Midterm Exam Two
Math 135-002 Engineering Calculus I
Colorado Mesa University Fall 2022

Name: _____

The specter of a graded exam unfortunately haunts us today.

Silence your phone during the exam. You may go to the restroom as you need, but leave your phone on your desk if you do.

Each page of this exam will be weighed roughly equally.

Answers expressed in decimal must be accurate to within $\pm 10^{-5}$.

Remember that the purpose of this exam is to provide a document to justify the letter grade you're awarded in this class to the University. Your instructor wants you to pass this course. Imagine that your instructor is a lawyer who has to argue to the University for the best grade they can give you with your exam in their hand. Provide them with all the evidence that you can that you've learned the material presented in this course.

Finally, note that since the purpose of this exam is to assign you a grade to rank you among your fellow students, it inherently undermines the ideal collegial nature of school by incentivizing competition rather than cooperation with your peers. Please understand that being graded is *not* an intrinsic element of education, and your instructor does so reluctantly.

1. Without the aid of technology, demonstrate how to find a formula for the derivative of each of the following functions. Express the derivative formula concisely.

(a) $f(t) = 3t^3 + 2t^2 + t + \pi - \dfrac{1}{t}$

(b) $g(x) = x^{33}\sin(x)$

(c) $h(\varphi) = \sqrt[3]{\csc(b\varphi)}$ where b is just some real number.

2. What expression is the result of evaluating this limit?

$$\lim_{h \to 0} \frac{\sec(x+h) - \sec(x)}{h}$$

3. Demonstrate how to calculate the derivative of the arctangent function knowing only that $\frac{d}{d\theta}\tan(\theta) = \sec^2(\theta)$, and express the derivative as a formula containing no trigonometric or arctrigonometric functions. (Recall arctangent is the inverse of the tangent function.)

4. What is an equation of the line tangent to the graph of the function $f(x) = \sqrt{4x^2 - 11} - 1$ at the point $(3, f(3))$? (Hint: This is the same question from the first exam, but now you should be able to answer it more quickly using what you've learned about derivatives.)

5. Here's the graph of a function $y = f(x)$.

$y = f(x)$ ————
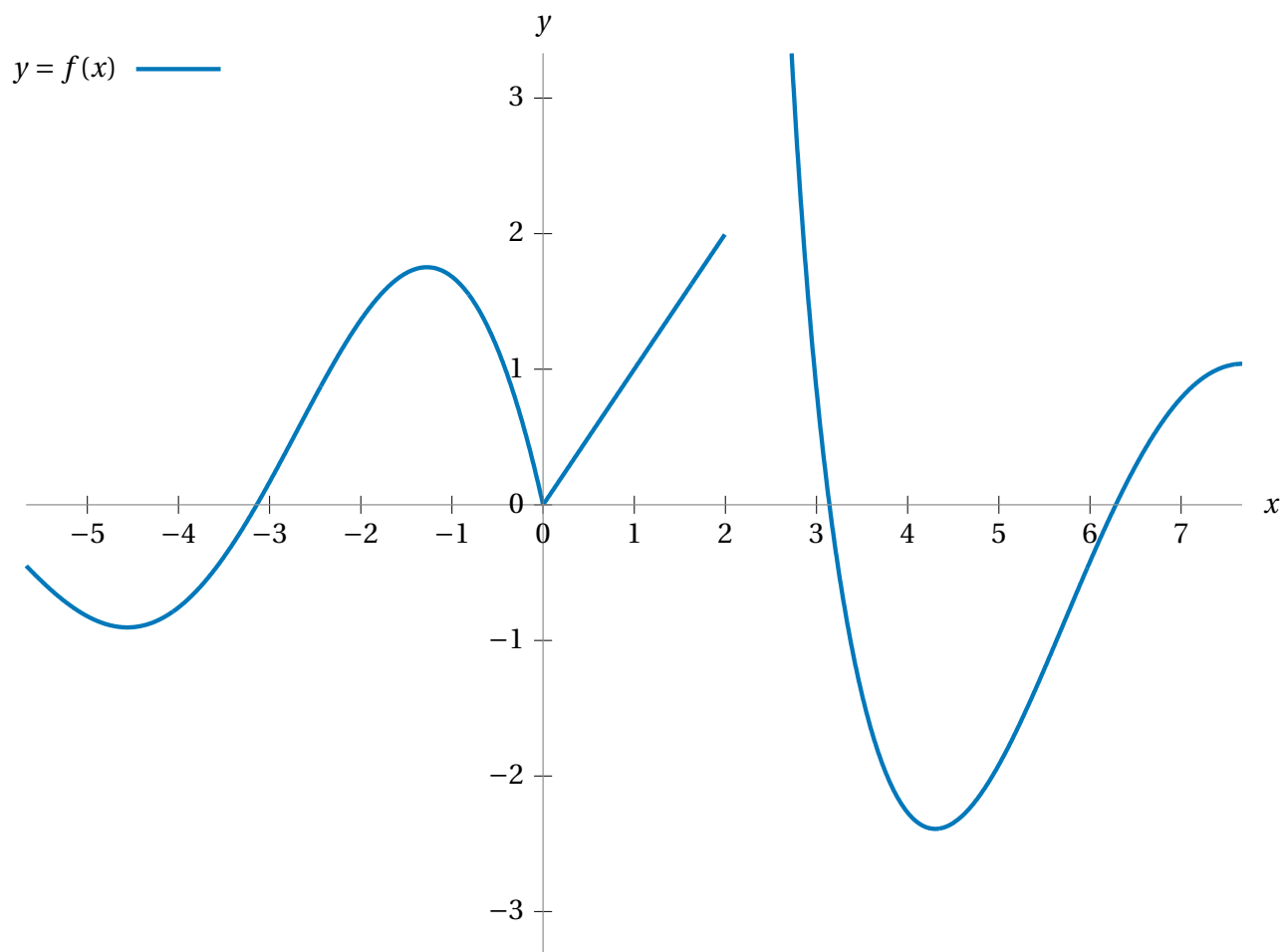
(a) On the same set of axis, sketch the graph of $y = f'(x)$.

(b) Draw small squares around every inflection point of f.

(c) Estimating, on what intervals is f concave up?

6. A Norman window has the shape of a rectangle surmounted by a semicircle. If you design a Norman window for a building, and you need the perimeter of the Norman window to be 30ft, what dimensions of the window will allow the greatest possible amount of light into the building?

7. A boat is being pulled into a dock by a rope tethered to the bow of the boat at one end and reeled into a crank anchored to the dock at the other end. The crank on the dock is 5ft higher than the bow of the boat. If the rope is being reeled in at a rate of 2ft/s, how fast is the boat approaching the dock when it is 23ft from the dock?