Math135 Engineering Calculus I

# First Midterm Exam

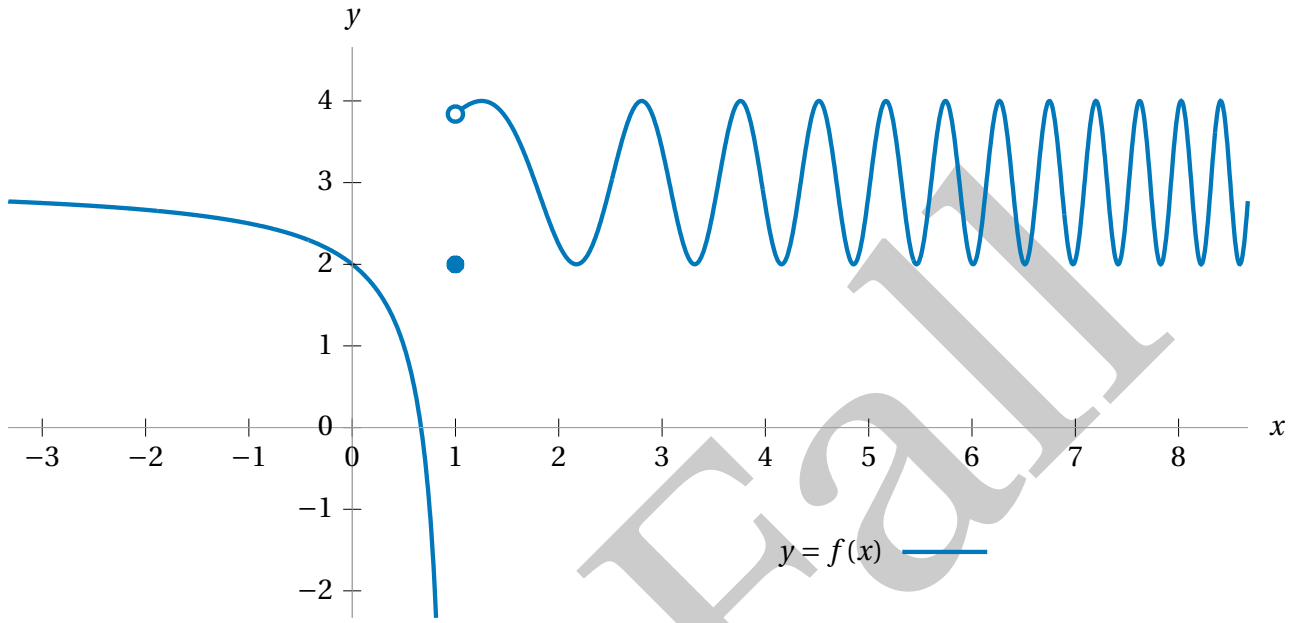
Colorado Mesa University 2024 Fall

NAME: _____

1. Explain, as if explaining to a peer in the class, what it means to say "$\lim_{x\to 3^-} f(x) = \infty$."

2. What is the precise, mathematical definition of a function $f$ being *continuous at a point*?

3. Based on this graph of $y = f(x)$ estimate the values of the following expressions. If the value of any limit is not defined (does not exist), indicate this by simply crossing the expression out.




(a) $\lim_{x \to -\infty} f(x)$

(b) $\lim_{x \to \infty} f(x)$

(c) $\lim_{x \to 1^-} f(x)$

(d) $\lim_{x \to 1^+} f(x)$

(e) $\lim_{x \to 1} f(x)$

(f) $f(1)$

4. Using technology, approximate the value of the following limit *accurate to five decimal places.*

$$\lim_{x \to \infty} \left(1 + \frac{2}{x}\right)^x$$

5. Sketch the graph of an example of a function $f$ that satisfies the following conditions.

$$\lim_{x\to-\infty} f(x) = \infty \qquad f(-8) = 0 \qquad \lim_{x\to-8} f(x) = -8$$

$$\lim_{x\to 8^-} f(x) = 0 \qquad \lim_{x\to 8^+} f(x) = f(8) = 8 \qquad \lim_{x\to\infty} f(x) = -8$$



6. What are the values of the following expressions? If the value of any limit is not defined (does not exist), indicate this by simply crossing the expression out.

(a) $\lim_{x\to 0} \dfrac{1}{42}$

(b) $\lim_{x\to 0} 42 - x$

(c) $\lim_{x\to 0} \dfrac{42}{x^2}$

(d) $\lim_{x\to 42} \dfrac{x^2 - 42^2}{x - 42}$

(e) $\lim_{x\to 42^+} \dfrac{42}{42 - x}$

(f) $\lim_{x\to\infty} \dfrac{x}{42 - x}$

7. *Demonstrate* how to manually (algebraically) determine the values of the following limits. If the value of any limit is not defined (does not exist), indicate this by simply crossing the expression out. You may freely use the fact that $\lim_{x\to 0} \sin(x)/x = 1$. You may use L'Hospital's rule for evaluating limits only if you can *prove* that L'Hospital's rule works.

   (a) $\lim_{x\to 0} \dfrac{\sin(x)}{42x}$

   (b) $\lim_{x\to 7} \dfrac{8-\sqrt{9x+1}}{x^2-7^2}$



* (TRIVIA) What 2004 comedy starring Tina Fey and Lindsay Lohan featured the following limit?

$$\lim_{x\to 0} \frac{\ln(1-x)-\sin(x)}{\sin^2(x)}$$

8. You are on a satellite in low synchronous orbit around the planet Jupiter. There is no artificial gravity on the satellite, so even the smallest movement is difficult against Jupiter's might. Standing in the satellite's spacious observation room, with it vaulted ceilings, massive portholes, and chronic echo, you can't help but feel lonely, but gazing out at the violent beauty of the planet's tumultuous surface below reminds you that you are not lost, sustaining your hope that one day you may be rescued.

   While you wait, you have a marble to play with. Despite its extreme weight, you manage to toss the marble straight up at a speed of 34.4 ft/s. The altitude (height) of the marble above you, in feet, measured $t$ seconds after you throw it is roughly modelled by the function $A(t) = 5.8 + 34.4t - 40.2t^2$

   (a) What is the maximum altitude the marble reaches?

   (b) How long after you throw it does the marble fall back down and hit the ground?

   (c) What is the average velocity of the marble between the moment it hits its highest altitude and the moment it hits the ground?