Start with the normalized SIR:

$$i = Bxy; \dot{x} = -i; \dot{y} = i - y$$

If $C(t)$ is the case reproductive number for an individual infected at time t, then the overall mean is:

$$\bar{C} = \frac{\int dt\, i(t)C(t)}{\int dt\, i(t)} = \frac{\int dt\, i(t)C(t)}{Z},$$

where Z is the size of the epidemic.

We can integrate over time since infection δ, and write:

$$C(t) = B \int d\delta\, x(t+\delta)\exp(-\delta)$$

If we also solve the $\dot{y}$ equation, by looking at who has come in τ time ago and how many of them survived, and write:

$$y(t) = \int d\tau\, i(t-\tau)\exp(-\tau)$$

and had any patience with integrals, we could presumably expand and substitute and get something like:

$$Z\bar{C} = B \int dt\, x(t)y(t) = \int dt\, i(t).$$

This seems to be what I wanted, but also seems to be a dead end. It's hard to imagine any tricks to try if we squared the whole integral for $C(t)$ and plugged *that* in.