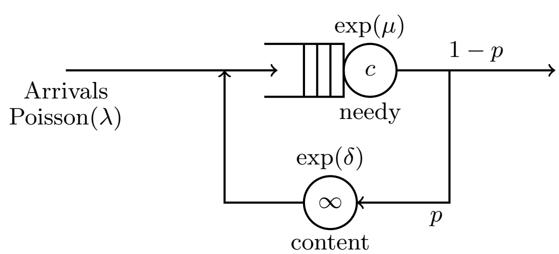
Decision Making Under Uncertainty

Assignment 2 (Discrete-Event Simulation and Optimal Network Design)
Deadline: December 6, 2019

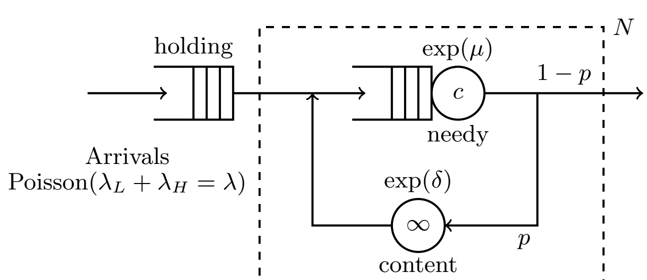
Consider a model where customers seek service from c servers that operate in parallel. Customers arrive according to a Poisson process with rate λ. After service is completed, with probability $1-p$ they leave the system and with probability p they return for further service, after a random exponential delay time (this can be modelled as an infinite server queue). We refer to the service phase as a *Needy state*, and to the delay phase as a *Content state*. Thus, during their stay in the system, customers start in a Needy state and then alternate between Needy and Content states. When customers become Needy and a server is idle, they are immediately treated by a server. Otherwise, customers wait in queue for an available server. The queueing policy is FCFS (First Come First Served). Needy service times are independent and exponentially distributed with mean $1/\mu$. Content times (delays) are independent and exponentially distributed with mean $1/\delta$. We further assume that the Needy and Content times are independent of each other and of the arrival process. Note that newly arriving customers always start in the Needy state.



Let $Q(t) = \{Q_1(t), Q_2(t)\}$ be a two-dimensional Markov processes with $Q_1(t)$ representing the number of Needy customers at time t and $Q_2(t)$ the number of Content customers at time t. Under our assumptions, the system is an open (product-form) Jackson network.

a. Derive the product-form solution for the stationary distribution of this Jackson network and determine the stability conditions (consult Lecture on 30 October).

Now consider the following extension of this system: At all times, only a maximum number of N customers can be admitted into the system. The customers that are initially blocked because the system if full when they arrive, wait in some (virtual) queue outside the system. Those initially blocked customers can enter the system immediately when there is a place available (when one of N customers already inside the system leaves the system). Moreover, we have two different customer priority types entering the holding queue. High priority customers precede low priority customer for system admission, where λ_H and λ_L are the respective arrival rates. This extension makes the system more complicated. In particular, the system is no longer a product-form network.



b. Write for this extended system a discrete-event simulation. In order to do this, you might need object-oriented programming.

We are of course interested in the impact of N on the system behavior, particularly in comparison with the open Jackson network (which is the case $N = \infty$). Please work from here onwards with a system with $c = 2$ servers and $\mu = 1$.

c. Use your discrete-event simulation to present performance measures (perhaps probability of blocking, probability of waiting for the different customer priority classes, mean queue length, mean waiting time,...) for the extended system for different values of λ_L, λ_H, p, and δ, and different values of N. Elaborate on your findings by creating a comprehensive management report including several sensitivity analyses.

d. Investigate the difference in performance between the original system ($N = \infty$) and the extended system (finite N). For instance, how fast does the behavior of the extended system mimic the original system behavior as N increases?