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ScienceDirect
European Journal of Operational Research 192 (2009) 692–699
EUROPEAN JOURNAL OF OPERATIONAL RESEARCH
www.elsevier.com/locate/ejor

Short Communication

Optimal building evacuation time considering evacuation routes

Salvador Casadesús Pursals [a,*], Federico Garriga Garzón [b]

[a] Department d'Estadística i Investigació Operativa, Escola Técnica Superior d'Enginyeria Industrial i Aeronáutica, Universitat Politécnica de Catalunya, Colom, 11, 08222 Terrassa, Barcelona, Spain

[b] Department d'Organització d'Empreses, Escola Técnica Superior d'Enginyeria Industrial i Aeronáutica, Universitat Politécnica de Catalunya, Colom, 11, 08222 Terrassa, Barcelona, Spain

Received 30 January 2007; accepted 3 October 2007
Available online 10 October 2007

Abstract

The main purpose of this work is to present a formulation of the building evacuation problem that incorporates evacuation routes and applies the functions developed by Nelson and McLennan [H.E. Nelson, H.A. McLennan (Eds.), Emergency Movement, The SFPE Handbook of Fire Protection Engineering, 1996, pp. 3.286–3.295 (Section 3/Chapter 14)] to model the movement of people. These considerations lead to significant changes in the form of the evacuation and inverse evacuation functions, so it is necessary to develop a new procedure for solving the building evacuation problem.

Keywords: Building evacuation; Evacuation optimization; Optimal building evacuation

1. Introduction

The first formulation of the building evacuation problem was developed by Francis [3,4]. Shortly after, Francis et al. [5] used flow networks to study the same problem. Later, Francis and Kisko designed the program EVACNET [6,7], an application developed in FORTRAN 77 that is used to determine evacuation times and the optimal distribution of occupants towards the exits. Choi et al. [2] further developed the research into deterministic models and flow networks and considered the problem in terms of flow networks with capacity restrictions and arcs with variable capacity; their work produced greedy algorithms for solving certain structures. In the 1990s, Hope and Tardos [14,15] made their own contribution to solving the problem by proposing polynomial algorithms based on the work of Minieka [25] and Megiddo [22,23]. Kostreva [16], Getachew [12,13] and Wiecek [42–44], carried out a general study in which building evacuation was viewed as a multi-objective problem. More recent proposals containing interesting heuristic solutions have been provided by Lu et al. [21].

The problem has also been studied by applying stochastic models, which are more realistic but also more complex. The most important studies to adopt this approach are those of Smith [34–37] and Lovas [17–20]. The first formulations used queuing networks and were solved using the algorithms developed by Resier and Lavenberg [32,33]. Smith and Lovas also provided their own solutions, some of which used simulations to solve the queuing networks.

The most notable proposals of recent years have been simulation-based studies, including the pioneering work of Weinroth [41], which highlighted the usefulness of other programs such as EXITT, EVACSIM, EXIT-89 and WAY OUT. The most recent contributions were made by Thomson [39,40] with program SIMULEX, and Galea [9–11] with program EXODUS.

* Corresponding author. Tel./fax: +34 93 739 8168.
E-mail address: Salvador.casadesus@upc.edu (S.C. Pursals).

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doi:10.1016/j.ejor.2007.10.004

2. Problem formulation

The present study is based on the initial formulation of the building evacuation problem made by Francis [3,4], who considered the evacuation optimization of an enclosure. By reducing the scope of the problem, however, we can use more precise models of the movement of people and introduce behavior models to provide a more in-depth analysis of the problem.

We consider an enclosure with k occupants (Fig. 1) who have n possible exits that are located in such a way that there is no interference in the movement of occupants to each one of them. The occupants are distributed in such a way that we can assume a constant flow through each exit. There is also an evacuation route to each exit, so for n exits there will be n different evacuation routes of length l_j and with an access area of a_j. The evacuation of the enclosure is complete when all the occupants have arrived at their destiny and the evacuation time is the duration of the longest route, indicated by the point at which the last occupant reaches his destiny.

The following model can be used to determine the number of people who must use each route, with the aim of minimizing the total evacuation time (z):

$$\text{Min } z = \text{Max}[t_j(x_j)] \quad j = 1, \ldots, n, \tag{1}$$

$$\text{s.t.} \quad \sum_{j=1}^{n} x_j = k, \tag{2}$$

$$x_j \geqslant 0, \tag{3}$$

where j indicates the number of the exit, x_j the number of people who will leave the enclosure by exit j, and $t_j(x_j)$ is the evacuation function that provides the time needed for x_j people to leave the enclosure. Although x_j should be an integer variable (number of people), in most problems with high value of k it is considered a real variable that is adjusted to the nearest integer value once the solution has been obtained. The constraint (2) dictates that all of the k people leave the enclosure, whereas the negativity condition of the variables (3) means that the number of people who use a certain route cannot be negative.

The underlying assumptions are that the occupants are uniformly distributed in the enclosure and begin the evacuation at the same moment, while their movement towards each of the exits is independent and the evacuation function for each exit is known.

3. Evacuation function

It is possible to determine analytically the number of people who can leave a building in a certain time by using models of people movement. The most suitable models of people movement are provided by Fruins [8], Pauls [27–30] and Predtechenskii and Milinskii [31], based on recent studies published by Melinek [24] and Nelson and McLennan [26].

The people movement models provide the magnitudes of the speed and the flow of people circulation. The specific flow f_j that is registered in an element of step j is defined as the number of people who cross a unit of width per unit of time, whereas the flow F_j is defined as the number of people who cross it in a unit of time. The flow F_j is equal to the product of the specific flow f_j and the width of the step element

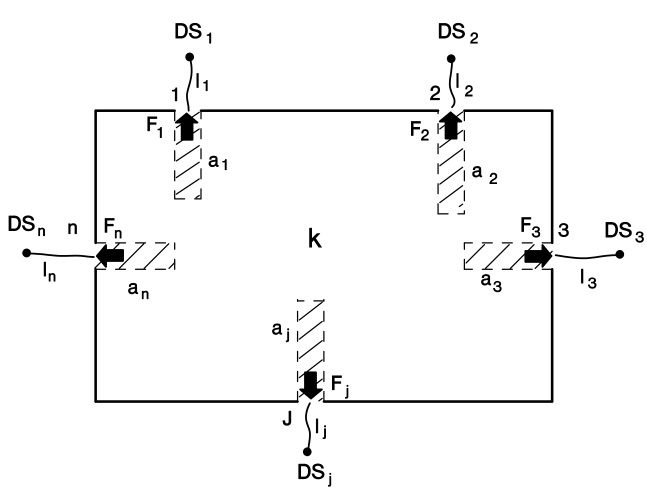

Fig. 1. Enclosure with k occupants and n exits.

$$F_j = f_j \cdot w_j. \tag{4}$$

The time needed for a certain number of people to leave the enclosure by a specific exit, if they do not have to follow a specific route, is inversely proportional to the flow recorded at this exit and directly proportional to the number of people who use exit j.

$$t_j(x_j) = \frac{x_j}{F_j}, \tag{5}$$

where F_j is the flow recorded at exit j, x_j the number of people who leave the enclosure through exit j, and $t_j(x_j)$ the time needed for x_j people to leave the enclosure by exit j.

If it is considered that the occupants reach the exit at exactly the moment at which the alarm signal sounds, the evacuation corresponds to the initial problem formulation proposed by Francis [3,4], who imposes the condition that the evacuation function for each exit is a strictly increasing convex function $t_j(x_j) \geqslant 0$ and in case $x_j = 0$ when $t_j(0) = 0$. This condition severely restricts the problem and makes it impossible to model delays and evacuation routes. Although this formulation is realistic for certain cases, it does not apply to the majority, because when the alarm signal sounds the occupants of the enclosure who go to each exit must generally follow a route until they reach the evacuation points.

The formulation proposed by Togawa [38] was used to remove this hypothesis and obtain a model that takes into account the route that must be followed to the exit of an enclosure in addition to the exit flows. In this case, an independent term must be added to the previous function (5) to represent the time the occupants take to reach the exit

$$t_j(x_j) = t_{1j}(x_j) + \frac{x_j}{F_j}, \tag{6}$$

where $t_{1j}(x_j)$ is the time that occupants who use route j will take to reach the exit. This term is directly proportional to the length of route l_j and inversely proportional to speed v_j as recorded on route j. The speed v_j will be a function of density d_j and of the number of people x_j who use route j. If the speed of circulation is constant, the function $t_j(x_j)$ is a linear function with an independent term.

This new formulation generalizes certain aspects of the problem. The original model assumed that the occupants were distributed uniformly in the enclosure; the speed of circulation and the flow at a given exit were constant and independent of the number of people using it. In this case it is assumed that all occupants are located in a position from which the distances to the various exits are represented by $l_1, l_2, \ldots, l_n$. Fig. 1 shows that certain areas of circulation are defined in the enclosure towards each exit in which the flow is formed.

Using the formulation of the problem we can calculate that, if x_j occupants follow route j, the resulting evacuation time will be z_j. For an enclosure with two exits i and j, if the evacuation time z_i to exit i is greater than the time z_j to exit j, the time needed to follow the route to exit i will be the overall evacuation time of the enclosure. In order to reduce the evacuation time it is necessary to reduce the number of occupants x_i assigned to route i and increase the allocation to route j and its corresponding evacuation time. The optimal allocation will be one that ensures the same evacuation time for each route. For the sake of accuracy, x_j should be a real variable, when x_j corresponds to the number of people (an integer variable), in enclosures with a large number of occupants the solution supposes similar evacuation times.

4. Occupation density function

In the building evacuation model proposed by Francis [3,4], the values of exit flows are obtained from the circulation model of Fruins [8]. In order to use this circulation model to define the flows and speeds of circulation, we must first predict the occupation densities at each exit and to consult the flow values in the corresponding table. Predicting the flows and speeds a priori is a complex task in most cases and conditions the result of the problem, as these values are used to determine the optimal allocation for each exit and the validity of the results therefore depends on the accuracy of the initial predictions.

If we consider the specific flow of people at each exit to be identical, according to (4) the flow at a given exit will be directly proportional to the minimum width of the exit. Therefore, we achieve optimal evacuation by ensuring that the number of occupants assigned to an exit is proportional to the relationship between the minimum exit width and the total exit width of the enclosure. This initial approach can be improved by considering the concept of effective passage width introduced by Pauls [27,30]. In this case, we do not consider the minimum passage widths that physically exist in the evacuation routes; instead, according to the passage element we establish certain separations to the physical limits whose values are recorded in tabs, see Pauls [28].

Once we know the total exit flow of the enclosure we can then determine the minimum evacuation time for the enclosure and the minimum evacuation time for each route. This enables us to calculate the optimal allocation of occupants to each exit. The optimal allocation is therefore given by the product of the evacuation time of the route and the exit flow and is directly proportional to both. This allocation is critical, since assigning more occupants to a certain exit will increase the overall evacuation time.

In contrast, if we used the equations proposed by Nelson and McLennan [26] to model the movement of people, there is no previous condition on the speed and flow of circulation at each exit. Instead, these variables are simply a function of the number of people who use each exit and the geometric characteristics of the circulation route. Fig. 2 shows the modeling principles of this configuration.

When the alarm signal sounds, x_j people move towards an exit and generate an occupation density d_j in the evacuation route. The speed and circulation flows are determined from the density and the geometric characteristics of the route. These values are defined using the model of people movement formulated by Nelson and McLennan [26]. The main characteristic of this model is the linear relationship between circulation speed and density. Therefore, when the occupation density is between 0.5382 and 3.5 people/m^2, the speed of circulation and the flow are functions of the occupation density. However, when the occupation density is lower than 0.5382 people/m^2, the movement of people depends on their personal characteristics. In this case, the variables that determine the speed of flow have a high degree of variability, so we use a constant speed of 1.1996 m/s for horizontal circulation, regardless of the density.

$$v(d_j) = \begin{cases} 0.8568 \cdot \lambda_j & 0 < d_j \leqslant 0.5382 \\ (1 - \alpha \cdot d_j) \cdot \lambda_j & 0.5382 < d_j \leqslant 3.5, \end{cases} \tag{7}$$

where v is the circulation speed, d_j is the density, λ_j defines the characteristics of evacuation route j whose value is tab for different passage shapes [26], and α is a constant that depends on the measurement units. For example, the λ parameter takes the value 1.40 m/second in the case of horizontal circulation, whereas $\alpha = 2.86$ when the speed is expressed in feet/minute and the occupant density is measured in people/square foot, and $\alpha = 0.266$ if the speed is given in m/second and the density in people/m^2.

Then, if the specific flow is determined by the product of the circulation speed and the density, the flow will be the product of the specific flow w_j, where w_j is the minimum effective width of the exit.

$$F(d_j) = \begin{cases} v(d_j) \cdot d_j \cdot w_j = 0.8568 \cdot \lambda_j \cdot d_j \cdot w_j & 0 < d_j \leqslant 0.5382 \\ v(d_j) \cdot d_j \cdot w_j = (1 - \alpha \cdot d_j) \cdot \lambda_j \cdot d_j \cdot w_j & 0.5382 < d_j \leqslant 3.5 \end{cases}. \tag{8}$$

The density d_j recorded at a given exit depends on the number of people who use it (x_j) and on the access area of the exit (a_j). If we assume that the density is directly proportional to the number of occupants assigned to the exit and inversely proportional to its access area:

$$d_j = \frac{x_j}{a_j}. \tag{9}$$

By incorporating this value into expression (6), we obtain the evacuation function that determines the time it will take for x_j people to leave the enclosure via route j:

$$t_j(x_j) = \begin{cases} \frac{1}{0.8568 \cdot \lambda_j}\left[l_j + \frac{a_j}{w_j}\right] & 0 < x_j \leqslant x_{j,I} \\ \frac{a_j}{(a_j - \alpha \cdot x_j) \cdot \lambda_j}\left[l_j + \frac{a_j}{w_j}\right] & x_{j,I} < x_j \leqslant x_{j,S} \end{cases}, \tag{10}$$

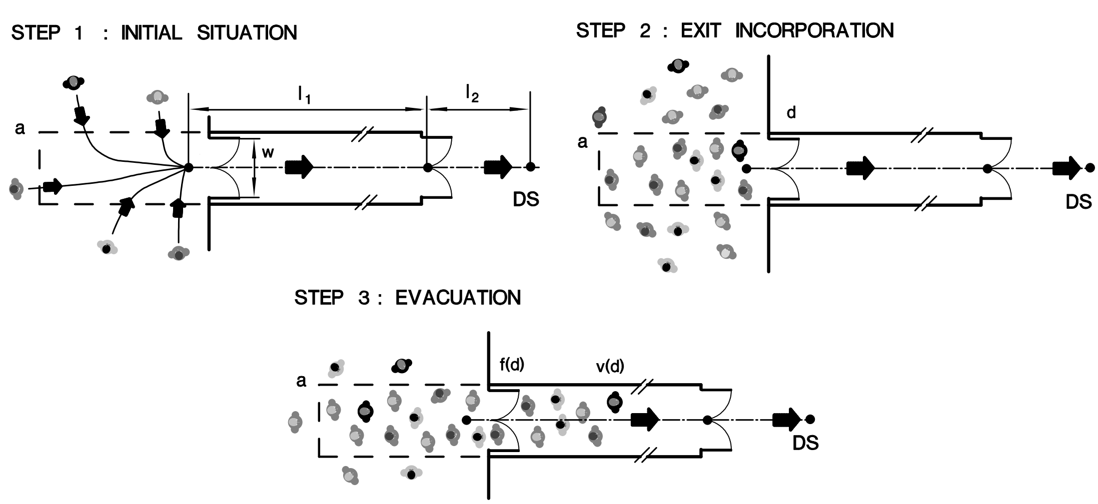

Fig. 2. Modeling of the incorporation to an evacuation route.

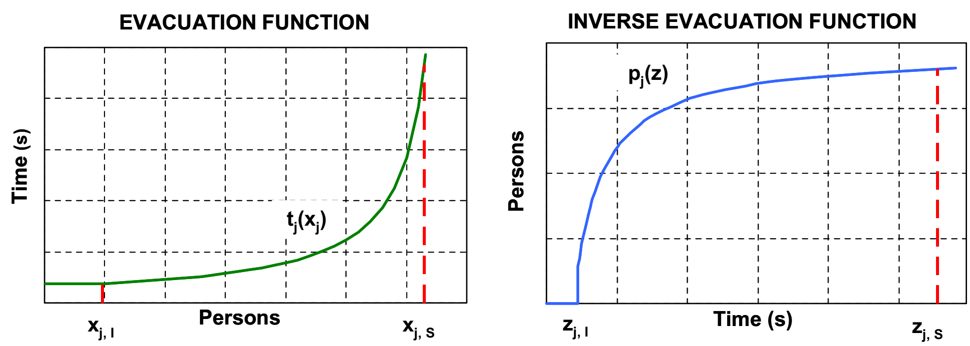

Fig. 3. Evacuation function and Inverse evacuation function.

where $x_{j,I}$ is the lower limit of the number of occupants assigned to exit j, and $x_{j,S}$ the upper limit of occupants assigned to exit j.

$$x_{j,I} = 0.5382 \cdot a_j \; x_{j,S} = 3.5a_j. \tag{11}$$

Fig. 3 shows the evacuation function. We can see that when $x_j < x_{j,I}$ the evacuation time is constant and independent of the enclosure occupation x_j. In this section the occupation density is below to 0.5382, which ensures a constant speed of circulation. If the circulation speed is constant, the flow variation depends exclusively on the density. Consequently, the evacuation times are equal, due to the enormous difference between the two exit flows. In the following section $x_{j,I} - x_{j,S}$ the growth of the function is hyperbolic. The initial section, in which up to $x_{\mathrm{j,I}}$ people could vacate the enclosure at the same time, is referred to as the fluency interval of the evacuation function, by the similarity to a phenomenon observed in stress deformation graphs of mechanical traction tests: once the elastic period is surpassed there is a horizontal section in the curve in which the material extends without the tension increasing.

5. Inverse evacuation function

The inverse evacuation function $p_j(z_j)$ for exit j establishes the number of people who can vacate the enclosure in a time z_j using exit j. It is given by the following expression:

$$p_j(z_j) = \begin{cases} 0 & 0 \leqslant z_j < z_{j,I} \\ 0 < p_j(z_j) \leqslant x_{j,I} & z_j = z_{j,I} \\ \frac{a_j}{\alpha}\left(1 - \frac{l_j \cdot w_j + a_j}{z \cdot \lambda_j \cdot w_j}\right) & z_{j,I} < z_j \leqslant z_{j,S} \end{cases}. \tag{12}$$

In the inverse evacuation function $p_j(z_j)$ the time z_j is the independent variable and the number of people assigned to an exit (x_j) the dependent variable:

$$z_j = t_j(x_j) \quad p_j(z_j) = x_j \tag{13}$$

By replacing the values $x_{j,I}$ and $x_{j,S}$ in the evacuation function we obtain the values $t_{j,I}$ and $t_{j,S}$, denoted by $z_{j,I}$ and $z_{j,S}$, where $z_{j,I}$ is the time it will take a number of occupants x_j with an occupation density of 0.5382 people/m^2 to reach exit j and leave the enclosure. Similarly, $z_{j,S}$ indicates the time it will take x_j occupants with an occupation density of 3.5 people/m^2 to reach exit j and leave the enclosure.

6. Proposed algorithm

The proposed algorithm for solving the problem of enclosure evacuation along independent and known routes is based on the work of Brown [1]. By considering evacuation routes and using the equations proposed by Nelson and McLennan we make significant changes to the form of the evacuation function and inverse evacuation function. These new considerations adapt the procedure to the solution proposed by Brown by generalizing it to obtain the following algorithm:

Step 1. For each exit, evaluate the intervals at which to apply the Nelson and McLennan equations [26] used to model the movement of people. From the density limits (11), determine the interval limits $x_{j,I}$ and $x_{j,S}$. Then, calculate the evacuation function $t_j(x_j)$ for each route j.

Step 2. Determine the points $z_{j,I}$ and $z_{j,S}$ of the inverse evacuation function $p_j(z_j)$. Obtain the inverse evacuation function for each exit. If all of the discontinuity points are different, we order the inverse evacuation function $p_j(z)$

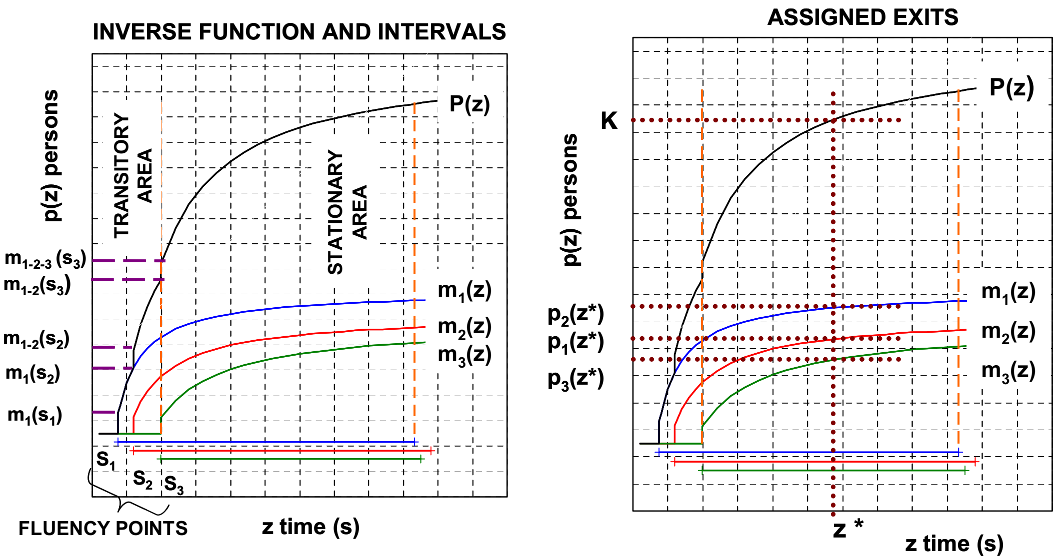

Fig. 4. Solution process.

for each exit according to the discontinuity points $z_{j,I}$, following the criterion from lower to higher fluency time $z_{j,I}$. From this, we obtain the inverse evacuation functions $m_j(z)$ ordered according to its fluency point s_j. Finally, we obtain the maximum value z_S at which the global inverse evacuation function $P(z)$ will be defined.

Step 3. Determine the global inverse evacuation function $P(z)$. This is done by adding the inverse evacuation functions of each exit, taking into account the discontinuities. The resulting situation is shown in Fig. 4.

$$P(z) = \begin{cases} 0 & 0 \leqslant z < s_1 \\ m_1(z) & s_1 < z < s_2 \\ \dots\dots\dots\dots\dots\dots\dots\dots & \dots\dots \\ m_1(z) + \ldots + m_{n-1}(z) + m_n(z) & s_n < z \leqslant z_S \end{cases}. \tag{14}$$

Step 4. Limit $P(z)$ at the fluency points, defining the global inverse evacuation function $P(z)$.

$$\begin{aligned} & z = s_1 \Rightarrow 0 \leqslant P(z) \leqslant m_1(s_1) \\ & \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ & z = s_n \Rightarrow m_{1-2-\dots-(n-1)}(s_n) \leqslant P(z) \leqslant m_{1-2-\dots-n}(s_n) \end{aligned} \tag{15}$$

Step 5. Calculate the optimal evacuation time z^*. This is done, by analyzing the number of enclosed people (k) in order to determine the section of the $P(z)$ function in which we will work. Fig. 4 shows the case of an enclosure with three exits. The situation of the enclosed people (k) gives rise to three possible scenarios: (1) k is in a fluency interval, (2) k is in the different stages of the transitory period and (3) k is in the stationary period.

Step 6. Once the value of z^* is known, determine the number of people who must use each exit and then replace z^* in the inverse evacuation function of each exit, considering the section in which k is located. This process is shown in Fig. 4, which is used to obtain the allocations of the three exits.

7. Computational results

People (610) are uniformly distributed in an enclosure with three independent exits. There are passage zones a_j towards each of the exits. These areas are free of obstacles and have a minimum width w_j. The main characteristics of the enclosure are shown in Table 1.

If we consider that circulation towards the exits takes place along a horizontal corridor, the value of the constants is $\alpha = 0.266$ m^2/person and $\lambda = 1.40$ m/second. We developed a computer program to carry out the calculations of the proposed algorithm. The solution is shown in Table 2.

Table 1
Numerical example characteristics

Exit	Minimum effective width of the exit w (m)	Length of the evacuation route of the exit l (m)	Surface of access to the evacuation route a (m^2)
1	2.0	0	90
2	1.6	25	75
3	1.2	60	70

Table 2
Numerical example solution

Stationary interval		$401.00 < k \leqslant 781.29$				
Exit	Persons	Speed of circulation (m/second)	Flow (pers./second)	Path time (second)	Wait time (second)	Total time (second)
1	275.86	0.259	1.585	0.00	174.04	174.04
2	198.78	0.413	1.751	60.54	113.51	174.04
3	135.36	0.680	1.578	88.25	85.80	174.04
Occupation	610.00		4.914			

8. Conclusions

In this study, we formulate the building evacuation problem and generalize the evacuation function to incorporate evacuation routes. In the original problem formulation, Francis imposed the condition that the evacuation function of each exit must be a strictly increasing function, which is a very strict condition that severely limits the problem and makes it impossible to model delays and evacuation routes.

Instead of adopting the initial model proposed by Francis, which is used to obtain the values of the exit flows from the circulation model developed by Fruins, we use the functions developed in the 1990s by Nelson and McLennan to model the movement of people, which enables us to carry out no supposition a priori of speeds and the circulation flows in each exit.

By considering evacuation routes and using the equations proposed by Nelson and McLennan, we made significant changes to the form of the evacuation functions and inverse evacuation functions. In order to do so it was necessary to generalize the procedure proposed by Brown.

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