

Pullbacks of bundles are Pullback squares:

pull
back
bundle:

$$\begin{array}{ccccc}
 \sum_{c \in B_2} f \circ s(c) & \xrightarrow{\lambda} & \sum_{b \in B_1} f(b) & & \\
 \downarrow p_2 & & \downarrow p_1 & & \\
 B_2 & \xrightarrow{s} & B_1 & \xrightarrow{f} & U_F
 \end{array}$$

$$\lambda(c, \mathcal{F}) = (s(c), \mathcal{F})$$

$$p_1(b, \mathcal{F}) = b$$

$$p_2(c, \mathcal{F}) = c$$

to show the square is a pull back square, we need to show

$$\sum_{c \in B_2} f \circ s(c) = \sum_{x \in \sum_{b \in B_1} f(b)} \sum_{c \in B_2} (s(c) = p_1(x))$$

$$\begin{array}{c}
 \curvearrowright g \\
 \curvearrowleft h
 \end{array}$$

define g by $g(c, \mathcal{F}) := (s(c), \mathcal{F}), c, \text{refl}_{s(c)}$ b/c $p_1(s(c), \mathcal{F}) = s(c)$

define h by $h((b, \mathcal{F}), c, r) \overset{\text{in } f(b)}{\curvearrowright} \overset{\text{in } (s(c) = p_1(b, \mathcal{F}))}{\curvearrowleft} \overset{(s(c) = b)}{\text{''}}$

$:= (c, \mathcal{F})$ \mathcal{F} in $f(b)$, but r is a witness that $(s(c) = b)$ so \mathcal{F} is in $f \circ s(c)$ as desired.

$$g \circ h((b, \mathcal{F}), c, r) = g(c, \mathcal{F}) = (s(c), \mathcal{F}), c, \text{refl}_{s(c)}$$

but $(b = s(c))$ is inhabited by r .

and $(r = \text{refl}_{s(c)})$ must be inhabited b/c there is no higher homotopy (since these are real cohesive types).