Quantum Mechanics Modeled in Sound Wave Mechanics

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Quantum mechanics has been becoming more widely accepted, and for good reason – quantum theory can explain many previously unsolved problems, such as the photoelectric effect. There exist many systems that can behave like quantum systems, and one such system is acoustics. In this experiment we propagate sound waves through one-dimensional and three-dimensional systems and measure properties of acoustic resonance with these systems. Our experiment analyzes and compares the results of acoustic resonance to quantum resonance in both the "particle-in-a-box" situation and in the case of the hydrogen atom. Using our experimental setup, we report the speed of sound to be $c = 343.92 \pm 0.60$ m/s. The comparisons of this experiment align with quantum cases and support the hypothesis that quantum mechanics is a complete theory of modern physics.

I. INTRODUCTION

The Schrödinger equation,

$$i\hbar\frac{d}{dt}|\Psi\rangle = \hat{H}|\Psi\rangle, \tag{1}$$

describes the wave function of a quantum-mechanical system[1]. Wave equations can describe classical systems as well, such as sound waves. As quantum mechanics begins to be investigated more thoroughly, comparisons can be drawn between wave functions of classical systems versus wave functions of quantum systems. The wave function Ψ describes the propagation of motion of a wave, and can give the speed of propagation[2].

One similarity between quantum and classical systems is the idea of resonance, or when a system has matching temporal or spatial periods with another object. In the case of this experiment, we say that the material is at resonance when the sound waves incident are in matching in temporal phase with the metal tube or sphere. Resonance was discovered to be possible in atomic cases, such as the hydrogen atom, or some diatomic molecule[3]. This phenomenon is called nuclear magnetic resonance (NMR), and in the case of the hydrogen atom, can be adequately compared with a metal sphere with incident sound waves.

In the propagation of sound waves through a medium, we say that the acoustic resonant frequency observed through air is described by Equation 2,

$$f = \frac{nc}{2L}, \tag{2}$$

where f is the observed frequency, n is the resonance number, c is the speed of sound through air, and L is length of the material. Note that n must take discrete values $n = 1, 2, 3, \ldots$. According to this equation, we would expect that n and f are directly related, which will be further discussed in Section III.

In this experiment, we compare the quantum phenomena of NMR of a hydrogen atom to resonance of a metal sphere with sound waves, and we compare the "particle in a box" 1-D situation of quantum mechanics to sound waves in a 1-D metal tube. We will compare spectral lines of sound waves to those of light spectroscopy as well. If quantum mechanics is a complete theory of physics, then we expect to see the same phenomena in both sound waves and in quantum scenarios.

II. METHOD

A diagram of the apparatus used for the experiment can be seen in Figure 1. The experiments carried were of four different forms.

The first form was the simple 1-D case. The single generator pushed a sine wave to the controller, and the controller emitted the resulting sound wave from the speaker within the aluminum tube. The sound wave traveled down the 60 cm tube to the microphone, which then converted the incident sound wave into an alternating current to be read on the oscilloscope. The frequency was slowly increased until the sound wave from the microphone was at a local maximum amplitude, which meant that the aluminum tube was at resonance. This process was repeated for the first twenty resonances, and a generous uncertainty of 7 Hz on each frequency was claimed.

The second form used the 1-D tube but with the oscilloscope measuring driving frequency on the x-axis and resulting amplitude on the y-axis for a 15 cm aluminum tube. With the oscilloscope in this mode, spectra were taken by observing peaks of amplitude across a range of frequencies. No uncertainties were reported for this form, since no quantitative analysis took place.

The third form was identical to the first form, except instead of the sound wave passing through the aluminum tube, it was passed through the aluminum sphere. The sphere could be adjusted by its equatorial angle, and the

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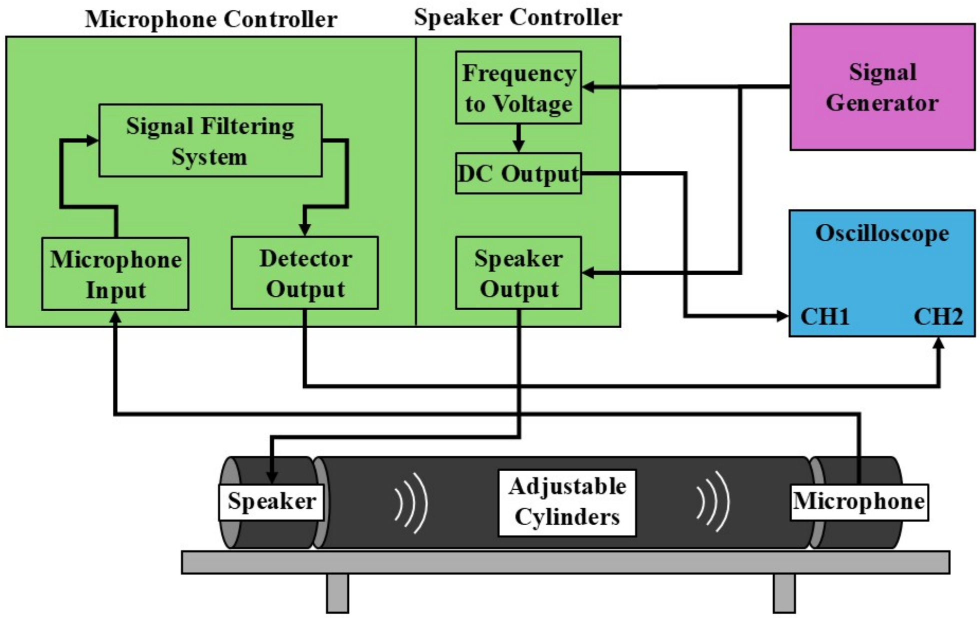

FIG. 1. The apparatus used for the experiment. The noise wave was generated as an oscillating sine wave with programmable frequency from the signal generator. This signal then traveled to the controller, which either pushed the wave to one of the speakers or to the frequency-to-voltage converter, which can then be monitored on the oscilloscope. The microphone recorded the amplitude and frequency of the incident sound waves and sent it back to the controller, which can adjust its output amplitude via the attenuator for easier viewing on the oscilloscope. The tube can be adjusted for different lengths, and the sphere can be rotated for different equatorial angles, which leads to different distances between the speaker and the microphone.

angle chosen for this experiment was 180°, which was chosen to be where the microphone and the speaker were a maximum of 10 cm apart. The resonant frequencies were identified in the same way as form 1. The first resonances within 0-8 kHz were measured, which resulted in measurements for the 8 first resonant frequencies. For this experiment, an uncertainty in frequency was claimed to be a generous 7 Hz.

In the fourth form, the setup was the same as the third form, however, the absolute noise amplitude was measured as the sphere swept from 0-180 degrees in equatorial coordinates, or 90-180 degrees in polar coordinates. This was done at some resonant frequencies measured previously. These resulting data were then compared to the Legendre polynomials for the sphere for qualitative analysis. The uncertainty on each measurement was claimed to be the square root of the data.

Important to note for each experiment is that the microphone is measuring the oscillations of the power wave produced, not the pressure wave. In each experiment, the temperature and humidity could not be exactly controlled, so each measurement has confounding error. This error was minimized by maintaining the state of the room where measurements were taken, such as keeping the door open to reduce humidity and attempt to achieve a temperature and humidity equilibrium.

III. RESULTS AND DISCUSSION

In Figure 2, we see the first twenty resonant frequencies of a 60 cm aluminum tube as a function of resonance number n. The data are fit with χ^2 to the direct relation seen in Equation 2, with a reduced form of $f = an$. The fit gives $\chi^2 = 22.1$ and $\chi^2_\nu = 1.2$, which suggests that this is a good fit. The fit returns a value of $a = 286.60 \pm 0.14$ Hz, which corresponds to a value for the speed of sound in air of $c = 343.92 \pm 0.60$ m/s, which is a reasonable value. There is a slight decreasing trend in the residual values, which is likely to be an artifact of the increasing humidity and temperature of the room that held the apparatus, and does not negate our results.

The results of the 1-D tube align with the expectations of the "particle-in-a-box" quantum system – the energies of the system can only take discrete values, which are integer multiples of the base-level energy. The wave function of this quantum system becomes like a standing wave at these discrete values, which is what we see during our validation of the resonant frequencies – as well as being at a maximum amplitude, the observed sound

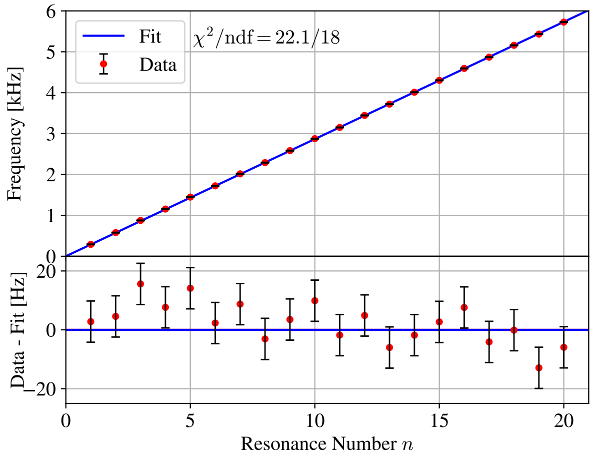

FIG. 2. The top portion of the plot shows resonant frequency values as a function of resonance number. The fit used is a direct fit with form $y = ax$, where $a = 286.60 \pm 0.14$ Hz. The bottom portion of the figure is the residual plot, and all error bars seen are ± 7 Hz.

Resonance n	Frequency [Hz]
1	407 ± 7
2	2285 ± 7
3	3666 ± 7
4	4947 ± 7
5	6182 ± 7
6	6514 ± 7
7	7387 ± 7
8	7990 ± 7

TABLE I. Resonant frequencies of the spherical aluminum shell.

wave is in phase with the produced sound wave.

In Figure 3, we see the spectrum of sound amplitude versus driving frequency for a 15 cm tube. This distribution is not symmetric, and this is because the natural distribution of the Lorenztian line shape is asymmetric. This distribution appears to match that of a Lorentzian distribution, which is also the case for light spectroscopy. In light spectroscopy, we see line broadening occur because of Heisenberg's uncertainty principle, which states that if the energy level (or frequency) is known precisely, then there must be some nonzero uncertainty in lifetime of the photon incident on the detector, resulting in broader spectral lines. The shape of lines due to the aforementioned broadening effect is described by a Lorentzian distribution, which looks near-identical to the distribution seen in Figure 3.

In Table I, we see the resonant frequency as a function of resonance number for the aluminum spherical shell. This relation is clearly not linear, and more modeling work must be done in order to determine the nature of the 3-dimensional case.

In Figure 4, we see the measured sound amplitude in

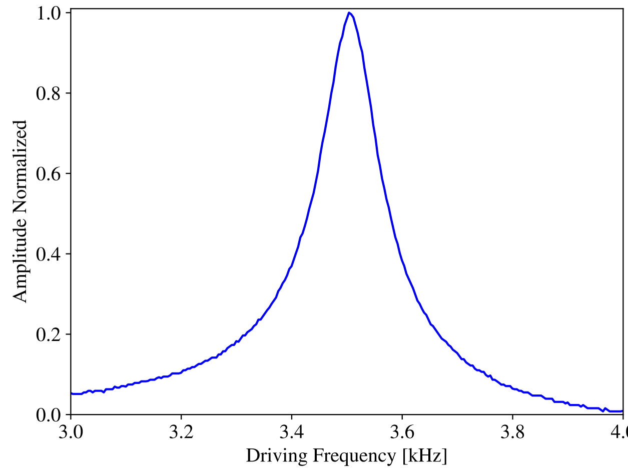

FIG. 3. A zoom of the resonant frequency of a 15 cm tube with the maximum amplitude frequency at the center. The data are taken from a still of the oscilloscope screen and scraped using `https://apps.automeris.io/wpd4/` to take approximate data points. The y-axis is normalized to the maximum amplitude of the feature.

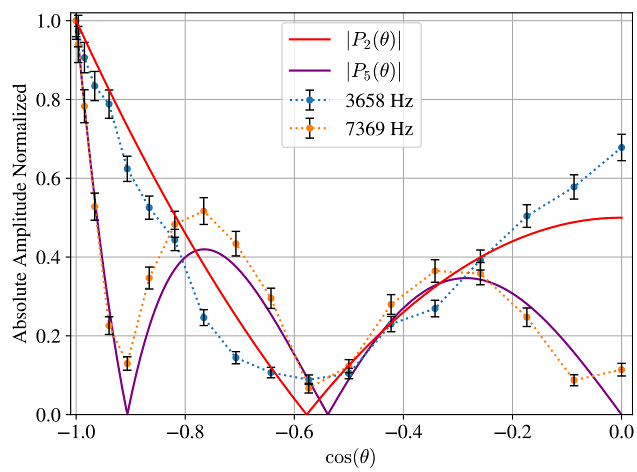

FIG. 4. The normalized sound intensity as a function of $\cos(\theta)$, where θ is the polar angle between the speaker and the microphone. Values were plotted for the aluminum sphere at two different resonant frequencies with the most similar Legendre polynomials overplotted. The error bars correspond to the square root of the value.

the spherical shell at different values of $\cos(\theta)$. The Legendre polynomials are overplotted, and they represent the expectations for a "pure state" of the sphere. We see that P_5 fits the data from the 7369 Hz measurements fairly well, with minima in the same place and the same overall shape. P_2 and the data from the 3658 Hz measurements do not fit as well, but they still have the same minima location. Furthermore, the shape of the data fits well with the left side of P_2 but does not fit the curved shape on the right side.

These Legendre polynomials do not match the data perfectly since the data is not in a pure state. The data is a combination of different Legendre polynomials and is subject to point spread, causing the resulting measurements of noise amplitude to be lower sensitivity than the theory. Despite the data sensitivity, the presence of the minima in the same place as the Legendre polynomials indicates that the polynomials can describe the sound waves. This is also a good descriptor of quantum systems, such as the particle-in-a-box and the hydrogen atom; oftentimes, the system can be in a mixed state, just like multiple Legendre polynomials added together.

IV. CONCLUSION

The purpose of this experiment was to compare acoustic resonance to quantum resonance in both one and three-dimensional cases. By applying the wave function, we were able to calculate the speed of sound through air, similar to the speed of light calculations. We showed that acoustic spectra have the same Lorentzian distribution as is seen in light spectroscopy, and we have seen that a sphere can have acoustic resonances and mixed states, like atoms and molecules have been shown to have. We have seen that every application of acoustic wave physics aligns with some quantum physics phenomenon, which can only further confirm the validity of quantum theory over classical theory.

V. LOG

The data and logbook can be found here: `https://docs.google.com/document/d/1r3ryUkV3QHzqqj05VwmxlcZ-FgnTCLEgWYZKRplA75Q/edit?usp=sharing`

VI. IMPROVEMENTS

1. Changed from a figure plotting the resonances of the 3D case to a table reporting the values.
2. Refined Figure 2 to include the χ^2 information and make the axes look nicer.
3. Added all data and discussion about Legendre polynomials and Figure 4.
4. Cleaned up the apparatus figure, Figure 1, and described the fourth experimental setup in the Methods section.
5.

[1] E. Schrödinger, Phys. Rev. **28**, 1049 (1926).

[2] J. L. R. D'Alembert, *Histoire de l'Académie royale des sciences et des belles-lettres de Berlin: avec les mémoires pour la même année, tirez des registres de cette Academie*, Vol. 3 (Ambroise Haude, 1749) pp. 214–219.

[3] I. I. Rabi, J. R. Zacharias, S. Millman, and P. Kusch, Phys. Rev. **53**, 318 (1938).