Yuchen Shi
6962247146

Experimental Project

1. Background:

π is a famous mathematical constant, and it is defined as the ratio of a circle's circumference to its diameter. It is an irrational number, which means it cannot be expressed exactly as a common fraction. It has infinite decimals, and its decimals have no pattern. This experimental project will address the question whether the decimal expansion of π appear in random order. In other words, as we get more and more digits in the expansion, is every of the ten digits equally likely to appear?

2. Assumptions:

Since the decimals are infinite, we cannot count all of the decimals. We could pick a sample of the decimals that could represent the whole set of decimals. I assume the distribution of the decimals is uniform, so that if we pick the first 1000 digits of π, it is equivalent as we randomly choose 1000 digits from all the decimals.

3. Null hypothesis: The digits in the decimal expansion of π appear in random order.
Alternative hypothesis: The digits in the decimal expansion of π do not appear in random order.

4. Experiment:

I would search online for the first 1000 digits of π, and then copy them into python. And then I would use a for loop to count the number of 0, 1, 2, 3…9 in this 1000 digits. Under my null hypothesis, I would expect the number of each digit to be 100. Because 1000 is a pretty large number, and the expected number of observations for each group is 100, which is also large enough, so I think this sample is representative. Combining with the result we get from python, I would use a chi-square test to test whether the distribution of digits fits the model that I expect. If the chi-squared statistic we calculated is larger than the critical value we get from the chi-square table, we could reject the null hypothesis and conclude the digits do not appear in random order. If the chi-squared statistic we calculated is not larger than the critical value, we fail to reject the null hypothesis and we could say the digits appear in random order.

5.

Python code:

```
digits="1415926535 8979323846 2643383279 5028841971 6939937510 5820974944 5923078164
0628620899 8628034825 3421170679 8214808651 3282306647 0938446095 5058223172
5359408128 4811174502 8410270193 8521105559 6446229489 5493038196 4428810975
6659334461 2847564823 3786783165 2712019091 4564856692 3460348610 4543266482
1339360726 0249141273 7245870066 0631558817 4881520920 9628292540 9171536436
7892590360 0113305305 4882046652 1384146951 9415116094 3305727036 5759591953
0921861173 8193261179 3105118548 0744623799 6274956735 1885752724 8912279381
8301194912 9833673362 4406566430 8602139494 6395224737 1907021798 6094370277
0539217176 2931767523 8467481846 7669405132 0005681271 4526356082 7785771342
7577896091 7363717872 1468440901 2249534301 4654958537 1050792279 6892589235
4201995611 2129021960 8640344181 5981362977 4771309960 5187072113 4999999837
```

2978049951 0597317328 1609631859 5024459455 3469083026 4252230825 3344685035
2619311881 7101000313 7838752886 5875332083 8142061717 7669147303 5982534904
2875546873 1159562863 8823537875 9375195778 1857780532 1712268066 1300192787
6611195909 2164201989"

```
num0=0
num1=0
num2=0
num3=0
num4=0
num5=0
num6=0
num7=0
num8=0
num9=0


for c in digits:
    if c == "0":
        num0 += 1
    elif c == "1":
        num1 += 1
    elif c == "2":
        num2 += 1
    elif c == "3":
        num3 += 1
    elif c == "4":
        num4 += 1
    elif c == "5":
        num5 += 1
    elif c == "6":
        num6 += 1
    elif c == "7":
        num7 += 1
    elif c == "8":
        num8 += 1
    elif c == "9":
        num9 += 1

print(num0)
print(num1)
print(num2)
print(num3)
print(num4)
```

```
print(num5)
print(num6)
print(num7)
print(num8)
print(num9)
```

Python output:

```
93
116
103
102
93
97
94
95
101
106
```

6. Test Statistic: X^2=(93-100)^2/100+(116-100)^2/100+(103-100)^2/100+(102-100)^2/100+(93-100)^2/100+(97-100)^2/100+(94-100)^2/100+(95-100)^2/100+(101-100)^2/100+(106-100)^2/100=4.74

7. P-value=P(X^2>4.74)=0.856 (degrees of freedom=10-1=9)
As a result, we fail to reject the null hypothesis at any significance level lower than 85.6%.

For example, if we choose significance level of 5%:
Critical value: X^2 with a=0.05, degree of freedom=10-1=9: 16.9190
Since the test statistic we calculated is smaller than the critical value, we fail to reject the null hypothesis at 5% significance level.

8. To sum up, we are confident to say the digits in the decimal expansion of π appear in random order. In other words, as we get more and more digits in the expansion, every of the ten digits 0, 1, …, 9 is equally likely to appear.