Wumpus World Formal Specification

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1 Introduction

This article describes a very (incomplete) formal specification of the Wumpus world game. The specification is defined in the Z language. The purpose of the specification is to formally define different types of interactions between the the environment in which the Wumpus lives in, and the way it interacts with an agent that is situated in this environment.

2 Definitions and Utility Functions

A function $uniform$ is defined that returns a uniform random number between x and y.

$$uniform : \mathbb{N} \times \mathbb{N} \rightarrow \mathbb{R}$$

$$\forall x, y : \mathbb{N} \mid y \geqslant x \bullet x \leqslant uniform(x, y) \leqslant y$$

A function $random$ is defined that returns a random natural number in the range specified by the parameters to the function.

$$random : \mathbb{N}_1 \times \mathbb{N}_1 \rightarrow \mathbb{N}_1$$

$$\forall x, y : \mathbb{N}_1 \mid y \geqslant x \bullet random(x, y) \in x..y$$

3 Agent

This section defines the details of the Hunter Agent which explores the cave world in the hope of finding the hidden treasure. The cave environment in which the Hunter Agent is situated in has a number of threats, including a number of bottomless pits and a monster called the wumpus.

The Hunter Agent can only travel from room to room in the cave in certain directions. He can travel only up or down and left or right. The agent cannot travel or move diagonally.

$$DIRECTION ::= Up \mid Down \mid Left \mid Right$$

The agent can receive a predefined number of percepts from the environment, while in any given room of the cave.

Stench If the agent smells a stench, indicates that the wumpus is one of the adjacent rooms (not diagonally).

Breeze If the agent feels a breeze, this indicates that there is pit in one of the adjacent caves (not diagonally).

Glitter If the agent sees glitter, then the agent has found the gold in the current room.

Bump If the agent perceives a bump that means it has hit the wall of the cave and cannot travel further in the direction he was travelling in.

Scream If the agent hears a scream, this means that the wumpus has been killed and any further stench can be ignored.

$$PERCEPT ::= Stench \mid Breeze \mid Glitter \mid Bump \mid Scream$$

The Hunter Agent only has five different types of actions available to him. He can either travel one room forward, turn left or right, shoot a single arrow in the direction the agent is facing, or climb out of the cave. The agent only has a single arrow available. If he shoots the arrow it flies in the direction in which the agent was facing until it hits the wall of the cave or until it hits the wumpus. If it hits the wumpus, the wumpus is killed. The agent can only climb out of the cave if there is an opening in the room. In this scenario, the only opening will be in the room in which the agent entered the cave; that is the start room.

$$ACTION ::= Forward \mid TurnRight \mid TurnLeft \mid Shoot \mid Climb$$

The agent schema has a sequence of $PERCEPT$ representing a list of the percepts the agent is currently perceiving (while in the current location). Similarly, the schema also has sequence of $ACTION$ to indicate the actions that it whishes to perform this time step. The schema indicates that the agent has two basic beliefs about the world. Firstly, it knows if it is alive, and secondly it knows if it has shot the arrow it starts out with. Note that the agent doesn't know anything about it's current position in the cave. In this particular case the agent isn't storing it's own local coordinates, but it still doesn't know about the cave's global coordinate system.

Agent

$$percepts : \text{seq } PERCEPT$$
$$actions : \text{seq } ACTION$$
$$alive : \mathbb{B}$$
$$shotArrow : \mathbb{B}$$

When the agent is initialised it has no percepts and no actions to perform (and hence the empty sequences). It also knows that it is alive and that it has yet to shoot the arrow it has.

AgentInit

$$\Delta Agent$$

$$percepts' = \langle\,\rangle$$
$$actions' = \langle\,\rangle$$
$$alive' = \text{true}$$
$$shotArrow' = \text{false}$$

4 Wumpus

All the wumpus knows in the wumpus schema is, if it is alive or not.

Wumpus

$$alive : \mathbb{B}$$

When the wumpus is initialised it knows it is alive.

InitWumpus

$$\Delta Wumpus$$

$$alive' = \text{true}$$

5 Environment

In this section the environment in which the wumpus and the agent are situated in is defined. The cave is represented as a two dimensional grid of rooms. Therefore, and particular room can be defined by its position in the grid by a tuple representing the grid coordinates. A $ROOM$ is defined

by its position coordinates, which is a relation between two natural numbers greater than one. Each room can then be refered to as a row/column tuple pair.

$$ROOM == \mathbb{N}_1 \leftrightarrow \mathbb{N}_1$$

One particular room is special in the cave. That is the entrance. The entrance is special because the agent can climb into and climb out of the cave only when at the entrance. The entrance is defined as the first row/column pair in the cave $(1, 1)$ or $\{1 \mapsto 1\}$.

$$\begin{array}{|l} entrance : ROOM \\ \hline entrance = \{1 \mapsto 1\} \end{array}$$

The cave schema defines the cave firstly by the number of rows and columns it has. This indirectly defines the number of rooms in the cave. A finite subset of these rooms will have bottomless pits in them. It is not possible for there to be a pit in the entrance room. The gold will be located in a room in the cave. The wumpus will also be located in a single room. It is entirely possible that wumpus and the gold are in the same room. The cave also contains the hunter agent. The cave schema also keeps track of the agent's location and the direction it is facing.

$$\begin{array}{|l} \textit{Cave} \\ \hline numRows : \mathbb{N}_1 \\ numColumns : \mathbb{N}_1 \\ pits : \mathbb{F}\, ROOM \\ gold : \mathbb{F}_1\, ROOM \\ wumpus : \mathbb{F}_1\, ROOM \\ agentLocation : \mathbb{F}_1\, ROOM \\ agentDirection : DIRECTION \\ agent : Agent \\ \hline \#pits \leqslant numRows * numColumns \\ \text{dom } pits \subseteq 1..numRows \\ \text{ran } pits \subseteq 1..numColumns \\ entrance \notin pits \\ \text{dom } gold \subseteq 1..numRows \\ \text{ran } gold \subseteq 1..numColumns \\ \#gold = 1 \\ \#wumpus = 1 \\ \text{dom } wumpus \subseteq 1..numRows \\ \text{dom } wumpus \subseteq 1..numColumns \\ \text{dom } agentLocation \subseteq 1..numRows \\ \text{ran } agentLocation \subseteq 1..numColumns \\ \hline \end{array}$$

The cave is initialised using three parameters. Firstly the number of rows and the columns in the cave is set, followed by the probability that there is a pit in any given room. The cave initialisation schema also places random pits in the cave, and also randomly places the wumpus and the gold. It is entirely possible that the gold ends up in a bottomless pit, in which case it is impossible for the agent to succeed at its mission. It is also possible for the gold and the wumpus to end up in the same room.

InitCave

$\Delta Cave$

$n?, m? : \mathbb{N}_1$

$pitChance? : \mathbb{R}$

$numRows' = n?$

$numColumns' = m?$

$0.0 \leqslant pitChance? \leqslant 1.0$

$\forall i, j : \mathbb{N}_1 \mid$

$\quad i \in 1..numRows \land j \in 1..numColumns \land \{i \mapsto j\} \neq entrance \bullet$

$\quad uniform(0, 1) \leqslant pitChance?$

$\quad \Rightarrow pits' = pits \cup \{i \mapsto j\}$

$\forall gx, gy : \mathbb{N}_1 \mid$

$\quad gx \in 1..numRows \land gy \in 1..numColumns \bullet$

$\quad gx = random(1, numRows) \land gy = random(1, numColumns)$

$\quad \Rightarrow gold' = \{gx \mapsto gy\}$

$\forall wx, wy : \mathbb{N}_1 \mid$

$\quad wx \in 1..numRows \land wy \in 1..numColumns \bullet$

$\quad wx = random(1, numRows) \land wy = random(1, numColumns)$

$\quad \Rightarrow wumpus' = \{wx \mapsto wy\}$

$agentLocation' = entrance$

$agentDirection' = Left$