

Problem Set #4

① a)

$$C'(t) = -k(C - 70^\circ\text{F})$$

made the constant the negative abs. value of k

~~$$C(t) = 70 + (C_0 - 70)e^{-kt}$$~~

b)

$$9^\circ\text{F} = -(141)(180 - 70)$$

$$k = \frac{9}{110}$$

c)

$$C'(120) = \frac{-9}{110}(120 - 70) = \frac{-9}{110}(50) = \frac{-9.5}{11} = \boxed{\frac{-45}{11}} \quad \boxed{\frac{45}{11}^\circ\text{F}}$$

d)

t	C	C'
0	180	-9
1	171	-8.26
2	162.7	-7.59
3	155.15	-6.97
4	148.18	-6.39
5	141.79	-5.87
6	135.91	-5.39
7	130.52	-4.95
8	125.57	-4.55
9	121.02	-4.17
10	116.85	

$$C'(C) = \frac{-9}{110}(C - 70)$$

$$C'(171) = \frac{-9}{110}(171 - 70) = \frac{-9 \cdot 101}{110} = -8.26$$

by hand

≈ 10 minutes
little less time

by Python

≈ 9.636 min
slightly larger

e)

saved as "coffeegraph.png"

more accurate b/c i used
a step of 10000

2) a) $\frac{dY}{dt} = k(Y)(1 - \frac{Y}{\text{Carry cap}})$

$$0.2 = k(1)(1 - \frac{1}{10})$$

$$\frac{0.2}{0.9} = k(1 - \frac{1}{10})$$

$$k = \frac{0.2}{0.9(1 - \frac{1}{10})} = 0.22$$

$$Y'(t) = 0.22Y(1 - \frac{Y}{10})$$

b) ~~2.1111111111111111~~ $t = 33,9000$ when within 1% of carry cap

c) $\frac{dY}{dt} = k(Y)(1 - \frac{Y}{\text{Carry cap}})$

$$0.1 = k(1)(1 - \frac{1}{10})$$

$$k = \frac{0.1}{\frac{9}{10}} = \frac{10}{9} = \frac{1}{9} = 0.11 = k$$

It takes abt 2x longer to do everything

d) $\frac{dA}{dt} = 0.05Y$

	2g1 (changed both)	2g2 (changed production)	2g3 (changed loss)
Time (hrs)	65,009	34,060	175,999
Alc Produced @ final (lbs)	17.917	4,569	13,316

e) $\frac{dY}{dt} = k(Y)(1 - \frac{Y}{\text{Carry cap}}) - (0.1A \cdot Y)$

$$\frac{dY}{dt} = \frac{2Y}{9}(1 - \frac{Y}{10}) - (0.1AY)$$

h) $S'(t) = -0.15 \cdot Y$

i) 25 lbs of sugar

$$\frac{dY}{dt} = \frac{2Y}{9}(1 - \frac{Y}{0.45}) - (0.1A \cdot Y)$$

f) The Yeast reaches its max value of 3.301 @ $t = 15.15$ (6.899 lbs from carrying cap)

Fermentation ends \approx when $t = 65,009$

j) Values @ final

Time	68.44 (hrs)
Alc	3.42 (lbs)
Yeast	0.009 (lbs)
Sugar	14.75 (lbs)