Analysis of Surface Cracks in Plates Loaded in Bending under Elastic-Plastic and Fully-Plastic Conditions

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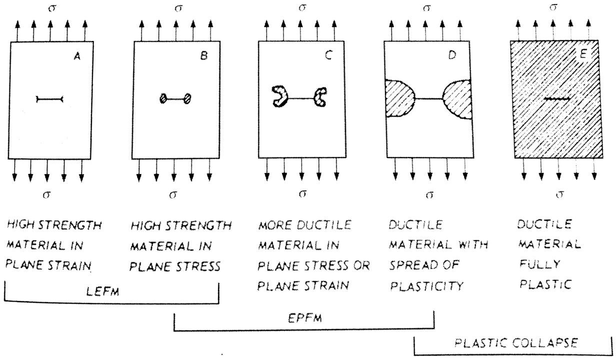
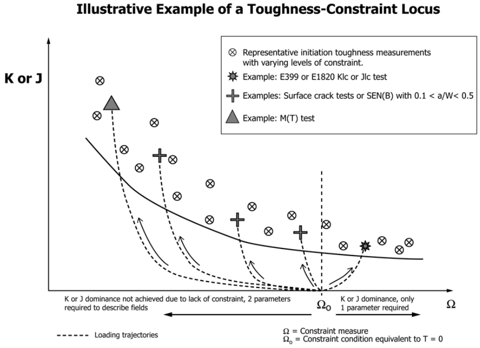
Introduction



Categories of Fracture Mechanics Behaviors

- Fracture mechanics regime classified by plastic zone size
- Linear elastic and fully-plastic are easier to model, but elastic-plastic is common in tough materials

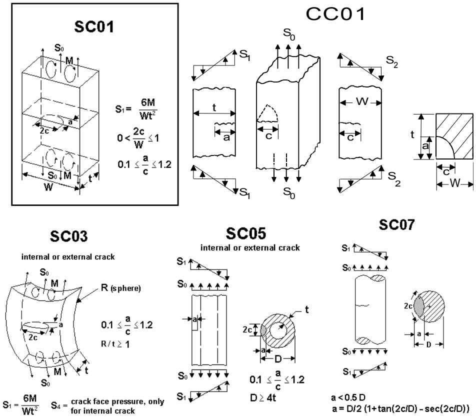


Effect of Constraint on Fracture Toughness

State of stress (constraint) near the crack tip can affect fracture toughness

- high constraint enables high stress without large plastic zone size, toughness is simple material property
- low (negative) constraint increases fracture toughness, toughness is property of material and geometry (or stress state)

- Constraint and stress states can get pretty involved.
- Semi-elliptical surface cracks (SC01) are among the simplest part-through crack cases, and are the subject of ASTM E2899.
- Handbook or curve-fit solutions exist for the other geometries, but only for linear elastic materials.
- NASA's TASC program covers elastic-plastic surface cracks in tension, but **not** in bending.



Research goals

- High quality set of elastic-plastic finite element analysis results as a basis for curve-fit or handbook calculations
- Modified TASC program for bending or tension analysis
- Investigation of EPRI and load separation estimation techniques

Literature Review

For a surface crack in bending, stress intensity at a given location:

$$K_\mathrm{I} = (H\sigma_\mathrm{b}F_\mathrm{b})\left(\frac{\pi a}{Z}\right)^{0.5}$$

$$\sigma_\mathrm{b} = \frac{6M}{Wt^2}$$

$$Z = 1 + 1.464\left(\frac{a}{c}\right)^{1.65}$$

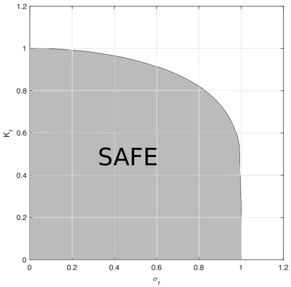
$$F_\mathrm{b} = \left[M_1 + M_2\left(\frac{a}{t}\right)^2 + M_3\left(\frac{a}{t}\right)^4\right] f_\phi f_\mathrm{wb} g$$

For surface cracks in tension, we need "only" 10 equations.

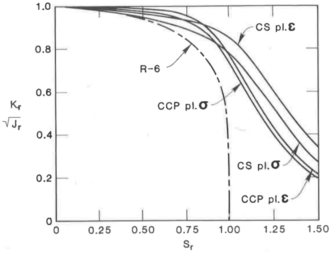
No such curve fit exists for elastic-plastic materials.

...plus another 12 equations, and that's just for linear elastic materials.

Failure assessment diagram: typically a graph of ratio of stress and stress intensity versus critical values



FAD using strip yield model

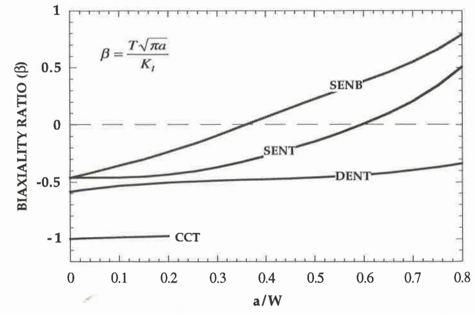


FAD for two geometries and strain-hardening material

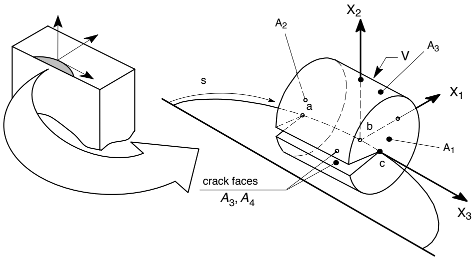
Williams T stress affecting normal stress in x direction (and z direction for plane strain):

$$\sigma_{ij} = \frac{K_\mathrm{I}}{\sqrt{2\pi r}} f_{ij}(\theta) + \begin{bmatrix} T & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \nu T \end{bmatrix}$$

affects plastic zone size and fracture toughness, varies with geometry and loading condition, applies to linear elastic materials (and is relatively accurate for elastic-plastic materials).



Biaxiality ratio for various specimens



Example of a domain integral volume for numerically calculating J

The contour integral form of J

$$J = \int_{\Gamma} \left(w\, dy - T_i \frac{\partial u_i}{\partial x}\, ds \right)$$

can be converted to a discretized version using a group of elements surrounding a point on the crack front

$$J = \sum_{\substack{\text{all elements}\\ \text{in domain}}} \sum_{p=1}^{m} f\left(\sigma, \epsilon, u\right)$$

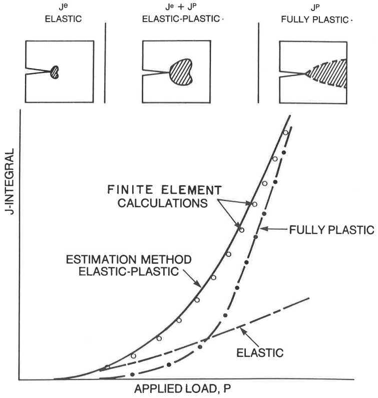
using m Gauss points in each element.

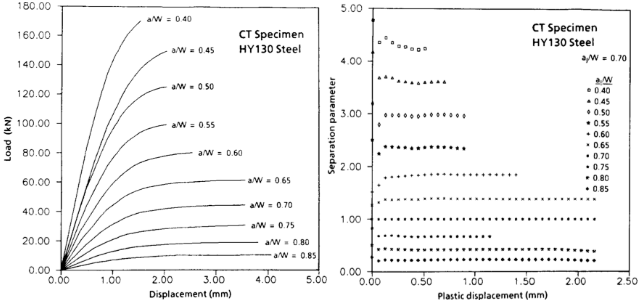
EPRI estimation:

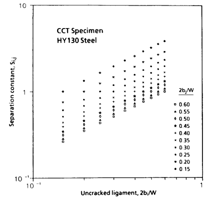
Elastic-plastic J as sum of elastic and fully-plastic J components:

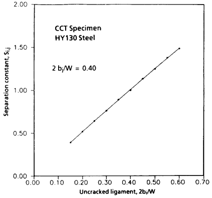
$$J = J_{\mathrm{el}} + J_{\mathrm{pl}}$$

- $J_{\mathrm{el}} = \frac{K^2}{E'}$ with apparent crack size
- $J_{\mathrm{pl}} = h_1\left(\frac{a}{W}, n\right)\left(\frac{P}{P_0}\right)^{n+1} \alpha\epsilon_0\sigma_0 b$ with Ramberg-Osgood material model and limit load



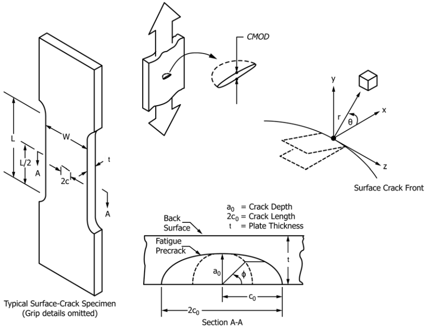




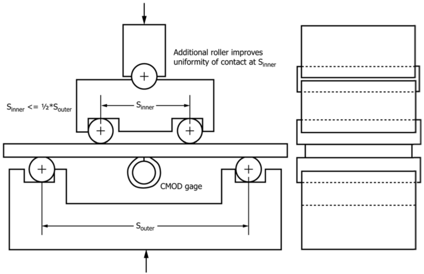


Load separation parameter versus plastic displacement for a compact tension specimen

ASTM E2899: Standard Test Method for Measurement of Initiation Toughness in Surface Cracks Under Tension and Bending



Test specimen and crack configurations



Four-point bend test configuration

- Starter crack machined into flat plate, fatigued to sharpen crack front
- CMOD monitored as tension or bending load increased monotonically
- Either specimen fails or start of stable crack tearing is detected
- Location where crack growth occurs is recorded
- Conditions classified as linear elastic, elastic-plastic, or fully-plastic
- If LEFM or EPFM, calculate constraint from tables
- If LEFM, calculate K from series of provided equations
- If EPFM, **use nonlinear FEA** to calculate J

Calculation or analysis requirements for other ASTM standards:

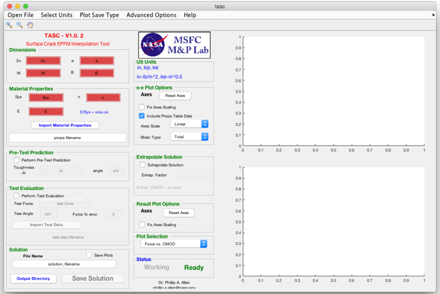
	Requirements	Tools
E8	Fit linear region of stress-strain curve, offset to find yield strength	Strip chart and calculator
E399	Fit linear region of force-CMOD curve, draw secant line with 5% less slope, calculate K with algebraic equations	Strip chart and calculator or spreadsheet
E1820	Calculate K with algebraic equations. EPRI method for J_{el} calculated from K, J_{pl} from area under load-displacement curve and algebraic equations	Spreadsheet

Tool for Analysis of Surface Cracks (TASC)

ASTM E2899 is unusual in its analysis requirements

- requires results of elastic-plastic finite element analysis
- other standards require much simpler calculations or graphical constructions
- NASA TASC program satisfies requirements, but only for tension

Mechanics of bending is more complex than for tension (constraint, crack closure)

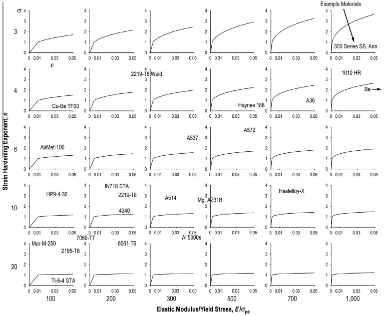


NASA TASC program

TASC: interpolation code using database of 600 EPFM results for flat plates in tension

20 normalized geometries:

$0.2 \leq \frac{a}{c} \leq 1.0,\ 0.2 \leq \frac{a}{t} \leq 0.8$



30 normalized materials:

$100 \leq \frac{E}{S_{ys}} \leq 1000,\ 3 \leq n \leq 20$

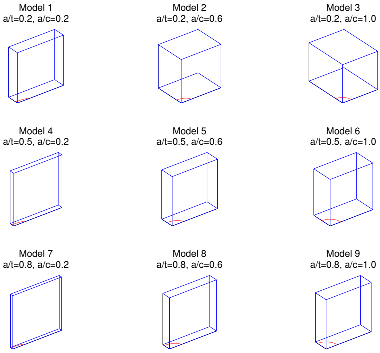
- Analytical and experimental methods both have inherent uncertainty
- Differences between predicted and measured behavior stem from two main causes:
  - repeatability and accuracy of measurements or calculations
  - using the correct models or procedures to include all significant physical effects

- First cause is resolved by performing **verification** tests (building the product right)
- Second cause is resolved by performing **validation** tests (building the right product)

Some V&V studies include:

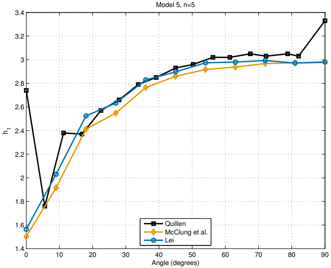
- Favanesi et al. (1994) comparing predictions from the NASCRAC software against FLAGRO and FRANC2D, and also against others' and their own experimental data
- Wilson (1995) identified problems with both NASCRAC and FLAGRO, including fracture properties that were treated as constants instead of functions of geometry and load
- McClung (2012) verified a K solution by comparing the FADD3D, FEACrack, and FLAGRO solvers—all were similar except for FEACrack at $\phi = 0$ and $\phi = \pi$

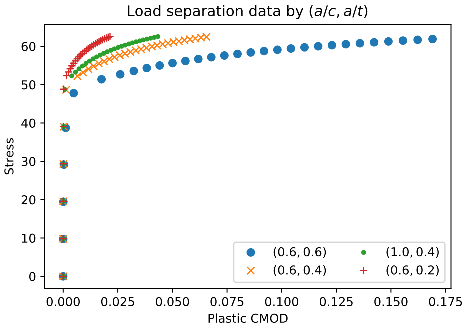
Modeling Preparation for Research Tasks



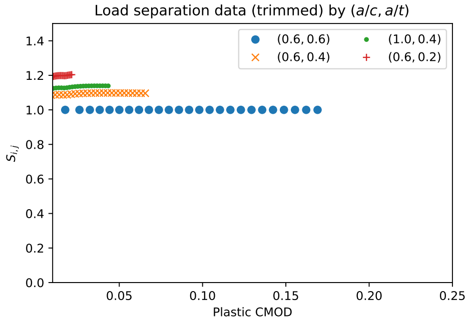
McClung et al. model geometry

$$h_1 = \frac{J_{\mathrm{pl}}}{\alpha \sigma_0 \epsilon_0 t \left(\frac{\sigma}{\sigma_0} \right)^{n+1}}$$



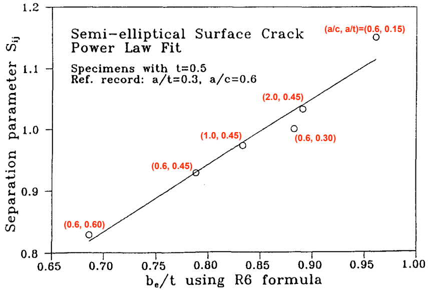


Tensile stress versus plastic CMOD

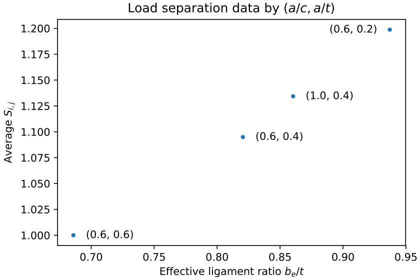


Separation parameter values versus plastic CMOD

$$\frac{b_e}{t} = 1 - \frac{\pi a}{2t\left[2 + \frac{a/c}{a/t}\right]}$$

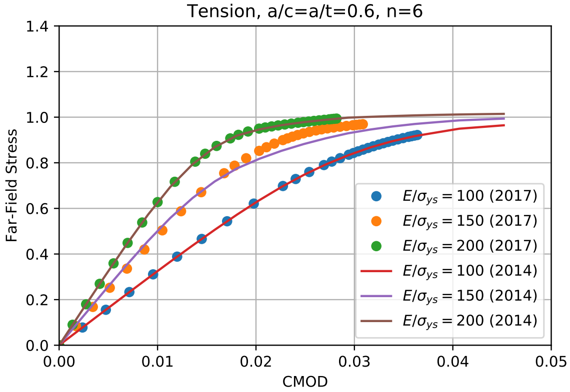


Annotated from Sharobeam and Landes, 1993



From current work

Applying Procedure to a New Material Model ($\frac{E}{\sigma_{ys}} = 150$)



Comparison of normalized FEA results, interpolated result, and TASC raw data

Research Plan for Bending Models and Modified TASC Program

$$t = 1 \quad 0.2 \leq \frac{a}{c} \leq 1.0 \quad 0.2 \leq \frac{a}{t} \leq 0.8$$
$$W = 5\max(c, t) \quad S_{\text{inner}} = W \quad S_{\text{outer}} = 2W \quad L = 1.1 S_{\text{outer}}$$

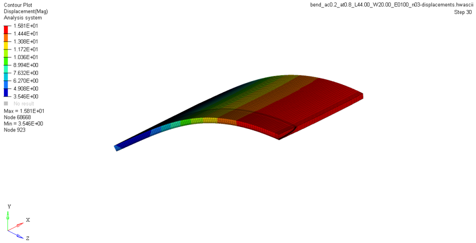


Plate model, $\frac{a}{c} = 0.2$, $\frac{a}{t} = 0.8$

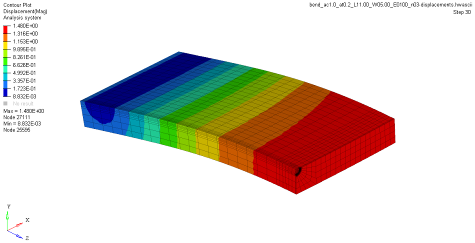
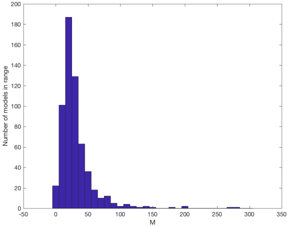
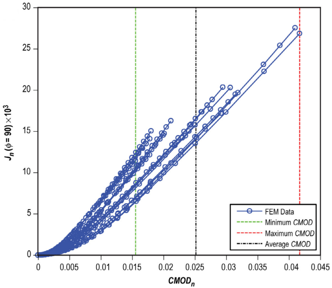


Plate model, $\frac{a}{c} = 1.0$, $\frac{a}{t} = 0.2$

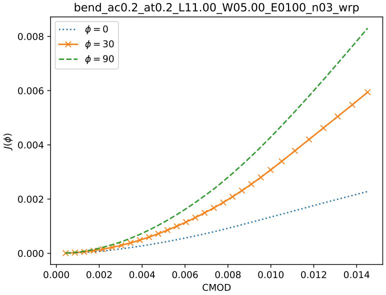
Allen and Wells (2014) reported $M = \frac{r_\phi \sigma_{ys}}{J} < 25$ for tension



Histogram of M results from TASC tension model database



J-CMOD graph used for extrapolation



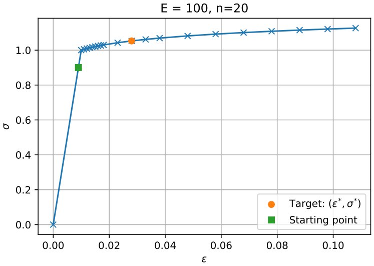
$\frac{a}{c} = 0.2,\ \frac{a}{t} = 0.2,\ E = 100,\ n = 3$

Adjust boundary conditions until

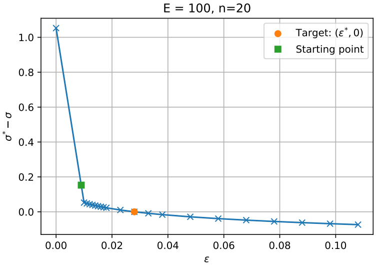
- slope of last 20% of J-CMOD curve is $20\times$ larger than initial slope
- slope of last 20% of J-CMOD curve is $< 10\%$ different than slope of previous 20%

at $\phi = 30^\circ$

Displacement control for tension models makes optimization easier

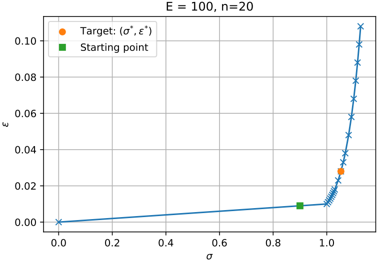


Example stress-strain curve using linear plus power law (LPPL) formulation

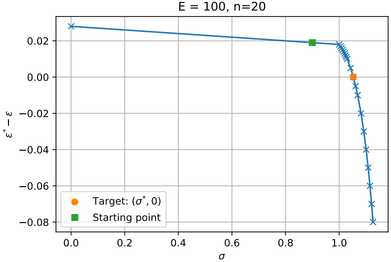


Transformed to find required strain level

Load control for bending models makes optimization more difficult

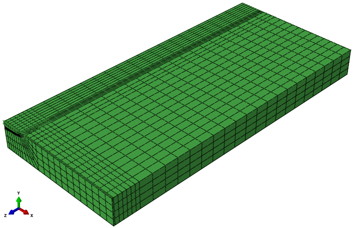


Example stress-strain curve using LPPL formulation, transformed to stress-controlled



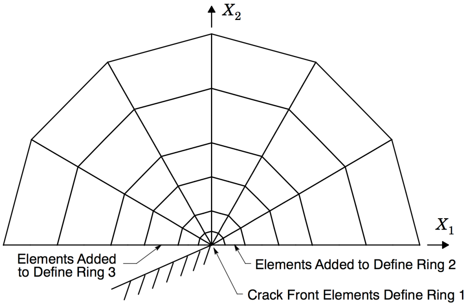
Transformed to find required stress level

Abaqus validation



Example Abaqus bending model from FEACrack

J convergence



Elements used in WARP3D J calculations

- Don't break anything already working for tension
- Make a `results_bending` database alongside the existing `results` database for tension
- Identify any equations only valid for tension models
- Replace with conditionals checking for model type, then use tension or bending equations as required
- Interpolation method should need no changes
- Validate a load-CMOD curve against existing bending experimental data

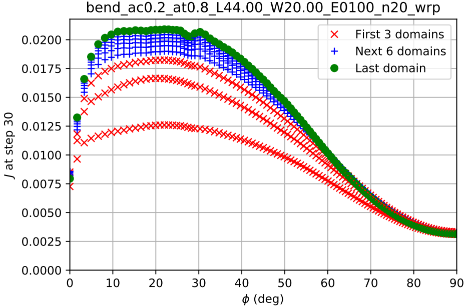
EPRI h_1

- Examine EPRI h_1 estimation method
- Abaqus with fully-plastic check
- WARP3D results intended for TASC

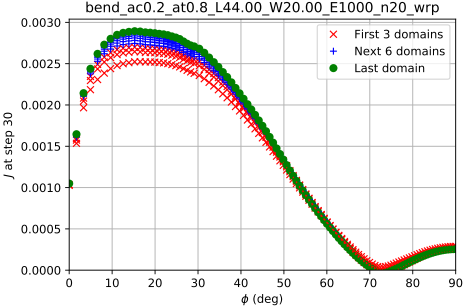
Load separation

- No published literature about load separation for surface cracks in bending
- Apply load separation to WARP3D results for $\frac{E}{\sigma_{ys}} = 500, n = 4$
- Use all 20 geometries for $0.2 \le \frac{a}{c} \le 1.0, 0.2 \le \frac{a}{t} \le 0.8$

Results and Discussion

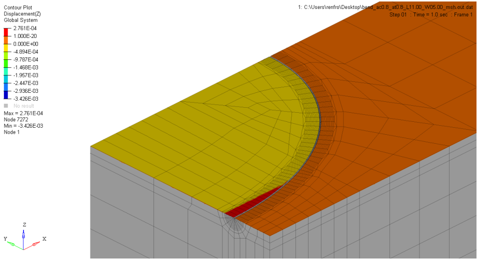


Convergence of J across 10 domains

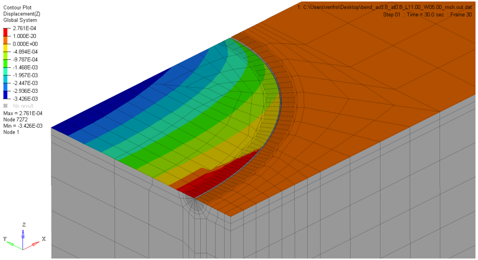


Anomalous J convergence graph

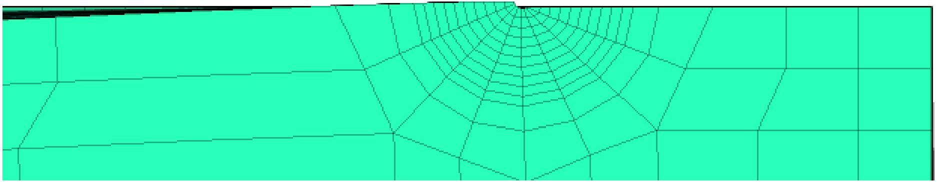
Why is J higher at $\phi = 90$ in some cases? What happens to plate deflection?



First load step



Last load step



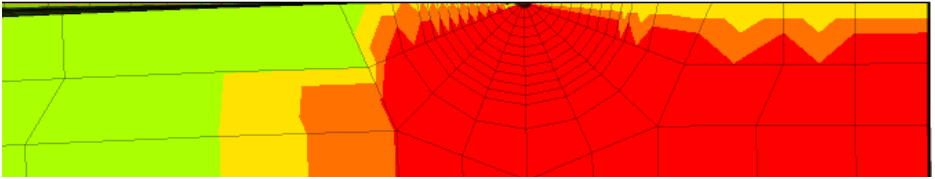
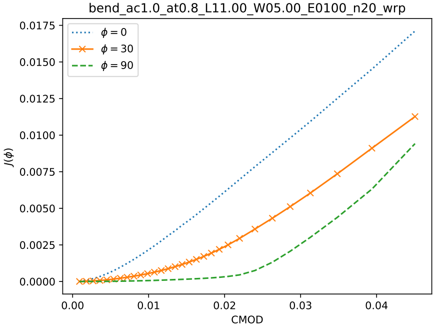
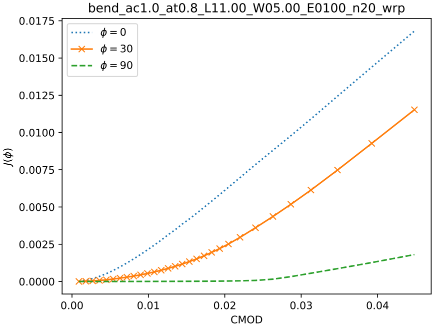


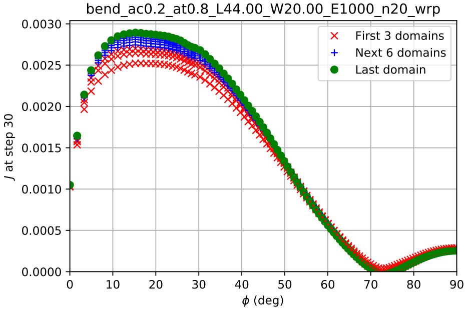
Plate deflection before and after addition of elastic boundary



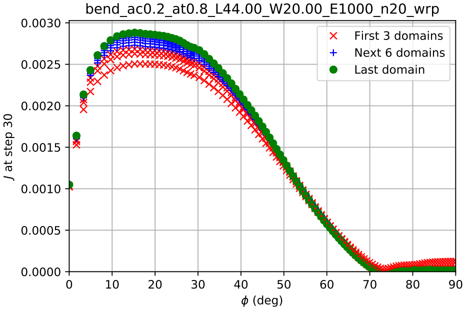
Before addition of elastic boundary



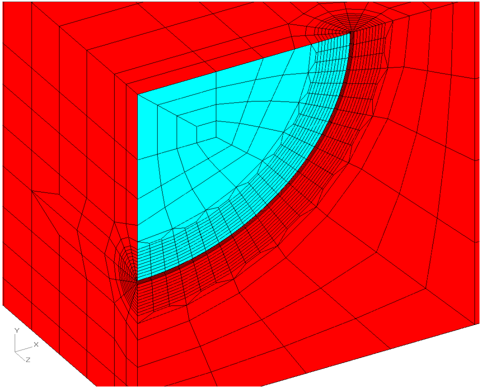
After addition of elastic boundary



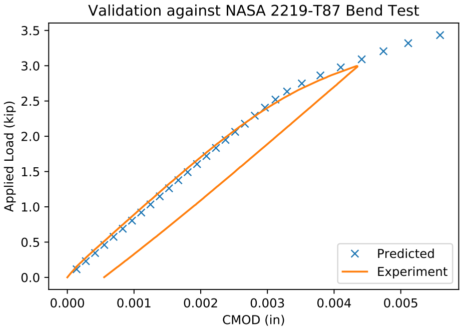
Before addition of elastic boundary



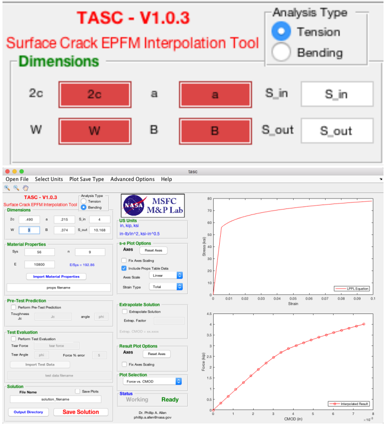
After addition of elastic boundary



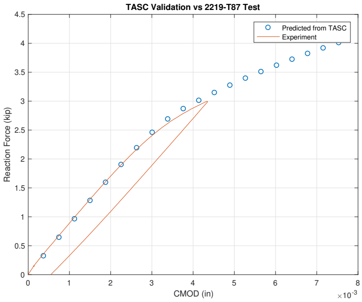
Crack front mesh of purpose-built experimental validation model



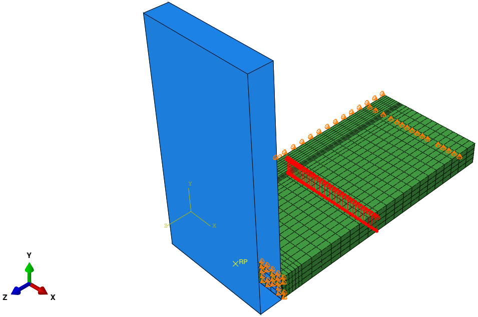
Comparison of predicted load and CMOD between purpose-built model and experiment



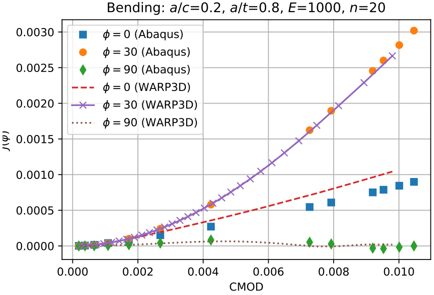
Modified TASC Program



Comparison of TASC and experiment

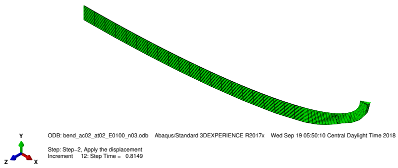


Sample Abaqus validation model

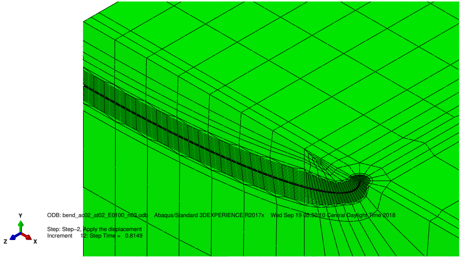


Sample J comparison between Abaqus and WARP3D

Previous tension cases used all elements in crack plane for fully-plastic check. Bending cases must use a smaller set since so many elements will be near the neutral axis.

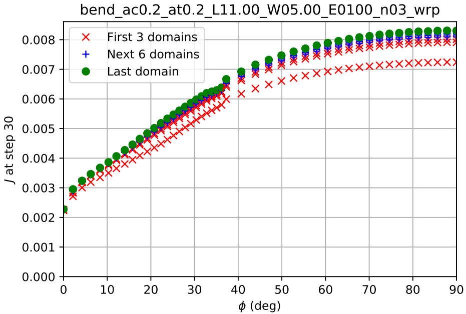


Element set used for fully-plastic check

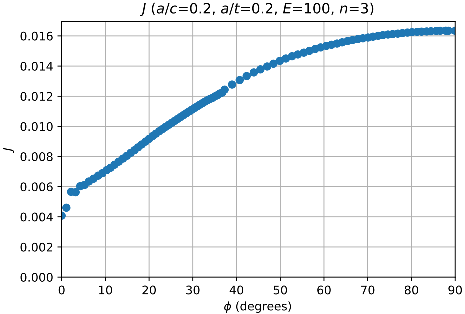


All elements shown for context

Abaqus J values are roughly double WARP3D values due to higher boundary condition values

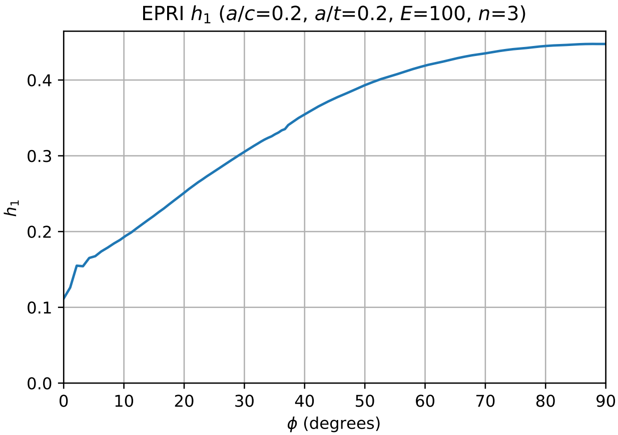


WARP3D

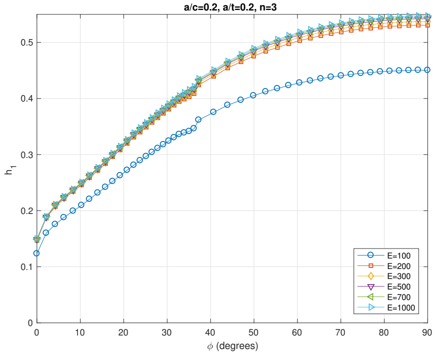


Abaqus with fully-plastic checks

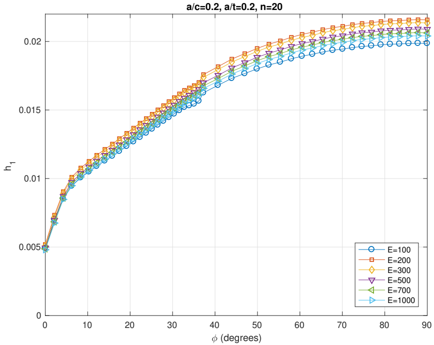
Abaqus h_1 values scale with J as expected



h_1 results from WARP3D are E-dependent to various degrees



$\frac{a}{c} = 0.2, \frac{a}{t} = 0.2, n = 3$

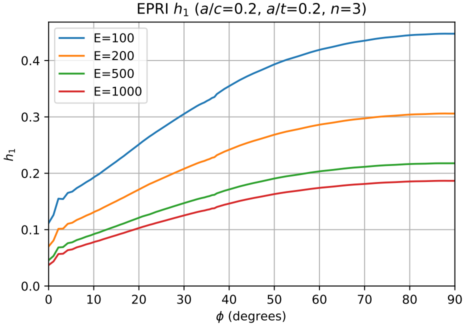


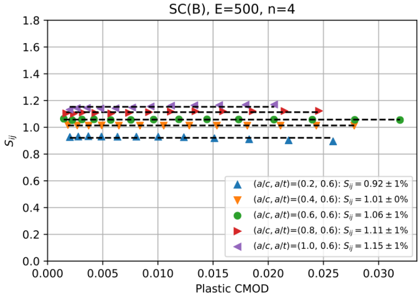
$\frac{a}{c} = 0.2, \frac{a}{t} = 0.2, n = 20$

Abaqus h_1 values with fully-plastic check are E-dependent

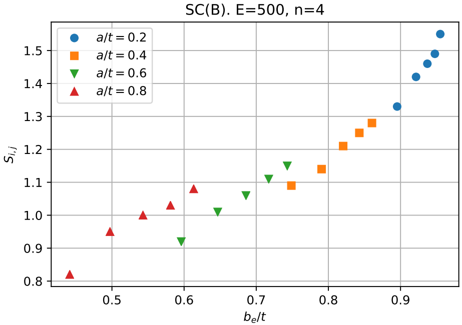
Needed higher stress to achieve fully-plastic state

- $E = 100, \sigma = 1.64$
- $E = 200, \sigma = 2.27$
- $E = 500, \sigma = 3.36$
- $E = 1000, \sigma = 4.41$

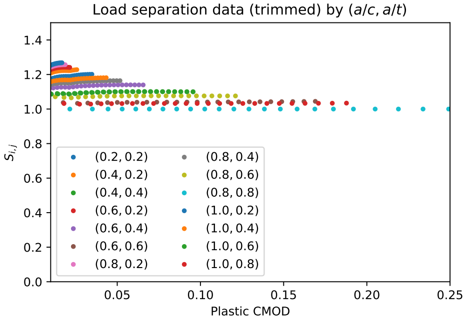




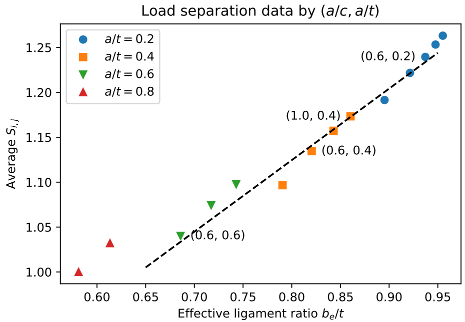
Variation of $S_{i,j}$ for cracked plates in bending, $\frac{a}{t} = 0.6$



Variation of $S_{i,j}$ versus effective uncracked ligament length (bending, $0.2 \le \frac{a}{t} \le 0.8$)



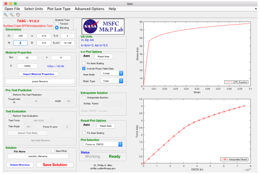
Separation parameters for cracked plates in tension

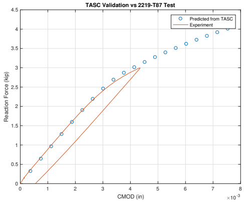


Variation in $S_{i,j}$ versus effective uncracked ligament length (tension, $0.2 \leq \frac{a}{t} \leq 0.8$)

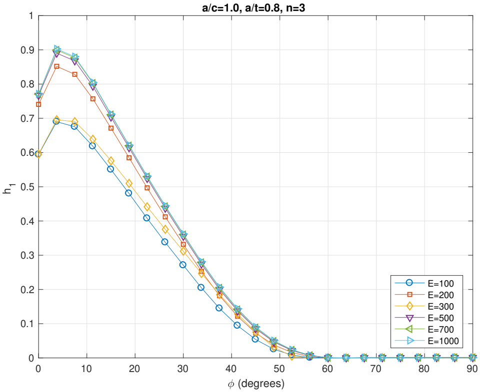
Conclusions and Recommendations for Future Work

1. Database of 600 elastic-plastic finite element results for surface cracks in bending
   - More challenging than tension models
   - Subset of results verified and validated against Abaqus and experimental data
   - Modified TASC program
   - Possible to satisfy requirements of ASTM E2899 for tension **and** bending without constructing purpose-built EPFM models
   - Greatly reduces analytical burden on anyone doing ASTM E2899 tests

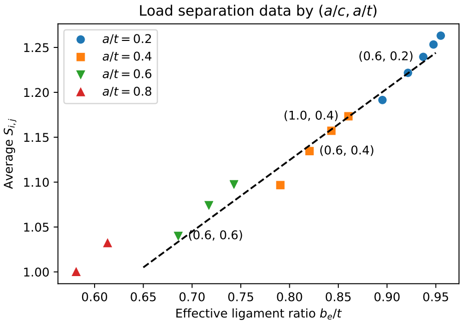




2. h_1 curve may not always be independent of E
   - But $h_1 = \dfrac{J_{\text{pl}}}{\alpha \sigma_0 \epsilon_0 t \left(\frac{\sigma}{\sigma_0}\right)^{n+1}}$ has no E, and neither do any of the handbook curves
   - Abaqus fully-plastic checks are not a cure-all for finding h_1



3. Load separation does not collapse to a single key curve
   - Load separation data is self-similar at a single depth, but not across multiple depths
   - Applies to surface cracks in both tension and bending



1. TASC and E2899
   - Finish integrating bend data into TASC, beyond load-CMOD validation
   - Additional values for material and/or crack geometry
   - Investigate other interpolation methods (piecewise cubic?)
   - Replace traction boundary conditions with rigid rollers and contact modeling
2. EPRI h_1
   - Further investigation into limits of h_1 estimates
   - Geometry, boundary conditions (types and magnitudes), materials
3. Load separation
   - Alternative to R6 $\frac{b_e}{t}$ that drives $S_{i,j}$ to a single curve
4. **More experimental data needed for all of these.**

Thanks to:

- Committee: Dr. Chris Wilson, Dr. Brian O'Connor, Dr. Sally Pardue, Dr. Guillermo Ramirez, Dr. Dale Wilson
- Center for Manufacturing Research (Mechanical Properties Testing, Materials Characterization, Computer Aided Engineering)
- Information Technology Services (High Performance Computing)
- Dr. Phillip Allen and ASTM Committee E08 on Fatigue and Fracture
- Friends and family

Appendix

- Introduction to Fracture Mechanics
- Initial Verification of Quillen Models
- Initial Verification of Two Tension Cases from Allen and Wells (2014)

Introduction to Fracture Mechanics

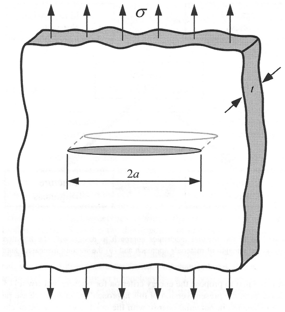
$$\mathcal{G} = -\frac{d\Pi}{dA} = \frac{\pi \sigma_{\mathrm{t}}^2 a}{E'} = \frac{K_{\mathrm{I}}^2}{E'}$$

where

$$A = 2at = \text{area of cracked surfaces}$$

$$E' = \begin{cases} E & \text{for plane stress} \\ \frac{E}{(1-\nu^2)} & \text{for plane strain} \end{cases}$$

$$K_{\mathrm{I}} = \sigma_{\mathrm{t}} \sqrt{\pi a}.$$



$$\sigma_{xx}(r,0) = \sigma_{yy}(r,0) = \sigma_1(r,0) = \sigma_2(r,0) = \frac{K_{\mathrm{I}}}{\sqrt{2\pi r}}$$

A through crack in an infinite plate loaded in tension

$$r_{\mathrm{y}} = \begin{cases} \frac{1}{2\pi}\left(\frac{K_{\mathrm{I}}}{\sigma_{\mathrm{ys}}}\right)^2 & \text{for plane stress} \\ \frac{1}{6\pi}\left(\frac{K_{\mathrm{I}}}{\sigma_{\mathrm{ys}}}\right)^2 & \text{for plane strain.} \end{cases}$$

$$r_{\mathrm{p}} = \begin{cases} \frac{1}{\pi}\left(\frac{K_{\mathrm{I}}}{\sigma_{\mathrm{ys}}}\right)^2 & \text{for plane stress} \\ \frac{1}{3\pi}\left(\frac{K_{\mathrm{I}}}{\sigma_{\mathrm{ys}}}\right)^2 & \text{for plane strain.} \end{cases}$$

$$a_{\mathrm{app}} = a + r_{\mathrm{y}} = a + \frac{r_{\mathrm{p}}}{2}$$

$$K_{\mathrm{Iapp}} = \sigma_{\mathrm{t}}\sqrt{\pi a_{\mathrm{app}}}$$

at $\phi = \frac{\pi}{2}$

$$K_\mathrm{I} = 1.1\sigma_\mathrm{t}\sqrt{\pi a}\left[F\left(\frac{a}{c}, \frac{\sigma_\mathrm{t}}{\sigma_\mathrm{ys}}\right)\right]$$

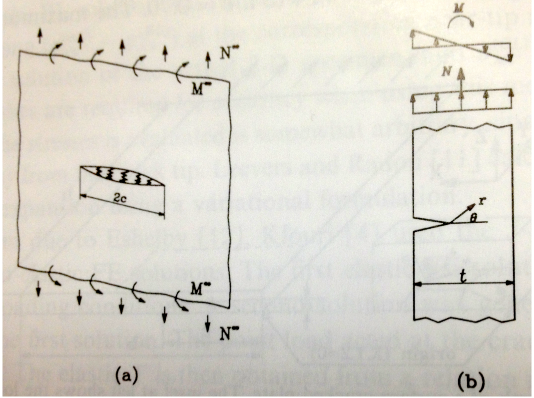
where

$$F\left(\frac{a}{c}, \frac{\sigma_\mathrm{t}}{\sigma_\mathrm{ys}}\right) = \left[\Phi^2 - 0.212\left(\frac{\sigma_\mathrm{t}}{\sigma_\mathrm{ys}}\right)^2\right]^{-0.5}$$

$$\Phi^2 = Q = 1 + 1.464\left(\frac{a}{c}\right)^{1.65}$$

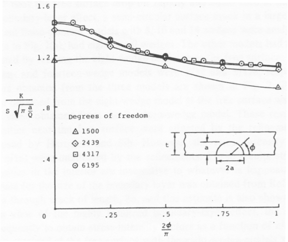
$$\Phi = \int_0^{\frac{\pi}{2}} \sqrt{\left[\sin^2\phi + \left(\frac{a}{c}\right)^2\cos^2\phi\right]}\, d\phi$$

$\frac{c}{W} \ll 1,\ \frac{a}{c} < 1,\ \frac{a}{t} < 0.5$

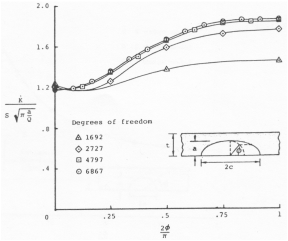


Line spring model (Wang and Parks, 1992). (a) Through-cracked plate with line-spring of width $2c$, (b) Part-through-cracked plane strain plate of comparable compliance to line-spring plate

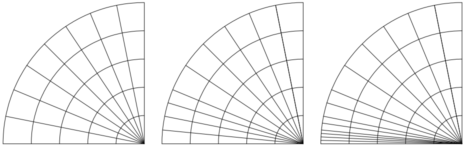
$$K = K_{\mathrm{I}} = \sigma_t \sqrt{\left(\pi \frac{a}{Q}\right) F\left(\frac{a}{t}, \frac{a}{c}, \phi\right)}$$



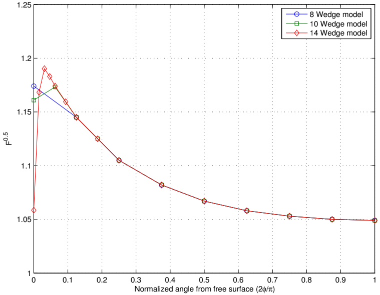
$Q = \frac{\pi^2}{4}$, $\frac{a}{t} = 0.8$



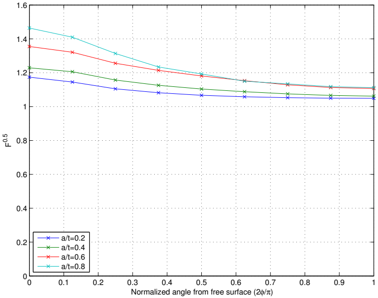
$Q = 1.104$, $\frac{a}{t} = 0.8$, $\frac{a}{c} = 0.2$



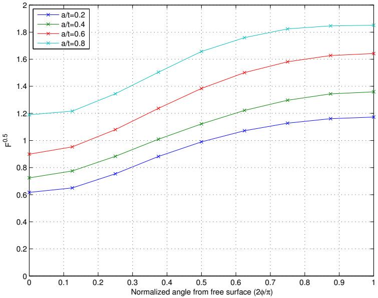
Comparison of wedge sizes used in estimating boundary-layer effect



Effects of mesh refinement near the free surface on the distribution of boundary-correction factors for semi-circular surface crack $\left(Q = \frac{\pi^2}{4};\ \frac{a}{t} = 0.2\right)$

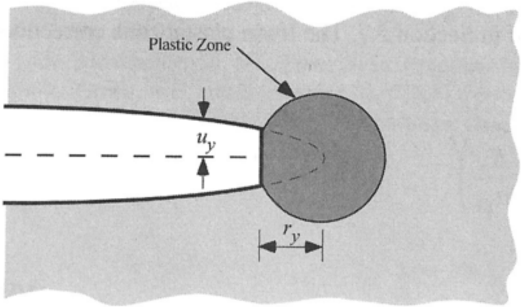


$$Q = \frac{\pi^2}{4}$$



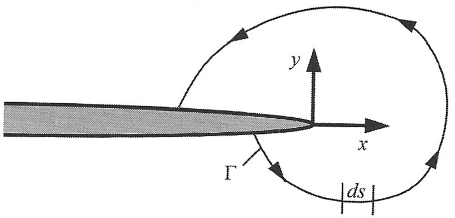
$$Q = 1.104, \ \frac{a}{c} = 0.2$$

$$K_{\mathrm{I}} = \sigma_{\mathrm{t}} \sqrt{F\left(\frac{a}{t}, \frac{a}{c}, \phi\right)\left(\pi \frac{a}{Q}\right)}$$



Estimation of CTOD from displacement field of crack with Irwin plastic radius correction

$$\delta_t = 2u_y = \frac{8}{E'} K_I \sqrt{\frac{r_y}{2\pi}}$$



$$J = \int_\Gamma \left(w\,dy - T_i \frac{\partial u_i}{\partial x}\,ds \right)$$

where

w = strain energy density

T_i = components of traction vector

u_i = components of displacement vector

ds = differential length along Γ

$$J = -\frac{d\Pi}{dA}$$

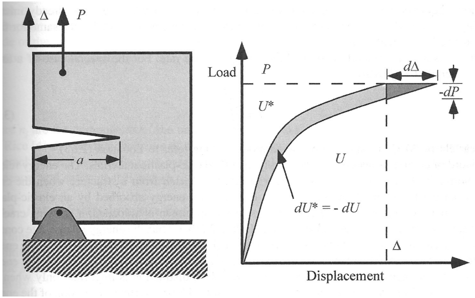
where

$$dA = \begin{cases} t\,da & \text{edge-cracked plate} \\ 2t\,da & \text{center-cracked plate} \\ 2\pi a\,da & \text{embedded circular crack} \end{cases}$$

and

$$\Pi = U - F$$

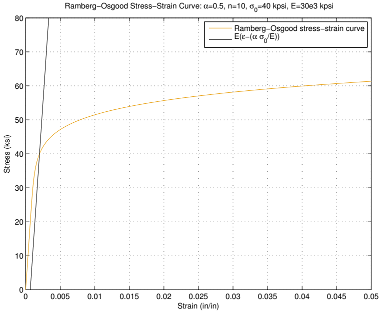
where U is the strain energy stored in the body and F is the work done by the applied load P.



$$\frac{\epsilon}{\epsilon_0} = \frac{\sigma}{\sigma_0} + \alpha \left(\frac{\sigma}{\sigma_0} \right)^n$$

where

$$\sigma_0 = \text{reference stress value}$$
$$\epsilon_0 = \frac{\sigma_0}{E}$$
$$\alpha = \text{yield offset parameter, often 0.5}$$
$$n = \text{strain hardening exponent}$$



Initial Verification of Quillen Models

```
! Define parameters
E=30.0e6
NU=0.3
! Set material properties
MP,EX,1,E
MP,PRXY,1,NU
```

APDL syntax

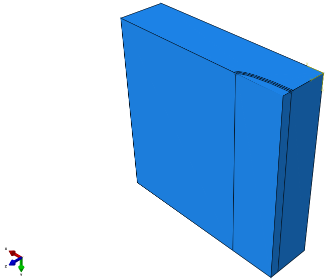
```
** Define parameters
*PARAMETER
E=30.0e6
NU=0.3
** Set material properties
*MATERIAL, NAME=STEEL
*ELASTIC
<E>, <NU>
```

Abaqus keyword syntax

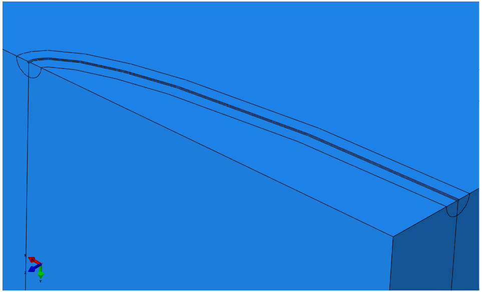
```
def setMaterial(model, E=200e9, nu=0.3): # a function
    model.Material(name='Steel')
    model.materials['Steel'].Elastic(table=((E, nu), ))
    # function ends here, main program follows

elasticModulus=30e6 # define parameters
PoissonRatio=0.3
# call functions to create model, set material
myModel = createModel(modelName='McClung-1')
setMaterial(E=elasticModulus,
            nu=PoissonRatio,
            model=myModel)
```

Abaqus Python syntax

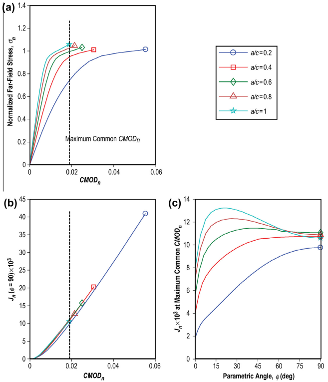


McClung et al. model 1

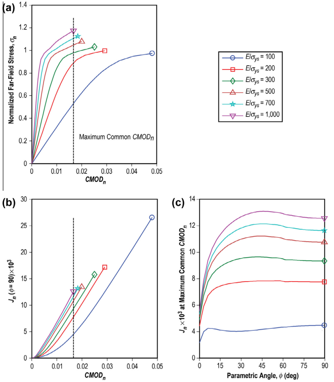


McClung et al. model 1 crack front detail

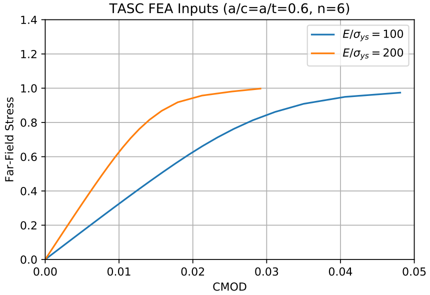
Initial Verification of Two Tension Cases from Allen and Wells (2014)



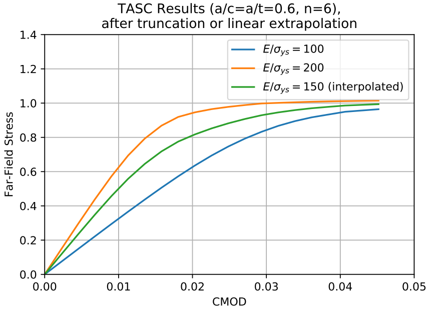
Gap in results for widest aspect ratios



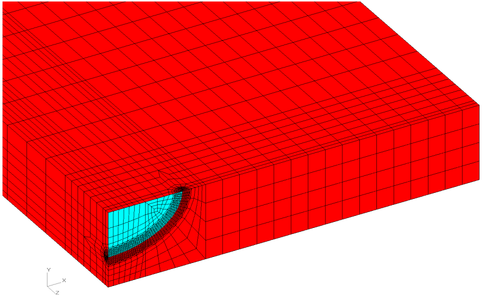
Gap in results for lowest E values



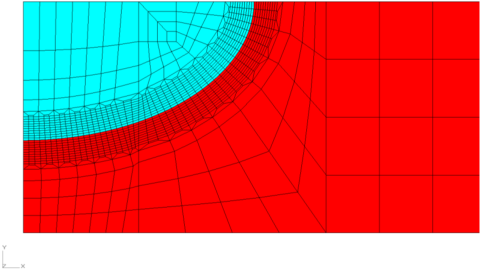
Raw FEA results used in TASC



Interpolated FEA results displayed by TASC



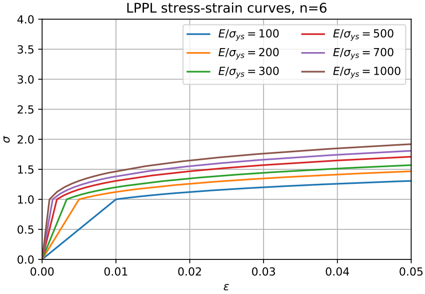
Isometric view of overall mesh



Detailed view of crack front

$$\frac{\epsilon}{\epsilon_{\mathrm{ys}}} = \begin{cases} \dfrac{\sigma}{\sigma_{\mathrm{ys}}}, & \epsilon \leq \epsilon_{\mathrm{ys}} \\ \left(\dfrac{\sigma}{\sigma_{\mathrm{ys}}}\right)^{n}, & \epsilon > \epsilon_{\mathrm{ys}} \end{cases}$$

where $\epsilon_{\mathrm{ys}} = \frac{\sigma_{\mathrm{ys}}}{E}$.



Set of LPPL stress-strain curves

$$M = \frac{r_\phi \sigma_{\text{ys}}}{J}$$

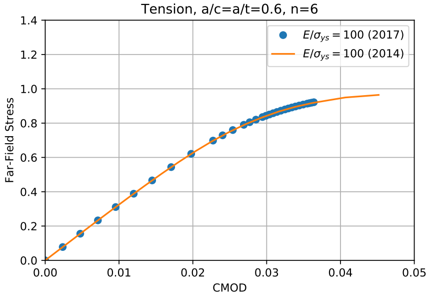
Applied displacement values for verification models

$\frac{E}{\sigma_{\text{ys}}}$	Displacement	ϕ	M using $r_{\phi a}$	M using $r_{\phi b}$
100	0.1028	30°	15.9833	36.4241
		90°	22.6234	15.0822
200	0.0550	30°	24.7288	56.3542
		90°	34.9604	23.3069

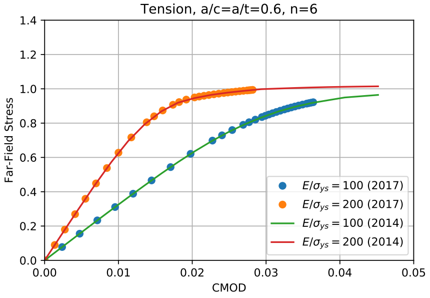
- 30 load steps
- `warp3d < file.inp > file.out`
- 21.6 minutes to solve on laptop, 2.2 minutes on HPC node

Python program

- run `packet_reader` to export displacements, forces
- extract node 1 z displacement, double to get CMOD
- identify nodes on $z = 0$ from input file
- extract z reactions from all identified nodes, sum to reaction force
- divide reaction force by plate cross section area to get stress



Verification of stress and CMOD relationship for first model



Verification of stress and CMOD relationship for second model