Question 2: Find the Circle

Equation of Circle: $(x - x_c)^2 + (y - y_c)^2 = r^2$

Equations:

1. $(0 - x_c)^2 + (0 - y_c)^2 = r^2$
2. $(-2 - x_c)^2 + (4 - y_c)^2 = r^2$
3. $(-2 - x_c)^2 + (-4 - y_c)^2 = r^2$

Reduce to Two Equations:

Equation 2 Minus Equation 1:

$$\begin{array}{l} (-2 - x_c)^2 + (4 - y_c)^2 = r^2 \\ (0 - x_c)^2 + (0 - y_c)^2 = r^2 \\ \hline (-2 - x_c)^2 + (4 - y_c)^2 - (-x_c)^2 - (-y_c)^2 = 0 \\ x_c^2 + 4x_c + 4 - x_c^2 + y_c^2 - 8y_c + 16 - y_c^2 = 0 \\ 4x_c + 4 - 8y_c + 16 = 0 \\ 4x_c - 8y_c + 20 = 0 \end{array}$$

Equation 3 Minus Equation 1:

$$\begin{array}{l} (-2 - x_c)^2 + (-4 - y_c)^2 = r^2 \\ (0 - x_c)^2 + (0 - y_c)^2 = r^2 \\ \hline (-2 - x_c)^2 + (-4 - y_c)^2 - (-x_c)^2 - (-y_c)^2 = 0 \\ x_c^2 + 4x_c + 4 - x_c^2 + y_c^2 + 8y_c + 16 - y_c^2 = 0 \\ 4x_c + 4 + 8y_c + 16 = 0 \\ 4x_c + 8y_c + 20 = 0 \end{array}$$

Resulting Two Equations:

1. $4x_c - 8y_c + 20 = 0$
2. $4x_c + 8y_c + 20 = 0$

Solve for Center Coordinates (x_c, y_c):

Solve for y_c:

$$\begin{array}{l} 4x_c - 8y_c + 20 = 0 \\ 4x_c + 8y_c + 20 = 0 \\ \hline -16y_c = 0 \\ y_c = 0 \end{array}$$

Sub $y_c = 0$, Solve for x_c:

$4x_c - 8(0) + 20 = 0$

$4x_c + 20 = 0$

$x_c = -5$

Center Coordinates: (-5, 0)

Solve for Radius:

$(0 - x_c)^2 + (0 - y_c)^2 = r^2$

$(0 - (-5))^2 + (0 - 0)^2 = r^2$

$5^2 + 0^2 = r^2$

$25 = r^2$

$r = 5$

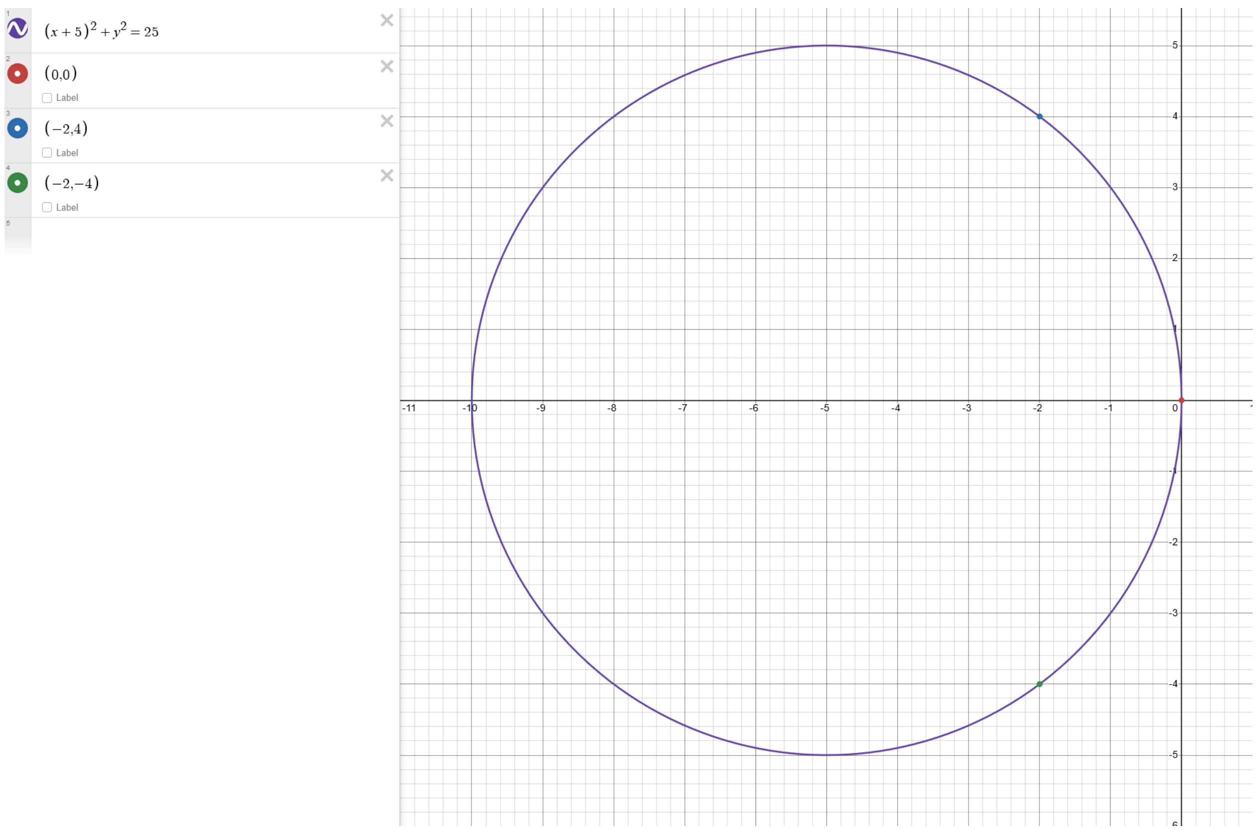
Circle:

Center: (-5, 0)

Radius: $r = 5$

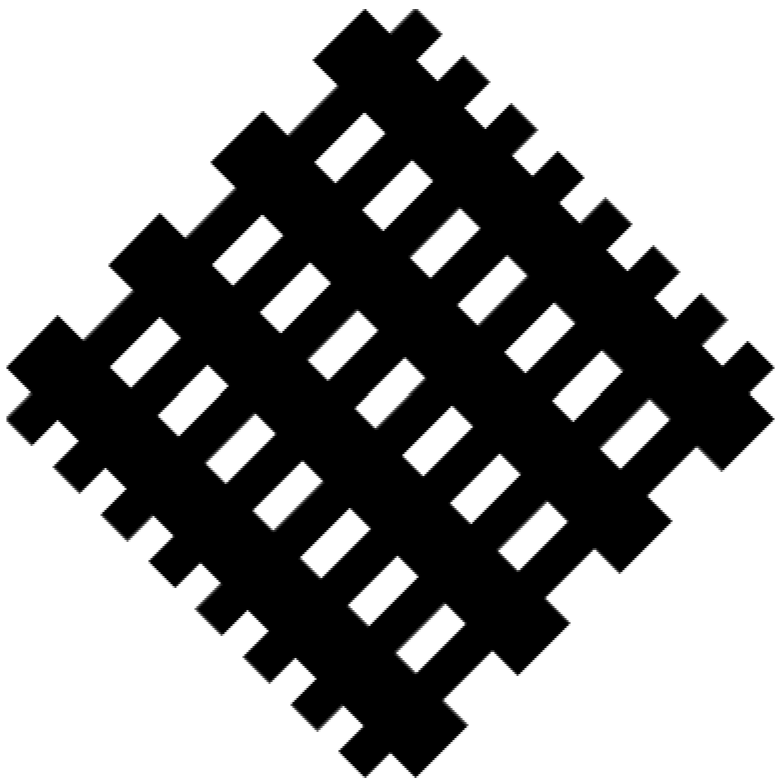
Equation: $(x + 5)^2 + y^2 = 25$

Graph:



Question 3: Fourier Trouble

(a) The two maxima in the first and third quadrants correspond to diagonal lines in "backslash" orientation. The other two maxima in the second and fourth quadrants correspond to diagonal lines in "forward slash" orientation. The maxima in the second and fourth quadrants are further away from the center of the Fourier-transformed image, representing higher frequencies that appear as more densely packed lines than the maxima of the first and third quadrants. The pattern is the superimposition of the two repetitive line patterns. Below is a rough sketch of the pattern, but the actual pattern would span indefinitely and have smoothed-out edges.



(b) The maxima are near the center of the magnitude spectrum, indicating lower frequencies that would appear as broad and wide-spaced lines in the original pattern. The maxima seem to be randomly placed but they cover many different angles. Thus, we would expect the pattern to have broad and wide-spaced lines positioned at a variety of angles.