

Midterm Exam Three

Math 135-002 Engineering Calculus I
Colorado Mesa University Fall 2022

Name: _____

The specter of a graded exam unfortunately haunts us today.

Silence your phone during the exam. You may go to the restroom as you need, but leave your phone on your desk if you do.

Answers expressed in decimal must be accurate to within $\pm 10^{-5}$.

Each page of this exam will be weighed roughly equally. It may be a good idea to not simply write calculations and answers, but to use your responses as a channel to communicate any insights you have. The grader is perceptive; provide them with evidence of your knowledge and your understanding.

Remember that the purpose of this exam is to provide a document to justify the letter grade you'll be awarded in this class to the University. Your instructor wants you to pass this course. Imagine that your instructor is a lawyer who has to argue to the University for the best grade they can give you with your exam in their hand. Provide them with all the evidence that you can that you've learned the material presented in this course.

Finally, note that since the purpose of this exam is to assign you a grade to rank you among your fellow students, it inherently undermines the ideal collegial nature of school by incentivizing competition rather than cooperation with your peers. Please understand that being graded is *not* an intrinsic element of education, and your instructor grades you reluctantly.

1. What is the result of evaluating these indefinite integrals? (Don't forget your +Cs.)

(a) $\int \sec^2(\theta) \, d\theta$

(b) $\int x^{-\frac{1}{2}} \sec^5(\sqrt{x}) \tan(\sqrt{x}) \, dx$

2. What is the value of

$$\int_0^4 |x-3| - 1 \, dx?$$

3. Given that $\int_0^4 f(x) \, dx = 5$, and given that f is an *even* function, what is the value of

$$\int_{-4}^4 3f(x) + x - 1 \, dx?$$

4. If $W(t)$ is a model for the rate, measured in ft^3/hr , at which water is being released from Lake Powell reservoir t hours after 12am this morning, what does the integral $\int_{12}^{13} W(t) \, dt$ represent?

5. Write down a calculation to verify that

$$\int \cos(\theta)(1 - \cos^2(\theta)) \, d\theta = \frac{1}{3} \sin^3(\theta) + C.$$

6. If $\omega'(t) = 21t^6$ and if $\omega(1) = \frac{5}{2}$, what must a formula for $\omega(t)$ be?

7. Consider the region in the plane bound between the graphs of

$$f(x) = \frac{1}{3}(x-5)^2 + 2 \quad \text{and} \quad g(x) = 6 - (x-3)^2.$$

(a) Write down an integral that represents the area of this region. It may be a good idea to sketch a picture of this region first.

(b) Write down an integral that represents the volume of the solid generated by revolving this region about the x -axis.

(c) Write down an integral that represents the volume of the solid generated by revolving this region about the line $x = -1$.

8. You are on the rooftop of a 500m tall building, and you are determined to pull this chimp back up onto the roof. The 40kg chimpanzee is strapped into a harness dangling at the end of a 200m long rope tethered to the rooftop. The rope itself has a mass of 1.16kg/m. Using 9.81 m/s^2 for earth's gravitational acceleration, write down an integral that represents the amount of work you've done after pulling ℓ meters of the rope up towards the roof.

9. The velocity of a car driving around a giant circular track is modelled by the equation $v(t) = t^3 + t + \sqrt{t}$ in miles per hour, t hours after it takes off from the starting line.

(a) How fast is the car going at $t = 4$ hours.

(b) How far does the car travel between $t = 0$ and $t = 4$ hours?

(c) What is the average speed of the car between $t = 0$ and $t = 4$ hours?