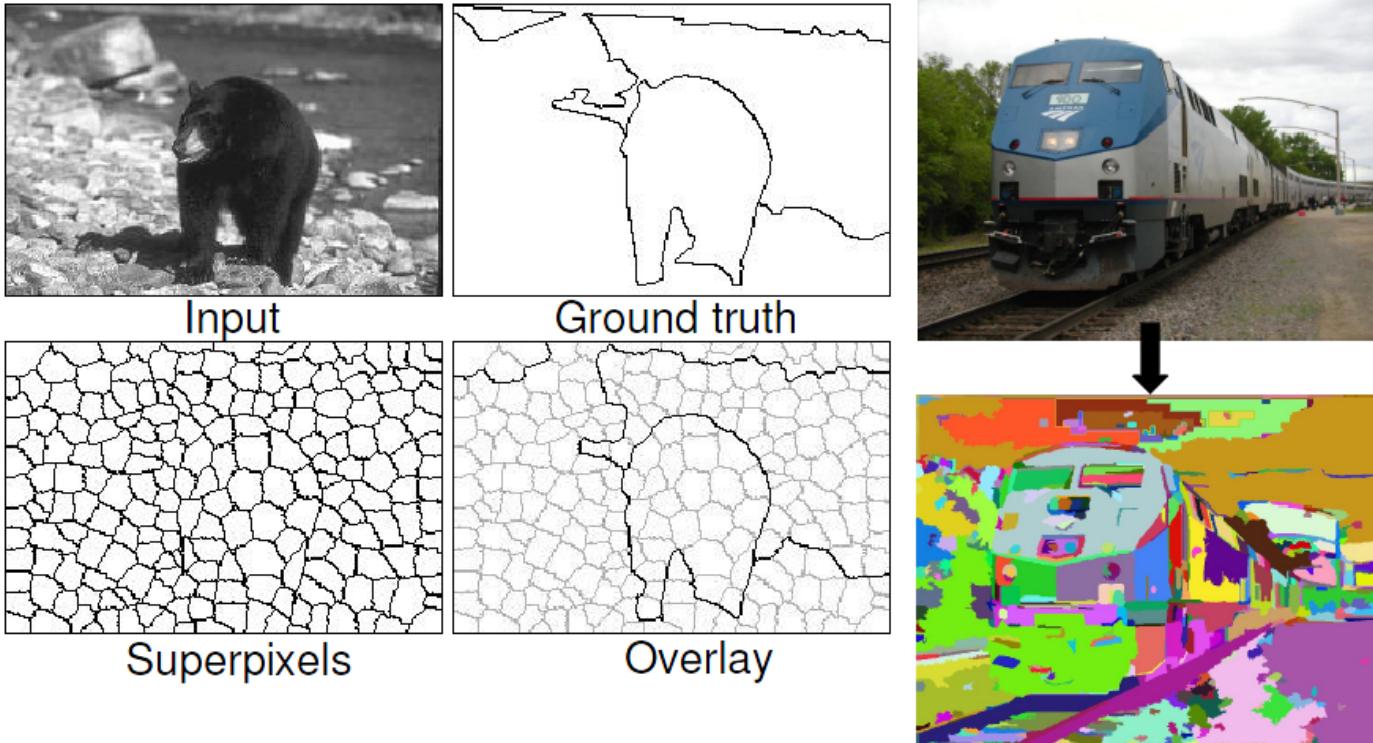
.892



Evaluation: Region overlap with ground truth

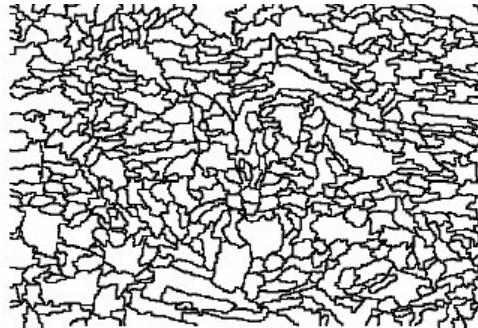


Second idea: Superpixels

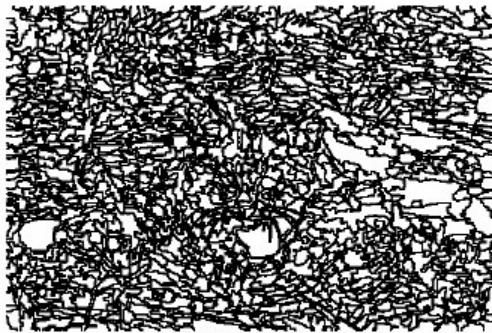


- ▶ Let's not even try to compute a “correct” segmentation
- ▶ Let's be content with an *oversegmentation* in which each region is very likely (formal guarantees are hard) to be uniform

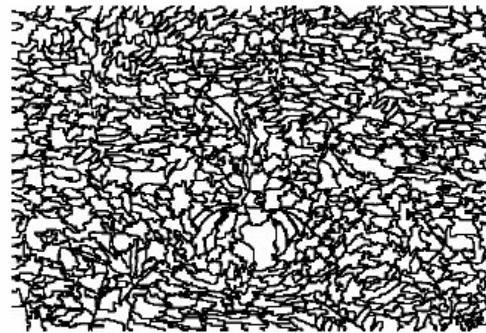
Second idea: Superpixels



Watershed



Mean shift

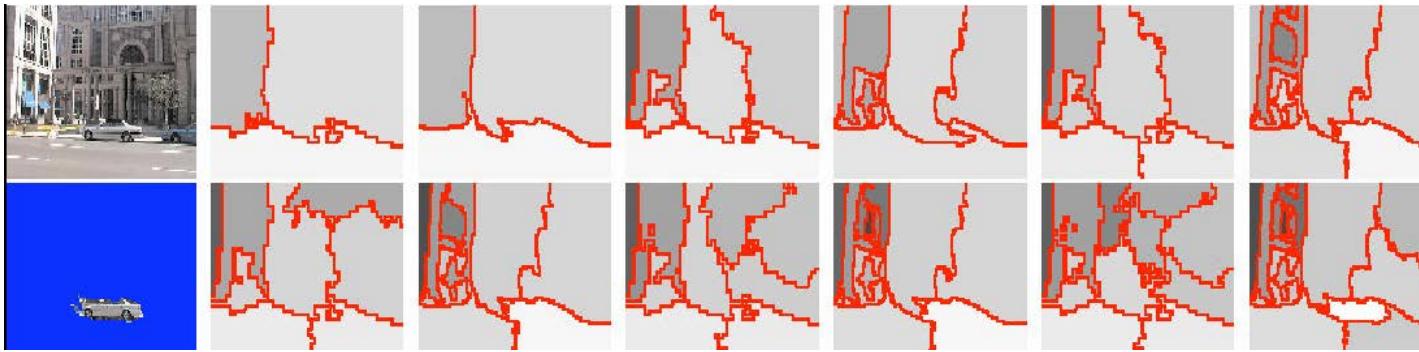


Graph-based

- ▶ Example from: How Do Superpixels Affect Image Segmentation?
- ▶ Progress in Pattern Recognition, Image Analysis and Applications. Springer LNCS. Volume 5197/2008.



Third idea: Multiple segmentations



- ▶ Generate many segmentations of the same image
- ▶ Even though many regions are “wrong”, some consensus should emerge

Example: Improving Spatial Support for Objects via Multiple Segmentations
Tomasz Malisiewicz and Alexei A. Efros. British Machine Vision Conference (BMVC), September, 2007.



Multiple segmentations: Example

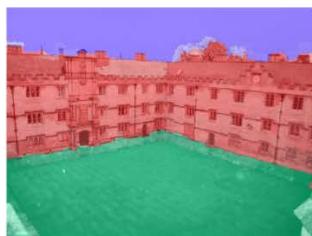


▶ Task:

Regions → Features → Labels
(horizontal, vertical, sky, etc.)



Input



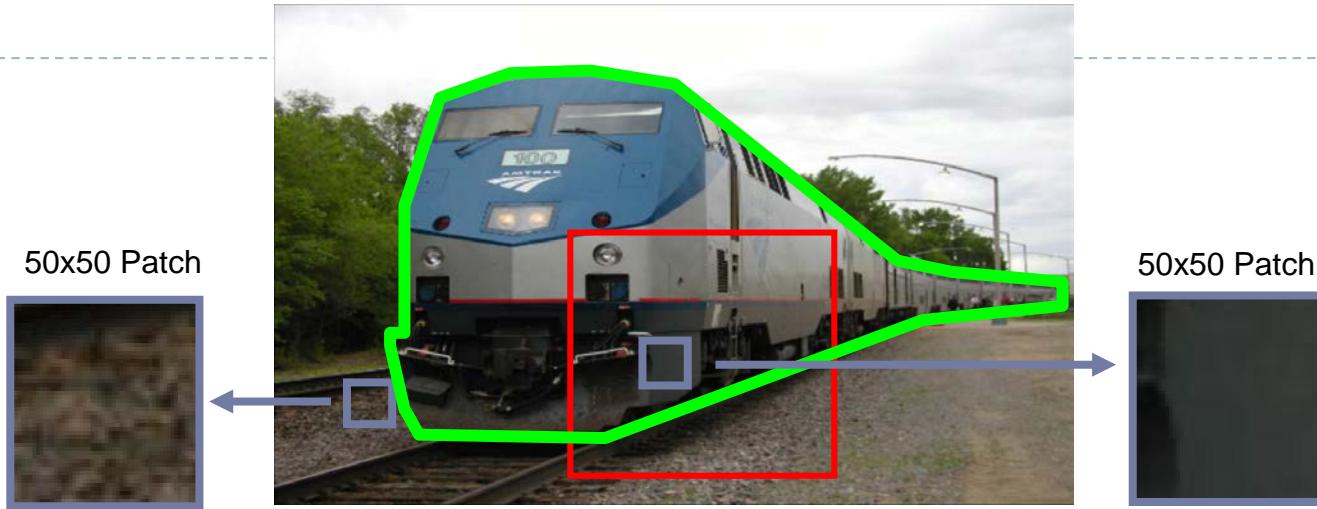
Labels



Novel View



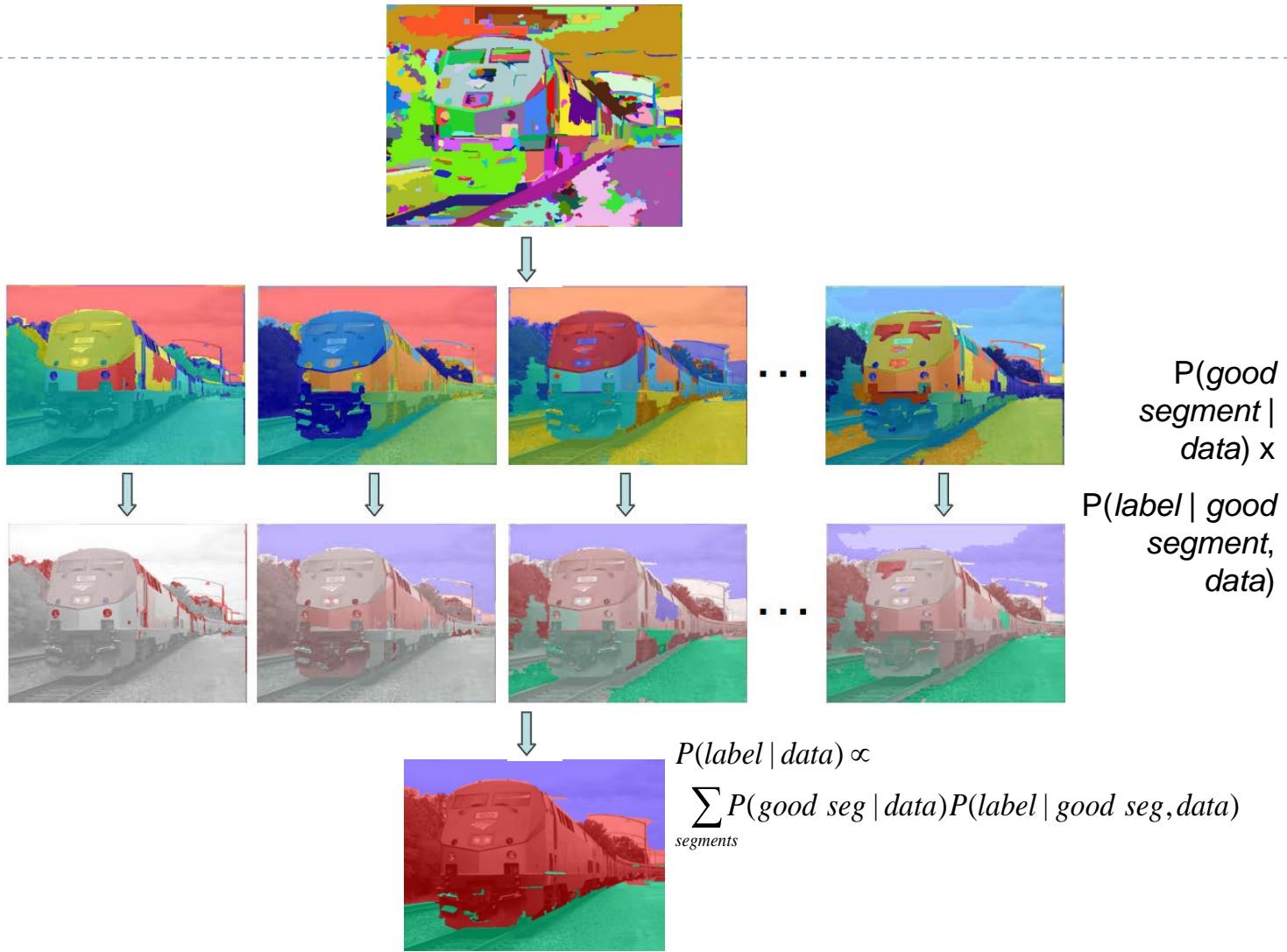
Novel View



- ▶ **Chicken and egg problem:**
 - ▶ If we knew the regions, we could compute the features and label the right regions
 - ▶ But to know the right regions we need to know the labels!
- ▶ **Solution:**
 - ▶ Generate lots of segmentations
 - ▶ Combine the classifications to get consensus

Recovering Surface Layout from an Image. D. Hoiem, A.A. Efros, and M. Hebert. IJCV, Vol. 75, No. 1, October 2007.



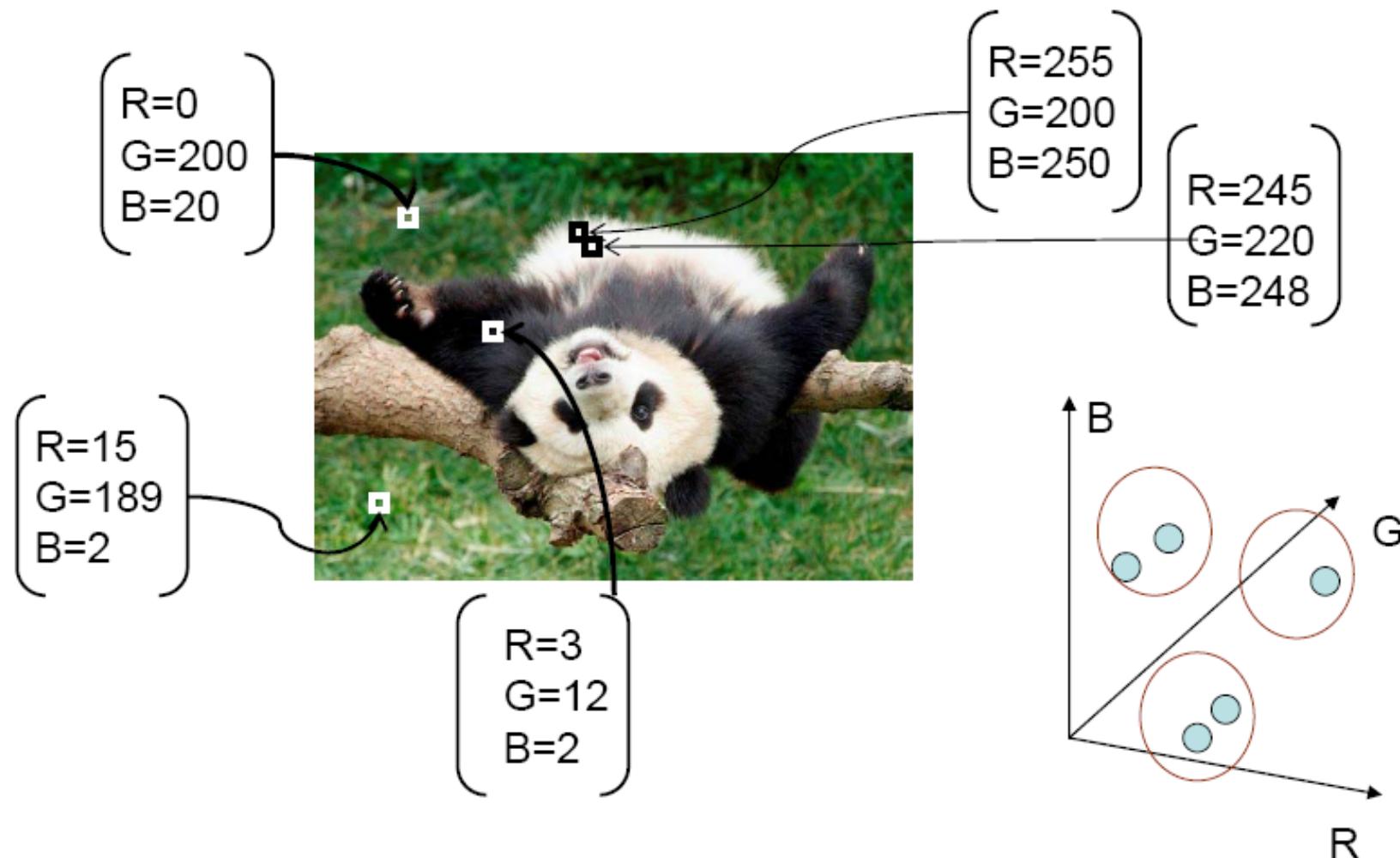


Segmentation using clustering

- ▶ Kmeans
- ▶ Mean-shift

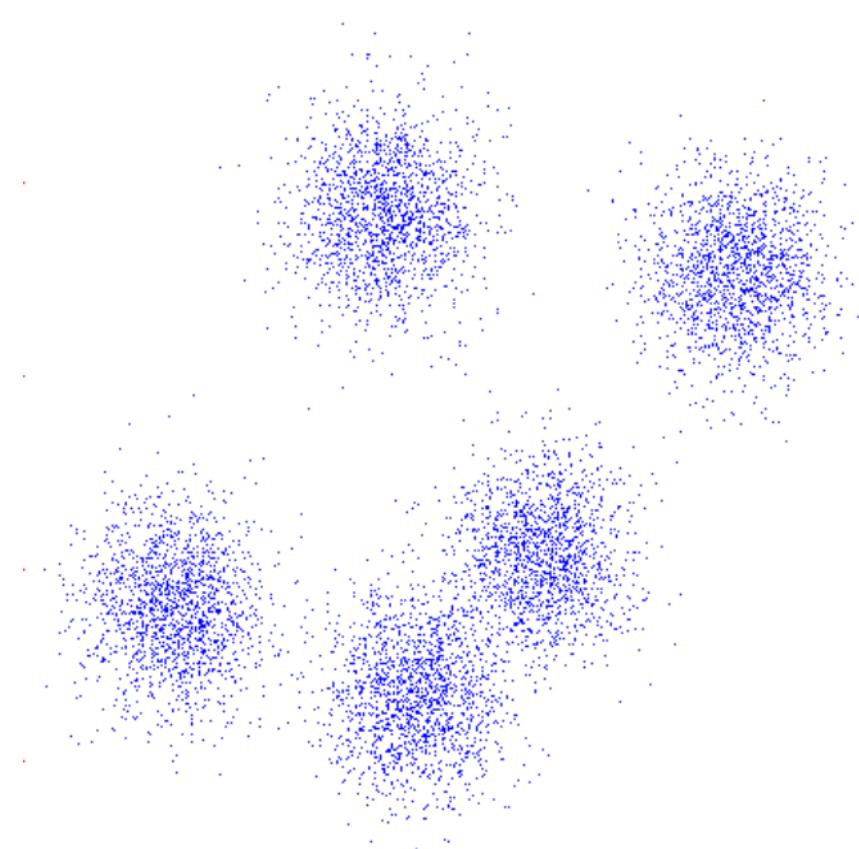


Feature Space

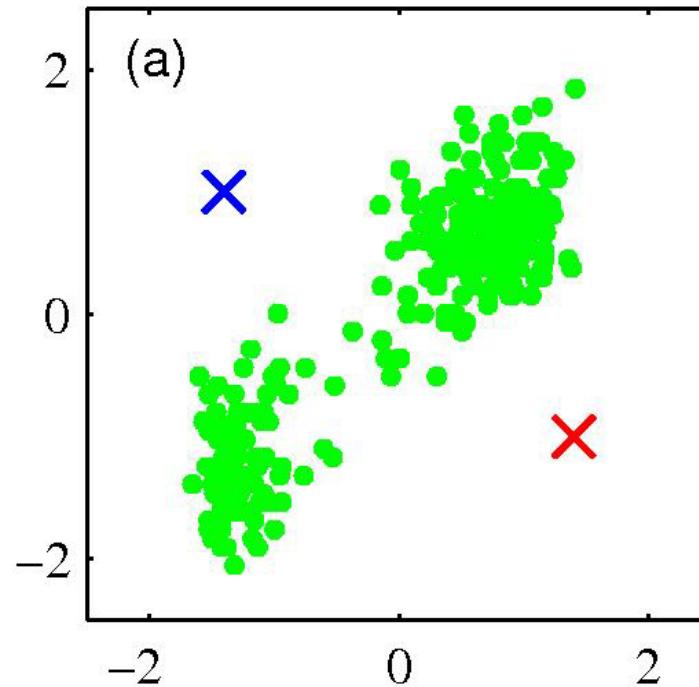


K-means

- An iterative clustering algorithm
 - **Initialize:** Pick K random points as cluster centers
 - **Alternate:**
 1. Assign data points to closest cluster center
 2. Change the cluster center to the average of its assigned points
 - Stop when no points' assignments change

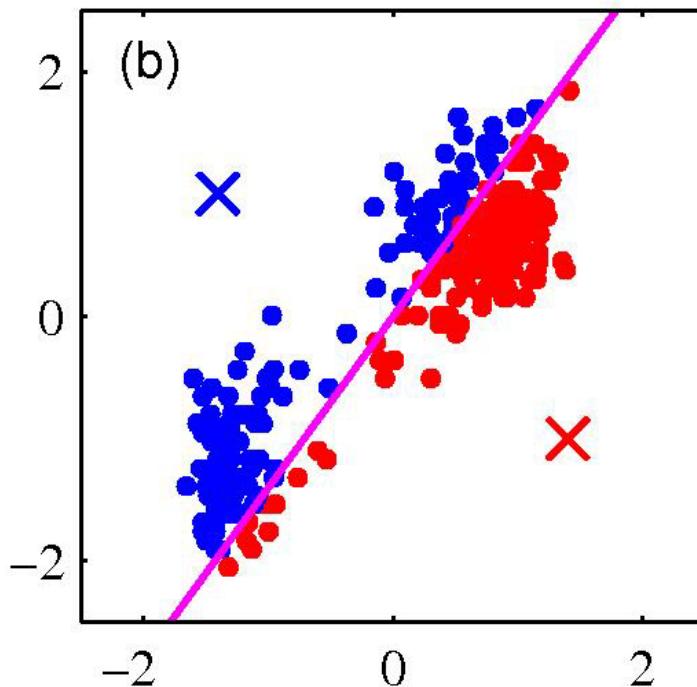


K-means



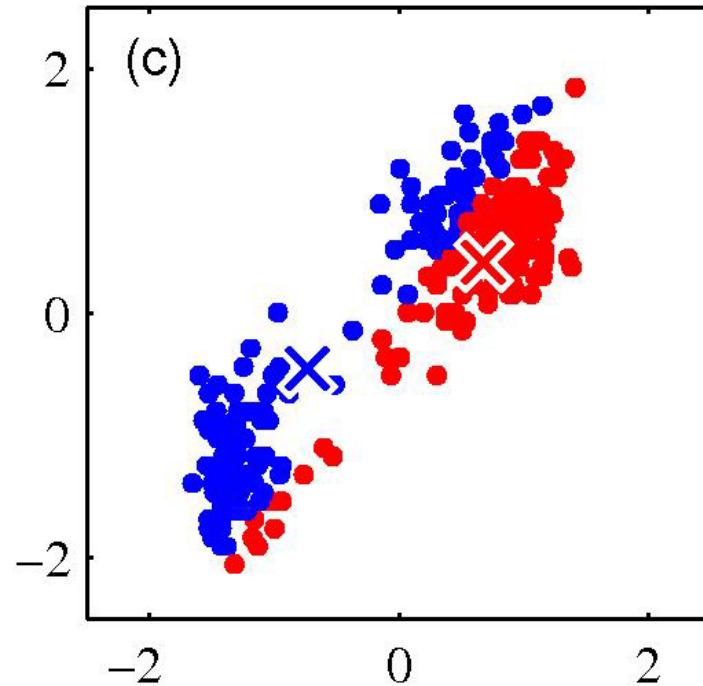
Pick K random points as cluster centers (means)
Shown here for $K=2$

K-means



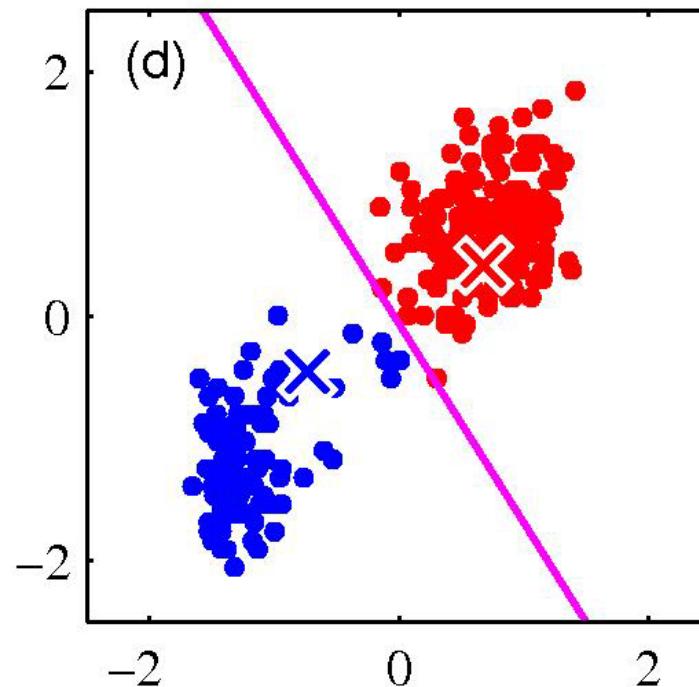
Assign data points to closest cluster center

K-means



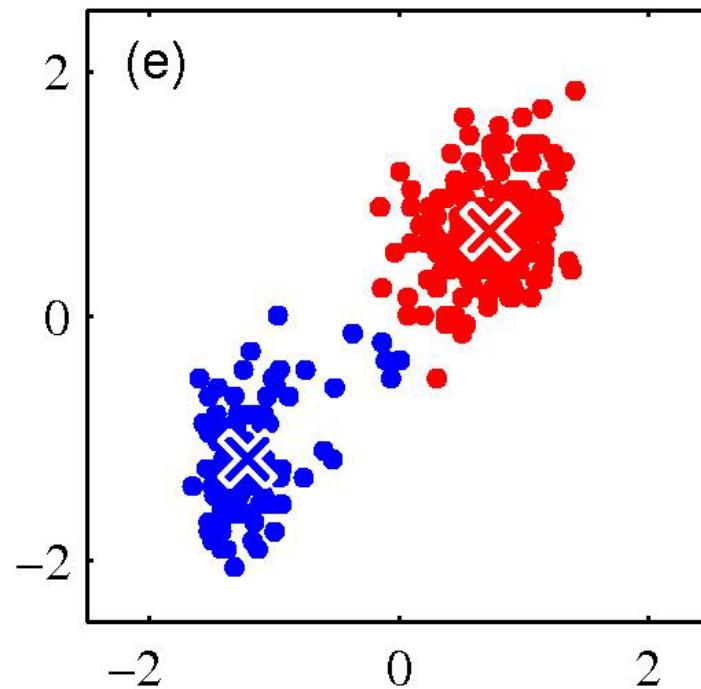
Change the cluster center to the average of the assigned points

K-means

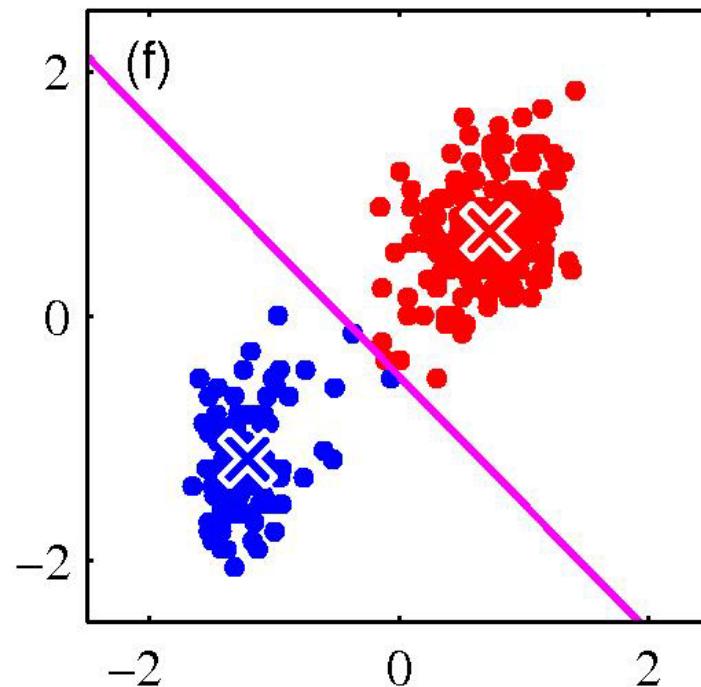


Repeat until convergence

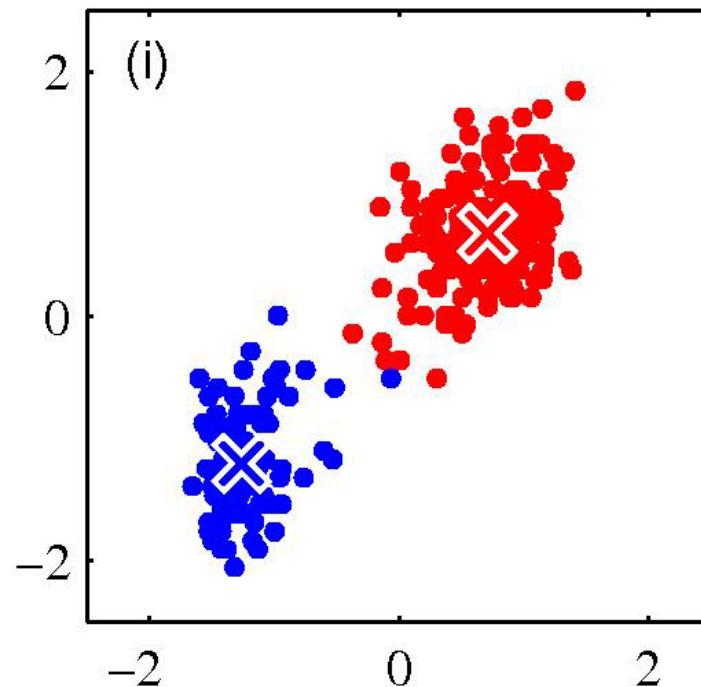
K-means



K-means



K-means



K-means clustering using intensity alone and color alone

Input image



Clusters on intensity

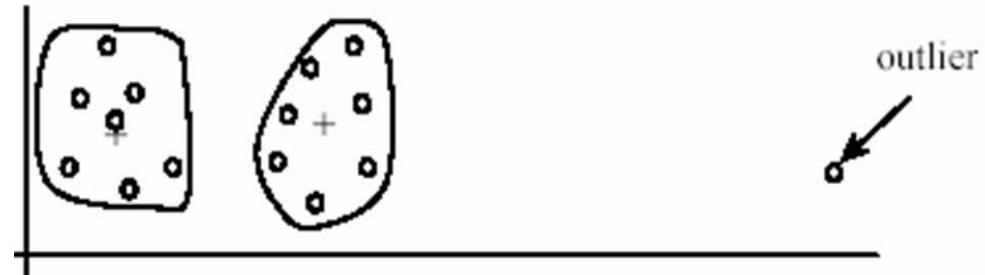


Clusters on color

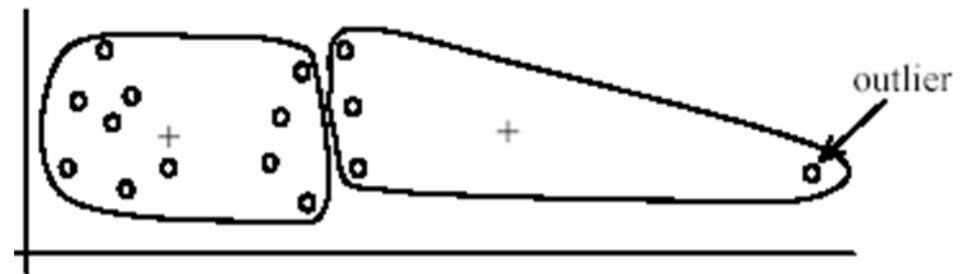


K-Means pros and cons

- **Pros**
 - Simple and fast
 - Easy to implement



(B): Ideal clusters



- **Usage**
 - Rarely used for pixel segmentation



Mean shift segmentation

- ▶ D. Comaniciu and P. Meer, Mean Shift: A Robust Approach toward Feature Space Analysis, PAMI 2002.
- ▶ Versatile technique for clustering-based segmentation

Segmented "landscape 1"

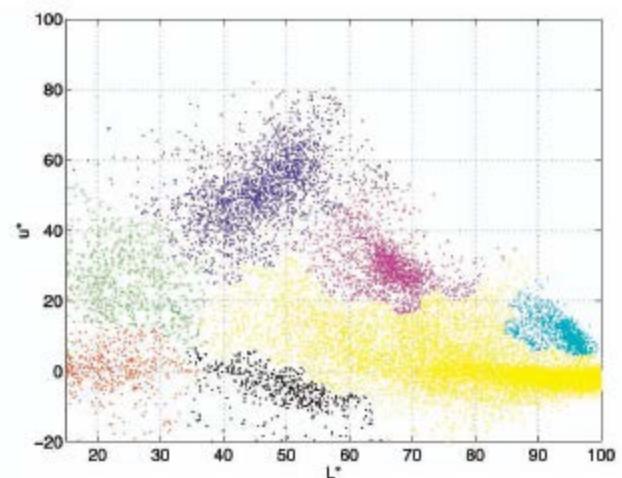
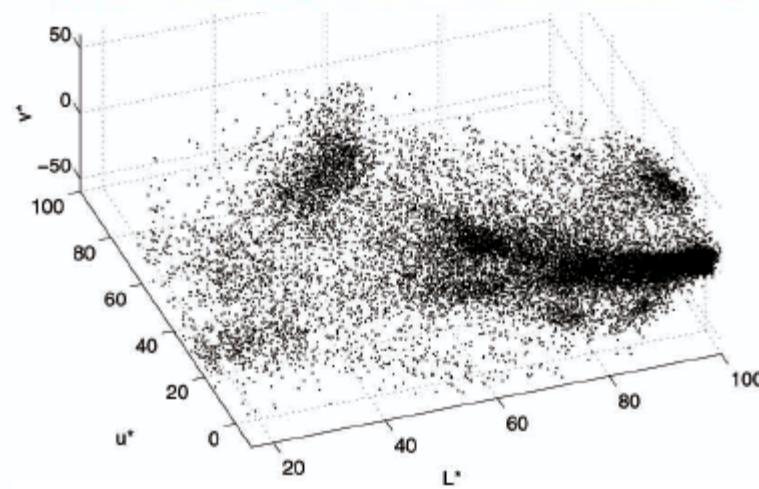
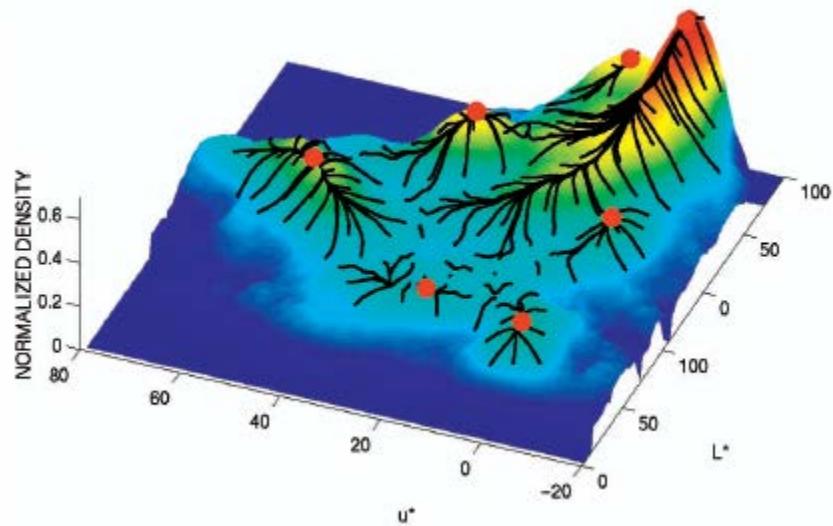


Segmented "landscape 2"

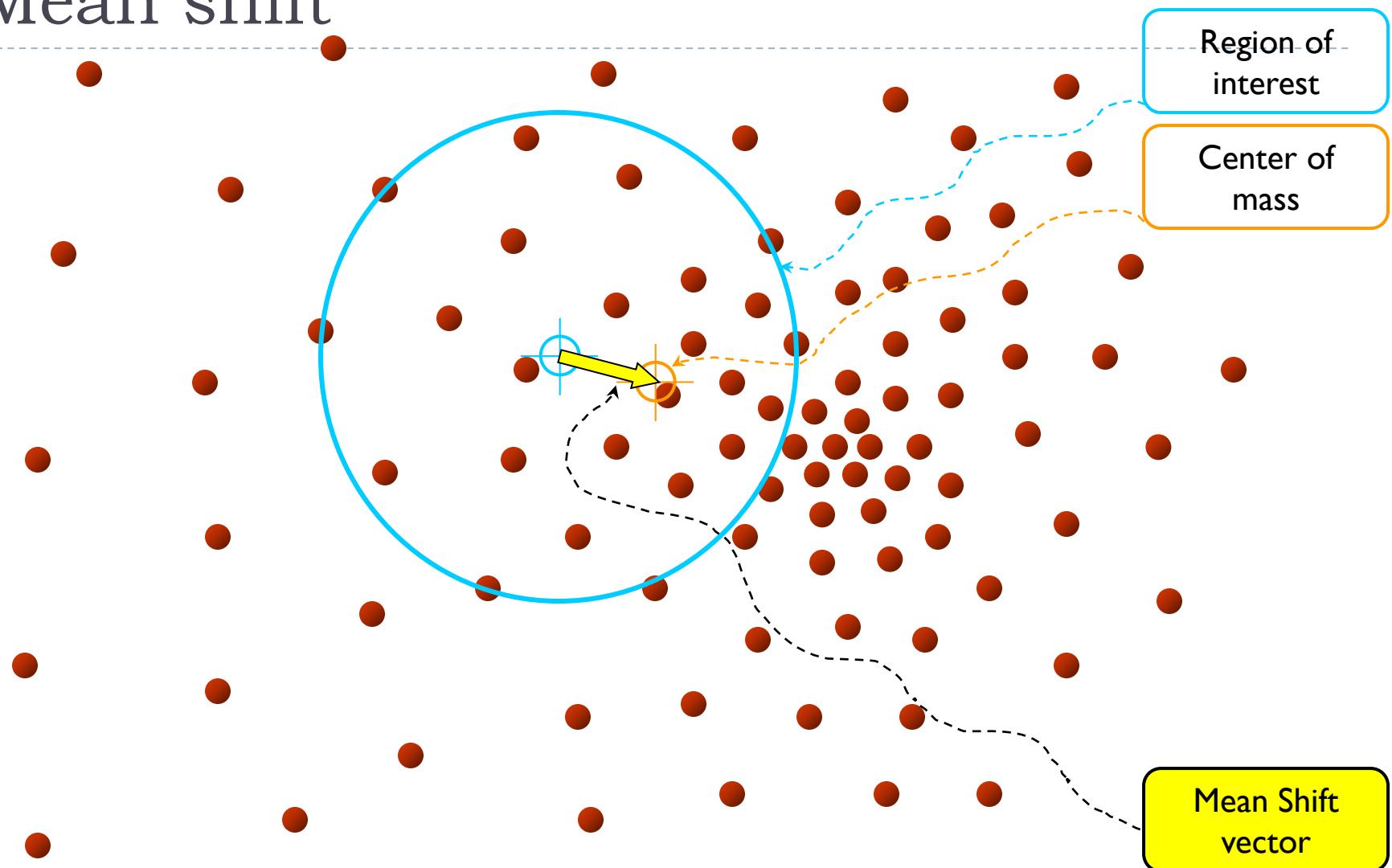


Mean shift algorithm

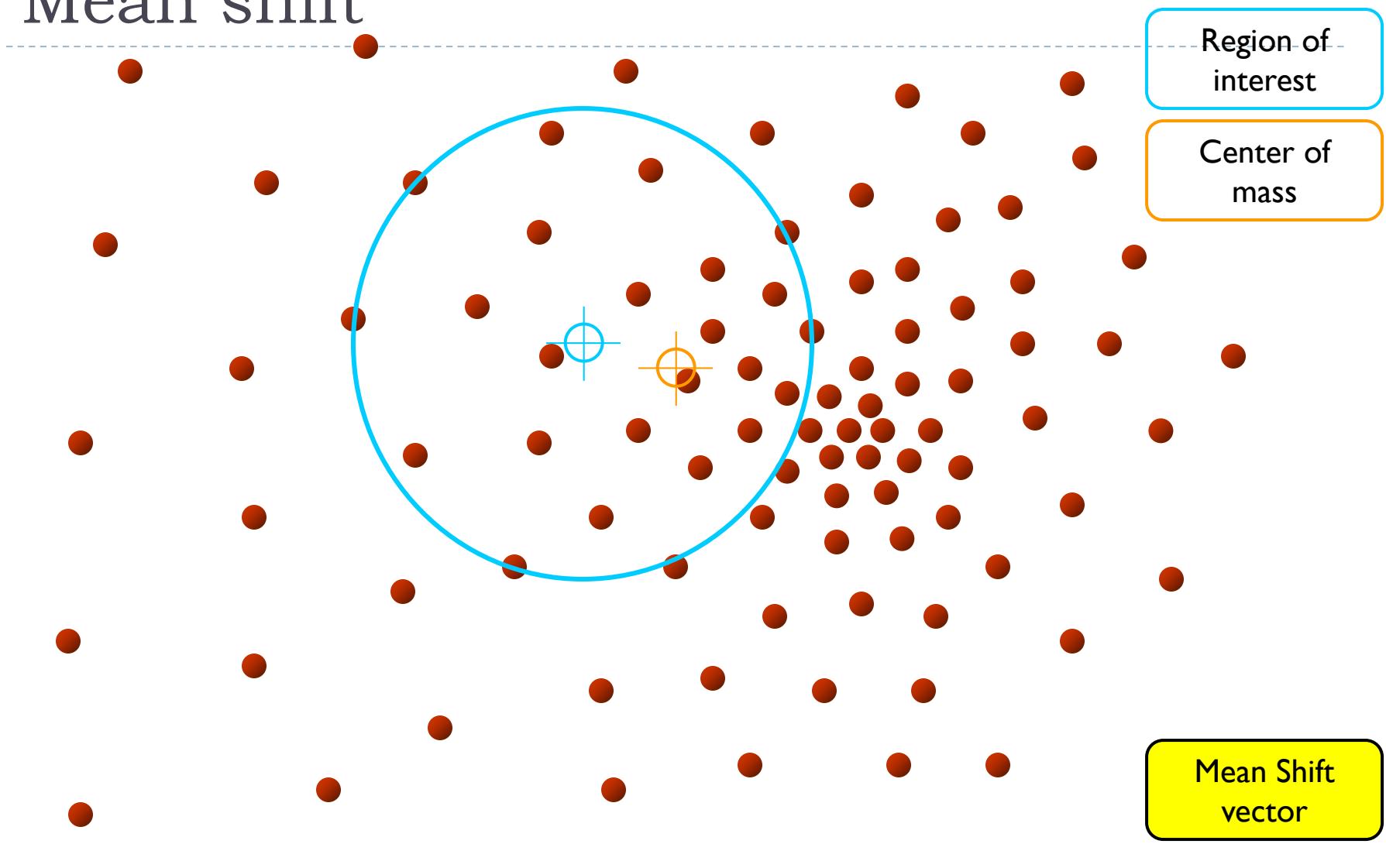
- ▶ Try to find *modes* of this non-parametric density



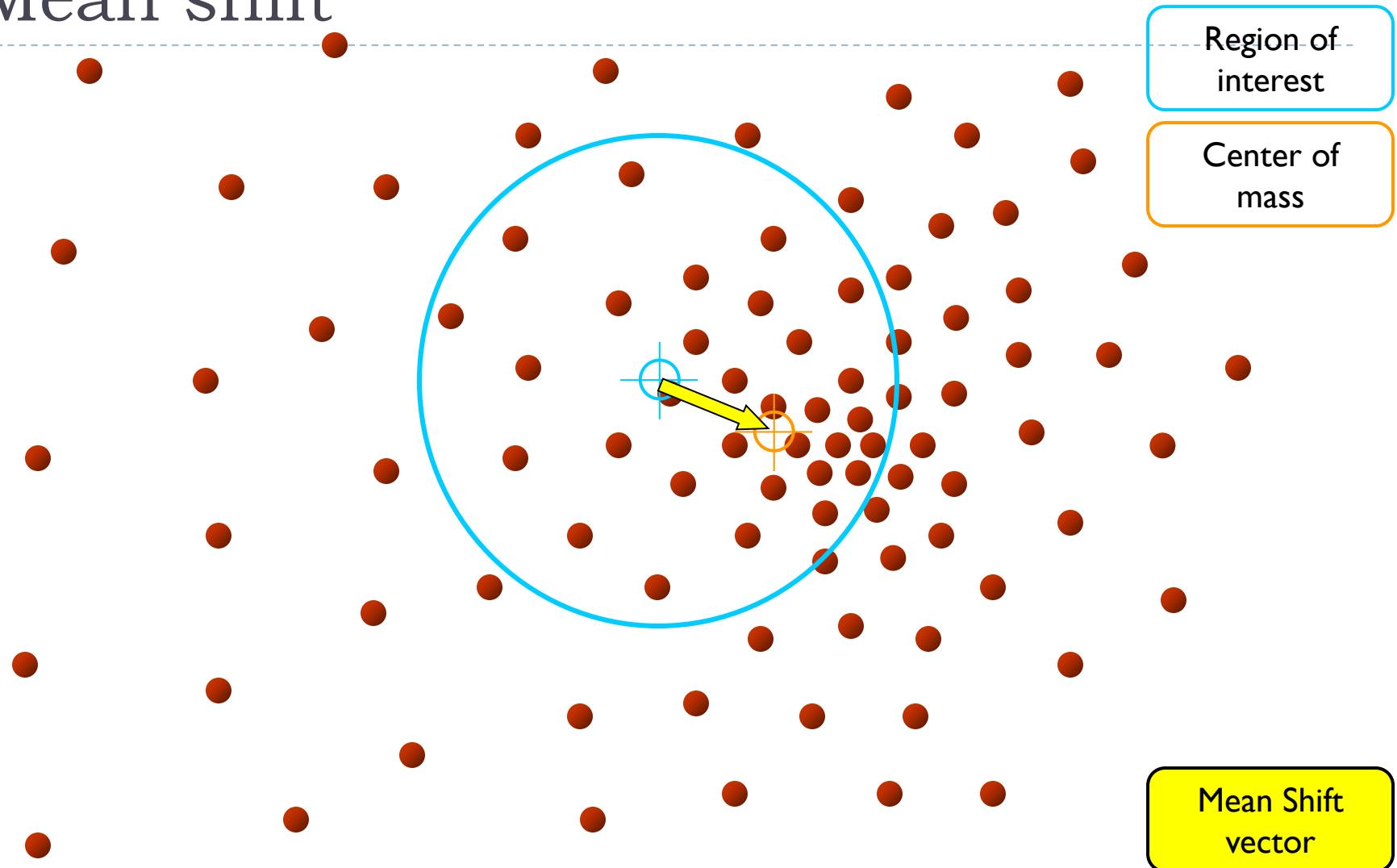
Mean shift



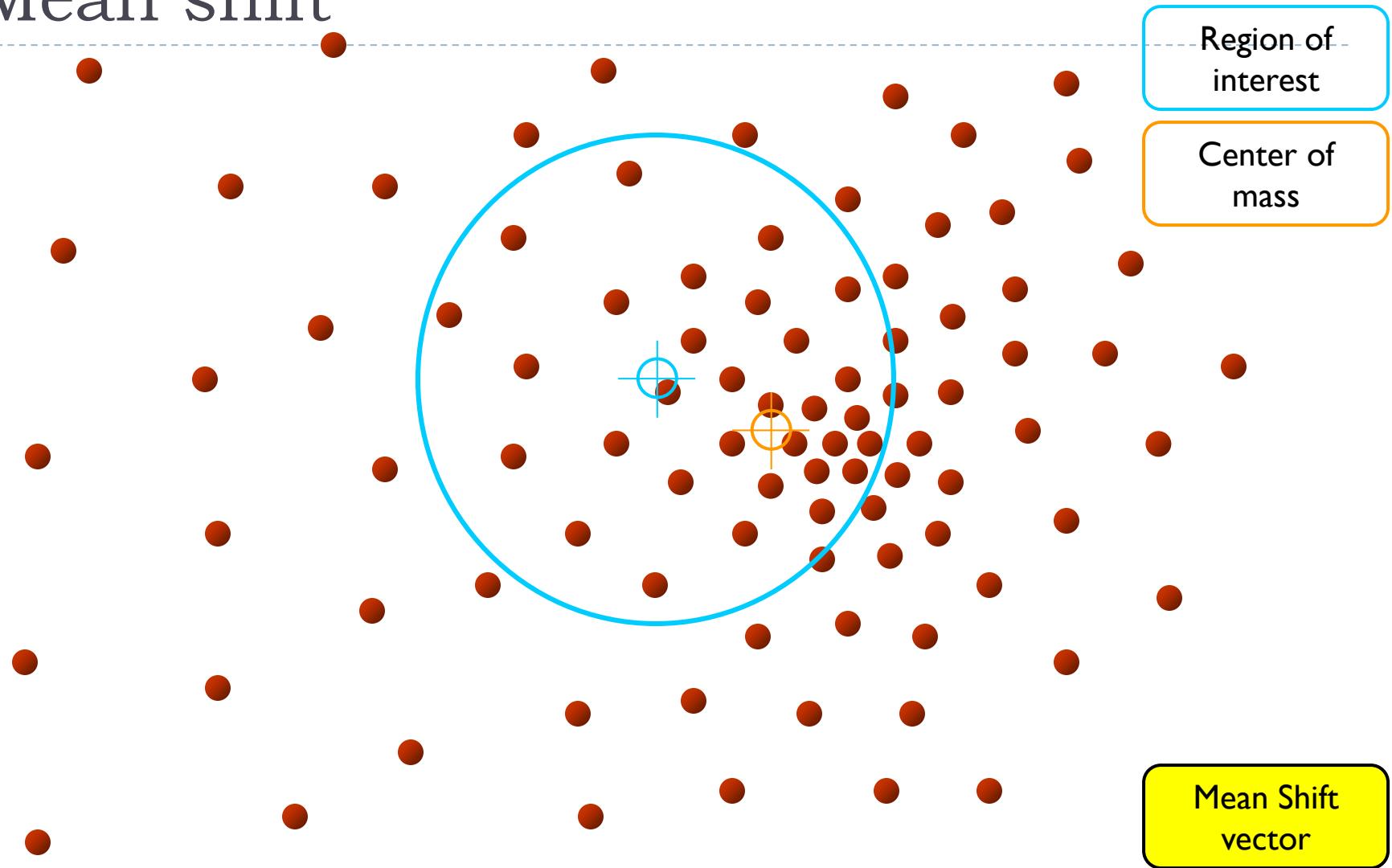
Mean shift



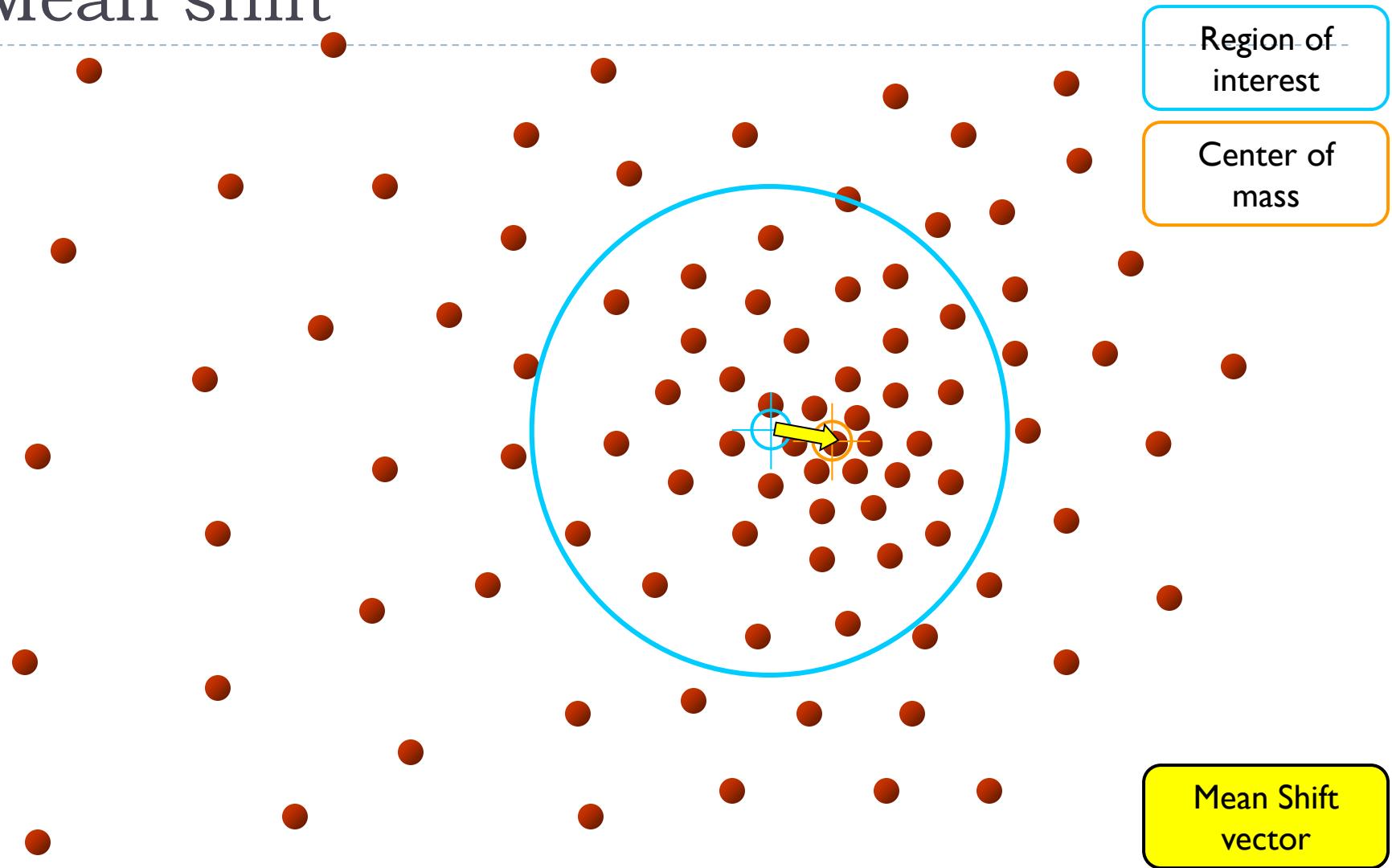
Mean shift



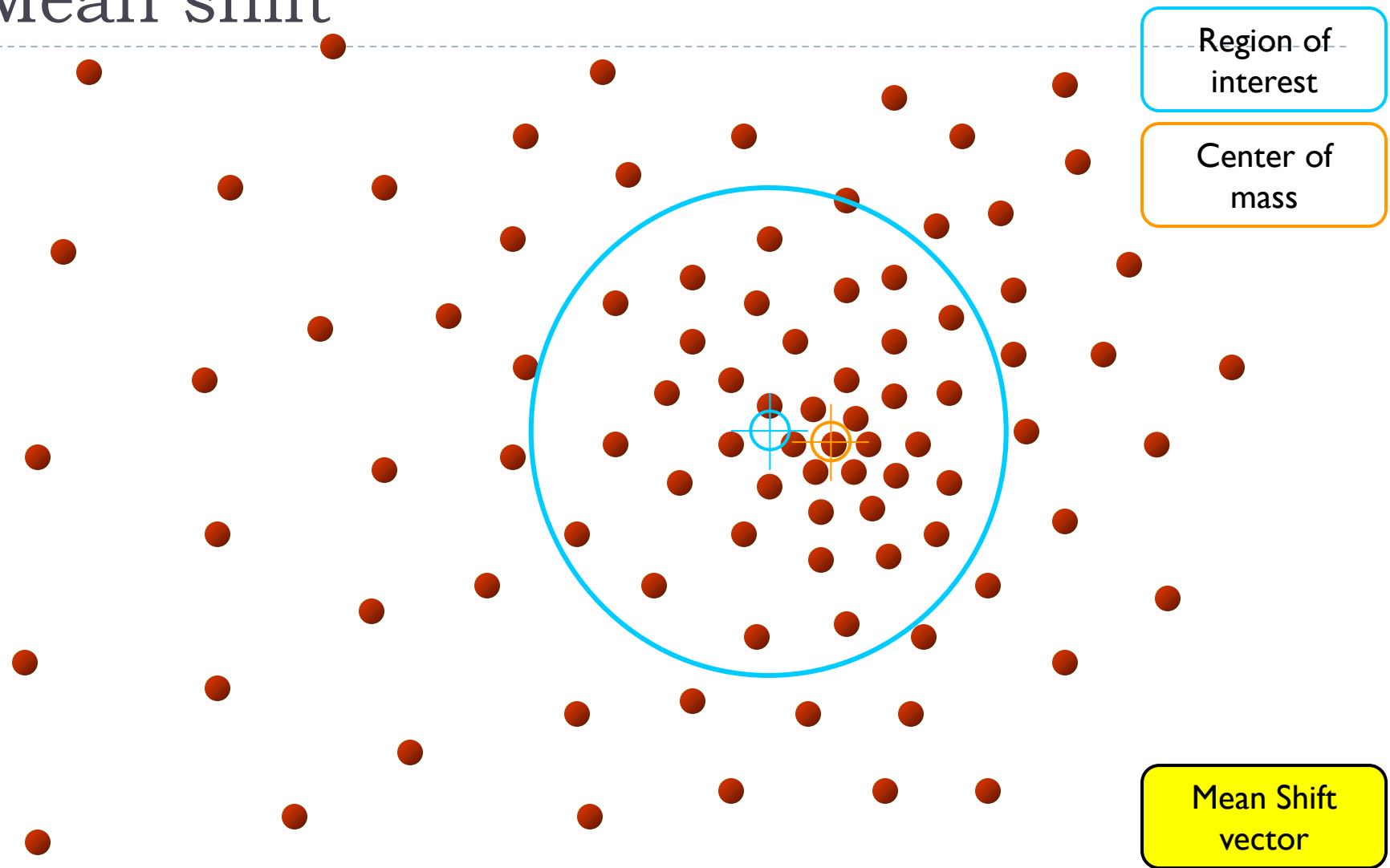
Mean shift



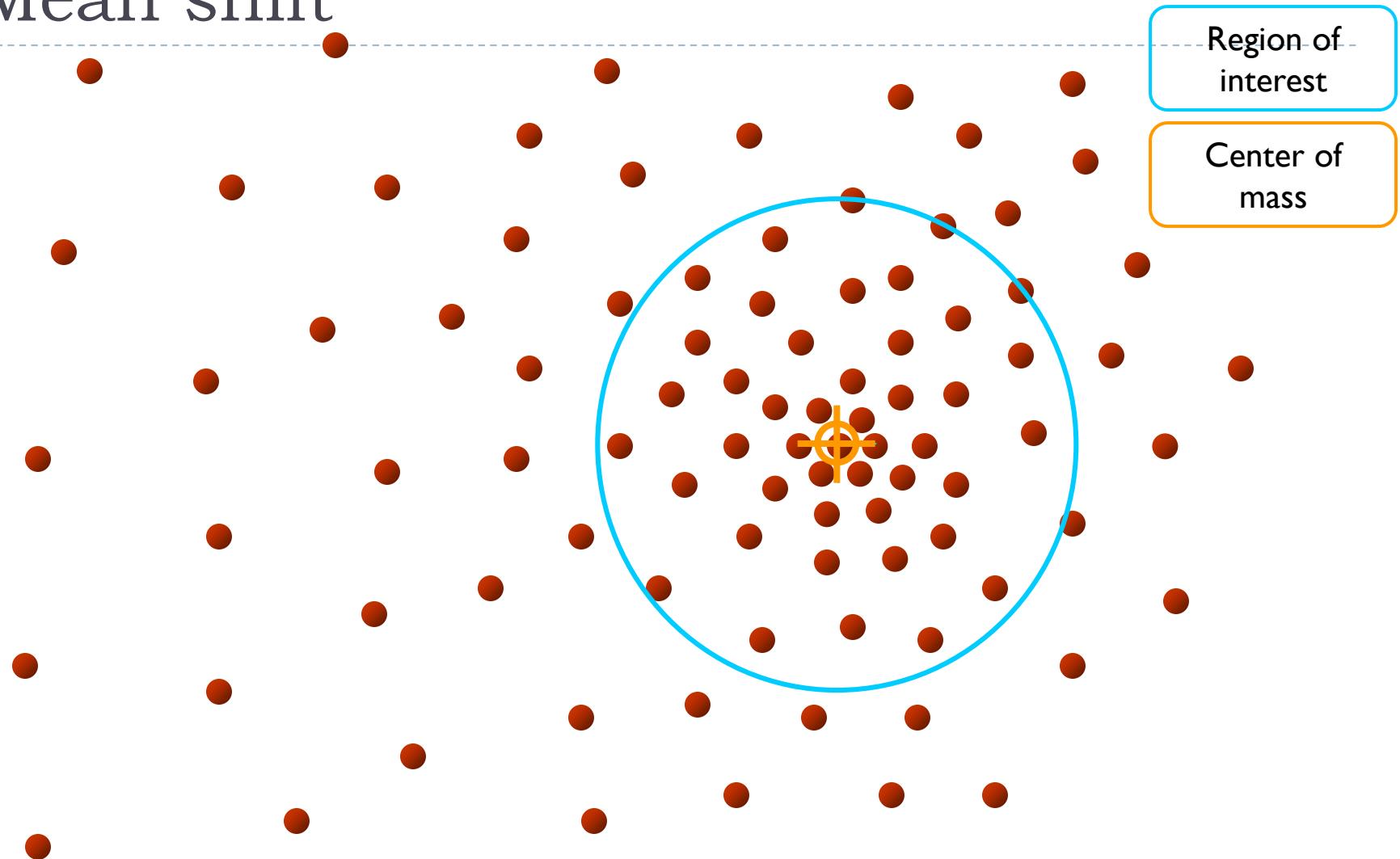
Mean shift



Mean shift



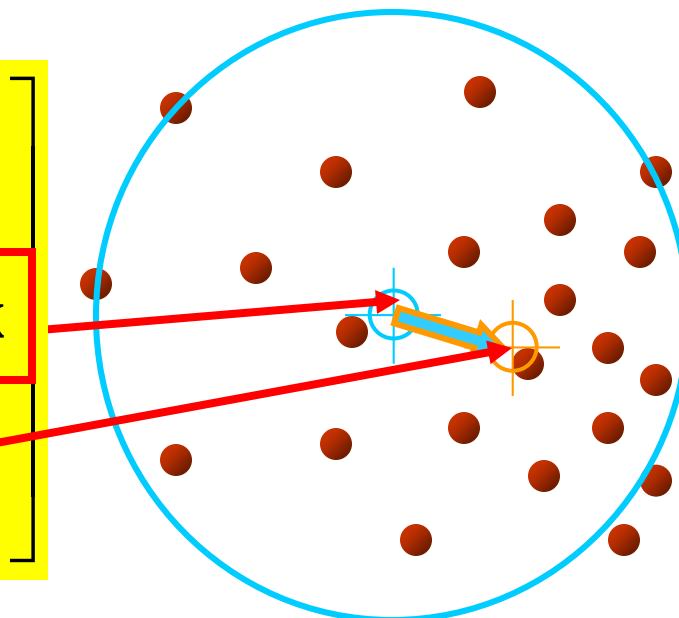
Mean shift



Computing the Mean Shift

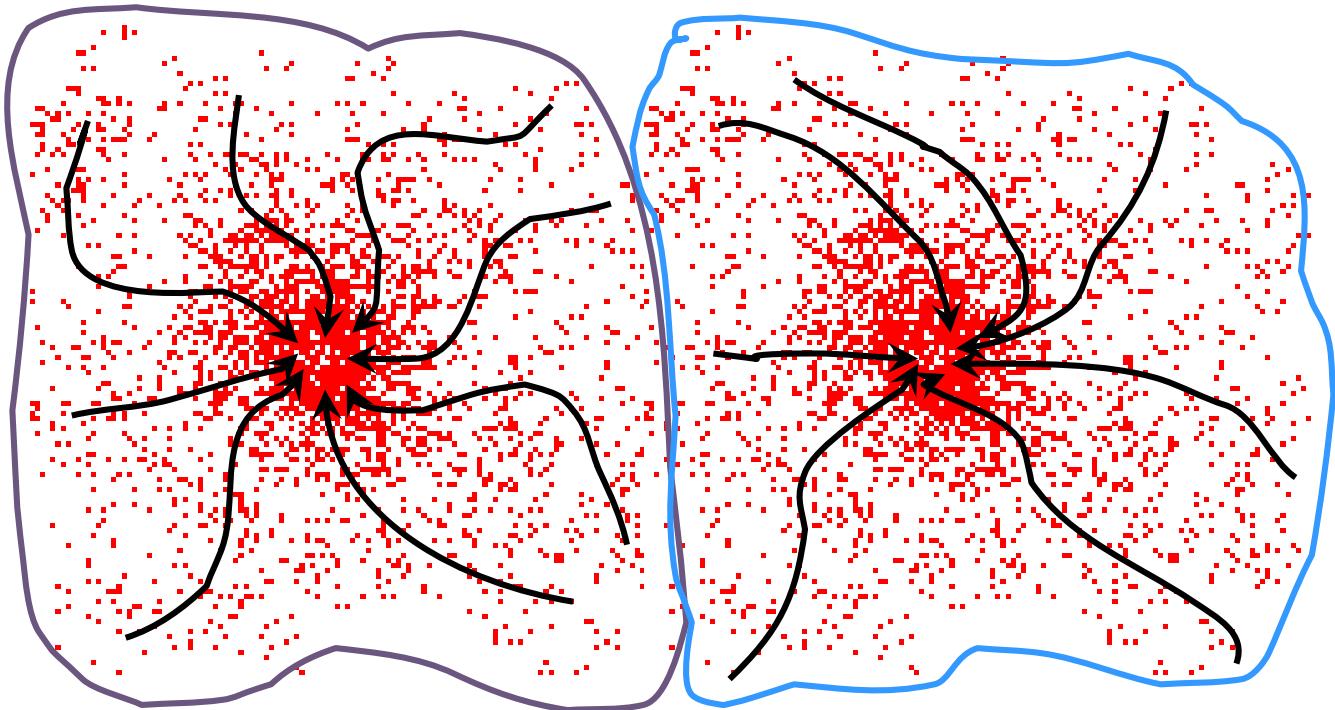
Simple Mean Shift procedure:

- Compute mean shift vector
- Translate the Kernel window by $\mathbf{m}(\mathbf{x})$

$$\mathbf{m}(\mathbf{x}) = \left[\frac{\sum_{i=1}^n \mathbf{x}_i g\left(\frac{\|\mathbf{x} - \mathbf{x}_i\|^2}{h}\right)}{\sum_{i=1}^n g\left(\frac{\|\mathbf{x} - \mathbf{x}_i\|^2}{h}\right)} \right]$$


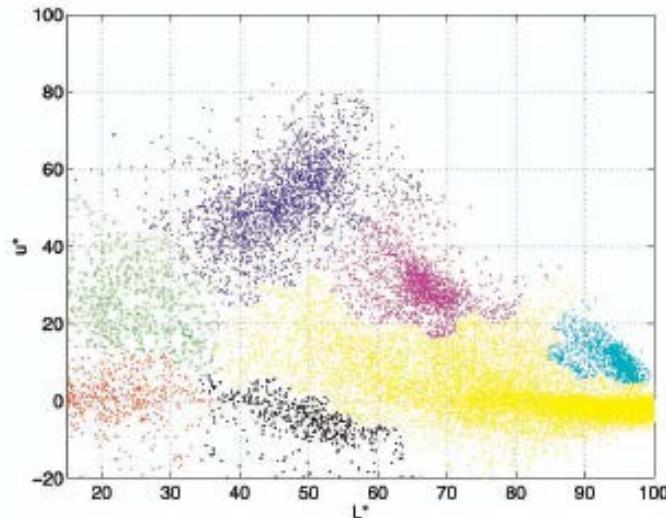
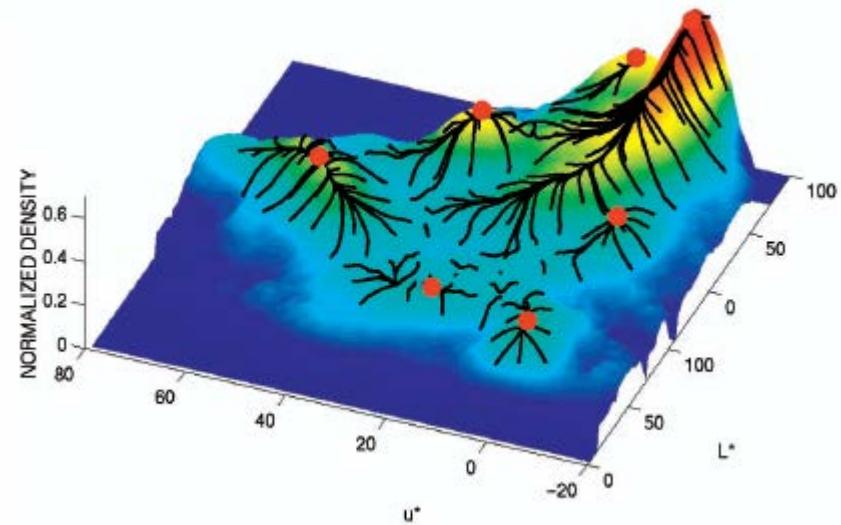
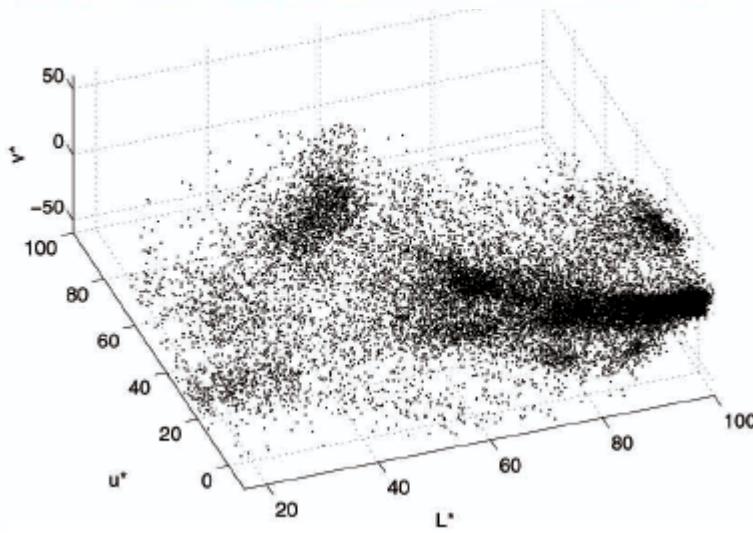
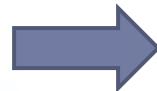
Attraction basin

- ▶ **Attraction basin:** the region for which all trajectories lead to the same mode
- ▶ **Cluster:** all data points in the attraction basin of a mode



Mean shift algorithm

- ▶ Try to find *modes* of this non-parametric density



Mean shift clustering

- ▶ The mean shift algorithm seeks *modes* of the given set of points
 - I. Choose kernel and bandwidth (kernel size for features K_f and position K_s)
 - 2. For each point:
 - a) Center a window on that point
 - b) Compute the mean of the data in the search window
 - c) Center the search window at the new mean location
 - d) Repeat (b,c) until convergence
 - 3. Assign points that lead to nearby modes (within width of K_f and K_s) to the same cluster

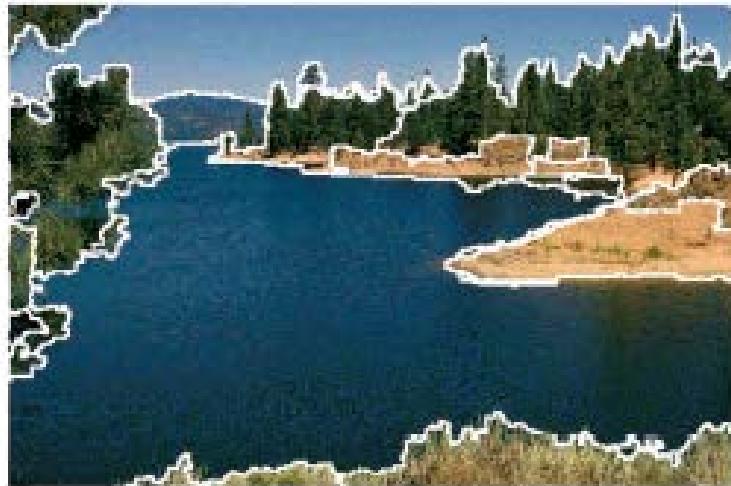


Mean shift segmentation results



<http://www.caip.rutgers.edu/~comanici/MSPAMI/msPamiResults.html>

Mean shift segmentation results



<http://www.caip.rutgers.edu/~comanici/MSPAMI/msPamiResults.html>



Mean-shift: other issues

- Speedups
 - Binned estimation – replace points within some “bin” by point at center with mass
 - Fast search of neighbors – e.g., k-d tree or approximate NN
 - Update all windows in each iteration (faster convergence)
- Other tricks
 - Use kNN to determine window sizes adaptively
- Lots of theoretical support

D. Comaniciu and P. Meer, Mean Shift: A Robust Approach toward Feature Space Analysis, PAMI 2002.



Mean shift pros and cons

- ▶ Pros
 - ▶ Good general-purpose segmentation
 - ▶ Flexible in number and shape of regions
 - ▶ Robust to outliers
 - ▶ General mode-finding algorithm (useful for other problems such as finding most common surface normals)
- ▶ Cons
 - ▶ Have to choose kernel size in advance
 - ▶ Not suitable for high-dimensional features
- ▶ When to use it
 - ▶ Oversegmentation
 - ▶ Multiple segmentations
 - ▶ Tracking, clustering, filtering applications
 - ▶ D. Comaniciu, V. Ramesh, P. Meer: *Real-Time Tracking of Non-Rigid Objects using Mean Shift*, Best Paper Award, IEEE Conf. Computer Vision and Pattern Recognition (CVPR'00), Hilton Head Island, South Carolina, Vol. 2, 142-149, 2000



Mean-shift reading

- Nicely written mean-shift explanation (with math)

<http://saravananthirumuruganathan.wordpress.com/2010/04/01/introduction-to-mean-shift-algorithm/>

- Includes .m code for mean-shift clustering

- Mean-shift paper by Comaniciu and Meer

<http://www.caip.rutgers.edu/~comanici/Papers/MsRobustApproach.pdf>

- Adaptive mean shift in higher dimensions

<http://mis.hevra.haifa.ac.il/~ishimshoni/papers/chap9.pdf>



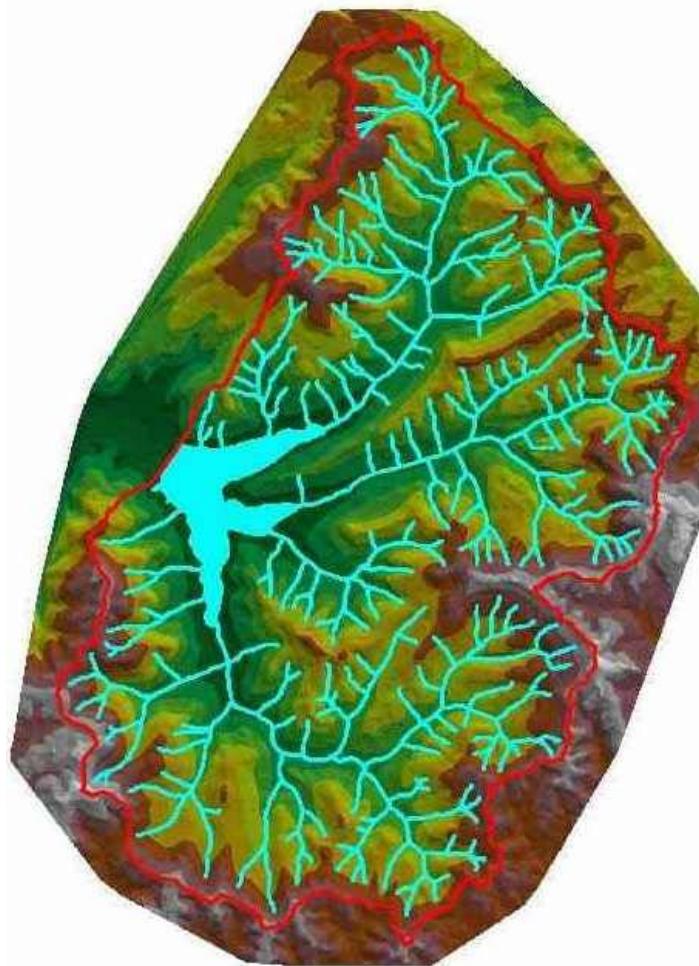
Image Over-Segmentation (Superpixels)

Superpixel algorithms

- ▶ Goal is to divide the image into a large number of regions, such that each regions lie within object boundaries
- ▶ Examples
 - ▶ Watershed
 - ▶ Felzenszwalb and Huttenlocher graph-based
 - ▶ Turbopixels
 - ▶ SLIC



Watershed algorithm



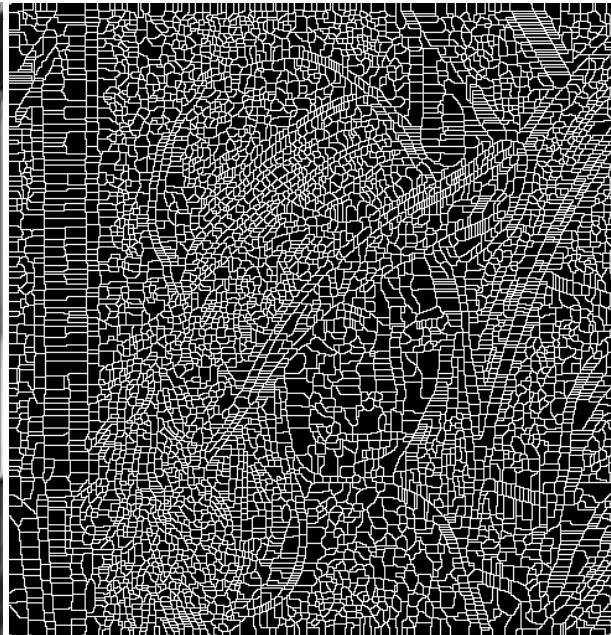
Watershed segmentation



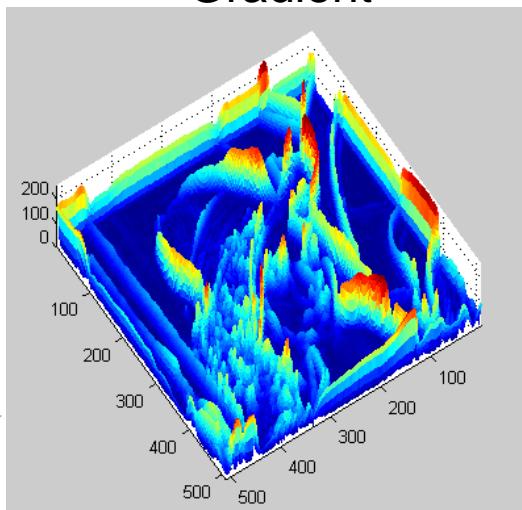
Image



Gradient



Watershed boundaries



Meyer's watershed segmentation

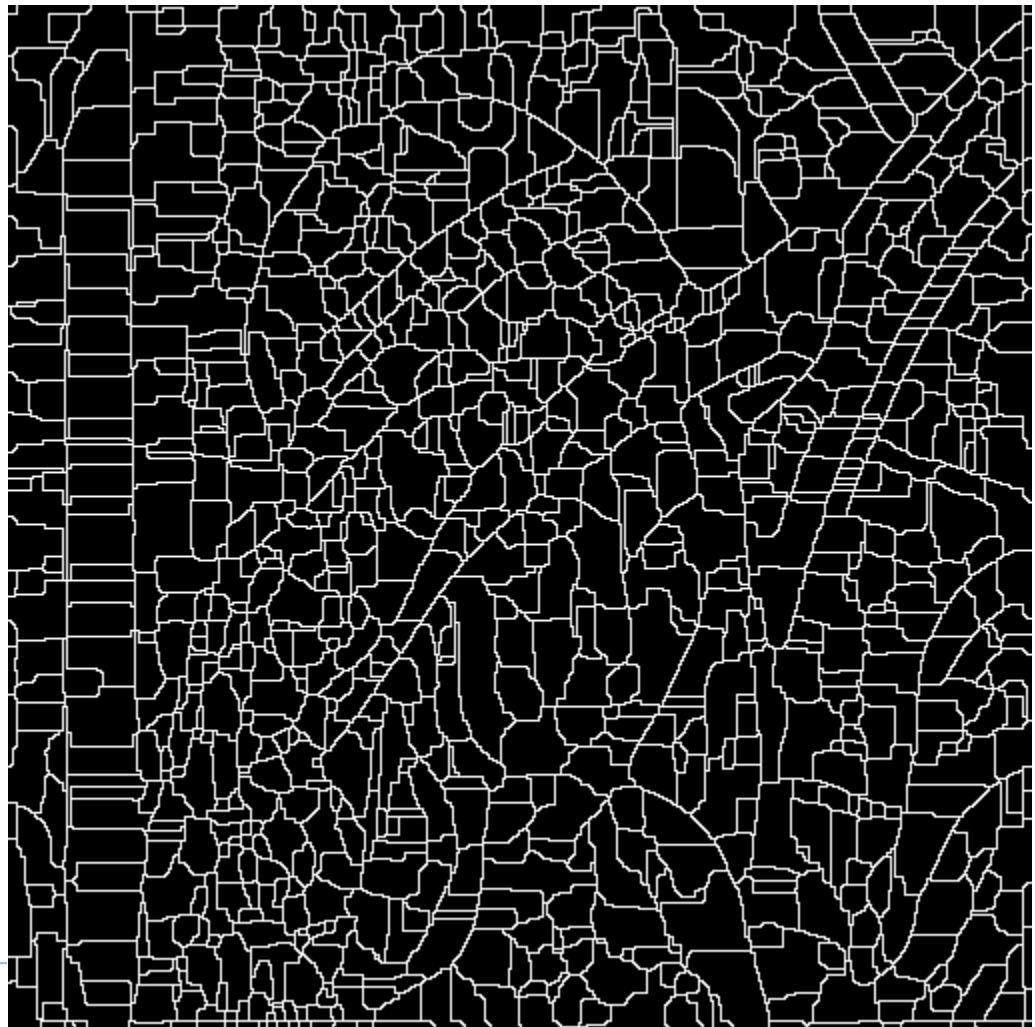
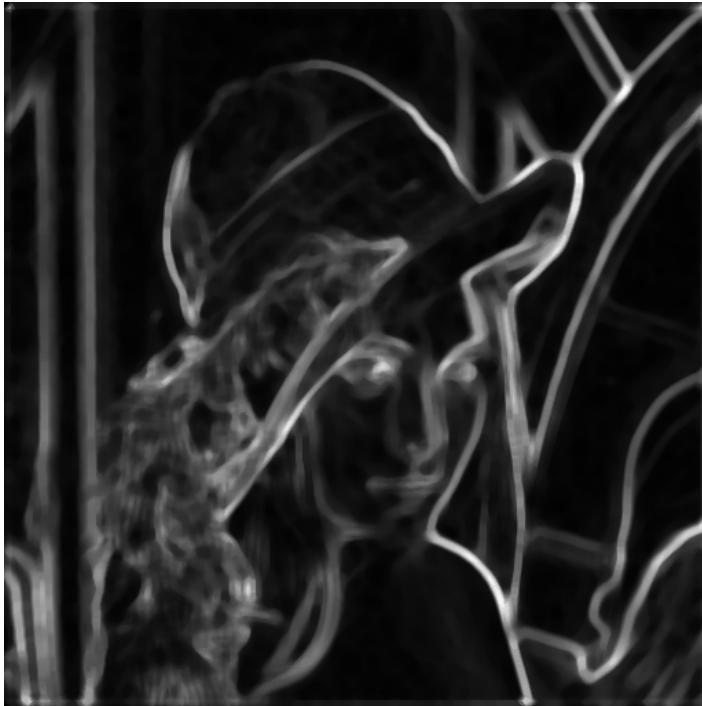
1. Choose local minima (in gradient image) as region seeds
2. Add neighbors to priority queue, sorted by value
3. Take top priority pixel from queue
 1. If all labeled neighbors have same label, assign that label to pixel
 2. Add all non-marked neighbors to queue
4. Repeat step 3 until finished (all remaining pixels in queue are on the boundary)

Matlab: `seg = watershed(bnd_im)`



Simple trick

- ▶ Use Gaussian or median filter to reduce number of regions



Watershed usage

- Use as a starting point for hierarchical segmentation
 - Ultra-metric contour map (Arbelaez 2006)
- Works with any soft boundaries
 - Pb (Berkeley contour detection) (w/o non-max suppression)
 - Canny (w/o non-max suppression)
 - Etc.



Watershed pros and cons

- **Pros**
 - Fast (< 1 sec for 512x512 image)
 - Preserves boundaries
- **Cons**
 - Only as good as the soft boundaries (which may be slow to compute)
 - Not easy to get variety of regions for multiple segmentations
- **Usage**
 - Good algorithm for superpixels, hierarchical segmentation



Superpixel algorithms

- ▶ Goal is to divide the image into a large number of regions, such that each regions lie within object boundaries
- ▶ Examples
 - ▶ Watershed
 - ▶ **Felzenszwalb and Huttenlocher graph-based**
 - ▶ Turbopixels
 - ▶ SLIC



Felzenszwalb and Huttenlocher: Graph-Based Segmentation

<http://www.cs.brown.edu/~pff/segment/>

Algorithm 1 *Segmentation algorithm.*

The input is a graph $G = (V, E)$, with n vertices and m edges. The output is a segmentation of V into components $S = (C_1, \dots, C_r)$.

0. Sort E into $\pi = (o_1, \dots, o_m)$, by non-decreasing edge weight.
1. Start with a segmentation S^0 , where each vertex v_i is in its own component.
2. Repeat step 3 for $q = 1, \dots, m$.



Felzenszwalb and Huttenlocher: Graph-Based Segmentation

<http://www.cs.brown.edu/~pff/segment/>

3. Construct S^q given S^{q-1} as follows. Let v_i and v_j denote the vertices connected by the q -th edge in the ordering, i.e., $o_q = (v_i, v_j)$. If v_i and v_j are in disjoint components of S^{q-1} and $w(o_q)$ is small compared to the internal difference of both those components, then merge the two components otherwise do nothing.

More formally, let C_i^{q-1} be the component of S^{q-1} containing v_i and C_j^{q-1} the component containing v_j . If $C_i^{q-1} \neq C_j^{q-1}$ and $w(o_q) \leq MInt(C_i^{q-1}, C_j^{q-1})$ then S^q is obtained from S^{q-1} by merging C_i^{q-1} and C_j^{q-1} . Otherwise $S^q = S^{q-1}$.

4. Return $S = S^m$.



Felzenszwalb and Huttenlocher: Graph-Based Segmentation



- + Good for thin regions
- + Fast
- + Easy to control coarseness of segmentations
- + Can include both large and small regions
- Often creates regions with strange shapes
- Sometimes makes very large errors



Superpixel algorithms

- ▶ Goal is to divide the image into a large number of regions, such that each regions lie within object boundaries
- ▶ Examples
 - ▶ Watershed
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 - ▶ Turbopixels
 - ▶ SLIC



Turbo Pixels: Levinstein et al. 2009

<http://www.cs.toronto.edu/~kyros/pubs/09.pami.turbopixels.pdf>

Tries to preserve boundaries like watershed but to
produce more regular regions

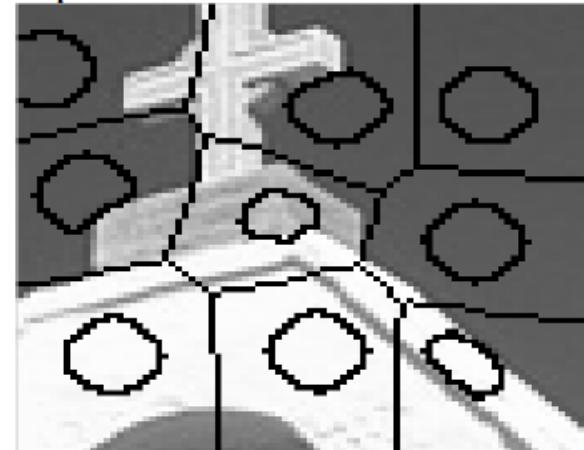
Step 1: (Section III-B)
Place K seeds



Step 2: (Section III-C)
Evolve T time-steps



Step 3: (Section III-D)
Update skeleton



Repeat until no evolution p



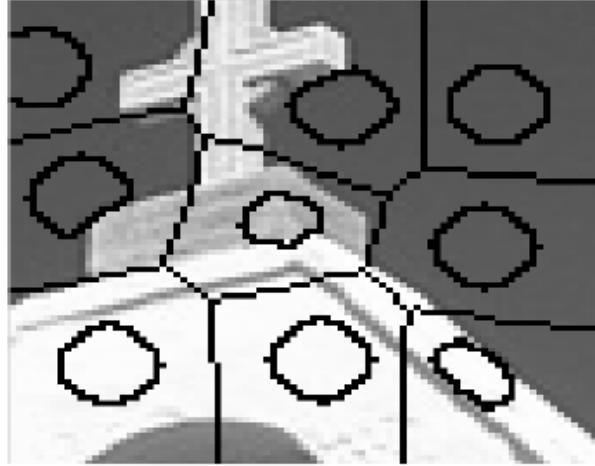
Turbo Pixels: Levinstein et al. 2009

<http://www.cs.toronto.edu/~kyros/pubs/09.pami.turbopixels.pdf>

Tries to preserve boundaries like watershed but to
produce more regular regions

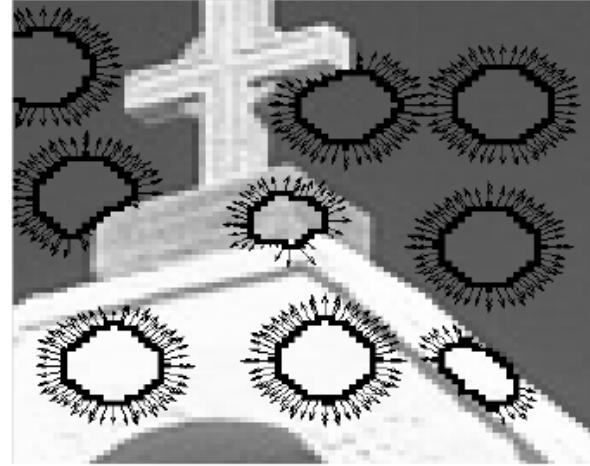
Step 3: (Section III-D)

Update skeleton



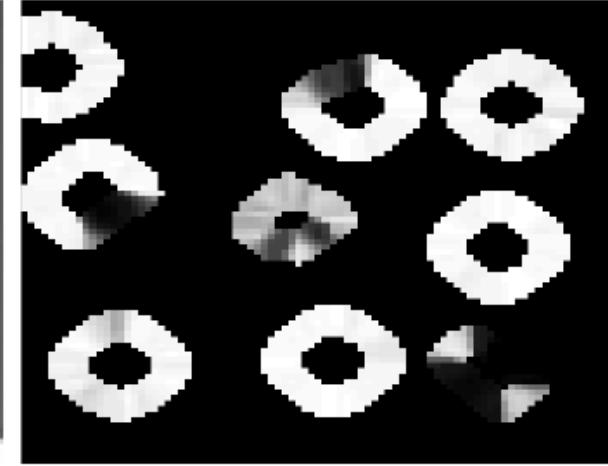
Step 4a: (Section III-E)

Update velocities



Step 4b: (Section III-F)

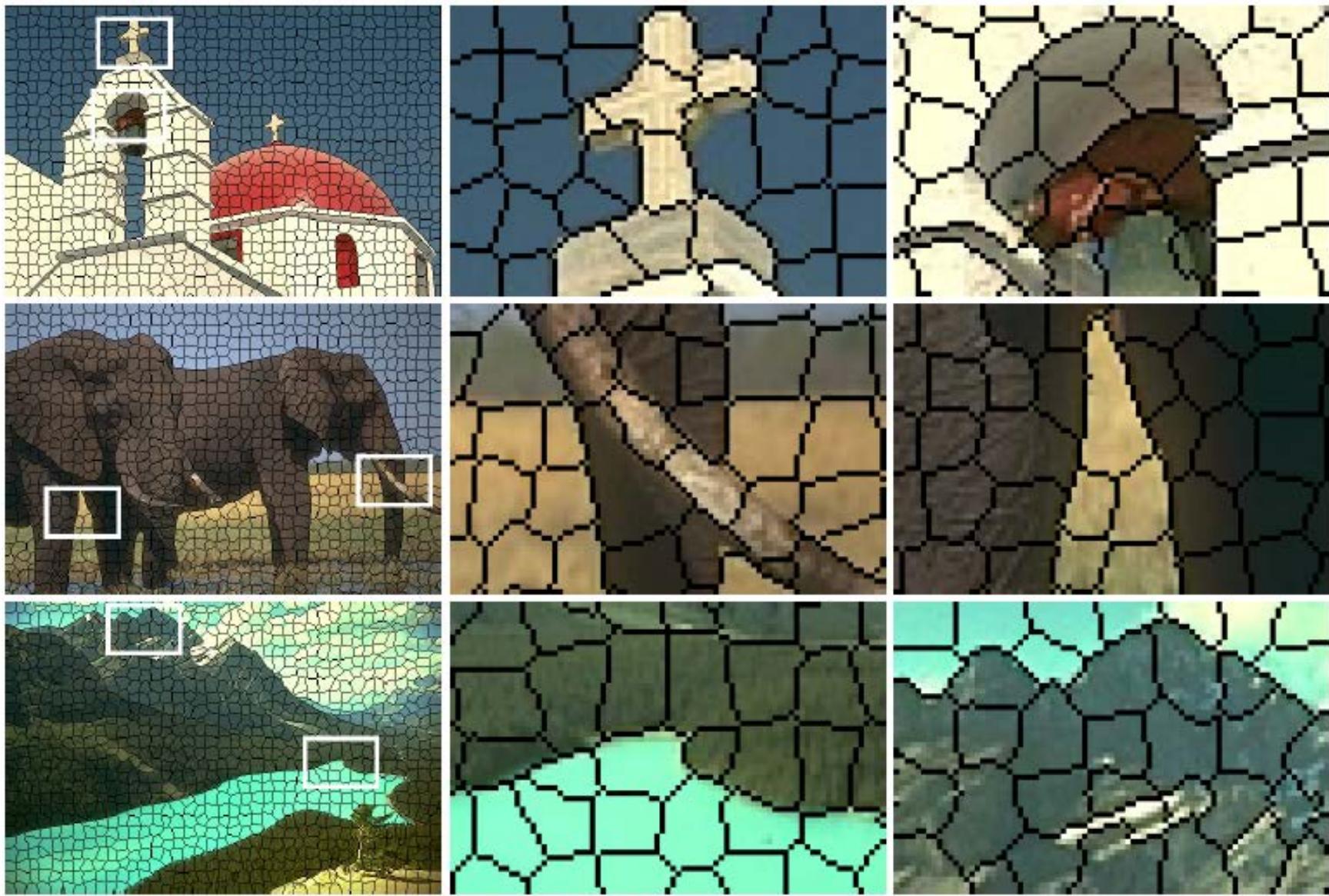
Extend velocities



Repeat until no evolution possible (Section 3.7)

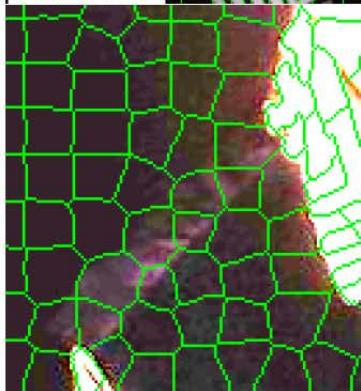
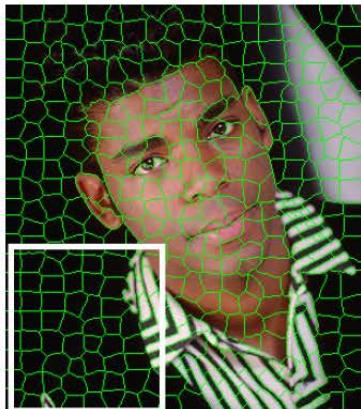


Turbo Pixels: Levinstein et al. 2009

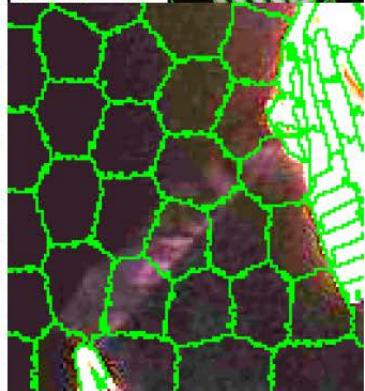
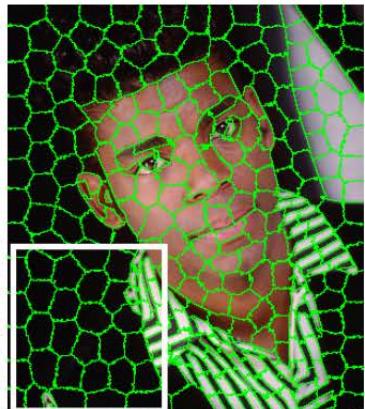


Turbo Pixels: Levinstein et al. 2009

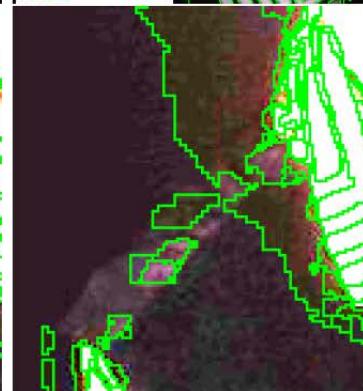
TurboPixels



N-Cuts



MeanShift



WaterShed

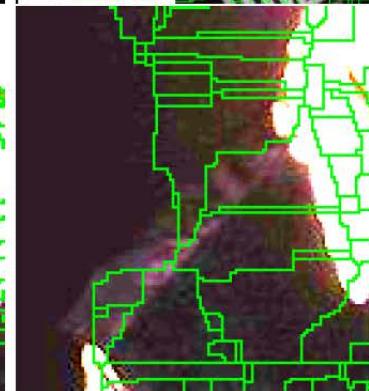


Fig. 1. Over-segments obtained with five algorithms: (a) TurboPixels (b) N-Cuts[27] (c) Local variation [6] (d) Mean-shift [3] (e) Watershed [7]. Each segmentation has (approximately) the same number of segments. The second row zooms in on the regions of interest defined by the white boxes.

Superpixel algorithms

- ▶ Goal is to divide the image into a large number of regions, such that each regions lie within object boundaries
- ▶ Examples
 - ▶ Watershed
 - ▶ Felzenszwalb and Huttenlocher graph-based
 - ▶ Turbopixels
 - ▶ SLIC



SLIC (Simple Linear Iterative Clustering)

[Achanta et al. PAMI 2012]

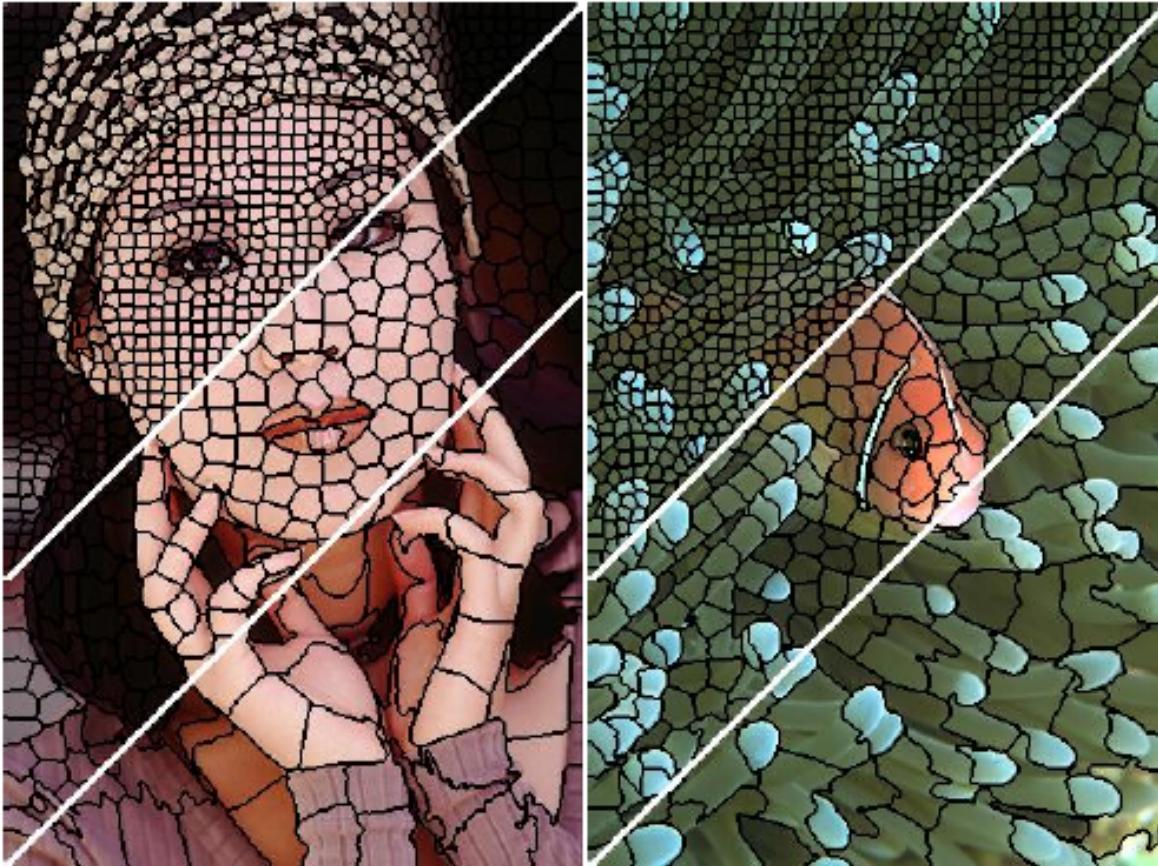
http://infoscience.epfl.ch/record/177415/files/Superpixel_PAMI2011-2.pdf

1. Initialize cluster centers on pixel grid in steps S
 - Features: Lab color, x-y position
2. Move centers to position in 3×3 window with smallest gradient
3. Compare each pixel to cluster center within $2S$ pixel distance and assign to nearest
4. Re-compute cluster centers as mean color/position of pixels belonging to each cluster
5. Stop when residual error is small



SLIC (Simple Linear Iterative Clustering)

[Achanta et al. PAMI 2012]



- + Fast 0.36s for 320x240
- + Regular superpixels
- + Superpixels fit boundaries
- May miss thin objects
- Large number of superpixels

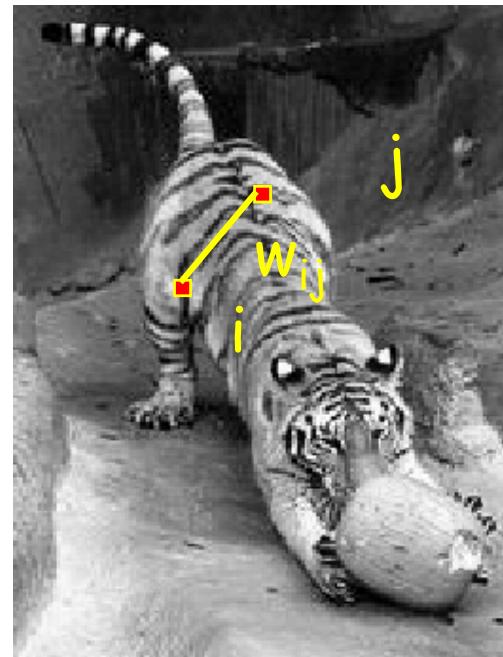
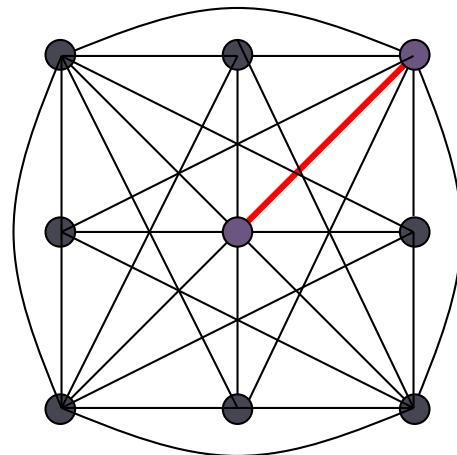


Graph Based Segmentation

- ▶ Normalized cuts
- ▶ Segmentation as energy minimization in a Markov random fields

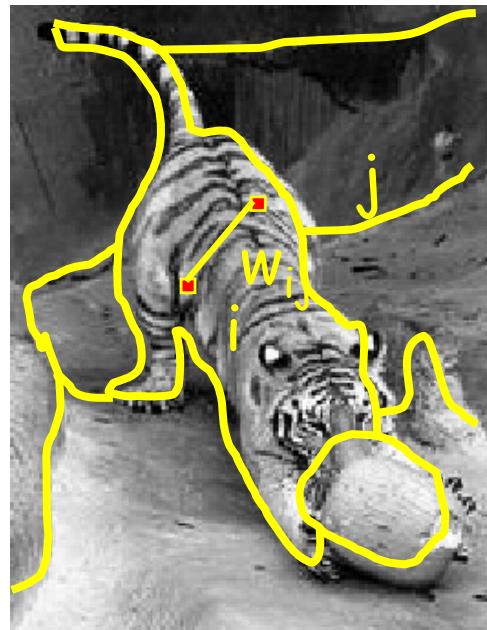
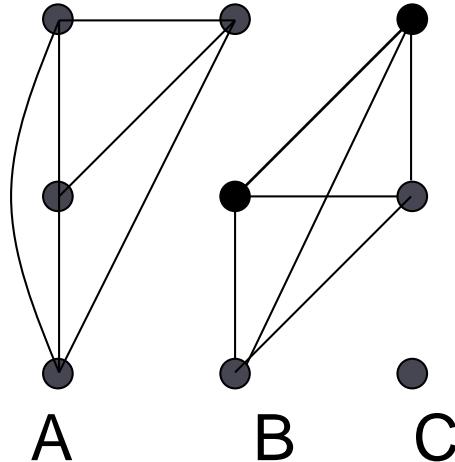
Images as graphs

- ▶ Node for every pixel
- ▶ Edge between every pair of pixels (or every pair of “sufficiently close” pixels)
- ▶ Each edge is weighted by the **affinity** or **similarity** of the two nodes



Segmentation by graph partitioning

- ▶ Break Graph into Segments
 - ▶ Delete links that cross between segments
 - ▶ Easiest to break links that have **low affinity**
 - ▶ similar pixels should be in the same segments
 - ▶ dissimilar pixels should be in different segments

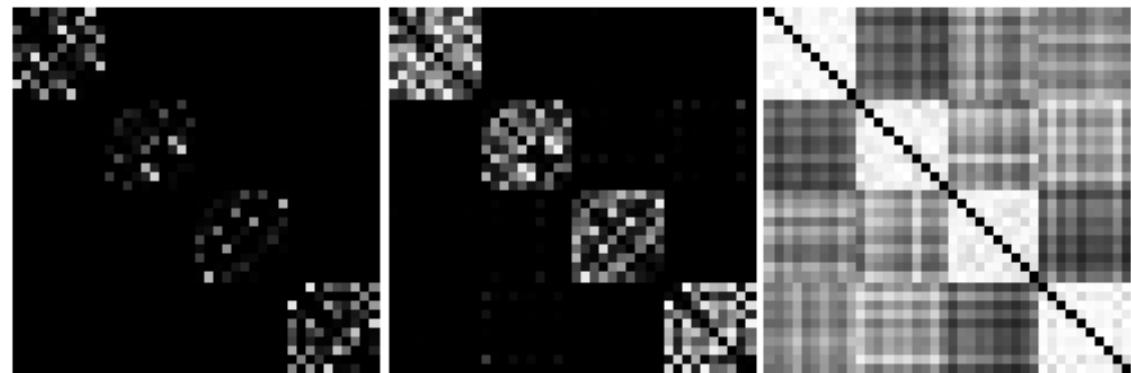
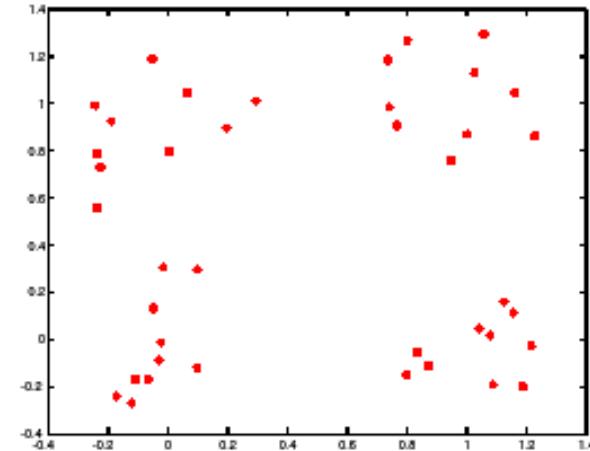
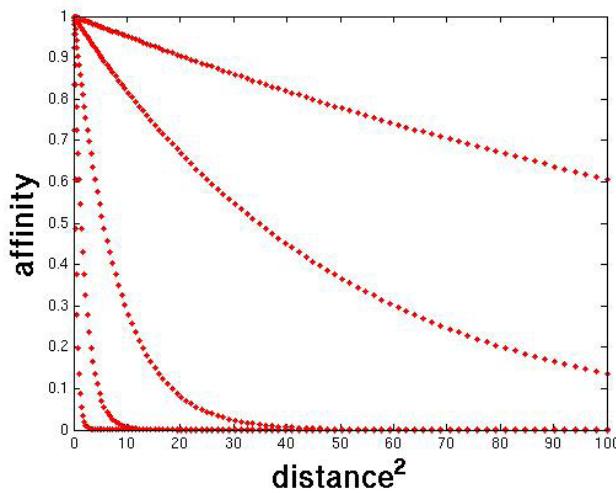


Measuring affinity

- **Distance**
$$aff(x, y) = \exp\left\{-\frac{1}{2\sigma_d^2} \|x - y\|^2\right\}$$
- **Intensity**
$$aff(x, y) = \exp\left\{-\frac{1}{2\sigma_d^2} \|I(x) - I(y)\|^2\right\}$$
- **Color**
$$aff(x, y) = \exp\left\{-\frac{1}{2\sigma_d^2} \underbrace{dist(c(x), c(y))^2}_{\text{(some suitable color space distance)}}\right\}$$
- **Texture**
$$aff(x, y) = \exp\left\{-\frac{1}{2\sigma_d^2} \underbrace{\|f(x) - f(y)\|^2}_{\text{(vectors of filter outputs)}}\right\}$$

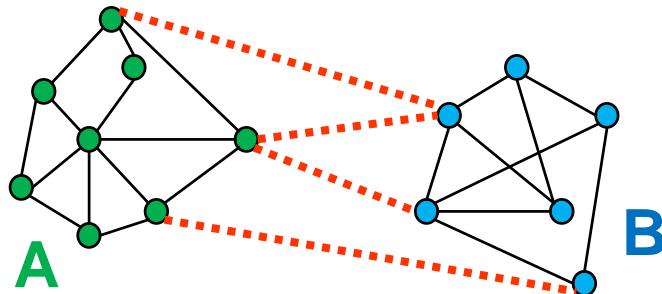
Scale affects affinity

- ▶ Small σ : group only nearby points
- ▶ Large σ : group far-away points



Graph cut

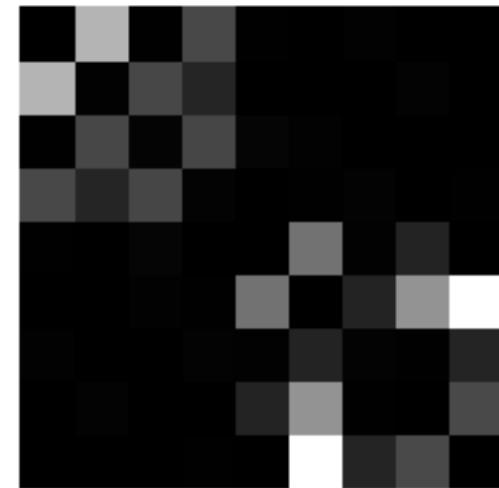
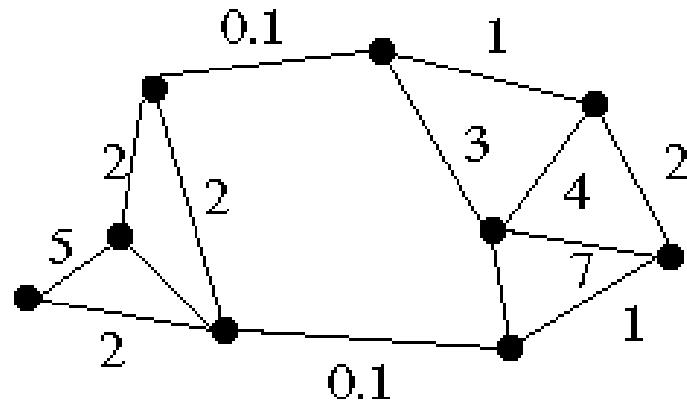
- ▶ Set of edges whose removal makes a graph disconnected
- ▶ **Cost of a cut: sum of weights of cut edges**
- ▶ A graph cut gives us a segmentation
 - ▶ What is a “good” graph cut and how do we find one?



Minimum cut

- We can segment by finding the **minimum cut** in a graph
 - ▶ Efficient algorithms exist for doing this

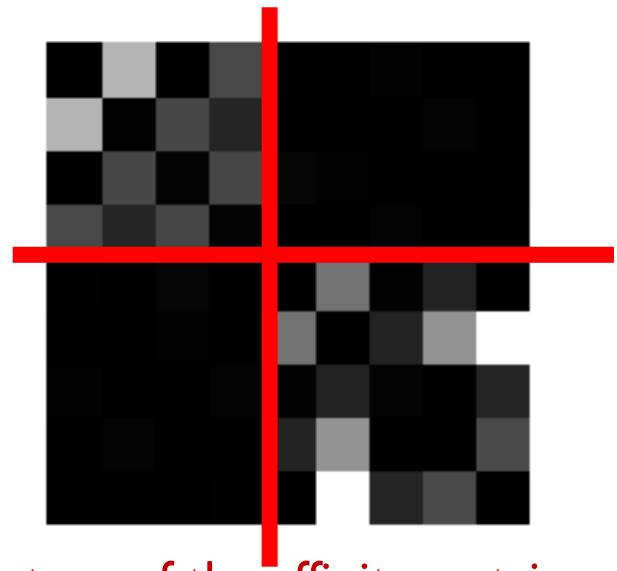
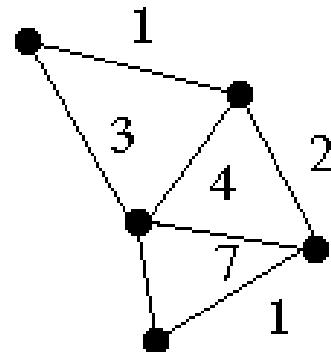
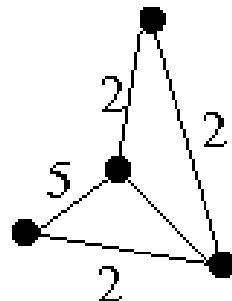
Minimum cut example



Minimum cut

- We can segment by finding the *minimum cut* in a graph
 - ▶ Efficient algorithms exist for doing this

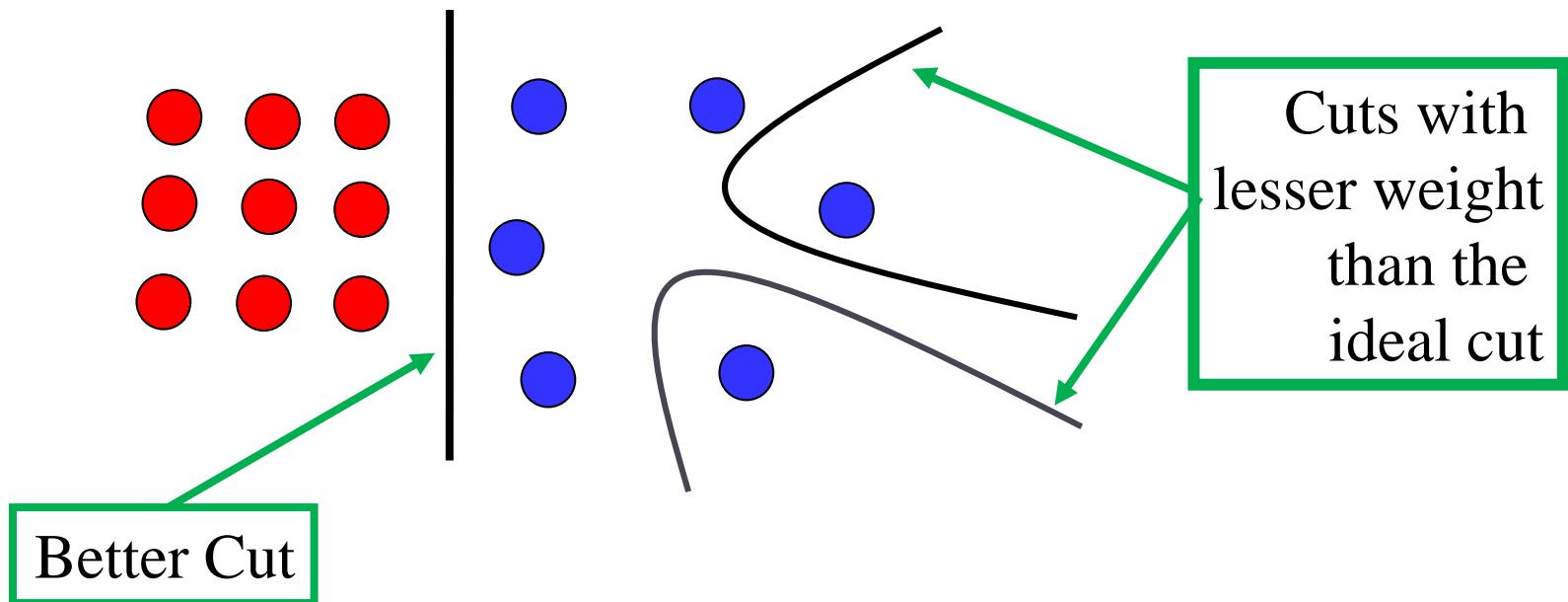
Minimum cut example



- The cut is defined by the block diagonal structure of the affinity matrix.
- Can this be generalized?

Minimum cut drawback

- ▶ Minimum cut tends to cut off very small, isolated components



Normalized cut (Ncut)

- ▶ Normalize the cost by the weights:
 - ▶ Within each cluster
 - ▶ Between clusters

Normalized cut (Ncut)

- ▶ Normalize the cost by the weights:
 - ▶ Within each cluster
 - ▶ Between clusters

$cut(A, B)$ = sum of weights of all edges **between A and B**

$assoc(A, V)$ = sum of weights of all edges **in A**

$$Ncut(A, B) = \frac{cut(A, B)}{assoc(A, V)} + \frac{cut(A, B)}{assoc(B, V)}$$

Ncut as a generalized eigenvector problem

- Let \mathbf{W} be the adjacency matrix of the graph

- .
- .

$$Ncut(A, B) = cut(A, B) \frac{1}{\sum_{p \in A} w_{p,q}} + \frac{1}{\sum_{q \in B} w_{p,q}}$$

Ncut as a generalized eigenvector problem

- Let \mathbf{W} be the adjacency matrix of the graph
- Let \mathbf{D} be the diagonal matrix with diagonal entries

$$\mathbf{D}(i, i) = \sum_j \mathbf{W}(i, j)$$

- Define the **cluster A vector**:

$$y(p) = \begin{cases} 1 & p \in A \\ negative & otherwise \end{cases}$$

- Then the normalized cut cost can be written as

$$Ncut(A, B) = \frac{y^T (D - W) y}{y^T D y}$$

Proof required! see paper

Ncut as a generalized eigenvector problem

- ▶ Problem:

Finding the exact minimum of the normalized cut cost is NP-complete (because y is discrete)

- ▶ Solution:

- ▶ Relax y to take on arbitrary values, then solved by a generalized eigenvalue problem

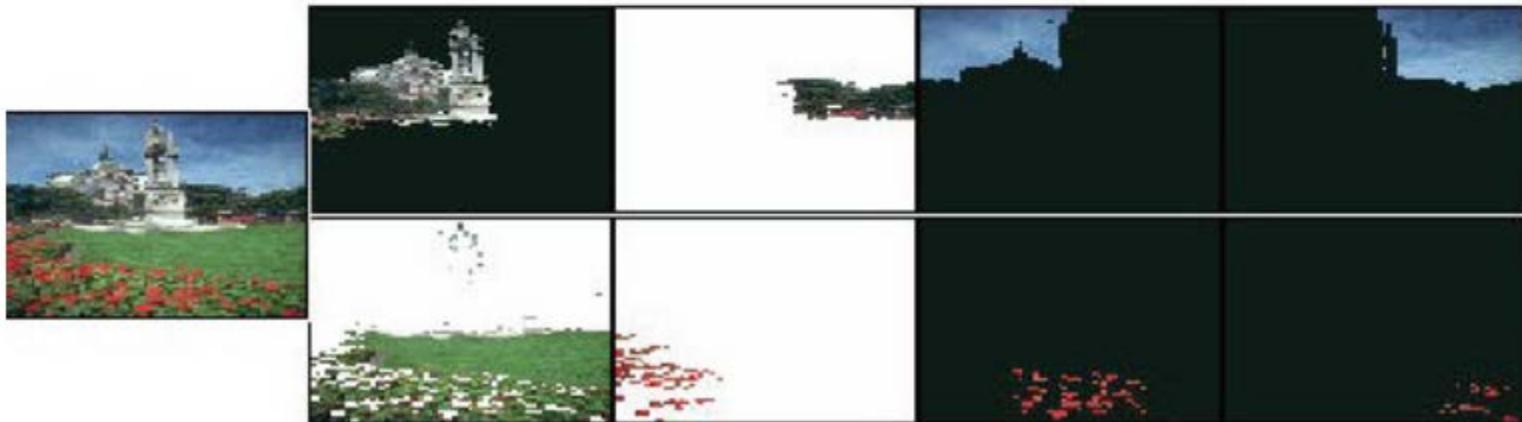
$$(D - W)y = \lambda Dy$$

- ▶ The solution y is given by the eigenvector corresponding to the second smallest eigenvalue
- ▶ Continuous results need to be converted into discrete → threshold

Normalized cut algorithm

- I. **Construct a weighted graph** $G = (V, E)$ from an image
2. Connect each pair of pixels, and **assign weights**
 $w(i, j) = \text{prob}(i, j \text{ belong to same region})$
3. Compute diagonal matrix $D(i, i) = \sum_j W(i, j)$
4. **Solve** $(D - W)y = \lambda Dy$ for the **eigenvector** with the second smallest eigenvalue
5. **Threshold eigenvector** to get a discrete cut
6. Recursively partition the segmented parts, if necessary

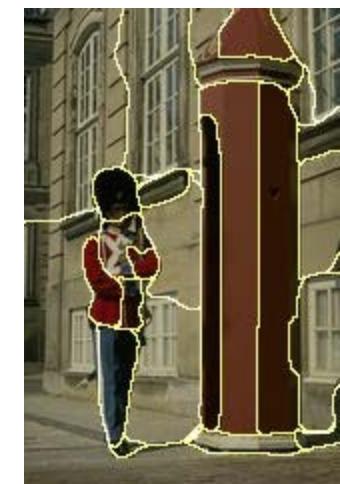
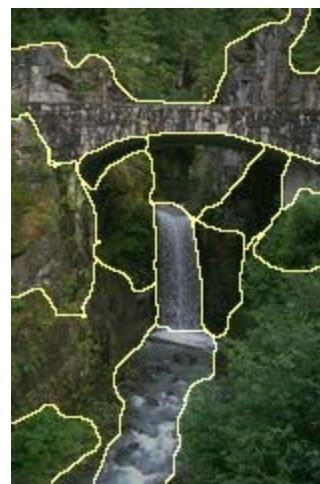
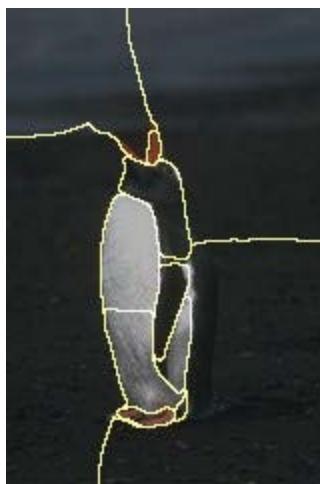
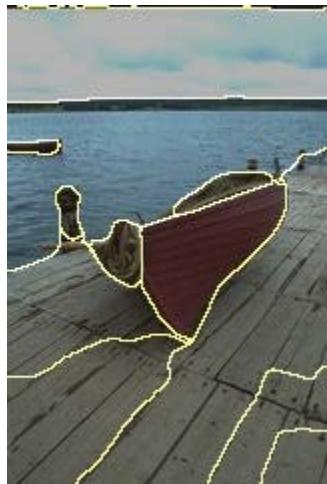
Example results



Example results



Results: Berkeley Segmentation Engine



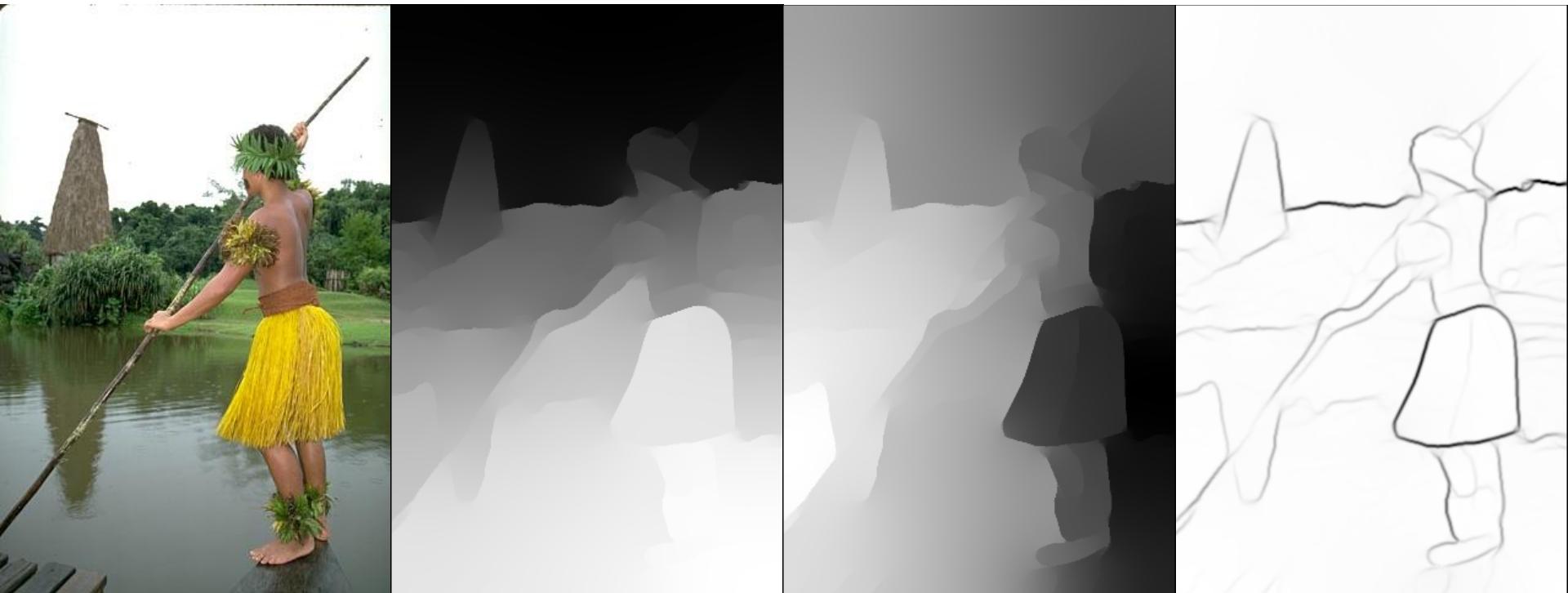
<http://www.cs.berkeley.edu/~fowlkes/BSE/>

Normalized cuts: Pros and cons

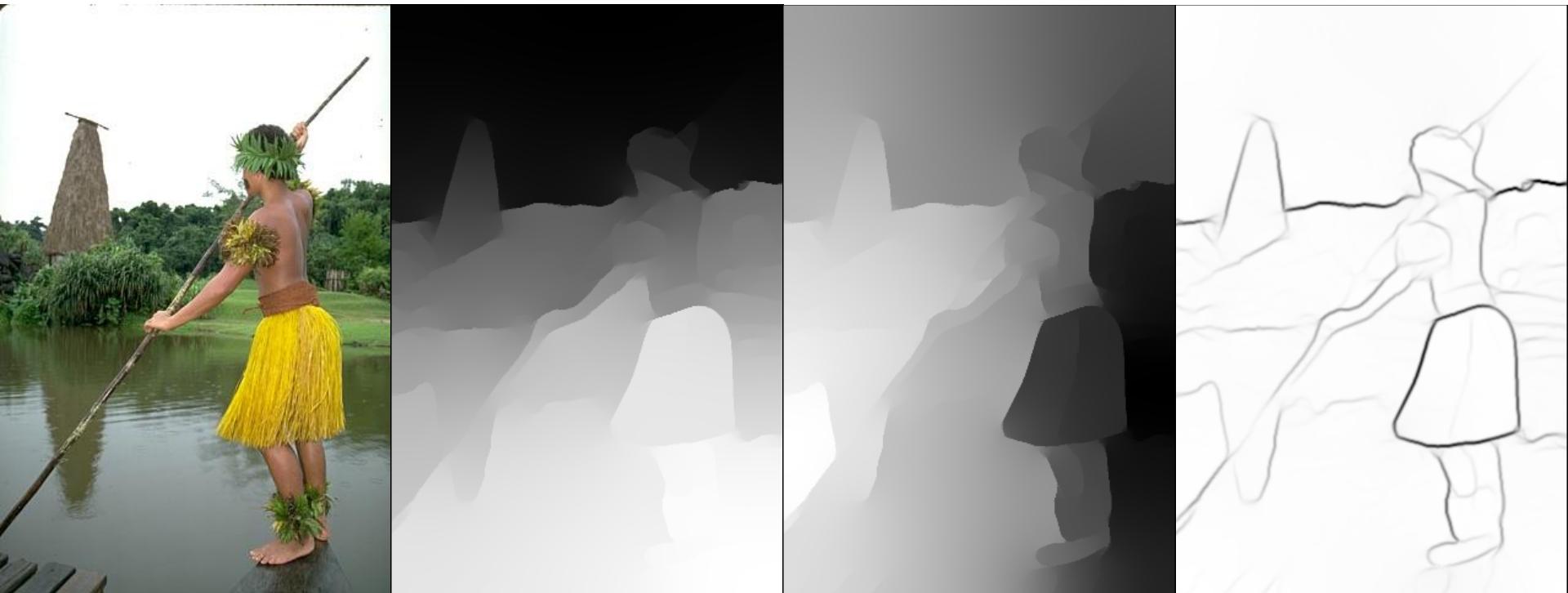
- **Pros**
 - ▶ Generic framework, can be used with many different features and affinity formulations
 - ▶ No model or data distribution
- **Cons**
 - ▶ High storage requirement and time complexity
 - ▶ Bias towards partitioning into equal segments



Eigenvectors carry contour information

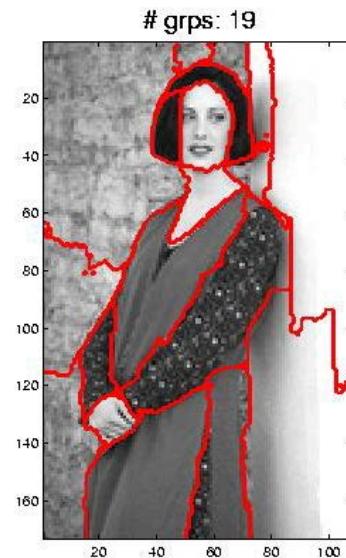
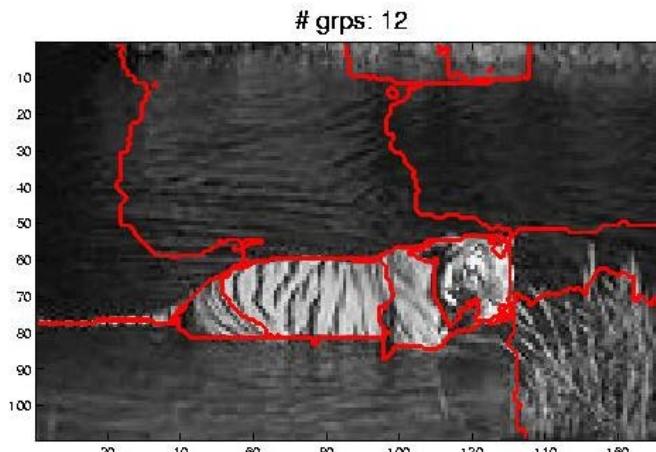
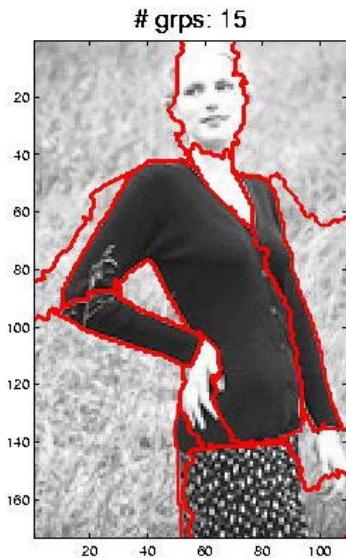


Eigenvectors carry contour information



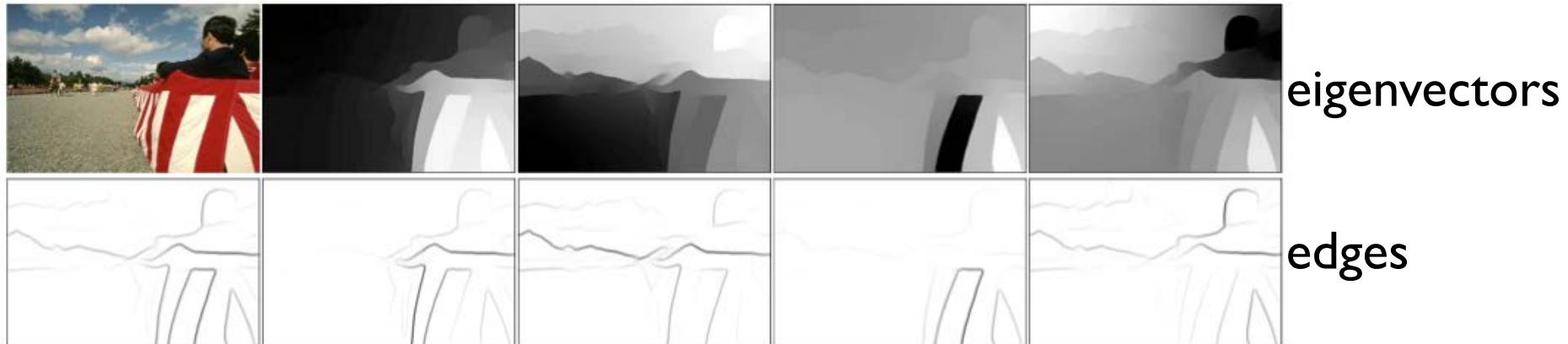
Avoiding Ncut drawback

- ▶ Do **not** try to find regions from the eigenvectors



Avoiding Ncut drawback

- ▶ Do **not** try to find regions from the eigenvectors
- ▶ Key idea:
 - ▶ Reshape eigenvectors into images
 - ▶ Compute edge probability P_b on eigenvector images
 - ▶ Final edge probability is the sum of all responses

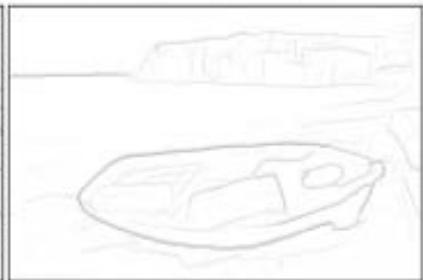
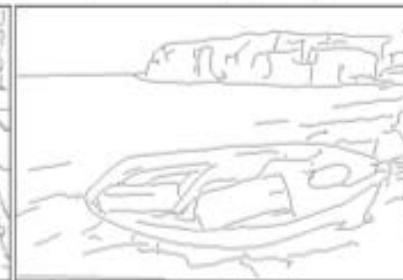
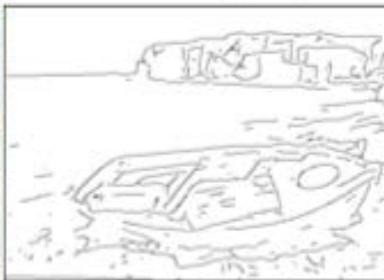
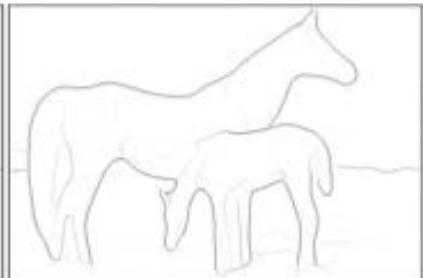
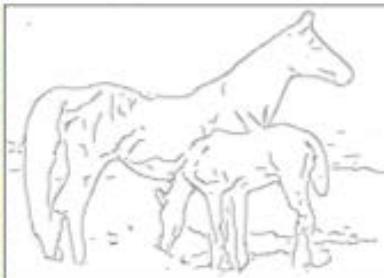


Example results

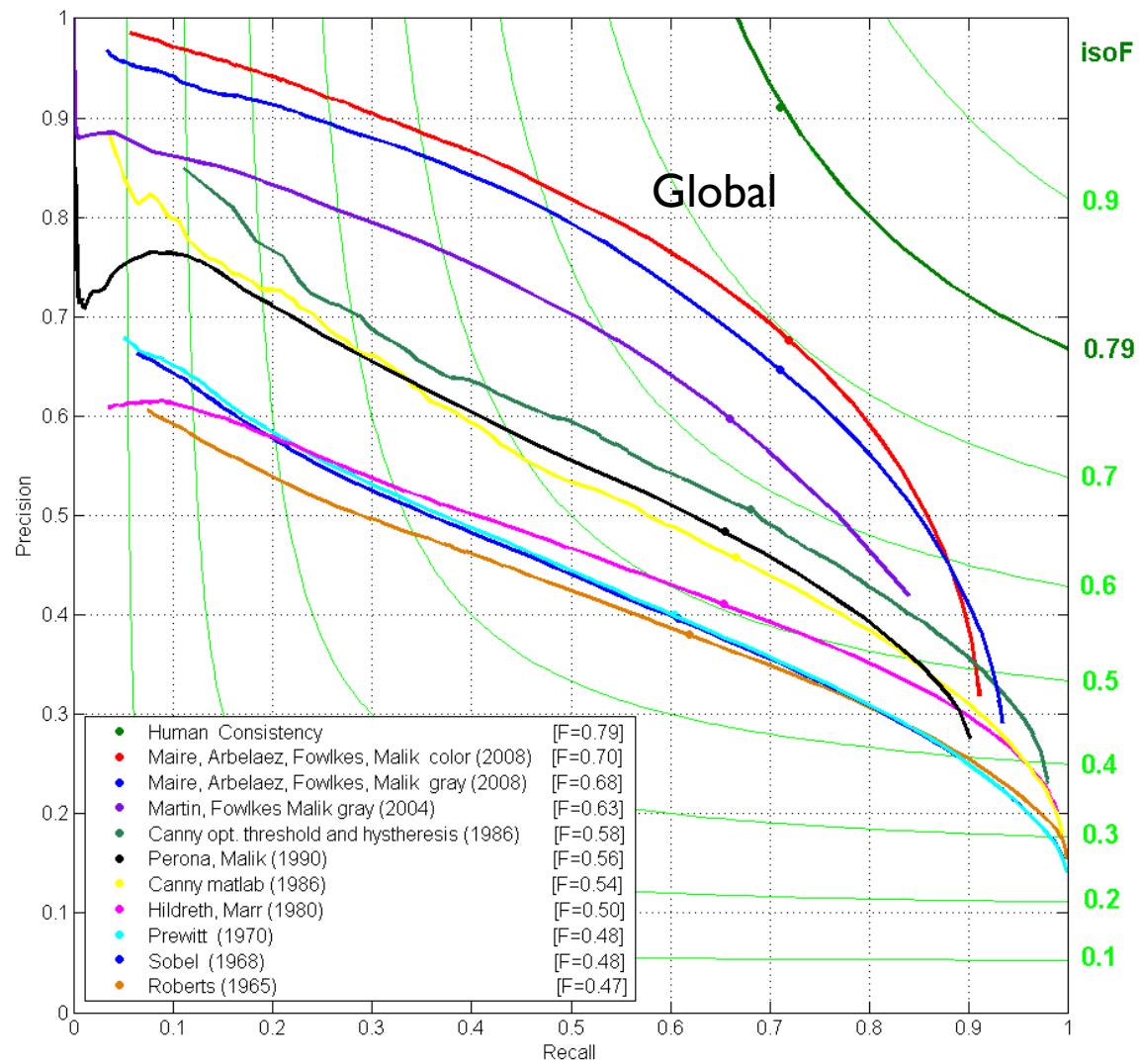
Final Pb

Pb

After threshold continuous



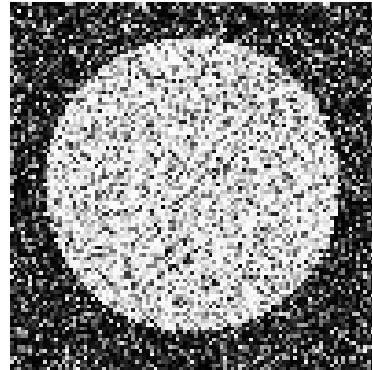
Contour detection ~2008 (color)



Graph Based Segmentation

- ▶ Normalized cuts
- ▶ Segmentation as energy minimization in a Markov random fields

Segmentation as energy minimization



$P(\text{foreground} \mid \text{image})$

Normalizing constant called “partition function”

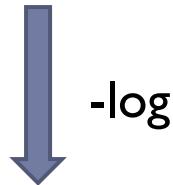
$$P(\mathbf{y}; \theta, data) = \frac{1}{Z} \prod_{i=1..N} f_1(y_i; \theta, data) \prod_{i, j \in edges} f_2(y_i, y_j; \theta, data)$$

Labels to be predicted Individual predictions Pairwise predictions



Writing Likelihood as an “Energy”

$$P(\mathbf{y}; \theta, data) = \frac{1}{Z} \prod_{i=1..N} p_1(y_i; \theta, data) \prod_{i, j \in edges} p_2(y_i, y_j; \theta, data)$$



$$Energy(\mathbf{y}; \theta, data) = \sum_i \psi_1(y_i; \theta, data) + \sum_{i, j \in edges} \psi_2(y_i, y_j; \theta, data)$$

Cost of assignment y_i

Cost of pairwise assignment y_i, y_j



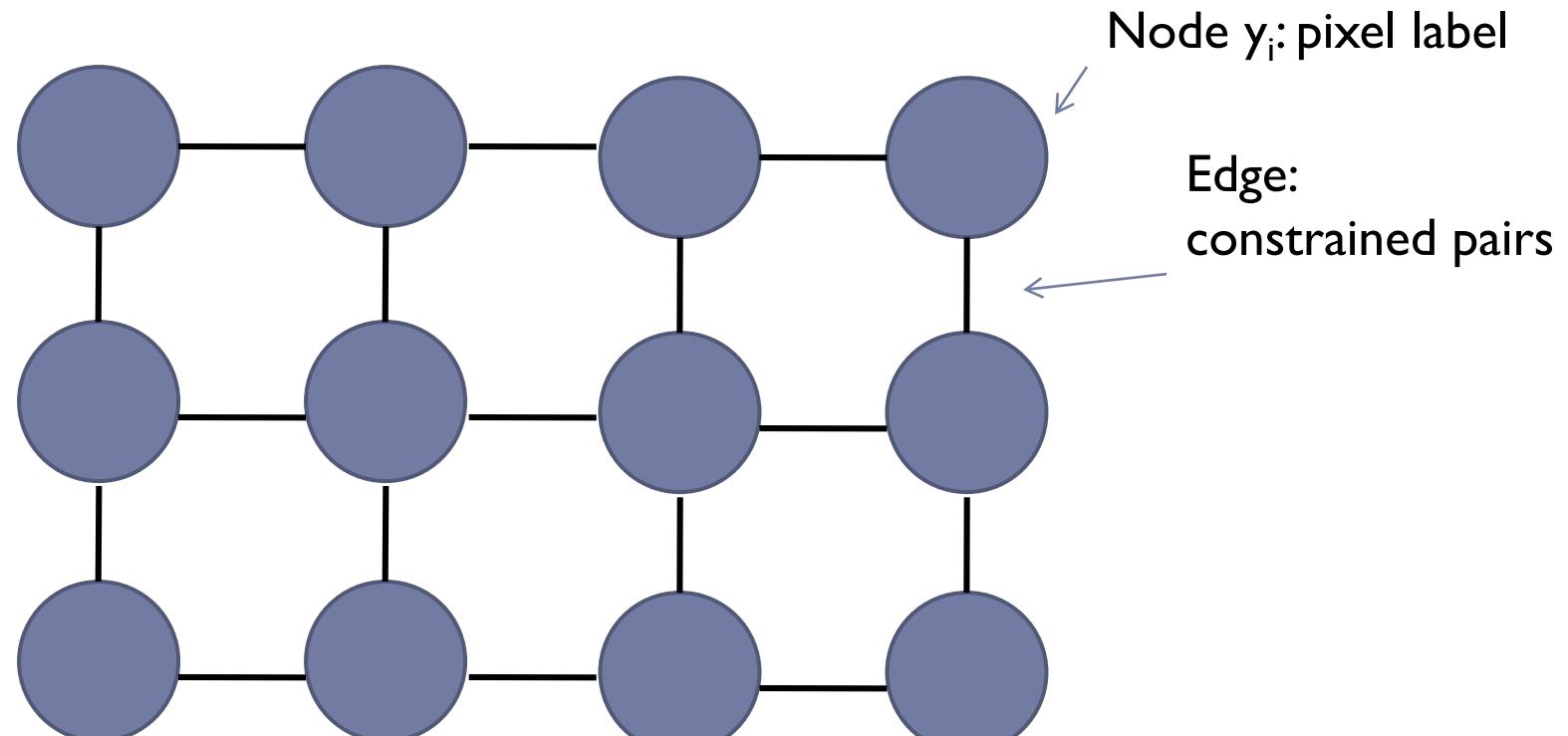
Notes on energy-based formulation

$$Energy(\mathbf{y}; \theta, data) = \sum_i \psi_1(y_i; \theta, data) + \sum_{i, j \in edges} \psi_2(y_i, y_j; \theta, data)$$

- ▶ Primarily used when you only care about the most likely solution (not the confidences)
- ▶ Can think of it as a general cost function
- ▶ Can have larger “cliques” than 2
 - ▶ Clique is the set of variables that go into a potential function



Markov Random Fields

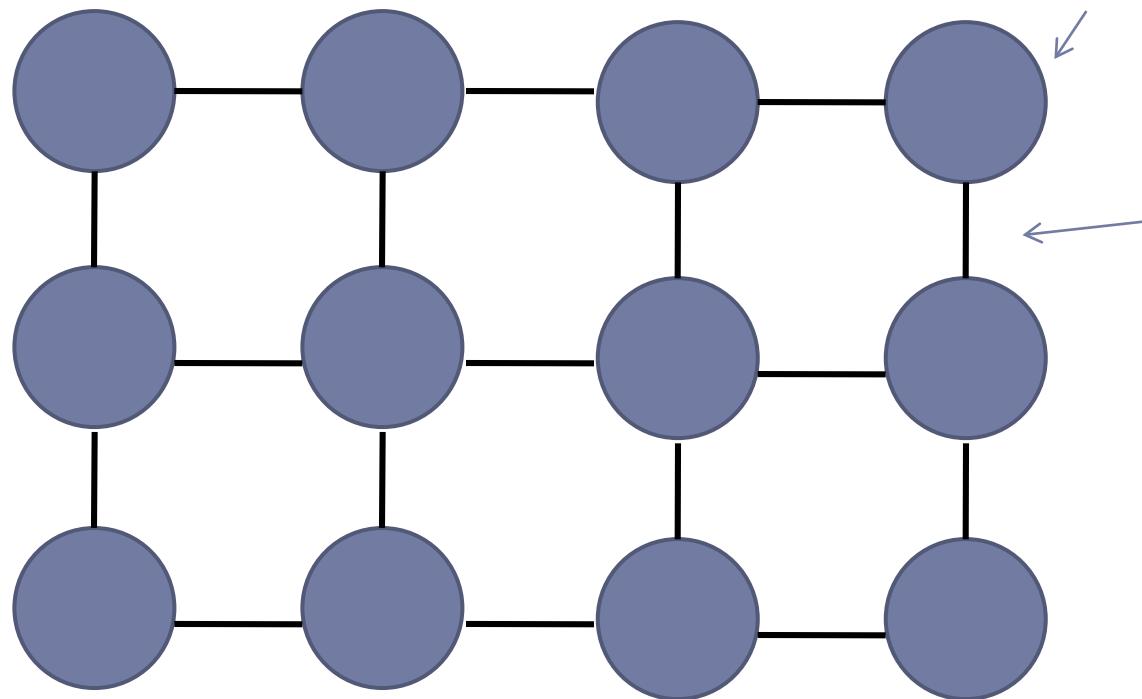


$$Energy(\mathbf{y}; \theta, data) = \sum_i \psi_1(y_i; \theta, data) + \sum_{i, j \in edges} \psi_2(y_i, y_j; \theta, data)$$



Markov Random Fields

- ▶ Example: “label smoothing” grid



Unary potential

$$\begin{aligned} 0: & -\log P(y_i = 0 \mid \text{data}) \\ 1: & -\log P(y_i = 1 \mid \text{data}) \end{aligned}$$

Pairwise Potential

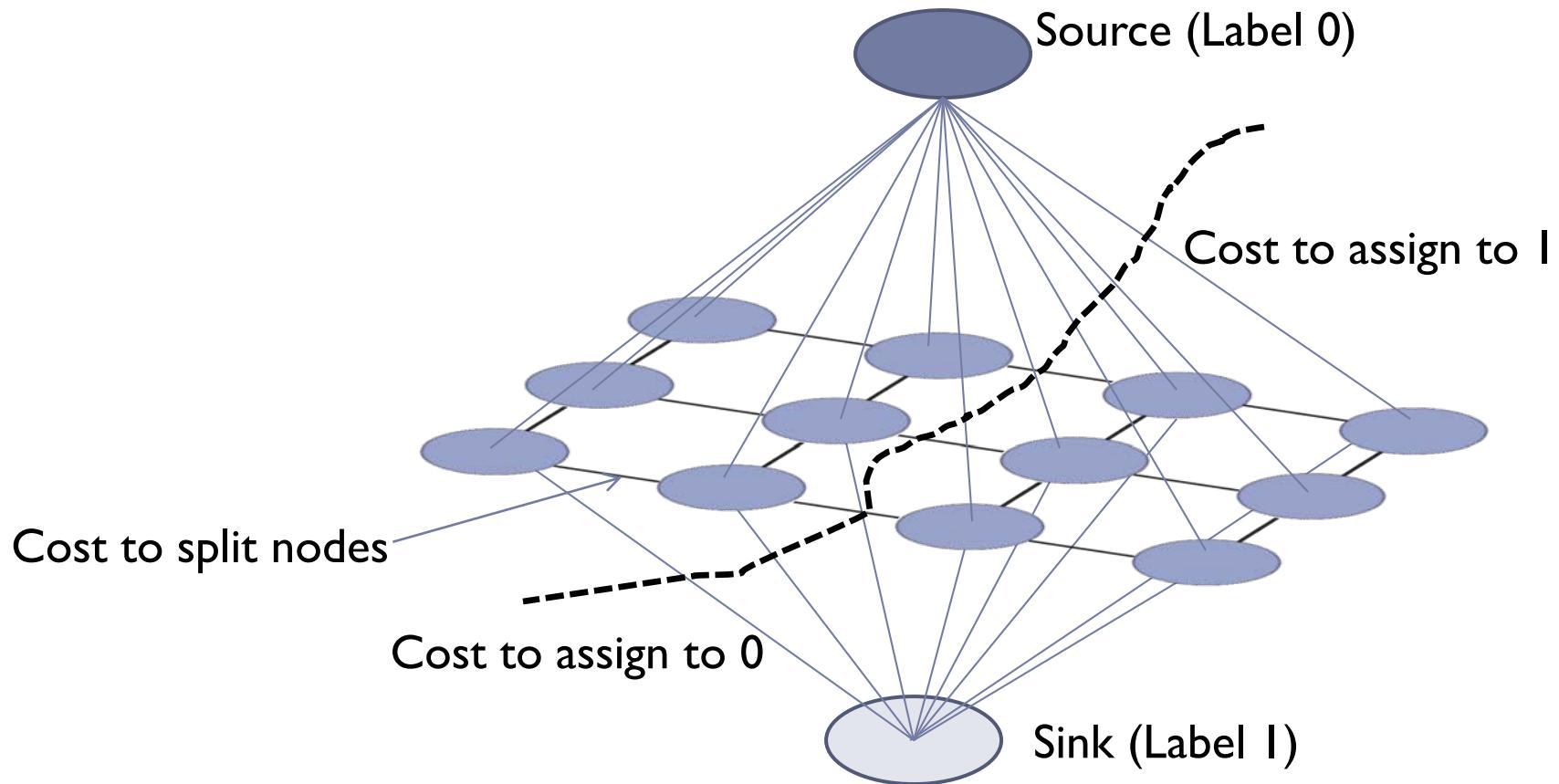
0	1
0	K
I	0

$$K > 0$$

$$\text{Energy}(\mathbf{y}; \theta, \text{data}) = \sum_i \psi_1(y_i; \theta, \text{data}) + \sum_{i, j \in \text{edges}} \psi_2(y_i, y_j; \theta, \text{data})$$



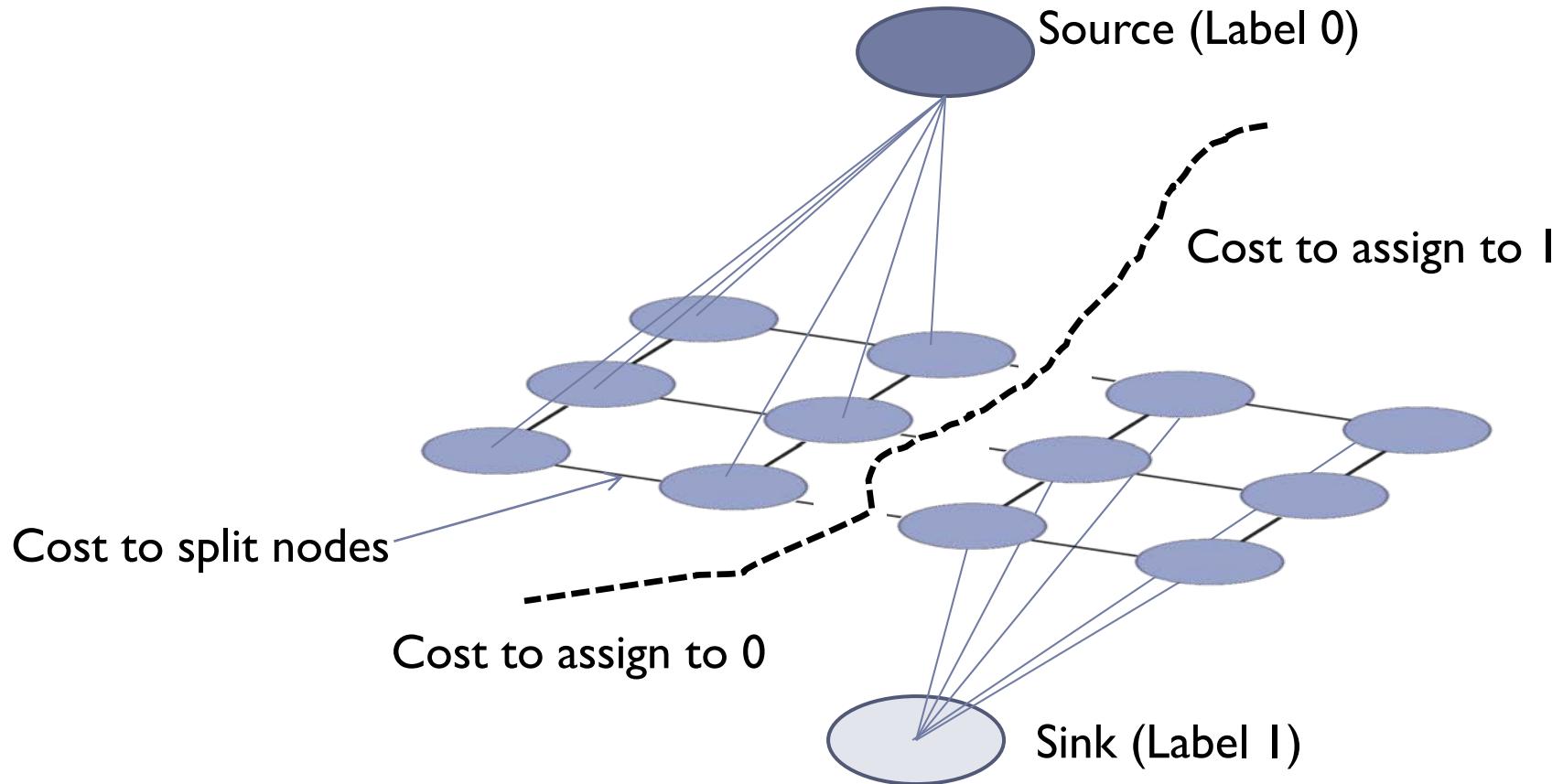
Solving MRFs with graph cuts



$$Energy(\mathbf{y}; \theta, data) = \sum_i \psi_1(y_i; \theta, data) + \sum_{i, j \in edges} \psi_2(y_i, y_j; \theta, data)$$



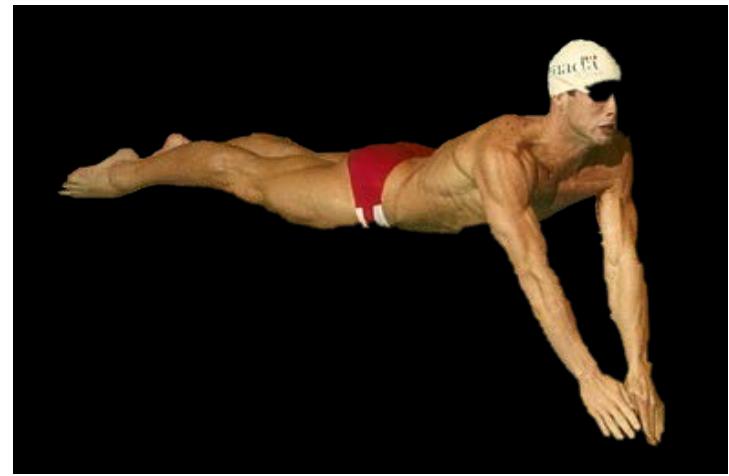
Solving MRFs with graph cuts



$$Energy(\mathbf{y}; \theta, data) = \sum_i \psi_1(y_i; \theta, data) + \sum_{i, j \in edges} \psi_2(y_i, y_j; \theta, data)$$



GrabCut segmentation



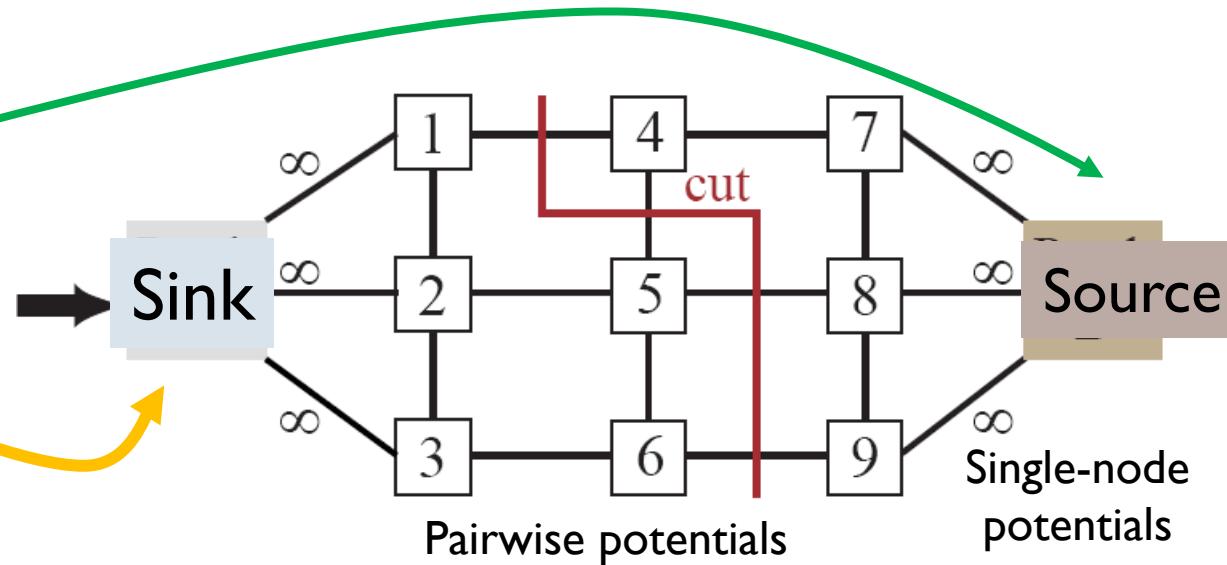
User provides rough indication of foreground region.

Goal: Automatically provide a pixel-level segmentation.



GrabCut

- ▶ Convert MRF into source-sink graph



- ▶ Minimum cost can be computed in polynomial time

[Kwatra et al., SIGGRAPH 2003]

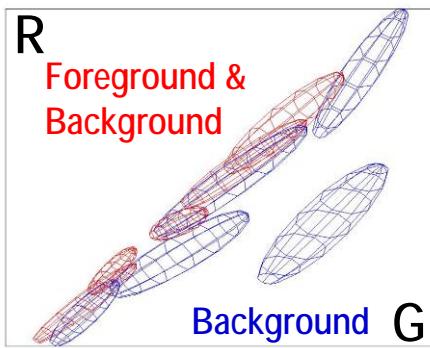
Binary segmentation as energy minimization

$$E(L) = E_d(L) + \lambda E_s(L)$$

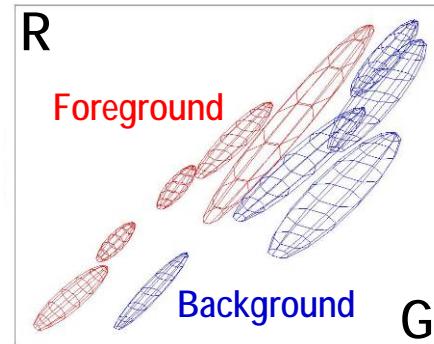
- ▶ For this problem, we can easily find the global minimum!
- ▶ Use max flow / min cut algorithm



Colour Model



Iterated
graph cut

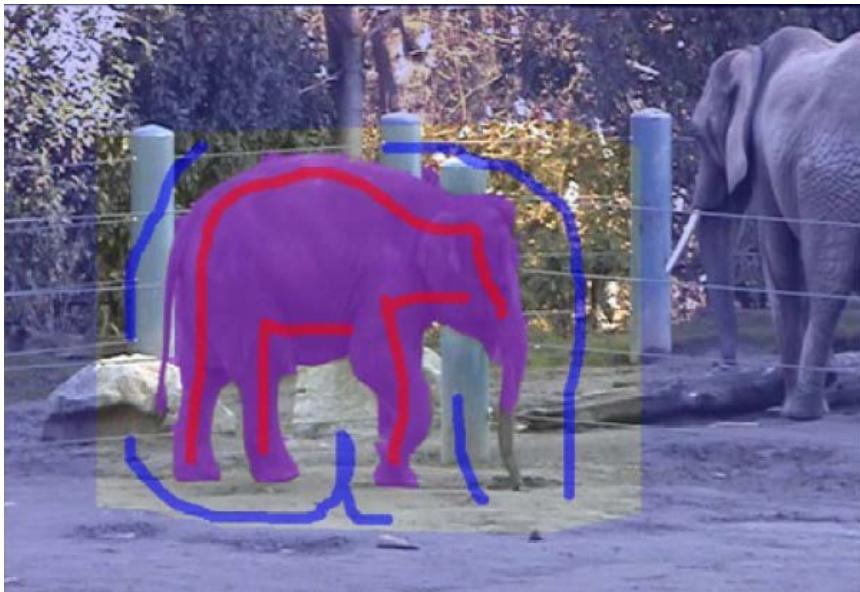


Gaussian Mixture Model (typically 5-8 components)

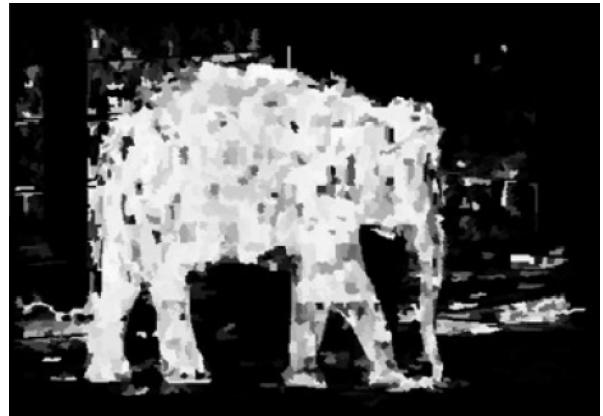


Source: Rother

$$E(L) = E_d(L) + \lambda E_s(L)$$



$$C'(x, y, 0)$$



$$C'(x, y, 1)$$



GrabCut segmentation

1. Define graph

- ▶ usually 4-connected or 8-connected
 - ▶ Divide diagonal potentials by $\sqrt{2}$

2. Define unary potentials

- ▶ Color histogram or mixture of Gaussians for background and foreground

$$\text{unary_potential}(x) = -\log \left(\frac{P(c(x); \theta_{foreground})}{P(c(x); \theta_{background})} \right)$$

3. Define pairwise potentials

$$\text{edge_potential}(x, y) = k_1 + k_2 \exp \left\{ \frac{-\|c(x) - c(y)\|^2}{2\sigma^2} \right\}$$

4. Apply graph cuts

5. Return to 2, using current labels to compute foreground, background models



GrabCut

Grabcut [Rother et al., SIGGRAPH 2004]



GrabCut

- ▶ Implemented in MS office Let's try it



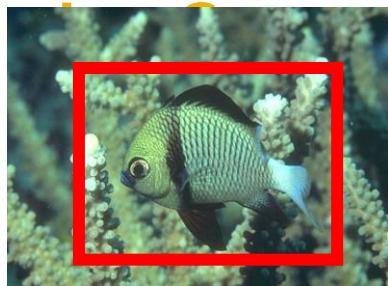
Reported results

Easier examples



More difficult Examples

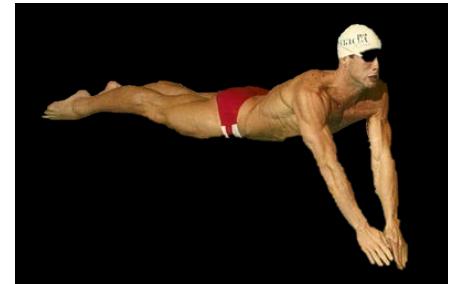
Camouflage &



Fine structure



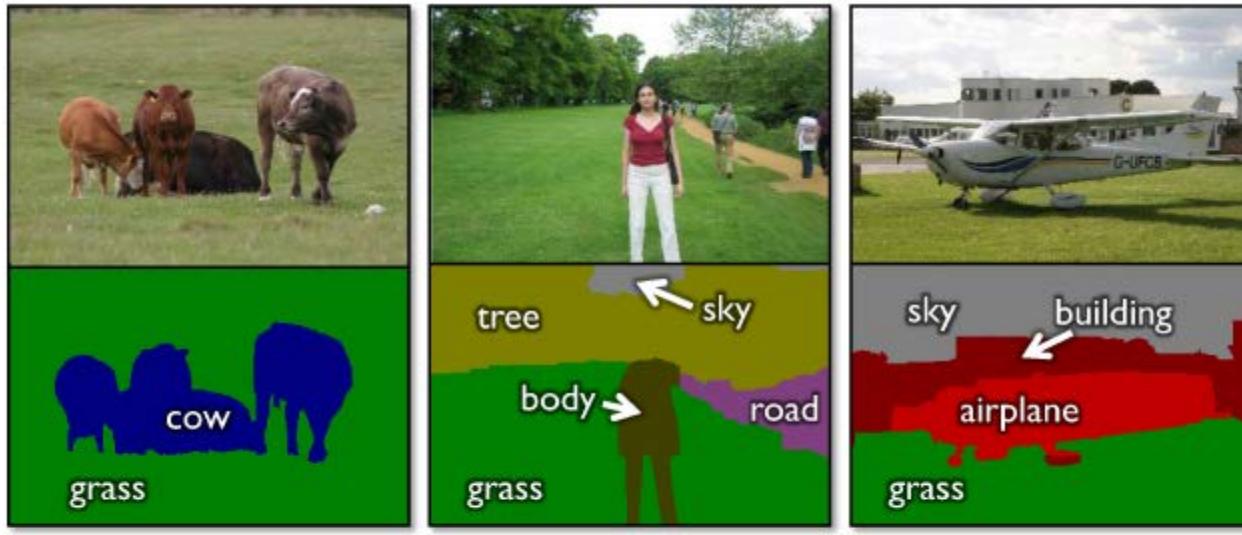
Harder Case



Other application: synthesis



Using graph cuts for recognition



object classes	building	grass	tree	cow	sheep	sky	airplane	water	face	car
bicycle	flower	sign	bird	book	chair	road	cat	dog	body	boat



Summary: MRF and graph-cuts

▶ Pros:

- ▶ Powerful, based on probabilistic model (MRF)
- ▶ Applicable to a wide range of problems
- ▶ Very efficient algorithms available for many problems
- ▶ Becoming a standard for segmentation

▶ Cons

- ▶ Graph-cuts can only solve a limited class of problems:
 - ▶ Sub-modular energy functions
 - ▶ Can only capture part of the power of MRF
- ▶ Only approximate solutions available for multi-label case

Graph cuts: Pros and Cons

- ▶ **Pros**
 - ▶ Very fast inference
 - ▶ Can incorporate data likelihoods and priors
 - ▶ Applies to a wide range of problems (stereo, image labeling, recognition)
- ▶ **Cons**
 - ▶ Not always applicable (associative only)
 - ▶ Need unary terms (not used for bottom-up segmentation, for example)
- ▶ **Use whenever applicable**



Further reading and resources

- ▶ **Graph cuts**
 - ▶ <http://www.cs.cornell.edu/~rdz/graphcuts.html>
 - ▶ Classic paper: [What Energy Functions can be Minimized via Graph Cuts?](#)
(Kolmogorov and Zabih, ECCV '02/PAMI '04)
- ▶ **Belief propagation**

Yedidia, J.S.; Freeman, W.T.; Weiss, Y., "Understanding Belief Propagation and Its Generalizations", Technical Report, 2001:
<http://www.merl.com/publications/TR2001-022/>

