Start with the normalized SIR:

$$i = Bxy; \dot{x} = -i; \dot{y} = i - y$$

If C is the case reproductive number for an individual (the same for all individuals infected at an instant), then the overall mean is:

$$\bar{C} = \frac{\int dt\, i(t) C(t)}{\int dt\, i(t)} = \frac{\int dt\, i(t) C(t)}{Z},$$

where Z is the size of the epidemic.

We can write

$$C(t) = B \int d\delta\, x(t + \delta) \exp(-\delta)$$

If we also solve the $\dot{y}$ equation and write

$$y(t) = \int d\tau\, i(t) \exp(-\tau)$$

and had any patience with integrals, we could presumably expand and substitute and get something like:

$$Z\bar{C} = B \int dt\, x(t) y(t) = \int dt\, i(t).$$

This seems to be what I wanted, but also seems to be a dead end. It's hard to imagine any tricks to try if we squared the whole integral for $C(t)$ and plugged *that* in.