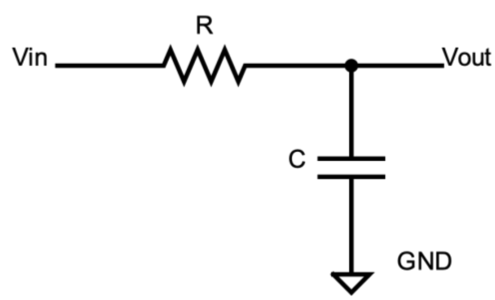
Homework 1: Time-Domain, Frequency-Domain, and Linear Circuit Review

**DUE MONDAY 2/13/2023 11:59PM**

The purpose of this assignment is to give you practice working with circuits in the time- and frequency-domains, and to give you more experience working with MATLAB and Simulink. If you are unfamiliar with Simulink, here is the link to the MathWorks Simulink Help Center and here is a video tutorial on how to specifically model electrical components in Simulink. You will model the circuits below in both code and a Simulink model so that you can check your own work as you go.

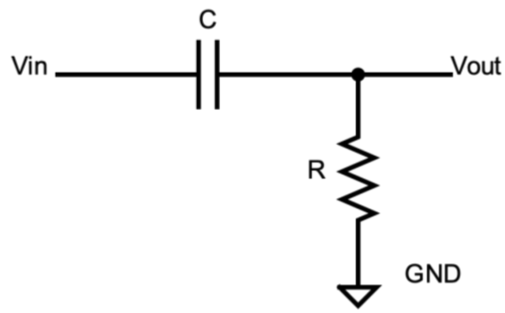
As stated in HW 0, remember that all homework must be submitted as a *single pdf* that contains all your work (including any handwritten calculations or drawings, any additional typed responses, your MATLAB code and figures, and your Simulink schematics and figures). Code should be included in this document as “searchable” text, which can be accomplished by publishing your code. All parts of your assignments should be clearly labeled. Figures should have titles, axes labels (with units if applicable), legends (if applicable), and be properly scaled.

Part I: Low-Pass Filter




1. Derive the differential equation that defines $\frac{dV_{out}}{dt}$ in terms of $V_{out}(t)$, $V_{in}(t)$, $R$, and $C$. You should do this outside of MATLAB and show your work.
2. Find a value for R such that the maximum current through the RC circuit is 1 mA when a step of $V_{in} = 1$ V is applied.
3. Using the value of R calculated above, find a value for C that makes the time constant $\tau = 0.25$ s.
4. With the equation for $\frac{dV_{out}}{dt}$ and the values for R and C that you determined above, find $V_{out}(t)$ when $V_{in}(t)$ is a waveform that starts at 0 V at $t = 0$ s, steps up to 1 V at $t = 0.1$ s, steps back down to 0 V at $t = 0.8$ s, and then ends at $t = 1.5$ s. Use the forward Euler’s method to find the solution (**not an ODE solver**). In one figure, plot $V_{in}(t)$ and $V_{out}(t)$ vs. $t$. Plot two x markers on $V_{out}(t)$ at the timepoints where the rising time constant occurs and where the falling time constant occurs. In a second figure, plot $I(t)$ (the current running through the RC circuit) vs. $t$.
5. Implement and show a schematic of the circuit in Simulink with the same R, C, and input $V_{in}(t)$ as defined above. Use your model to create the same two figures as described in question 4.
6. Do your results match?

Part II: High-Pass Filter

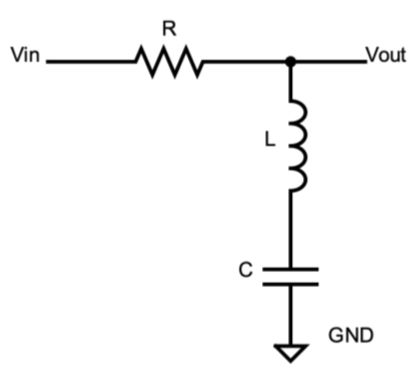



1. Derive the equation that defines $\frac{dV_{out}}{dt}$ in terms of $V_{out}(t)$, $V_{in}(t)$, $R$, and $C$. You should do this outside of MATLAB and show your work.
2. With the equation for $\frac{dV_{out}}{dt}$ you determined above and the same values for R and C from Part I, find $V_{out}(t)$ when $V_{in}(t)$ is a waveform that starts at 0 V at $t = 0$ s, steps up to 1 V at $t = 0.1$ s, steps back down to 0 V at $t = 1.6$ s, and then ends at $t = 3.1$ s. Use the forward Euler's method to find the solution (**not an ODE solver**). In one figure, plot $V_{in}(t)$ and $V_{out}(t)$ vs. $t$. Plot two x markers on $V_{out}(t)$ at the timepoints where the rising time constant occurs and where the falling time constant occurs. In a second figure, plot $I(t)$ (the current running through the RC circuit) vs. $t$.
3. Implement and show a schematic of the circuit in Simulink with the same R, C, and input $V_{in}(t)$ as defined above. Use your model to create the same two figures as described in question 3.
4. Do your results match?

Part III: Circuit Design

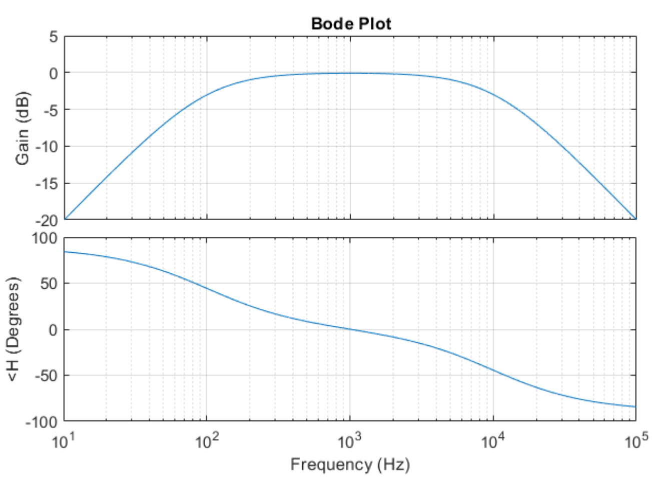
1. Let R and C be equal to the same values determined above. Can you add a single resistor to the circuit from Part 1 such that the maximum $V_{out}(t)$ of the new circuit is half of the maximum $V_{out}(t)$ of the original circuit when the same input waveform is applied? Implement and show a schematic of the new circuit in Simulink. In one figure, plot $V_{in}(t)$ and $V_{out}(t)$ vs. $t$. In a second figure, plot $I(t)$ (the current running through the RC circuit) vs. $t$.
2. Does the time constant change in this new circuit? Justify your answer.

Part IV: Band-Stop Filter



1. Derive the frequency-domain transfer function $H(j\omega)$ for $\frac{V_{out}}{V_{in}}$. You should do this outside of MATLAB and show your work.
2. Given $C = 1\ \mathrm{nF} = 10^{-9}\ \mathrm{F}$, and starting from the transfer function you derived, find values for $R$ and $L$ such that the center frequency is 50 kHz and the bandwidth is 20 kHz. Note, the bandwidth is defined as the difference between the upper and lower cutoff frequencies (-3 dB points). The center frequency is located halfway between the upper and lower cutoff frequencies and corresponds to the frequency with the most attenuation. You should do this outside of MATLAB and show your work. Hint – you should be able to solve this using only the material you learned in the lectures. Any additional equations you use should be explained/derived accordingly.
3. In MATLAB, use the transfer function and the values of $R$, $L$, and $C$ from above to solve for $H(j\omega)$ for frequencies $f = 10^{2:0.001:8}$ Hz. In one or two figures, create a bode plot including a plot of the magnitude (as a decimal between 0 and 1) vs. frequency (in Hz) and a second plot of the phase shift (in Hz) vs. frequency (in Hz). In both plots, the x-axis should be displayed on a log scale. Function hints – **abs**, **angle**, **semilogx**.
4. Open the provided Simulink model for this circuit. This model has specific timing, timestep, and scope parameters set to make sure you are able to do some calculations based on the output plot. Update the values for the resistor and inductor in the model to the $R$ and $L$ values established above. In this model, $V_{in} = \sin(2\pi(40{,}000))$. Both the $V_{in}$ and $V_{out}$ are displayed on the scope. **Use this scope output** (not the derived transfer function) to calculate the gain and phase shift of $V_{out}$ at this frequency. You should do these calculations outside of MATLAB and show your work. You don't need to include a snapshot of the model, but you should include a picture of the scope output. Note – if your MATLAB/Simulink version is giving you issues to open the file, please email celia@jhmi.edu with your MATLAB/Simulink version and we'll help you open the model.
5. Plot your calculated gain and phase shift for $f = 40{,}000$ Hz from Q4 as an 'x' on both plots from Q3. Do these points fall on the bode plot lines?

Part V: Band-Pass Filter




1. Design a circuit with only resistors and capacitors that will produce the bode plot above. Show your work and explain your design process.