Math135 Engineering Calculus I

Third Midterm Exam

Colorado Mesa University 2024 Fall

NAME: _____

1. What is the result of evaluating these indefinite integrals? Don't forget your $+C$s!

   (a) $\int t^9 + \sqrt[9]{t} + \frac{1}{t^9}\,\mathrm{d}t$

   (b) $\int 9 + 9\sin(\theta)\,\mathrm{d}\theta$

   (c) $\int \sec(9t)\tan(9t)\,\mathrm{d}t$

2. If $g'(t) = t^{1.618}$ and if $g(37) = 4882.6$, what must a formula for $g(t)$ be?

3. Given that $\int_1^3 f(x) + 1 \, dx = 7$ and $\int_3^1 2g(x) \, dx = 6$, what must the value of the following integral be?

$$\int_1^3 3f(x) + g(x) \, dx$$

4. What is the *exact* value, not just a decimal approximation, of this integral?

$$\int_0^{10} \sqrt{x^2 - 100}\,\mathrm{d}x$$

5. Demonstrate how to calculate the *exact* value of this definite integral.

$$\int_0^{\frac{\pi}{4}} \frac{\sqrt{1-\tan(9t)}}{\cos^2(9t)}\,\mathrm{d}t$$

6. Consider the graphs of the functions f and g defined by the formulas

$$f(x) = x \quad \text{and} \quad g(x) = (x-6)^2 + 4,$$

and let $\mathscr{A}$ denote the region in the (x, y)-plane bounded by these graphs.

(a) Demonstrate how to calculate the area of the region $\mathscr{A}$.

(b) Consider the solid generated by revolving $\mathscr{A}$ about the y-axis. Write down an integral that expresses the volume of this solid of revolution.

7. You are in the parking lot just outside St Mary's Medical Center, which, at 45 m high, is the tallest building in Grand Junction. There is a spring-loaded cannon bolted to the parking lot pavement, attached to which is a crank that you can turn to depress the spring. At rest, the length of the spring in the cannon is 2 m. A 37 kg chimpanzee climbs into the cannon with an aspiration to be launched onto the roof of St Mary's. (For the following calculations ignore air resistance, and use $9.797\,\mathrm{m/s^2}$ for gravitational acceleration near the earth's surface here in Grand Junction.)

   (a) The chimp climbs into the cannon and sits on the platform atop the spring within the bore of the cannon. The chimp's weight causes the spring to depress by a length of 65.672 cm. Now according to Hooke's law there is some constant by which the length a spring becomes depressed and the force required to keep it depressed are proportional. Calculate this constant.

   (b) You begin turning the crank, depressing the string and thereby "loading the cannon", until the spring is a length of 1 m. How much total *work* did you do turning the crank to depress the spring?

   (c) (SUPER BONUS) Upon releasing the spring, "firing" the cannon, how high into the air will the chimpanzee be launched? Will it make it onto the roof of St Mary's? (Make any assumptions about the spring you'd like.)