

**The square of a directed  $G=(V,E)$  is the graph  $G^2=(V,E^2)$  such that  $\langle u,v \rangle \in E^2$  if and only if  $G$  contains a path with at most two edges between  $u$  and  $v$ . Describe efficient algorithms for computing  $G^2$  from  $G$  for the adjacency list and adjacency matrix representations for  $G$ . Analyze the running time of your algorithm.**

### **Adjacency Matrix:**

In order to compute the adjacency matrix for  $G^2$ , you would need to square the adjacency matrix of  $G$ .

For  $i = 1$  to  $n$

    For  $j = 1$  to  $n$

$G^2[i][j] = 0$  # Create adjacency matrix  $G^2$

For  $i = 1$  to  $n$

    For  $j = 1$  to  $n$

        If  $G[i][j] == 1$

            For  $k = 1$  to  $n$

                If  $(g[j][k] == 1)$

$G^2[i][k] == 1$

The first 2 loops will run with complexity  $O(n^2)$ . The next 2 loops will also run in  $O(n^2)$ , but with the added  $n$  iterations to test all edges needed. The final running time of this algorithm will be  $O(n^3)$ .

### **Adjacency List:**

For vertex in Adjacency List  $U$

    For vertex in Adjacency List  $V$

        If  $\text{edge}(u, v)$  exists in  $E^2$

            Insert  $v$  into Adjacency List  $U^2$

For every edge in the Adjacency List  $U$ , we can up to  $v$  vertexes. The final running time of this algorithm is  $O(V \cdot E)$ .