MEETING NOTES

1. Dec 1 2017

1.1. **Chandrika's notes.**

1.1.1. *Rushed Introduction and summary.* Consider the following diagram, where A, B, C are sets, and D is the homotopy pushout of B and C along A.

$$\begin{tikzcd} A \arrow[r, hook, "f"] \arrow[d, "g"] & B \arrow[d] \\ C \arrow[r] & D \end{tikzcd}$$

We would like to show that D is a homotopy set as well using the encode-decode method. Therefore, we defined $\mathrm{code} : D \to D \to \mathrm{Type}$ so that $\mathrm{code}(x, y)$ is a proposition.

Amelia spoke of a picture to think of for D. This is my understanding of the picture. D is the following topological space ("the double mapping cylinder"). $(B \cup A \times [0,1] \cup C)/\sim$, where $\sim$ is defined by:

- $(a, 1) \sim b \iff f(a) = b$, $\forall a \in A$, $b \in B$
- $(a, 0) \sim c \iff g(a) = c$, $\forall a \in A$, $c \in C$

We have attempted to define code(x,y) to be the homotopy structure of the paths from x to y in this double mapping cylinder.

In Utah, we sucessfully defined encode and began to define decode : code $\to \mathrm{Id}_D$. We defined $\mathrm{decode}(\mathrm{code}(x, y))$ for all $x, y \in (B \cup C) \subset D$.

1.1.2. *What's next?* It remains to define $\mathrm{ap}_{\mathrm{decode}}$. I think that these are the types of the various parts of $\mathrm{ap}_{\mathrm{decode}}$.

$$\mathrm{ap}_{\mathrm{decode}}\Big(\mathrm{ap}_{\mathrm{code}}(\mathrm{inl}b, \mathrm{glue}(a))\Big) : \Bigg(\mathrm{decode}\Big(\mathrm{code}\big(\mathrm{inl}b, \mathrm{inl}f(a)\big)\Big) = \mathrm{decode}\Big(\mathrm{code}\big(\mathrm{inl}b, \mathrm{inr}g(a)\big)\Big)\Bigg)$$

$$\mathrm{ap}_{\mathrm{decode}}\Big(\mathrm{ap}_{\mathrm{code}}(\mathrm{glue}(a), \mathrm{inl}b)\Big) : \Bigg(\mathrm{decode}\Big(\mathrm{code}\big(\mathrm{inl}f(a), \mathrm{inl}b\big)\Big) = \mathrm{decode}\Big(\mathrm{code}\big(\mathrm{inl}g(a), \mathrm{inl}b\big)\Big)\Bigg)$$

$$\mathrm{ap}_{\mathrm{decode}}\Big(\mathrm{ap}_{\mathrm{code}}(\mathrm{glue}(a), \mathrm{inr}c)\Big) : \Bigg(\mathrm{decode}\Big(\mathrm{code}\big(\mathrm{inl}f(a), \mathrm{inr}c\big)\Big) = \mathrm{decode}\Big(\mathrm{code}\big(\mathrm{inl}g(a), \mathrm{inr}c\big)\Big)\Bigg)$$

$$\mathrm{ap}_{\mathrm{decode}}\Big(\mathrm{ap}_{\mathrm{code}}(\mathrm{inr}c, \mathrm{glue}(a))\Big) : \Bigg(\mathrm{decode}\Big(\mathrm{code}\big(\mathrm{inr}c, \mathrm{inl}f(a)\big)\Big) = \mathrm{decode}\Big(\mathrm{code}\big(\mathrm{inr}c, \mathrm{inr}g(a)\big)\Big)\Bigg)$$

$$\mathrm{ap}_{\mathrm{decode}}\Big(\mathrm{ap}_{\mathrm{code}}(\mathrm{glue}(a), \mathrm{glue}(a))\Big) : \mathrm{isContr}\Bigg(\mathrm{decode}\Big(\mathrm{code}\big(\mathrm{inl}f(a'), \mathrm{inl}f(a)\big)\Big) = \mathrm{decode}\Big(\mathrm{code}\big(\mathrm{inr}g(a'), \mathrm{inr}g(a)\big)\Big)\Bigg)$$