Decision Making under Uncertainty
(350934-M-6)
Fall 2019-2020

Assignment 1

The Stochastic Knapsack Problem (SKP) requires choosing a subset from among N items $(i = 1, \ldots, N)$ to allocate to a knapsack of capacity K. Each item i has a random size w_i. We assume that item sizes are statistically independent. If the collective size of the selected items exceeds the capacity, a penalty of p is assessed per unit excess. We assume that item i has an associated per-unit revenue r_i (with $r_i < p$). The binary decision variable x_i takes a value of 1 if item i is allocated to the knapsack, and a value of 0 otherwise. All items are considered simultaneously, and the values of their sizes are unknown before selection decisions are made.

Generation of Problem Instances

Part 1. (1 points)

Consider the SKP with 10 items with independent and Bernoulli distributed resource requirements. Note that in this case the item size can take only two values, namely (d_l_i and d_h_i) with probabilities $P(w_i = d\_l_i) = (1 - \pi_i)$ and $P(w_i = d\_h_i) = \pi_i$. Generate 10 random problem instances ($j = 1, \ldots, 10$) using Table 1 and Table 2. Note that for each random instance, you only need to generate d_l_i, d_h_i, π_i and r_i for items $i = 1, 2, \ldots, 10$ and compute p and K for your group.

Table 1. Settings for item parameters

i	$i = 1, 2, \ldots, 10$
d_l_i	Min $\{\gamma_i$, 10 $\}$ Let γ_i be a random number following Poisson (λ_i) where $\lambda_i = \lceil i/2 \rceil$
d_h_i	Triangular distribution with ○ Lower value $90 + g - i$ ○ Mode $100 + g - i$ ○ Upper value $110 + g - i$ You may round up/down to the closest integer (g is your group number.)
π_i	$0.5 + 0.05i - 0.001$
r_i	$51 - i$

Table 2. Base settings for other parameters

		Base
Risk Parameter	α	0 (EV model), 0.95 (CVaR model)
Unit excess weight penalty	p	$\left\lfloor 60 + \frac{g}{10} \right\rfloor$
Capacity	K	$400 + 4g$

Heuristic Algorithm

Part 2. (1 points)
Develop a simple greedy heuristic algorithm to suggest an item selection strategy for the SKP problem described above. No need to develop a complex solution method as that is not the focus.

Monte Carlo Simulation

Part 3. (2 points)
For the first instance (j =1), apply your greedy heuristic algorithm and deliver decision variables x_i. For this selection vector (x), run Monte Carlo simulation to generate item sizes based on the Bernoulli distribution to calculate the profit for the stochastic knapsack problem. Choose the number of runs (n) carefully and come up with 95% confidence interval for the profit.

Stochastic Programming Models

Part 4. (1 point)

Model EV: Formulate the SKP as a mixed integer linear programming (MILP) problem with an objective of maximizing expected profit.

Model CVaR: Formulate the SKP as a mixed integer linear programming (MILP) problem with an objective of maximizing CVaR.

Note: You may consider a finite set of scenarios U, with P_u denoting the probability of scenario $u \in U$. Note that there exists 2^N scenarios for the Bernoulli assumption. Letting $\Delta_u = \left(\sum_{i=1}^{N} w_{iu} x_i - K\right)^+$ denote the excess weight in scenario u, where w_{iu} denotes the weight of item i in scenario u, then the profit in scenario u can be represented by $\psi_u = \sum_{i=1}^{N} r_i w_{iu} x_i - p\Delta_u$.

Part 5. (2 points)
Solve the EV and CVaR models optimally for 10 random instances created in Part 1.

Part 6. (2 points)
Analyse the effects of $\alpha\ (risk),\ p$ and K on the objective function value of the CVaR model and item selection strategy. Choose sensible levels, explain your approach and discuss your results.

Sample Average Approximation

Part 7.
For the first instance (j =1) (and base setting for the remaining parameters of $\alpha,\ p$ and K), design and implement a Sample Average Approximation (SAA) algorithm to solve the EV and CVaR problems.

a. **(2 points)** Explain the SAA setup in detail. Provide the mathematical programming model for the SAA. Discuss the scenario generation process, number of samples, etc.

b. **(2 point)** Provide statistical estimates for lower bounds, upper bounds and the gap (also include variances) for the SAA scheme you designed.

Analysis

Part 8. (1 point)
Compare the solution provided by the SAA scheme with the heuristic and exact solution found earlier for the first instance (*j*)

Part 9. (1 point)
Discuss advantages and disadvantages of solution approaches for varying problem sizes and random distributions. Can you solve the EV problem for larger instances ($N > 20$) using solution approaches? What happens if the item weights follow Normal distribution (or other distributions Poisson, etc.)?

Bonus (1 point) Discuss how "Antithetic variates" can be adopted within the SAA scheme for the EV problem. Support your discussion with some results if possible.

Make necessary assumptions, report significant results using appropriate tables, figures etc. Provide discussions and draw conclusions when necessary. Follow a report format with introduction, models, results, etc.

Due date: October 25, 2019
Canvas Submission: Soft copy before the midnight on due date.

References

Stochastic Knapsack Problem

Y Merzifonluoğlu, J Geunes, HE Romeijn (2012) The static stochastic knapsack problem with normally distributed item sizes. Mathematical Programming 134 (2), 459-489.

Sample Average Approximation

A Shapiro and A Philpott (2007) A Tutorial on Stochastic Programming.

Variance reduction for stochastic optimization

T Homem-de-Mello, G Bayraksan (2014) Monte Carlo sampling-based methods for stochastic optimization. Surveys in Operations Research and Management Science 19 56–85.