Final Exam

Math 135-002 Engineering Calculus I
Colorado Mesa University Fall 2022

Name: _____

The specter of a graded exam unfortunately haunts us today.

Silence your phone during the exam. You may go to the restroom as you need, but leave your phone on your desk if you do.

Be sure to use your calculator effectively, storing intermediate computations to the full precision your calculator will allow. Answers expressed as a decimal approximation must be accurate to within $\pm 10^{-5}$.

When scored, each page of this exam will be weighed roughly equally. It may be a good idea to not simply write calculations and answers, but to use your responses to communicate insights. The grader is perceptive; provide them with evidence of your knowledge and understanding.

The purpose of this exam is to provide your instructor a document to justify to the University the letter grade you'll be assigned for this class. *Your instructor wants you to pass this course.* Imagine that your instructor is a lawyer who must represent you to the University with this exam in hand, advocating that you receive the highest letter grade possible. Provide them with all the evidence you can that you've learned the material presented in this course.

Finally, note that since the purpose of this exam is to assign you a grade to rank you among your peers, it inherently undermines the ideal collegial nature of school by incentivizing competition rather than cooperation. Please understand that being graded is *not* an intrinsic element of education, and your instructor grades you reluctantly.

1. What is the value of the limit

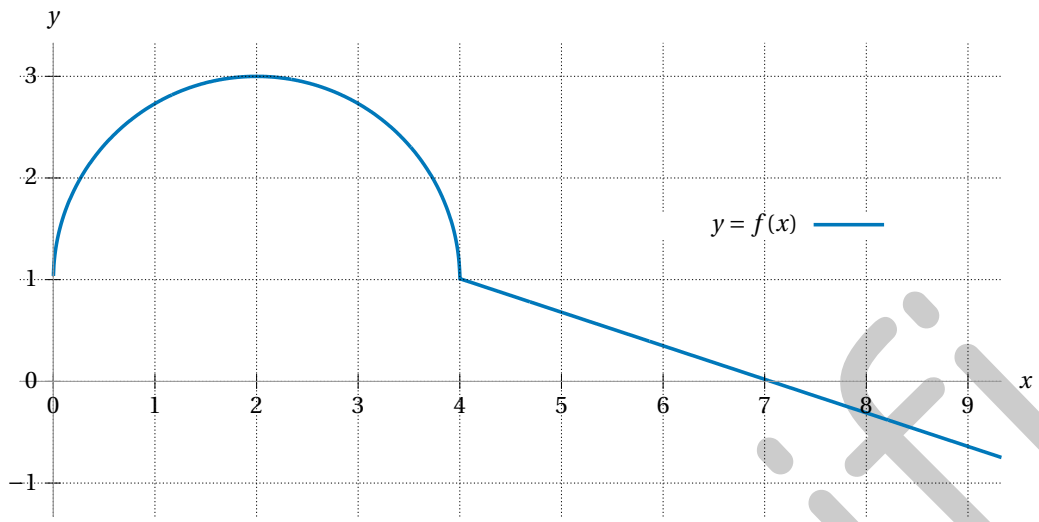
$$\lim_{x \to 2022^-} \frac{x - 2022}{|x - 2022|}?$$

2. What's a formula for the derivative of

$f(x) = x^3 \sin\left(x^4\right) + x^3 + 2022$?

3. What's a formula for the function F such that $F'(x) = x^3 \sin\left(x^4\right) + x^3 + 2022$ and $F(0) = \frac{3}{4}$?

4. Below is the graph of a function f.



(a) What is the exact value of

$$\lim_{x \to 0} \frac{f(5+h) - f(5)}{h}?$$

(b) What is the exact value of $\int_0^9 f(x)\,\mathrm{d}x$?

(c) What is the *average* value of f on the interval between $x = 0$ and $x = 9$?

(d) Estimating, for what value(s) of b does it appear that $3 = \int_1^b f(x)\,\mathrm{d}x$?

(e) Estimating, for what value(s) of x does it appear that $f'(x) = -1$? (Bonus: What is the exact value?)

(f) On this same set of axis, sketch the graph of $y = f'(x)$.

5. Consider the two functions

$$f(x) = x^2 + \frac{1}{\sqrt{x}} \qquad \text{and} \qquad g(x) = \frac{1}{\sqrt{x}} - x^2 + 5x - 4,$$

defined on the domain $x > 0$, and their graphs $y = f(x)$ and $y = g(x)$.

(a) What is an equation of the line tangent to the graph of f at $x = 1$?

(b) The graphs of f and g don't intersect, and $f(x) > g(x)$ on their domain. For which $x > 0$ is the vertical distance between the graphs of f and g minimal?

(c) The graph of g has a single inflection point. What is the x-coordinate of this inflection point?

6. What is the value of $\frac{\mathrm{d}y}{\mathrm{d}x}$ for the equation $\sec(xy) = \frac{2}{\sqrt{3}}$ at the point $(x, y) = \left(\frac{1}{2}, \frac{\pi}{3}\right)$?

7. A 13ft ladder is propped against a vertical wall. Not having enough traction though, the ladder begins to slip, with the top of the ladder sliding down the wall at a constant rate of 7ft/sec.

   (a) How fast is the base of the ladder sliding away from the base of the wall at the moment the tip of the ladder is 12ft above the ground?

   (b) How fast is the *angle* the ladder makes with the ground decreasing at this same moment?

8. Write down a calculation to verify that

$$\int \frac{\tan^2(x)}{\cos^2(x)}\,\mathrm{d}x \quad = \quad \frac{1}{3}\tan^3(x) + C\,.$$

9. Consider the region in the (x, y)-plane bound between the curve $y = \frac{1}{2}x - 3$, the x-axis, and the line $x = 4$, and consider the solid generated by revolving this region about the line $y = 2$. Write down *two* integrals that compute the volume of this solid: an integral with respect to x and another with respect to y.