Midterm Exam One

Math 135-002 Engineering Calculus I
Colorado Mesa University Fall 2022

NAME: _____

The specter of a graded exam unfortunately haunts us today.

Silence your phone during the exam. You may go to the restroom as you need, but leave your phone on your desk if you do.

You may use the blank final page as scratch paper, but note that the grader will only be looking for a response to a prompt on the page where the prompt is printed. Each page of this exam will be weighed roughly equally.

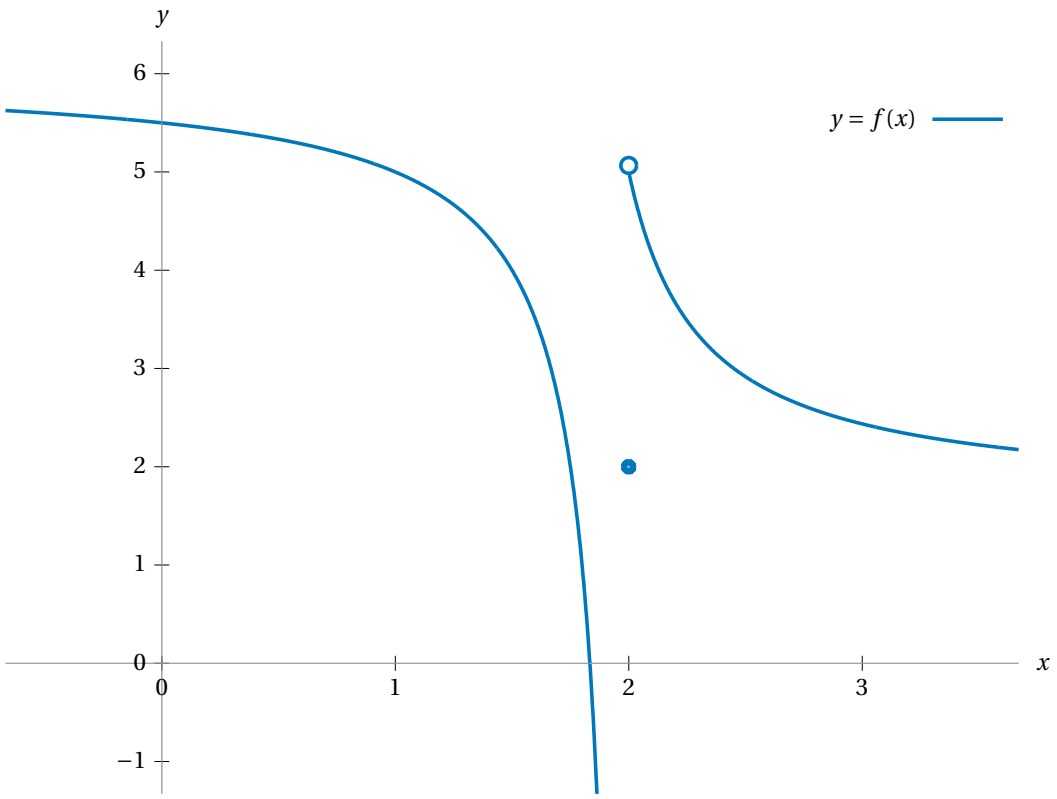
Remember that the purpose of this exam is to provide a document to justify the letter grade you're awarded in this class to the University. YOUR INSTRUCTOR WANTS YOU TO PASS THIS COURSE. Imagine that your instructor is a lawyer who has to argue to the University for the best grade they can give you with your exam in their hand. Provide them with all the evidence that you can that you've learned the material presented in this course.

Finally, note that since the purpose of this exam is to assign you a grade to rank you among your fellow students, it inherently undermines the ideal collegial nature of school by incentivizing competition rather than cooperation with your peers. Please understand that being graded is *not* an intrinsic element of education, and your instructor does so reluctantly.

1. You are standing on the surface of Venus. It is hot. Despite your extreme discomfort you toss a stone into the air at 80 ft/s. The height of the stone (in feet) above you t seconds after you throw it is roughly modelled by the function $h(t) = 80t - 15t^2$. What's the highest the stone gets? What is the average velocity of the stone in the two second just before it hit this highest point? Express your final calculations either precisely as a fraction, or as a decimal number accurate to six decimal places.

2. What is an equation of the line tangent to the graph of the function $f(x) = \sqrt{4x^2 - 11} - 1$ at the point $(3, f(3))$? (HINT: It's a very reasonable idea to use a calculator to assist you with answering this question.)

3. Based on this graph of $y = f(x)$ what are the values of the following expressions? If the value of any expression is not defined, indicate this by simply crossing the expression out.



(a) $\lim_{x\to 2^-} f(x)$ (b) $\lim_{x\to 2^+} f(x)$ (c) $\lim_{x\to 2} f(x)$ (d) $f(2)$

4. What are the values of the following expressions? If the value of any expression is not defined, indicate this by simply crossing the expression out.

(a) $\lim_{x\to -4} 3$ (b) $\lim_{x\to 3} 3x$ (c) $\lim_{x\to -1} \frac{(x-1)(x+1)}{x}$

(d) $\lim_{x\to 0} \frac{1}{x}$ (e) $\lim_{x\to 3^+} \frac{-1}{(x-3)^2}$ (f) $\lim_{x\to 0} \frac{\ln(1-x)-\sin(x)}{\sin^2(x)}$

5. Demonstrate how to find the values of the following limits manually (algebraically) without having to appeal to the use of technology. If the value of any expression is not defined, indicate this by simply crossing the expression out. You may freely use the fact that $\lim_{x\to 0} \sin(x)/x = 1$. You may use L'Hôpital's rule for evaluating limits only if you can *prove* that L'Hôpital's rule works.

   (a) $\lim\limits_{x\to 2} \dfrac{x^2+5x-14}{x-2}$

   (b) $\lim\limits_{x\to 0} \dfrac{\sin(3x)}{2x}$

   (c) $\lim\limits_{x\to 2} \dfrac{2-x}{\sqrt{4x+1}-3}$

6. One milliliter (mL) of water is about twenty drops. Water has been dripping from a leaky faucet into a plugged bathtub at a rate of about one drop every 3 seconds since about noon yesterday. Let $V(t)$ equal the volume of water in the tub, measured in mL, t seconds after 6am this morning. Do you think V a continuous function? Explain your reasoning. (HINT: it would probably help your reader understand your explanation if you include an illustration of, like, a sketch of V's graph or something.)

7. Let g be the function defined piecewise as

$$g(x) = \begin{cases} -1 & \text{for } x < -3 \\ f(x) & \text{for } -3 \le x < 1 \\ -x^2 + 5 & \text{for } x \ge 1 \end{cases}$$

Write down *any* formula $f(x)$ that defines a function f such that g will be continuous. Make it clear that your definition of f satisfies the requirement.

* (OPTIONAL CHALLENGE) Building off that last question, can you write down a formula $f(x)$ for a function f such that g will be continuous everywhere *except* at $x = 0$?