Bayesian Network Fundamentals

Help

Warning: The hard deadline has passed. You can attempt it, but **you will not get credit for it**. You are welcome to try it as a learning exercise.

Please check our grading policy under "Course Logistics" before submitting the quiz. The quiz isn't timed - you can save your answers halfway and come back again later.

☐ **In accordance with the Coursera Honor Code, I (Mike Ryan) certify that the answers here are my own work.**

Question 1

Factor product. Let X, Y and Z be binary variables.

If $\phi_1(X, Y)$ and $\phi_2(Y, Z)$ are the factors shown below, compute the selected entries (marked by a '?') in the factor $\psi(X, Y, Z) = \phi_1(X, Y) \cdot \phi_2(Y, Z)$, giving your answer according to the ordering of assignments to variables as shown below.

Separate each of the 3 entries of the factor with spaces, e.g., an answer of

0.1 0.2 0.3

means that $\psi(1, 1, 1) = 0.1$, $\psi(1, 2, 1) = 0.2$, and $\psi(2, 2, 2) = 0.3$.

X	Y	$\phi_1(X, Y)$
1	1	0.8
1	2	0.5
2	1	0.5
2	2	0.6

$\times$

Y	Z	$\phi_2(Y, Z)$
1	1	0.2
1	2	0.2
2	1	0.9
2	2	1.0

$=$

X	Y	Z	$\psi(X, Y, Z)$
1	1	1	?
1	1	2	
1	2	1	?
1	2	2	
2	1	1	
2	1	2	

2	2	1	
2	2	2	?

Question 2

Factor reduction. Let X, Z be binary variables, and let Y be a variable that takes on values 1, 2, or 3.

Now say we observe $Y = 2$. If $\phi(X, Y, Z)$ is the factor shown below, compute the missing entries of the reduced factor $\psi(X, Z)$ given that $Y = 2$, giving your answer according to the ordering of assignments to variables as shown below.

As before, you may separate the 4 entries of the factor by spaces.

X	Y	Z	$\phi(X, Y, Z)$
1	1	1	14
1	1	2	60
1	2	1	40
1	2	2	27
1	3	1	42
1	3	2	85
2	1	1	4
2	1	2	59
2	2	1	54
2	2	2	3
2	3	1	96
2	3	2	30

X	Z	$\psi(X, Z)$
1	1	?
1	2	?
2	1	?
2	2	?

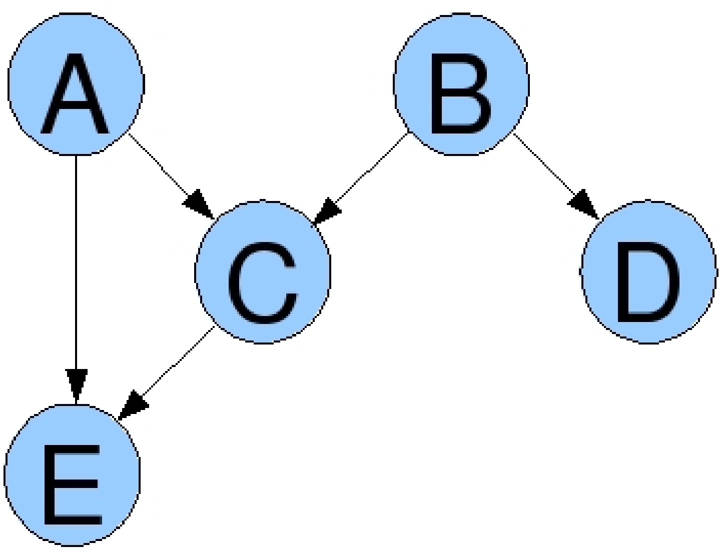
Question 3

Properties of independent variables. Assume that A and B are independent random variables. Which of the following options are always true? You may select 1 or more options (or none of them, if you think none apply).

- [ ] $P(A) + P(B) = 1$
- [ ] $P(A, B) = P(A) \times P(B)$
- [ ] $P(A|B) = P(A)$
- [ ] $P(A) \neq P(B)$

Question 4

Independencies in a graph. Which pairs of variables are independent in the graphical model below, given that none of them have been observed? You may select 1 or more options (or none of them, if you think none apply).

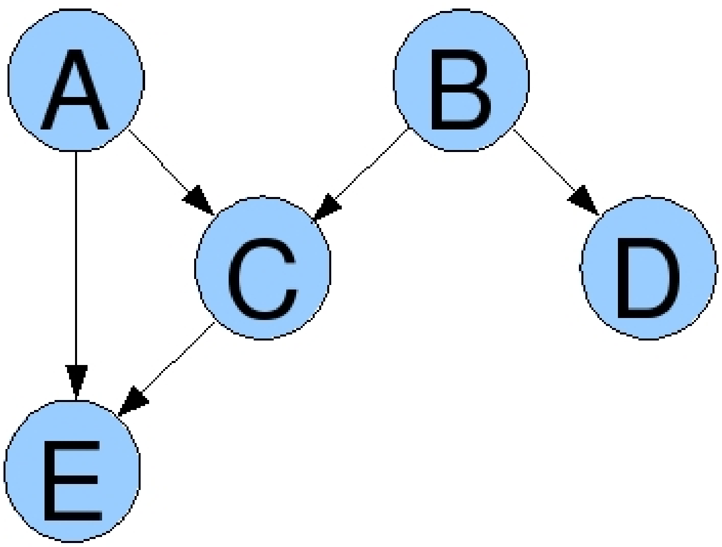


- [ ] None - there are no pairs of independent variables.
- [ ] A, E
- [ ] A, D

- C, D
- A, C

Question 5

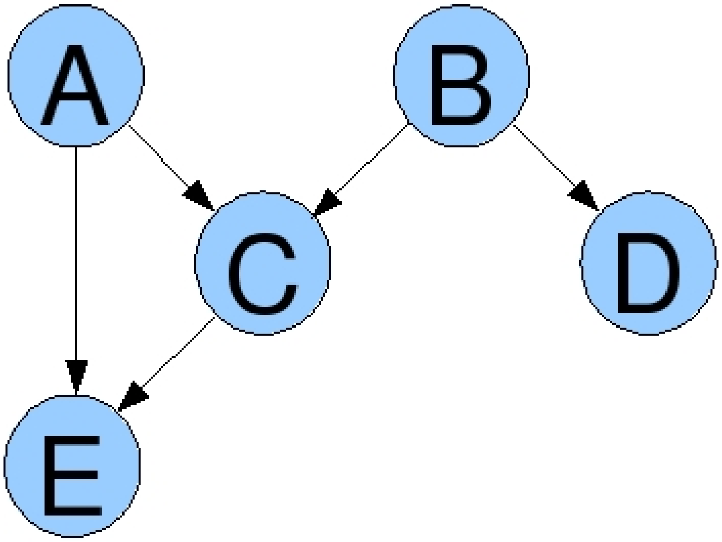
***Independencies in a graph.** (An asterisk marks a question that is more challenging. Congratulations if you get it right!) Now assume that the value of E is known. (E is observed. A, B, C, and D are not observed.) Which pairs of variables (not including E) are independent in the same graphical model, given E? You may select 1 or more options (or none of them, if you think none apply).



- D, C
- A, D
- A, C
- None - given E, there are no pairs of variables that are independent.
- A, B
- B, D
- B, C

Question 6

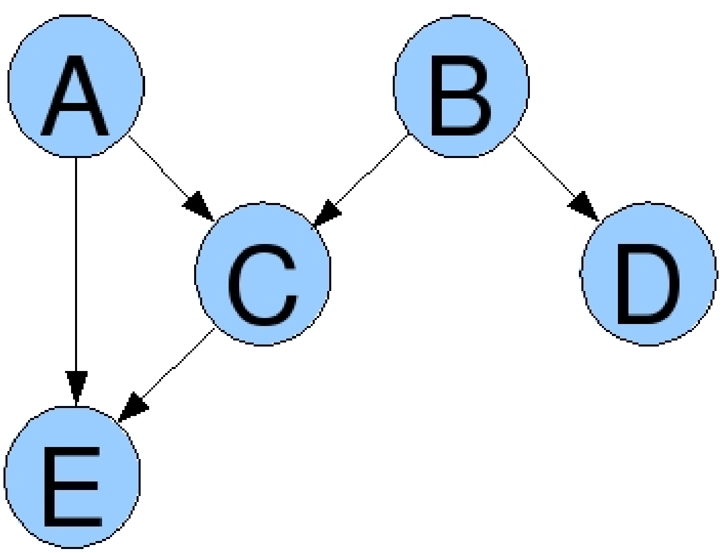
Factorization. Given the same model as above, which of these is an appropriate decomposition of the joint distribution $P(A, B, C, D, E)$?



- $P(A, B, C, D, E) = P(A)P(B)P(C|A)P(C|B)P(D|B)P(E|A)P(E|C)$
- $P(A, B, C, D, E) = P(A)P(B)P(A, B|C)P(B|D)P(A, C|E)$
- $P(A, B, C, D, E) = P(A)P(B)P(C)P(D)P(E)$
- $P(A, B, C, D, E) = P(A)P(B)P(C|A, B)P(D|B)P(E|A, C)$

Question 7

Independent parameters. How many independent parameters are required to uniquely define the CPD of C (the conditional probability distribution associated with the variable C) in the same graphical model as above, if A, B, and D are binary, and C and E have three values each?



If you haven't come across the term before, here's a brief explanation: A multinomial distribution over m possibilities $x_1, \ldots, x_m$ has m parameters, but $m-1$ independent parameters, because we have the constraint that all parameters must sum to 1, so that if you specify $m-1$ of the parameters, the final one is fixed. In a CPD $P(X|Y)$, if X has m values and Y has k values, then we have k distinct multinomial distributions, one for each value of Y, and we have $m-1$ independent parameters in each of them, for a total of $k(m-1)$. More generally, in a CPD $P(X|Y_1, \ldots, Y_r)$, if each Y_i has k_i values, we have a total of $k_1 \times \ldots \times k_r \times (m-1)$ independent parameters.

Example: Let's say we have a graphical model that just had $X \to Y$, where both variables are binary. In this scenario, we need 1 parameter to define the CPD of X. The CPD of X contains two entries $P(X=0)$ and $P(X=1)$. Since the sum of these two entries has to be equal to 1, we only need one parameter to define the CPD.

Now we look at Y. The CPD for Y contains 4 entries which correspond to: $P(Y=0|X=0), P(Y=1|X=0), P(Y=0|X=1), P(Y=1|X=1)$. Note that $P(Y=0|X=0)$ and $P(Y=1|X=0)$ should sum to one, so we need 1 independent parameter to describe those two entries; likewise, $P(Y=0|X=1)$ and $P(Y=1|X=1)$ should also sum to 1, so we need 1 independent parameter for those two entries.

Therefore, we need 1 independent parameter to define the CPD of X and 2 independent parameters to define the CPD of Y.

- 8
- 3
- 6
- 4
- 12
- 11
- 7

Question 8

I-maps. I-maps can also be defined directly on graphs as follows. Let $I(G)$ be the set of independencies encoded by a graph G. Then G_1 is an I-map for G_2 if $I(G_1) \subseteq I(G_2)$.

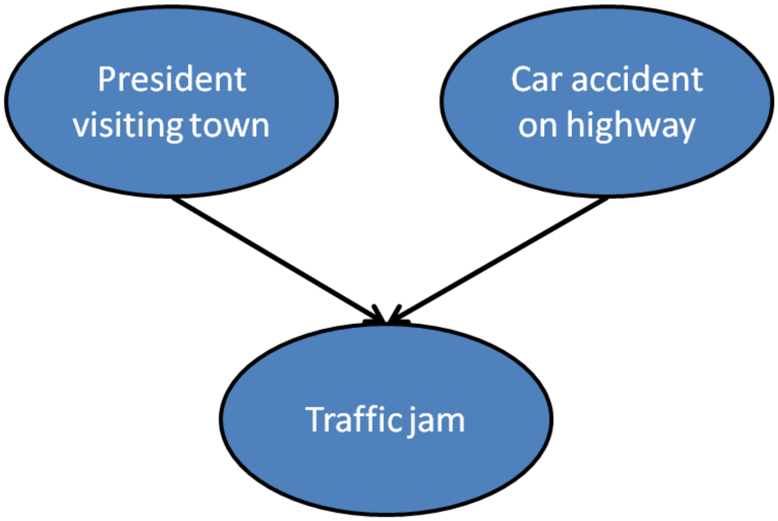
Which of the following statements about I-maps are true? You may select 1 or more options (or none of them, if you think none apply).

- [ ] A graph K is an I-map for a graph G if and only if all of the independencies encoded by K are also encoded by G.
- [ ] An I-map is a function f that maps a graph G to itself, i.e., $f(G) = G$.
- [ ] A graph K is an I-map for a graph G if and only if K encodes all of the independences that G has and more.
- [ ] I-maps are Apple's answer to Google Maps.
- [ ] A graph K is an I-map for a graph G if and only if K encodes exactly the same independencies as G.

Question 9

***Inter-causal reasoning.** Consider the following model for traffic jams in a small town, which we assume can be caused by a car accident, or by a visit from the president (and the accompanying security motorcade).

P(President = 1) = 0.01   P(Accident = 1) = 0.1



P(Traffic = 1 | President = 0, Accident = 0) = 0.1
P(Traffic = 1 | President = 0, Accident = 1) = 0.5
P(Traffic = 1 | President = 1, Accident = 0) = 0.6
P(Traffic = 1 | President = 1, Accident = 1) = 0.9

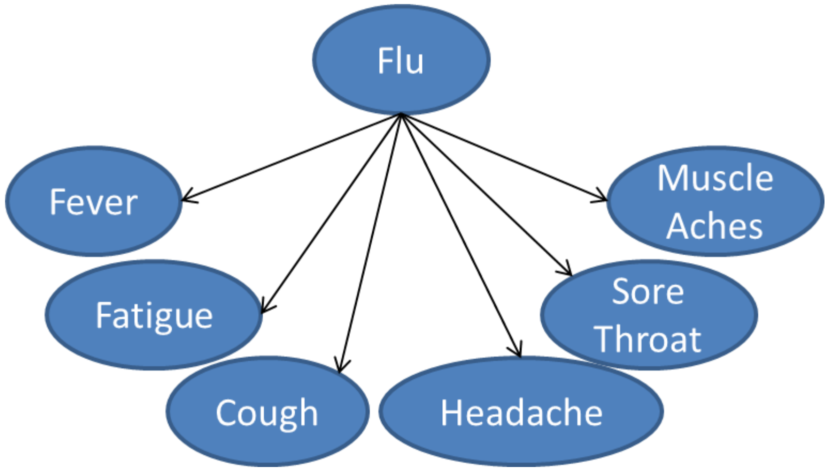
Calculate P(Accident = 1 | Traffic = 1) and P(Accident = 1 | Traffic = 1, President = 1). Separate your answers with a space, e.g., an answer of

0.15 0.25

means that P(Accident = 1 | Traffic = 1) = 0.15 and P(Accident = 1 | Traffic = 1, President = 1) = 0.25. Round your answer to two decimal places.

Question 10

***Naive Bayes.** Consider the following Naive Bayes model for flu diagnosis:



Assume a population size of 10,000. Which of the following statements are true in this model?

You may select 1 or more options (or none of them, if you think none apply).

- [ ] Say we observe that 1000 people have a headache (and possibly other symptoms), out of which 500 people have the flu (and possibly other symptoms), and 500 people have a fever (and possibly other symptoms). Without more information, we cannot estimate how many people with a headache also have both the flu and a fever.
- [ ] Say we observe that 1000 people have the flu, out of which 500 people have a headache (and possibly other symptoms) and 500 have a fever (and possibly other symptoms). We would expect that approximately 250 people with the flu also have both a headache and fever.
- [ ] Say we observe that 1000 people have the flu, out of which 500 people have a headache (and possibly other symptoms) and 500 have a fever (and possibly other symptoms). We can conclude that exactly 250 people with the flu also have both a headache and fever.
- [ ] Say we observe that 500 people have a headache (and possibly other symptoms) and 500 people have a fever (and possibly other symptoms). Without more information, we cannot estimate how many people have both a headache and fever.

- [ ] **In accordance with the Coursera Honor Code, I (Mike Ryan) certify that the answers here are my own work.**

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