Evacuation planning (Problem definition)

What is evacuation?

Evacuation as an aspect of emergency management can be explained as the act of leaving a danger zone as quickly as possible in an ordered fashion. The threat or occurrence of a disastrous event (e.g. bomb, fire, flood, industrial accidents and hurricane) may be the reason for the evacuation of different types of systems. These systems include buildings, transportation carriers (e.g. ships, planes) or districts, regions. Recent disasters like the terrorist attack on World Trade Center or the hurricanes Rita and Katrina highlighted once more the importance of evacuation planning.

In this work our focus is on building evacuation. Practice evacuations are being done every now and then in various buildings however most of the times they are not taken very seriously by the occupants. The following citations from the *eyewitness report from the 11 September 2001 disaster at the World Trade Center, New York, USA*, point out how serious the conditions may get during a building evacuation in a disaster.

JOE CRIMMINS, of Hoboken, N.J., was on the 43rd floor in the cafeteria of the World Trade Center tower hit by the first airplane. “There was an explosion,” Crimmins said. “The building shook. Within seconds, you could see debris coming towards the window. So we just ran toward the emergency exit. It took about 20 minutes to a half hour to get out. The stairways were crowded and smoky. The lower 10 to 15 floors were filled with water, so we were walking through water as firemen were walking up.

Michael Hingson, the 51-year-old has been blind since birth. Michael was on the seventy-eight floor of the World Trade Center, the one building, the north tower. He was guided out by his guide dog Roselle and another colleague, Frank. "The crowds weren't huge at first," Hingson said. "But as we started making our way down, they got bigger." It was getting hot, too, with temperatures in the stairwell climbing higher than 90 degrees. Hingson was sweating and Roselle was panting. "We moved fairly swiftly until we hit about the 40th floor. Then things got kind of jammed, a lot of stopping and starting."

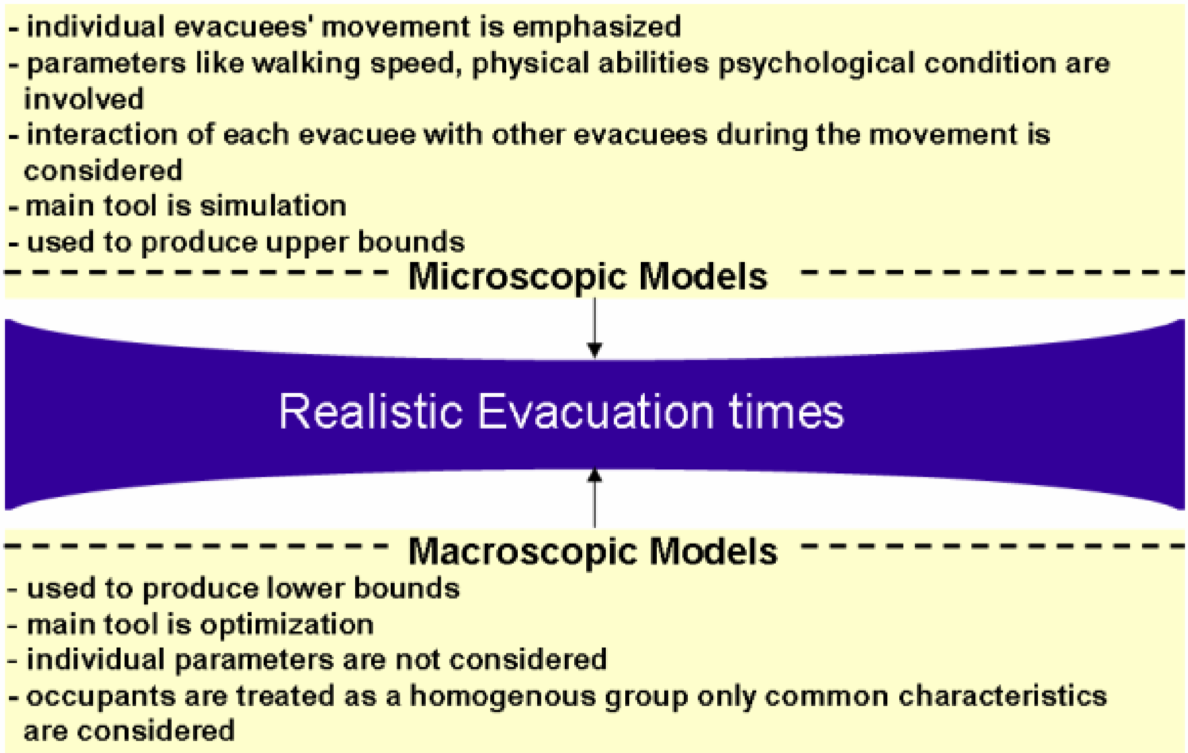
Ideal information

Referring to the paper Mathematical Modelling of Evacuation Problems: A State of Art written by Horst Hamacher and Stevanus Tjandra, 2001 and the PhD thesis Dynamic network optimization with application to the evacuation problem written by Stevanus Tjandra, 2003, the information which would be ideal to have in evacuation planning can be listed as follows:

- Type of system defined by layout/geographic information and familiarity, for example : office building, shopping mal and airport
- Behaviour estimation of the occupants under panic situation.
- Occupants distribution (includes age, gender and disability).
- Source and location of hazard, hazard propagation speed/characteristics and factors affecting the hazard propagation.
- Safe destinations (refuge places).
- Availability of emergency service facilities and personel.
- Analysis of evacuee's movement distribution to determine the evacuation time

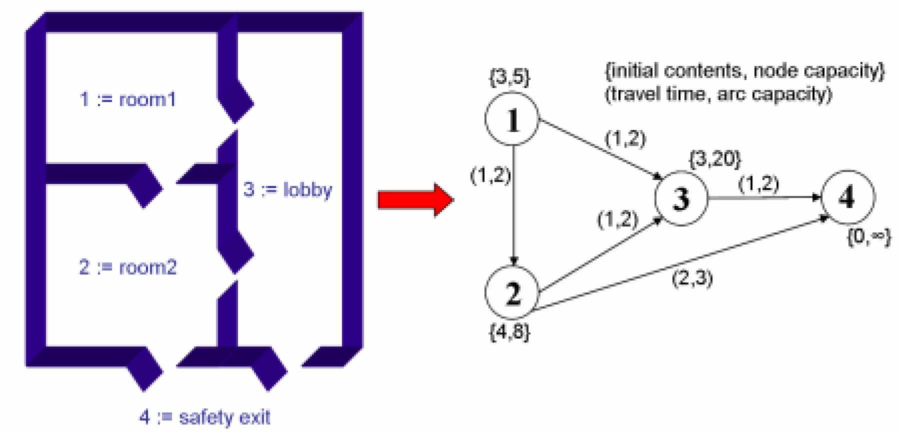
Macro- and microscopic models

In general, two approaches are used to model evacuation problems which emphasize on the estimation of the egress time (The time evacuees need to move towards the safety area). These are macro- and microscopic models.



Static networks

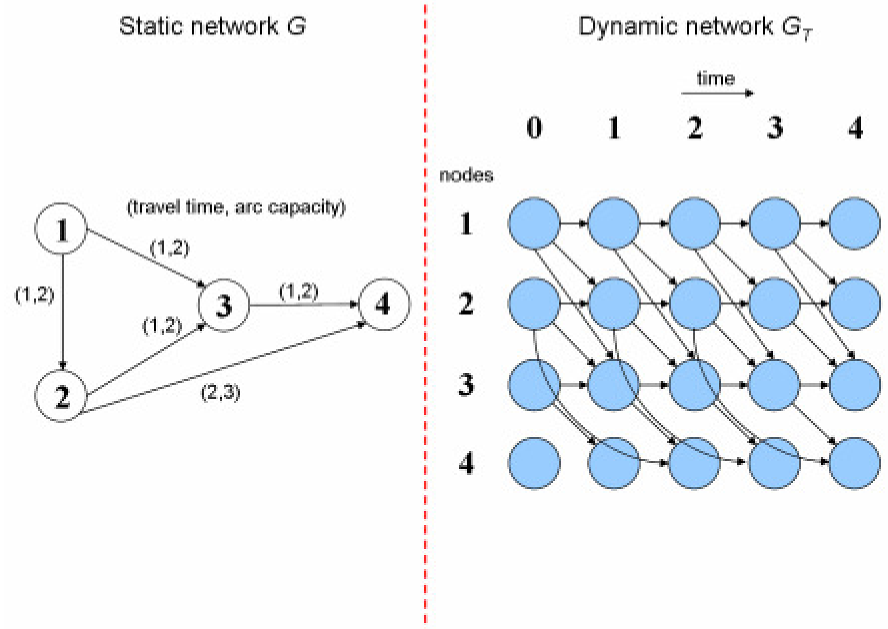
In this work the emphasis is on the macroscopic model. Most macroscopic models represent a building and the attributes of the building's components in a static network *G.* An informative introduction on Network Flow Problems can be found in the book Linear and Network Optimization by Horst Hamacher and Kathrin Klamroth, Vieweg, 2000. The following example is taken from the paper Mathematical Modelling of Evacuation Problems: A State of Art



In the static network ***G = (N, A)***, the set of ***nodes N*** correspond to the respective rooms, lobbies and safety areas; the set of ***arcs A*** is used to model corridors, stairways or hallways. Routes are modelled by ***paths*** of the graph. A ***path*** of the graph is composed by nodes and arcs; each arc connecting two adjacent nodes. For example the arc between the nodes 1 and 2 represents the possibility of moving from room1 to room2 during the evacuation. The nodes which are assumed to be occupied by evacuees are the source nodes. The supply of a source node is the estimated number of occupants in the corresponding room. In addition each node in ***G*** has a capacity representing the maximum number of evacuees simultaneously allowed to stay in the room. The ***sink node*** in the evacuation problem is the node obtained by connecting all the exit nodes. The total number of evacuees is assigned to this node as the demand value. Arcs have attributes like arc capacity and travel time. Arc capacity is the upper bound of the number of evacuees that can traverse the arc per unit time. The travel time which is the time needed to travel form one node to another is one of the important parameters involved in evacuation problems.
In order to model the time dependence of the evacuation problem dynamic network flows should be introduced.

Dynamic networks

In a real life situation, the evacuees have a finite walking speed. Moreover the connections between two positions may become inaccessible after some time due to blocking by fire or smoke. In that case the capacity of the representing arc can be set to zero after some time. Such conditions can not be expressed by a static network. To model the time dependency of the evacuation problem a dynamic network G_T (the time expanded version of G) is used. T represents the time horizon which is discretized into time periods t. A static network G and its time expanded version G_T with T=4 is illustrated below. It is assumed that travel times, arc capacities, are constant. Dynamic network problems are more complex than the static network problems since they consider multiple copies of the nodes (therefore network size increases) and keep track of information like the unit of flow travels through an arc at any time.

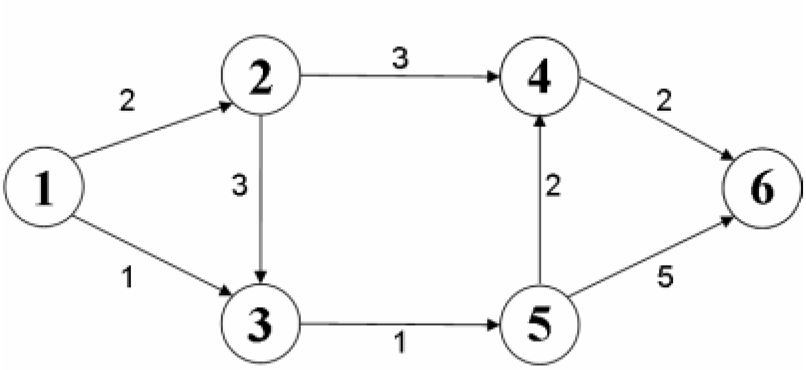


The objective of the evacuation problem can be stated as to minimize the time needed to evacuate a building or to maximize the number of people which can be evacuated in a given time horizon T. The latter can equally be written as to maximize the dynamic flow arriving at the sink for a given time horizon T. In other words the problem is to find the maximum dynamic flow which can flow in the network in T periods.

Shortest dipath problem

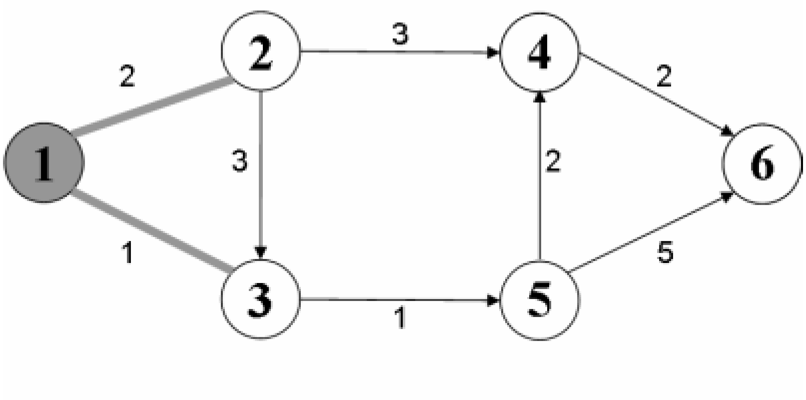
In this section we present the solution approaches developed for discrete time dynamic network flow problem and the maximum dynamic flow problem. We start with some concepts and well known problems from the graph theory and network flows. Our approach is to demonstrate simple examples to introduce the algorithms. For the theoretical work we refer to the references listed in the additional information.

The directed graph (Graph G = (N,A): Finite nonempty set of nodes N= $\{n_1,...,n_k\}$ and arcs A = $\{a_1 ,...,a_m\}$. Each arc $a \in A$ is defined by two endpoints and is denoted by a = $[n_p, n_q]$ or a = [p, q]. Directed graph G = (N, A) is a graph but now a direction is associated with every arc a = (p, q)) shown below represents an evacuation area consists of rooms and corridors. The rooms and corridors are modelled as nodes and arcs respectively. The routes that an evacuee may follow are modelled with paths (Path P is a sequence of nodes with the property that there is an arc from each of its nodes to the next node in the sequence) in the graph. If we assume that we have only one evacuee the evacuation problem can be reformulated as finding the route which minimizes the evacuation time. In graph theory this corresponds to the problem known as ***Shortest dipath problem (SDP)***. The *(SDP)* can be solved by using Dijkstra's algorithm. An implementation of this algorithm is given below. This example was taken from the book Linear and Network Optimization by Horst Hamacher and Kathrin Klamroth, Vieweg, 2000.

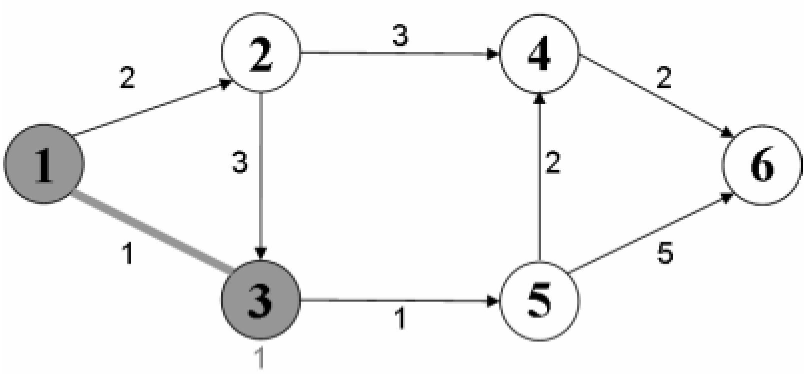


Let us assume that in case of a disaster our evacuee is in room 1 (modelled by node 1) of the building which has to be evacuated. The safety exit is modelled by node 6. Note that the numbers on the arcs denote the travel times (in time periods) between the subsequent nodes. Dijkstra's algorithm finds the shortest routes from node 1 to any other node. We are especially interested in the shortest path between nodes 1 and 6.

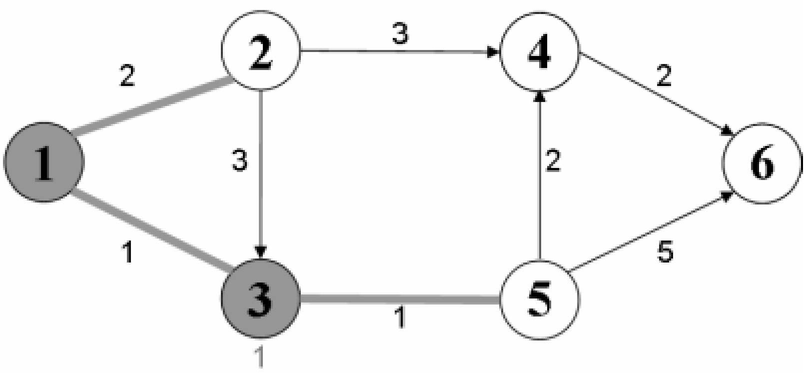
Shortest dipath problem



Dijkstra's algorithm starts with the source node, in our case node 1. Node 1 is coloured with gray to indicate that it is available for the search of shortest paths. Then the algorithm checks all the travel times from node 1 to the connected nodes in order to find the closest neighbour of node 1.

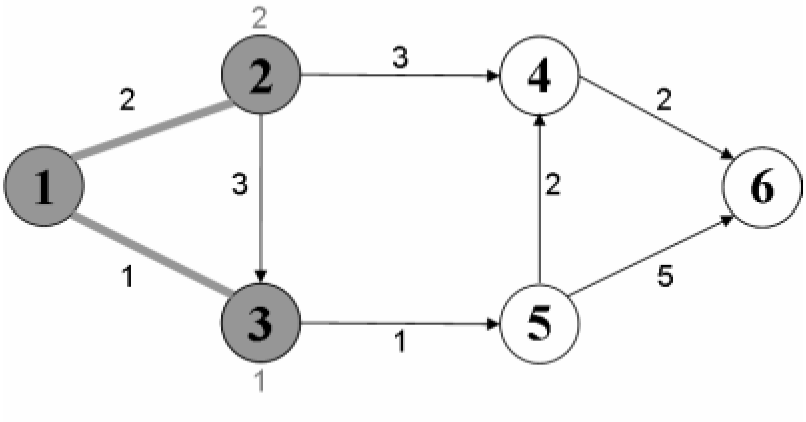


Node 3 is found as the closest neighbour of 1. Now it is also available meaning that we can search the neighbourhood of node 3 as well. The total travel time 1 is written under the node in gray.

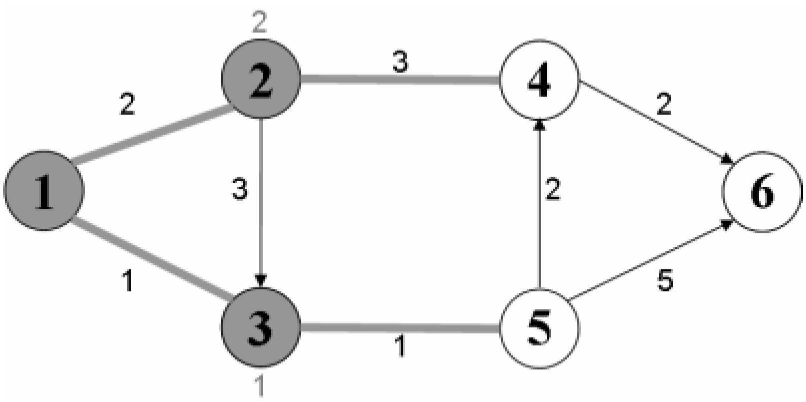


The algorithm continues by checking the travel times from the available nodes to the connected nodes. Here node 1 is connected to node 2 and node 3 is connected to node 5. Since the total travel times equals to 2 in both cases, the algorithm can either choose the arc between node 1 and node 2 or the arc between node 3 and node 5.

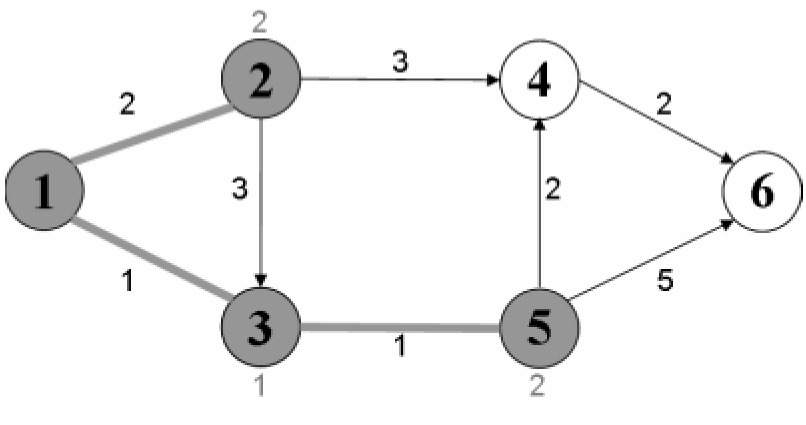
Shortest dipath problem



Node 2 is chosen as the new available node with the total travel time 2. Node 5 could be chosen as well. This would have no effect on the end result.

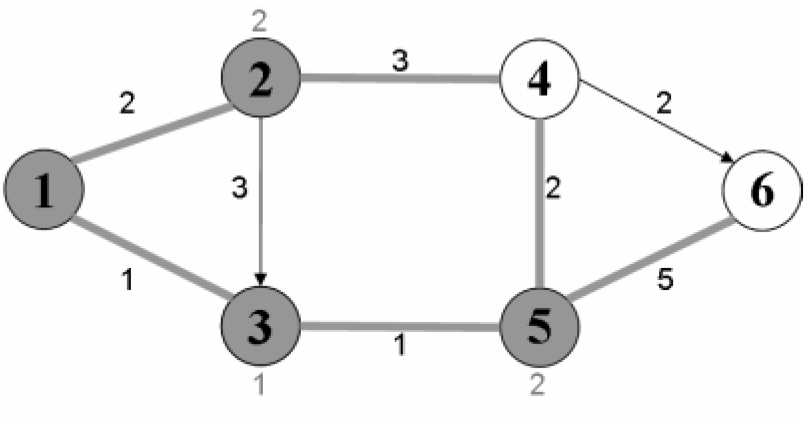


From the nodes 1, 2 and 3 we can reach the nodes 4 and 5. The shortest travel time to node 2 is 2. This is already shown in the figure. 2 + 3 (travel time from node 2 to 4) = 5 is the total travel time from node 1 to node 4 in this iteration. Likewise total travel time from node 1 to node 5 can be calculated as 2. The algorithm chooses the node with the minimum total travel time.

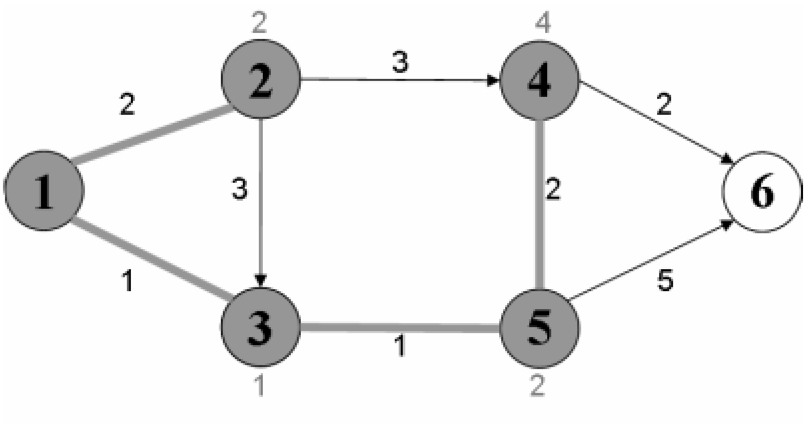


Node 5 becomes available with the total travel time 2.

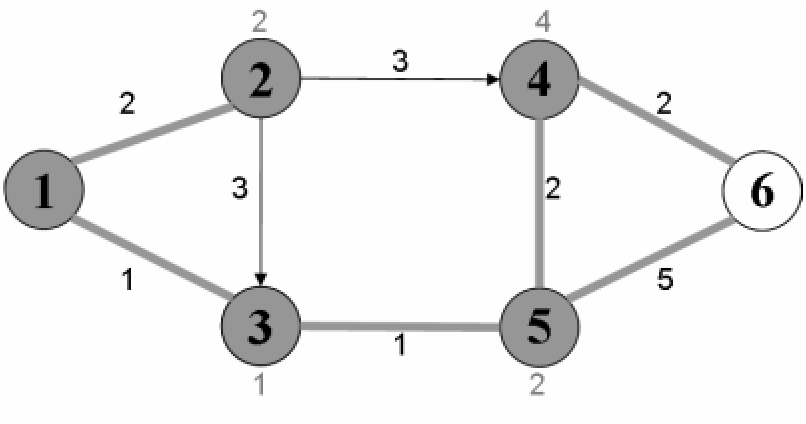
Shortest dipath problem



The connecting arcs between available nodes and the remaining ones are shown in the figure.

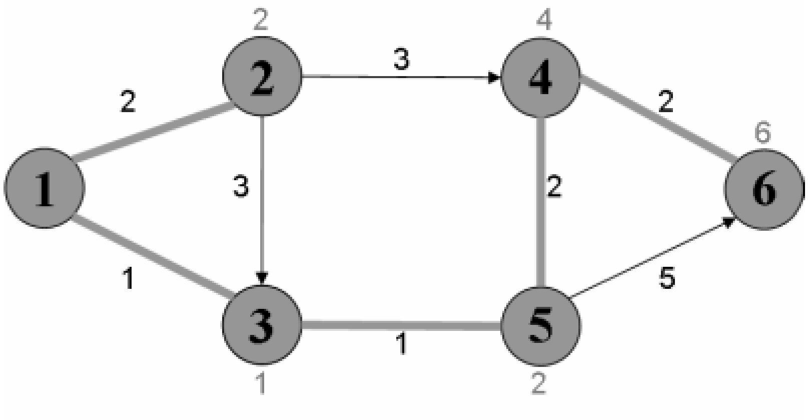


In this step node 4 becomes available.

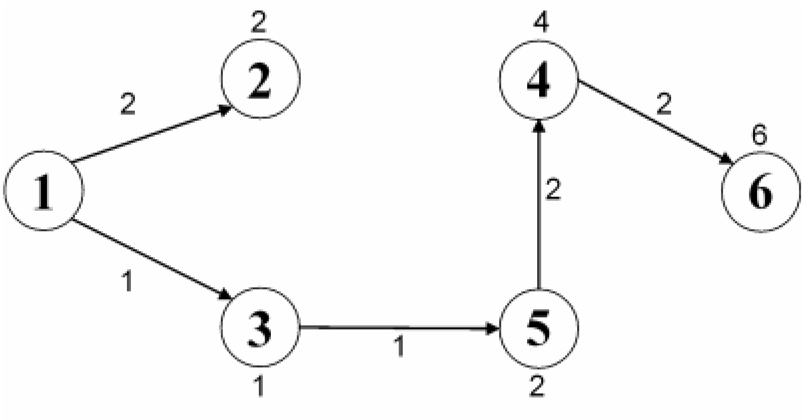


The algorithm continues until all nodes become available.

Shortest dipath problem



The shortest travel time between nodes 1 and 6 is found as 6 time periods following the route {1, 3, 5, 4, 6}.

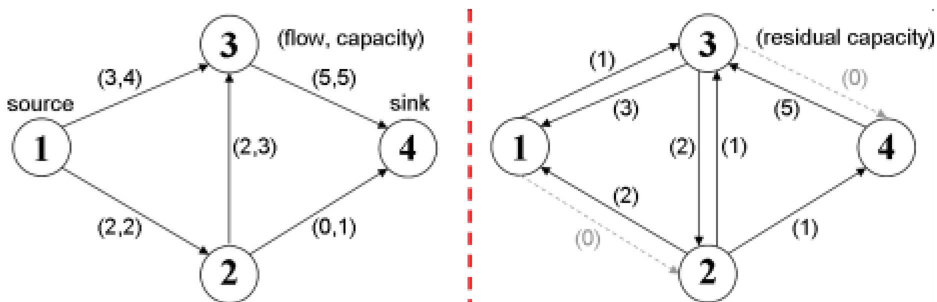


Shortest routes and travel times from node 1 to any other node are shown in the figure.

Residual networks and augmenting paths

Having introduced the shortest dipath problem, we consider the case that there is more than one evacuee in room 1 (source node) and we try to send as much evacuees as possible to the safety exit (sink node) without exceeding the capacities of the corridors. This problem can be interpreted as a maximum flow problem. Comprehensive information about shortest dipath problems and maximum flow problems (including the example illustrated below) can be found in the book Network flows written by Ravindra K. Ahuja, Thomas L. Magnanti, and James B. Orlin, 1993.

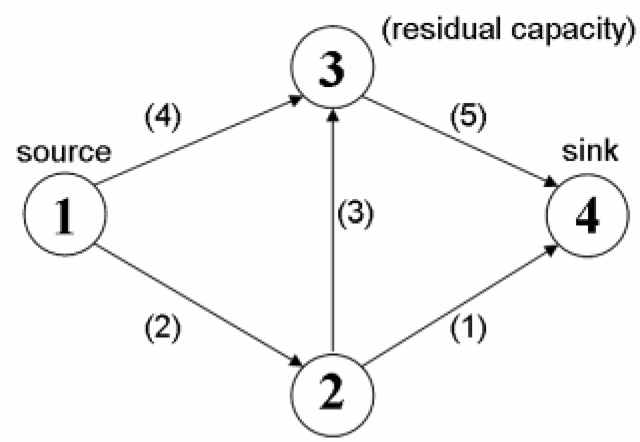
Maximum flow problems can be solved by using several algorithms. One of the simplest algorithms is the *generic augmenting path algorithm*. The generic augmenting path algorithm makes use of the concepts residual network and augmenting path. Let the graph shown in part (a) of the figure be our original network ***G***. Residual capacity of an arc is the difference between the current flow and the capacity of an arc. A residual network G_f of the network ***G*** is obtained by using the nodes of the original network and the residual capacities of the arcs. This is illustrated in part (b) of the figure. An augmenting path is any directed path from the source node to the sink node in the residual network. In our example there is exactly one augmenting path 1-3-2-4. In general only the arcs with non-zero capacities are included in the residual network. Therefore the arcs with zero capacities are shown in gray and will not be included in the residual network in what follows.



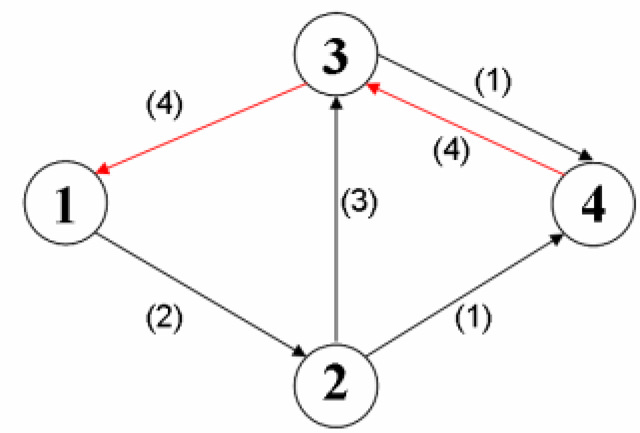
(a) Original network (b) Residual network

Generic augmenting path algorithm

The augmenting path algorithm starts with a residual network and searches it for augmenting paths. If the residual network contains an augmenting path that means more flow can be sent from the source node to the sink node. After increasing the flow along the augmenting path the residual network is updated. The algorithm continues until the residual network contains no augmenting paths. A demonstration of the generic augmenting path algorithm is given below. Detailed information about the algorithm including the example presented here can be found in the book Network flows written by Ravindra K. Ahuja, Thomas L. Magnanti, and James B. Orlin, 1993.

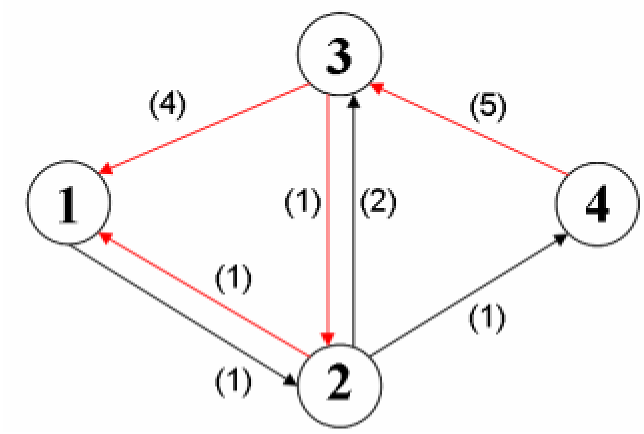


We start with a residual network for zero flow.

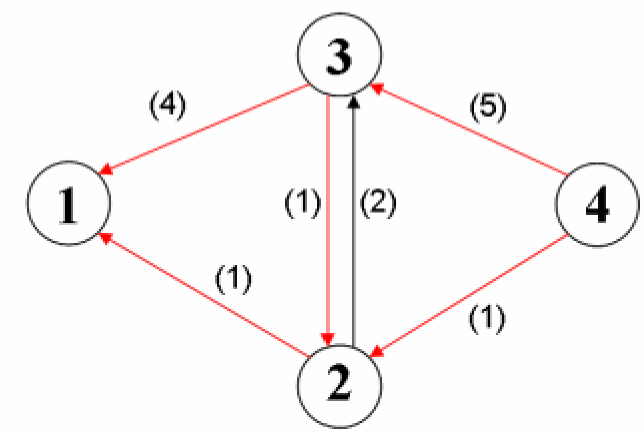


First we send 4 units along the path 1-3-4. After sending the flow the residual network is updated.

Generic augmenting path algorithm



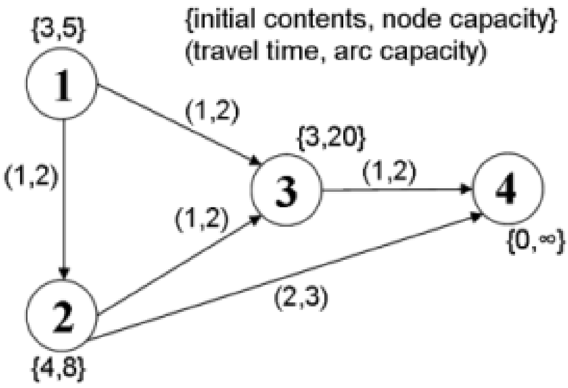
The next augmenting path is 1-2-3-4. One unit flow can be sent along this path.



The last augmenting path in this residual network is 1-2-4. After increasing one unit of flow along this path the algorithm stops. Maximum flow is found.

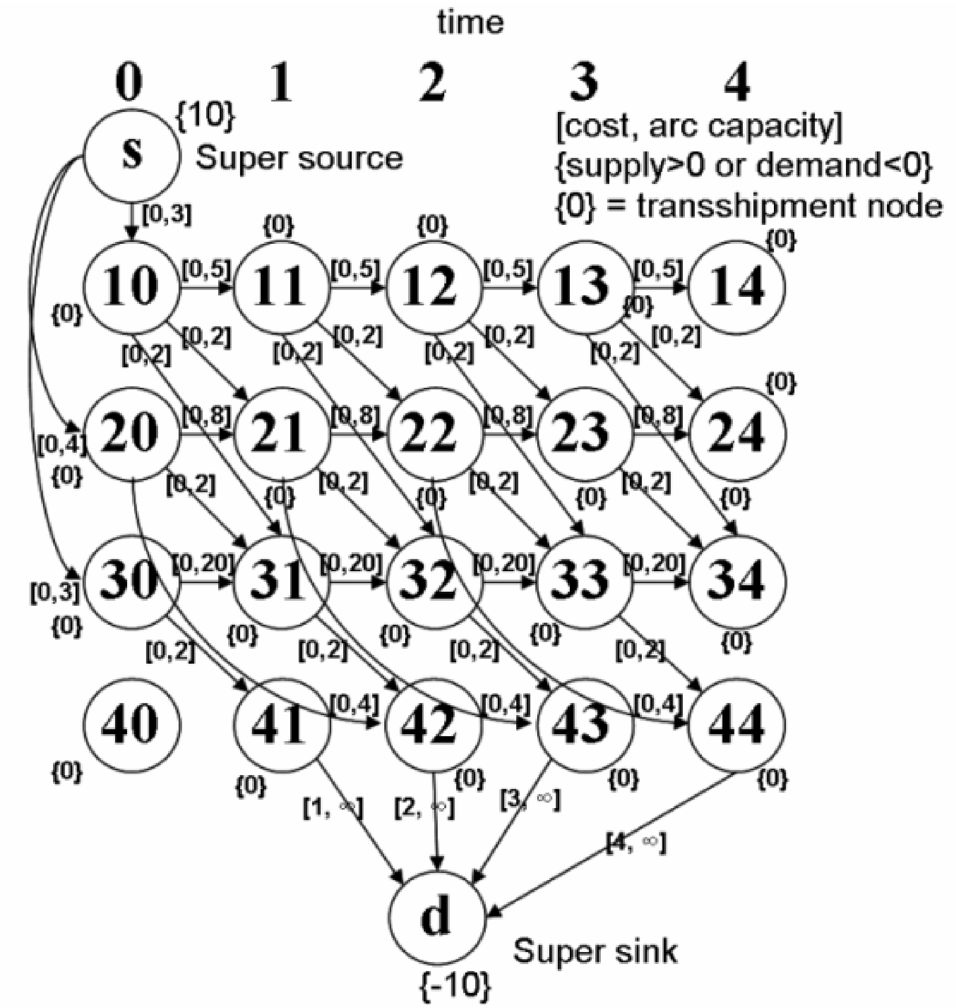
Discrete time dynamic network flow model

In discrete time dynamic network flow model, first the evacuation area is mapped into a static network. Rooms, lobbies, safety areas etc. are represented with nodes of the network; corridors, stairways are represented with arcs. If there are evacuees in the rooms at the beginning of the evacuation, corresponding nodes are considered as supply points. Safety areas are the demand points. There may exist some points that have neither supply nor demand and used for the transfer of the evacuees between some locations. These points are called transhipment points and have zero supply. The routes an evacuee can follow are modelled with paths. Information like initial contents of the rooms, capacities of corridors, travel times of the evacuees etc. are transferred into static network. Static networks can not model evacuation problems completely since the time component is missing. Therefore a time expanded version of the static network problem is constructed in which the time horizon is divided into predetermined periods and the nodes are copied for each period. Using the time expanded network, a dynamic network flow problem can be solved as a static flow problem. In the following we demonstrate an algorithm which can be used to solve the static network flow problem.



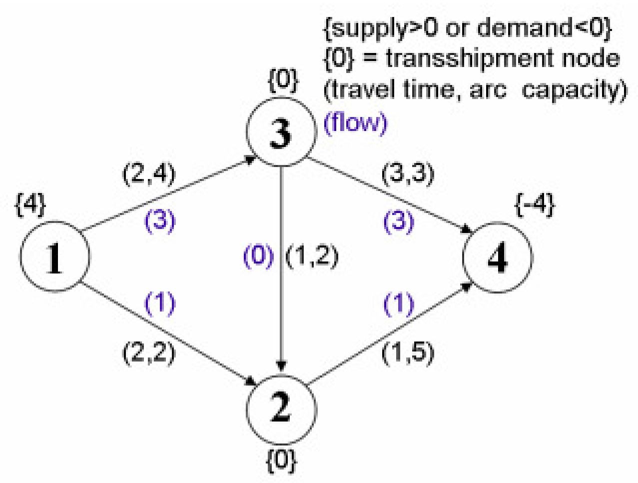
Discrete time dynamic network flow model

Dynamic network G_T of the static network G:

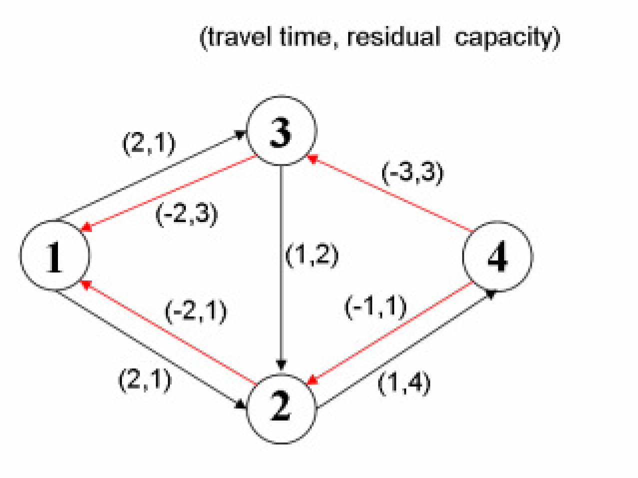


Discrete time dynamic network flow model

The dynamic or time expanded network introduced previously can be treated as a static network and therefore any minimum cost network flow algorithm can be applied to solve the problem (Note that, in evacuation problems costs are defined using the travel times). One of these algorithms is the cycle-canceling algorithm. This algorithm starts with a feasible flow (in general by solving a maximum flow problem) then determines the negative cost directed cycles in the residual network and increases the flows in these cycles. The cycle-cancelling algorithm stops when there exists no negative cost directed cycle. Once more we refer to the book Network flows for the example demonstrated below.

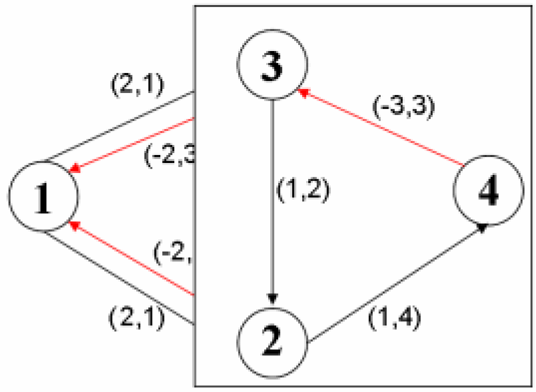


The numbers in blue stand for a feasible flow in the network.

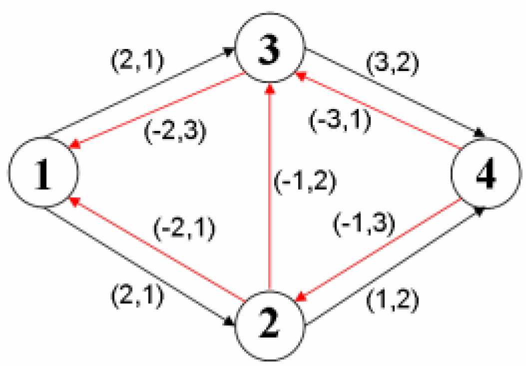


Corresponding to the flow the residual network is depicted. Note that if, for example, the cost of sending one unit of flow from node 3 to node 4 is 3 cost units then the cost of sending one unit flow from node 4 to node 3 is -3 cost units.

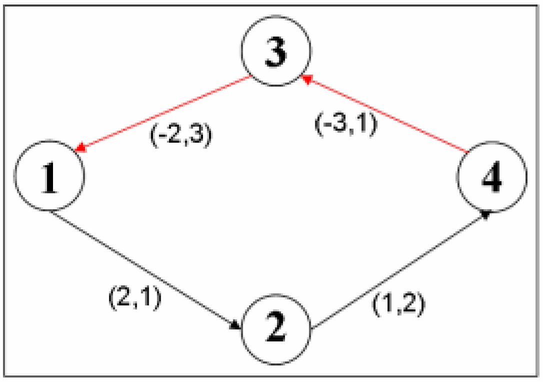
Discrete time dynamic network flow model



The path 3-2-4-3 is a negative cost directed cycle since -3 + 1 + 1 = -1.

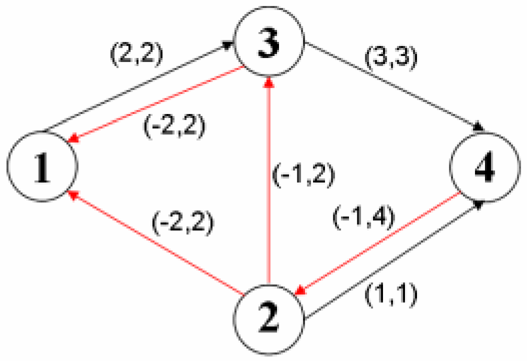


After sending 2 units (the minimum of the arc capacities) of flow along this cycle the residual network is updated.



The next negative cost directed cycle is determined as 4-3-1-2-4.

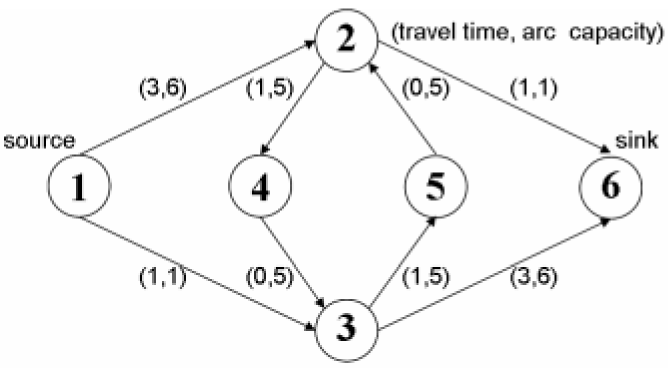
Discrete time dynamic network flow model



One unit of flow is sent along this cycle and the residual network is updated. The cycle canceling algorithm stops since there exists no other negative cost directed cycle.

Maximum dynamic flow problem

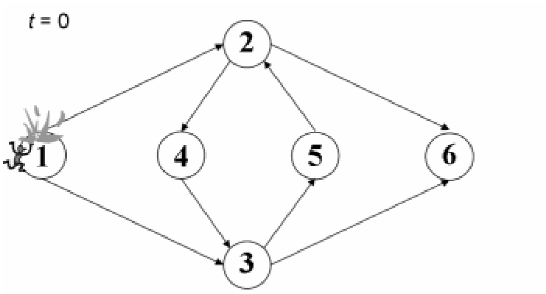
In some evacuation problems it is difficult to estimate the number of occupants in the evacuation area (i.e city hall). In these cases the evacuation problem can be formulated as maximizing the number of people which can be evacuated in a given time horizon ***T***. Therefore such an evacuation problem can be treated as a maximum dynamic flow problem. In the paper Mathematical Modelling of Evacuation Problems: A State of Art written by Horst Hamacher and Stevanus Tjandra, 2001 it is shown that the maximum dynamic flow problem can be solved by an approach called temporally repeated flow technique. In this approach first a minimum cost flow algorithm is applied to the original static network. Then the optimal flow is decomposed into chain (a sequence of nodes with no repeated nodes) flows. Finally each chain flow is repeated from time 0 to time ***T*** - {total travel time on the chain}.

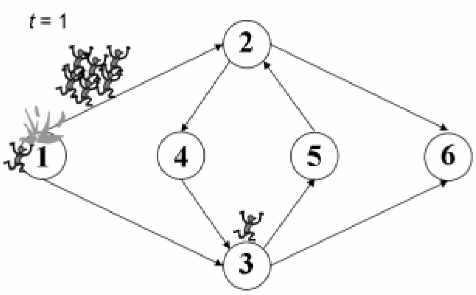


Arc	(1,2)	(1,3)	(2,4)	(2,6)	(3,6)	(4,3)
Flow	6	1	5	1	6	5

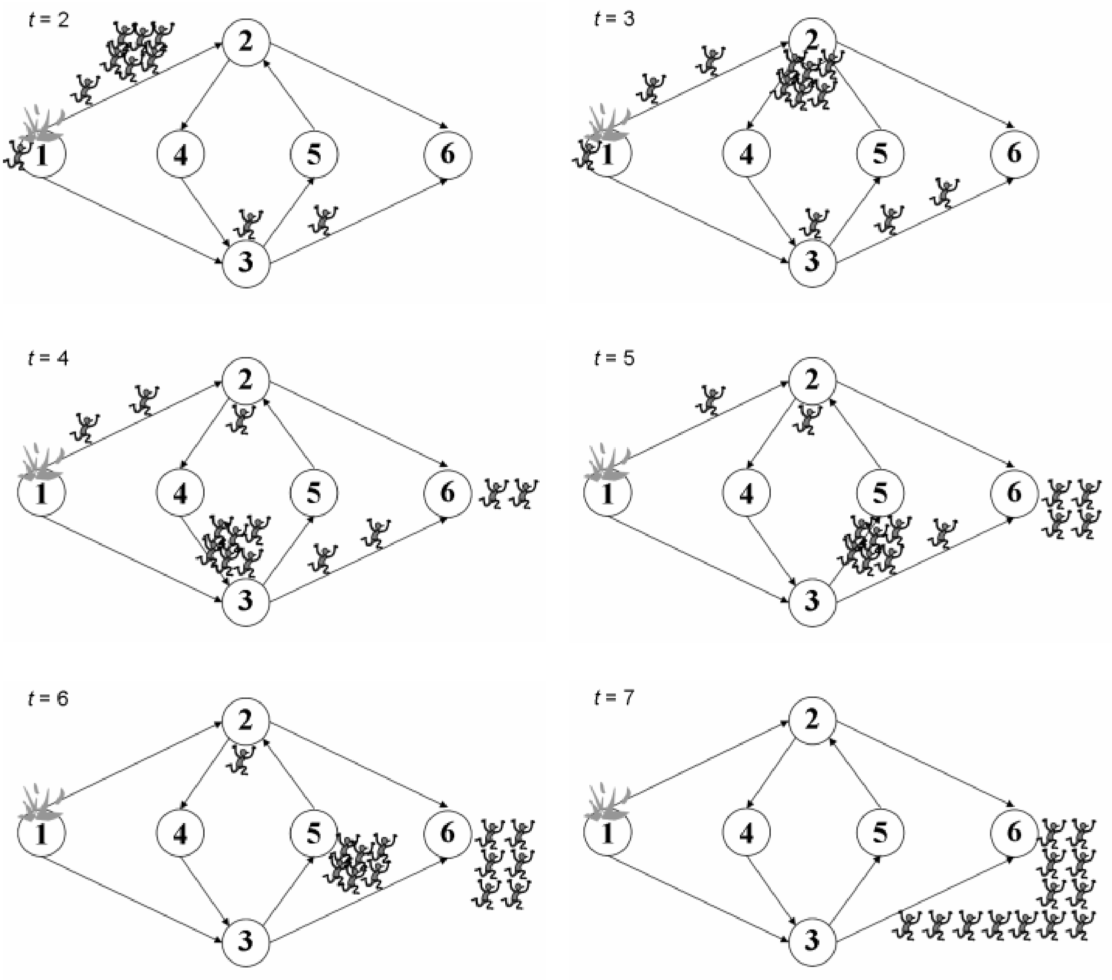
Optimal maximum flows for static network G

For the static network ***G*** (this example was taken from abovementioned paper) depicted left the optimal maximum flows are shown in the table. This leads to the chain flows $\boldsymbol{P_1}$ = {1, 2, 6} with total travel time 4 and capacity 1; $\boldsymbol{P_2}$ = {1, 2, 4, 3, 6} with total travel time 7 and capacity 5; $\boldsymbol{P_2}$ = {1, 3, 6} with total travel time 4 and capacity 1. ***T*** is given as 7 time periods. The flow chains $\boldsymbol{P_1}$*,* $\boldsymbol{P_2}$*,* $\boldsymbol{P_3}$ must be repeated 4 (at times 0, 1, 2, 3, 4), 1 and 4 times respectively. (The reader is recommended to verify the results.) The solution is demonstrated in the figures below. Each time 2 evacuees arrive at the safety exit at ***t***=4, 5, 6. At time 7 the number of the evacuees reaching the safety exit is 7. Therefore the total dynamic flow for ***T***=7 is equal to 13.





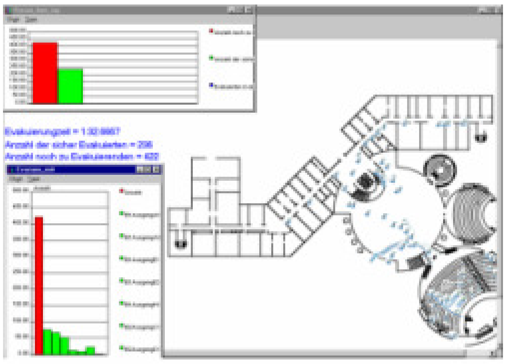
Maximum dynamic flow problem



Evacuation planning (Solution approach)

A case study at the university of Kaiserslautern

In the PhD thesis of Stevanus Tjandra, Dynamic network optimization with application to the evacuation problem, an algorithm is proposed to solve the Quickest Flow Problem with Time-Dependent Attributes. This algorithm is applied to find the lower bound of the evacuation time of Building 42 in the University of Kaiserslautern, Germany. This is a six stored building composed of lecture rooms, offices, auditoriums, halls and library. The floor plans and network representations are included in the PhD thesis. In the frame work of the case study also a real life evacuation of the biggest auditorium in Building 42 is rehearsed. A sample output from the simulation program based on the algorithm and the evacuation rehearsal (available only in German) can be viewed on the video files below.



Sample output of the simulation program
Downloadable (AVI [1.6MB])

Practice evacuation of Building 42
Downloadable (AVI [1.6MB])

Enhancements in dynamic network flow model

In the solution approach we concentrated on the problems with constant travel times where we discretized the time horizon $\boldsymbol{T}$ into predetermined periods. Two possible extensions to evacuation problems are *dynamic network with density dependent travel time* and *continuous time dynamic network model*.

Dynamic network with density dependent travel time

A more realistic approach to model evacuation problems is to use travel times dependent on the density of the flow. In this approach it is taken into consideration that during the evacuation movement the speed will first grow with higher density until a slow down occurs (queuing phenomenon) at a certain degree. Density dependent travel times are in general nonlinear. This is one of the reasons which makes the dynamic network problem with density dependent travel time more difficult to solve. A collection of the related literature is given below. Some of the references are from the field of traffic assignment.

[1] Carey, M. and Subrahmanian, E., An Approach To Modelling Time-varying Flows on Congested Networks, *Transportation Research B*, 34: 157-183(2000).

[2] Jayakrishnan, R., Tsai, W.K. and Chen, A., A Dynamic Traffic Assignment Model With Traffic-Flow Relationships, *Transportation Research C*, 3(1): 51-72 (1995).

[3] Kaufman, D.E., Nonis, J. and Smith, R.L., A Mixed Integer Linear Programming Model For Dynamic Route Guidance, *Transportation Research B*, 32(6): 431-440 (1998).

[4] Ran, B. and Boyce, D. , *Modeling Dynamic Transportation Networks*, Springer, Heidelberg (1996).

[5] Tjandra, S.A., *Dynamic Network Flow Models for Evacuation Problems*, Ph.D Thesis, Department of Mathematics, Technische Universität Kaiserslautern, Kaiserslautern, Germany (2001).

Continuous time dynamic network model

In discrete time dynamic network model of the evacuation problem the length of the time periods plays an important role. Decreasing the length of the time period increases the accuracy of the model. However choosing the lengths of the time periods too small enlarges the dynamic network and thus increases the complexity of the problem. The length of the time period is determined as a result of a trade-off between accuracy of the model and the problem size. In order to remedy the deficiencies caused by a discrete time approach the continuous time dynamic network model is proposed. Considering the accuracy of the mathematical model continuous approach is preferable to discrete models. However for large scale problems continuous time dynamic network models are difficult to handle. A collection of the literature related to continuous time dynamic network problems is given below.

[1] Anderson, E.J, Nash P., and Philpott, A.B., A Class of Continuous Network Flow Problems, *Mathematics of Operation Research* , 7 : 501-514 (1982).

[2] Anderson, E.J, Nash P., and Perold, A.F., Some Properties of a Class of Continuous Linear Programs, *SIAM Journal Control and Optimization*, 21(5) : 758-765 (1983).

[3] Anderson, E.J and Philpott, A.B., A Continuous-time Network Simplex Algorithm, *Networks* , 19 : 395-425 (1989)

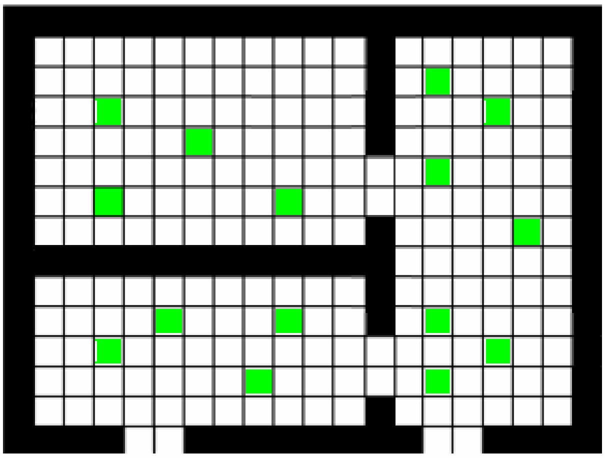
[3] Philpott, A.B., Continuous-Time Flows in Networks, *Mathematics of Operation Research*, 15(4) : 640-661 (1990).

[4] Pullan, Malcolm C., An Algorithm For a Class of Continuous Linear Programs, *SIAM Journal Control and Optimization*, 31(6) : 1558-1577 (1993).

[5] Tjandra, S.A., *Dynamic Network Flow Models for Evacuation Problems*, Ph.D Thesis, Department of Mathematics, Technische Universität Kaiserslautern, Kaiserslautern, Germany (2001).

Microscopic models

Another well known approach to model evacuation process is the microscopic approach. In microscopic models each individual is modelled as a separate flow object and provided with attributes like gender, age, walking speed, psychological condition, reaction time etc. Besides these attributes, responses of the evacuee to others or changed conditions like blockage of some corridors or high congestion in a stairway are considered as factors influencing an evacuees' movement. Since there exists a huge amount of data to be managed, microscopic models usually employ simulation mostly based on cellular automata (CA). A cellular automaton (cellular automata in plural) is a discrete dynamic system which consists of an infinite, regular grid of cells, each having a discrete step time evolution. The state of a cell is one of finitely many values at time ***t***. It depends on the states of the neighbouring cells at time ***t-1***.



The evacuation area is divided into cells each of fixed and equal size. The figure is a 2-dimensional illustration of an evacuation area with 2 rooms and a corridor. Borders of the rooms are shown with black lines, green squares represent the evacuees. By simultaneously updating the states of the cells, diverse movements of evacuees are simulated. More comprehensive explanation on microscopic models and cellular automata is given in Mathematical Modelling of Evacuation Problems: A State of Art.

Below is a list of related links to evacuation software, CA and simulation.

EXODUS developed by FIRE SAFETY ENGINEERING GROUP at the UNIVERSITY of GREENWICH

BYPASS developed by Institute of Physics University Duisburg-Essen

Chair of Transportation Econometrics and Traffic Modelling TU Dresden

References:

More information about evacuation planning can be found in the following links.

Network flows
(http://www.ise.ufl.edu/ahuja/NetworkFlowsBookCenter.htm)

Linear and Network Optimization
(http://ecampus.com/book/3528031557)

Mathematical Modelling of Evacuation Problems: A State of Art (PDF)
(http://kluedo.ub.uni-kl.de/frontdoor.php?source_opus=1673)

Dynamic network optimization with application to the evacuation problem (PDF)
(http://kluedo.ub.uni-kl.de/frontdoor.php?source_opus=1585)

Evacnet (evacuation software, University of Florida)
(http://www.ise.ufl.edu/kisko/files/evacnet/EVAC4UG.HTM#sec_1_0)

Aggregation of Large-Scale Network Flow Problems with Application to Evacuation Planning at SAP (PDF)
(http://kluedo.ub.uni-kl.de/volltexte/2005/1863/)

TraffGo HT, Pedestrians
(http://www.traffgo-ht.com/en/pedestrians/bibliography/index.html)

Panic: A Quantitative Analysis (Budapest University)
(http://angel.elte.hu/~panic/)

For detailed information about this project contact: tanatmis@mathematik.uni-kl.de