Hill_postulae

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Overview

Some of what makes one ℓ value more predictive of function than another is about biology, other parts are about math. This is a live document to record some quantitative relationships (both formal and informal) about Hill diversity that may lead to clearer interpretations of empirical findings.

Hill number definition

We define Hill diversity D as the mean species rarity in the assemblage

$$D = \left(\sum_{i=1}^{S} p_i (r_i)^{\ell} \right)^{1/\ell} \tag{1}$$

where D is diversity or mean rarity, p_i is the relative abundance of species i, r_i is the rarity of species i (defined as the reciprocal of p_i), S is the total species richness, and ℓ is the scaling exponent that determines the type of mean computed [@Roswell2021].

Hill diversity is more commonly written as

$$D = \left(\sum_{i=1}^{S} p_i^{q} \right)^{1/1-q} \tag{2}$$

When $\ell = 0$ ($q = 1$), these equations are defined by their limit.

Equations 1 and 2 are equivalent when $\ell = 1 - q$.

Ranges of Hill numbers (assuming perfect observation)

For a given combination of S, N (i.e. number of individuals), and ℓ, there is a range of values that D can possibly take. When ℓ is large, this range is also large, and when it is a large negative number, the range is much smaller, when $\ell = 1$ this range is at its minimum, $D = S$, for any N, distribution of relative abundance.

For $\ell \geq 1$, the minimum value D can take is S (maximum evenness). As $\ell \to \infty$, the maximum value of D grows to the maximum species rarity, i.e. N (maximum dominance).

For $\ell < 1$, the *maximum* value D can take is S (maximum evenness). As $\ell \to -\infty$, the minimum value of D shrinks to the minimum species rarity, i.e. something nearing 1 when $N >> S$ (maximum dominance).

D-flipping

For a given S, N, the spearman rank correlation between the Hill diversity of a set of assemblages when $\ell = \infty$ and when $\ell = -\infty$ is -1. More simply, the even assemblage has maximum diversity for $\ell < 1$ and minimum diversity when $\ell > 1$.

interpretations

When $\ell \to \infty$, D conveys information primarily about *abundance*

When $\ell \to -\infty$, D conveys information primarily about *dominance* (but not really evenness, which is complicated)

When the predictive ability of D is maximized when $\ell \neq (-\infty, 1, \infty)$ it is hard to say what aspects of the SAD are most salient.