1. Data Preprocessing on the Age List

Ages (sorted):

13, 15, 16, 16, 19, 20, 20, 21, 22, 22, 25, 25, 25, 25, 30, 33, 33, 35, 35, 35, 35, 36, 40, 45, 46, 52, 70

(a) Smoothing by Bin Means (bin depth = 3)

There are 27 values \Rightarrow 9 bins of size 3. Compute each bin's mean and replace members by that mean.

B1: $[13,15,16] \rightarrow 14.67$

B2: [16,19,20] → 18.33

B3: [20,21,22] → 21.00

B4: $[22,25,25] \rightarrow 24.00$

B5: $[25,25,30] \rightarrow 26.67$

B6: $[33,33,35] \rightarrow 33.67$

B7: [35,35,35] → 35.00

B8: $[36,40,45] \rightarrow 40.33$

B9: $[46,52,70] \rightarrow 56.00$

Smoothed sequence:

14.67, 14.67, 14.67, 18.33, 18.33, 18.33, 21, 21, 21, 24, 24, 24, 26.67, 26.67, 26.67, 33.67, 33.67, 33.67, 35, 35, 35, 40.33, 40.33, 40.33, 56, 56

Comment: Smoothing reduces local noise and flattens within-bin variation. It pulls extremes toward their bin's center, preserving trends but losing fine detail.

(b) Outliers via IQR

$$Q1 = 20.5$$
, $Q3 = 35.0 \Rightarrow IQR = 14.5$

Lower fence = $Q1 - 1.5 \times IQR = -1.25$

Upper fence = $Q3 + 1.5 \times IQR = 56.75$

Values > 56.75 are outliers $\Rightarrow 70$ is an outlier.

(c) Min-max normalize age=35 to [0,1]

min=13. max=70:

(35 - 13)/(70 - 13) = 0.386

(d) z-score normalize age=35

Mean \approx 29.96; population std \approx 12.70

 $z = (35 - 29.96)/12.70 \approx 0.40$

(e) Decimal scaling

 $Max = 70 \Rightarrow divide by 100 \Rightarrow 35 \rightarrow 0.35$

2. Function for General Min–Max Normalization Formula:

```
x' = \text{new\_min} + (x - \text{old\_min})/(\text{old\_max} - \text{old\_min}) \times (\text{new\_max} - \text{new\_min})
```

This rescales any variable to a desired range such as [0,1] or [5,10].

3. Two-Level Decision Tree using Information Gain

Dataset includes attributes: department, age, salary, and class (status).

Root Entropy:

 $P(junior)=113/165, P(senior)=52/165 \Rightarrow H(Y)=0.899$

Compute IG for each attribute:

IG(department) ≈ 0.049 IG(age) ≈ 0.425 IG(salary) ≈ 0.538 \leftarrow Highest

Root split = salary.

For salary=46–50K (mixed branch: 23 junior, 40 senior; H≈0.947): Splitting by department (or age) gives pure leaves.

Final tree:

Root: salary

- 26–30K → junior
- 31–35K → junior
- 36-40K → senior
- 41–45K → junior
- 46-50K → split by department
- sales → senior
- systems → junior
- marketing → senior
- 66-70K → senior

4. If-Then Rules

- 1. IF salary ∈ 26–30K THEN status = junior
- 2. IF salary ∈ 31–35K THEN status = junior
- 3. IF salary ∈ 36–40K THEN status = senior
- 4. IF salary ∈ 41–45K THEN status = junior
- 5. IF salary ∈ 66–70K THEN status = senior
- 6. IF salary ∈ 46–50K AND department = sales THEN status = senior
- 7. IF salary ∈ 46–50K AND department = systems THEN status = junior
- 8. IF salary ∈ 46–50K AND department = marketing THEN status = senior

This two-level decision tree provides 100% classification purity.