Synthesizing Indirect Effects in Mediation Models with Meta-Analytic Methods: Supplementary Materials 1

Mike Cheung

January 26, 2021

Contents

- This file demonstrates how to compute effect sizes and their sampling covariance matrix with two approaches using the delta method. The first one uses a numeric approach with the structural equation modeling (SEM) framework. The second approach computes the sampling covariance matrix with the symbolic calculations.

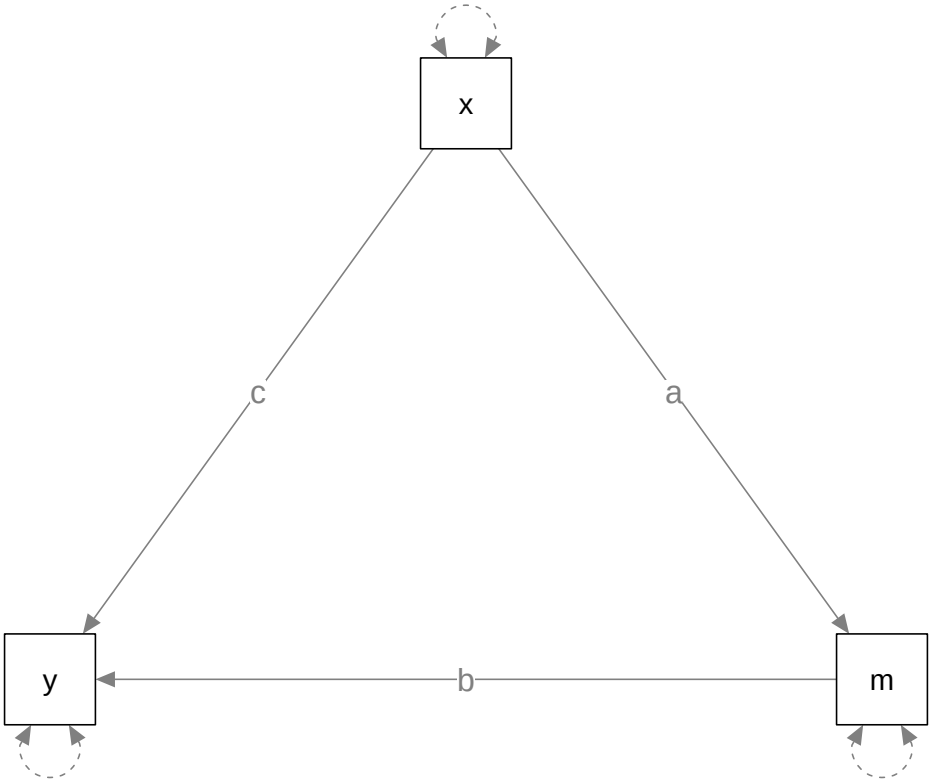
Numeric calculations with the SEM approach

One mediator

```
library(metaSEM)

## Model with one mediator: x -> m -> y
model1 <- "y ~ c*x + b*m
           m ~ a*x
           # Define indirect and direct effects
           Indirect := a*b
           Direct := c"

## Diplay the model
plot(model1)
```




```
## Sample correlation matrix
my.cor <- matrix(c(1, .5, .3,
                   .5, 1, .4,
                   .3, .4, 1),
                 nrow = 3,
                 ncol = 3,
                 dimnames = list(c("y", "m", "x"),
                                 c("y", "m", "x")))
my.cor
```

```
##     y   m   x
## y 1.0 0.5 0.3
## m 0.5 1.0 0.4
## x 0.3 0.4 1.0
```

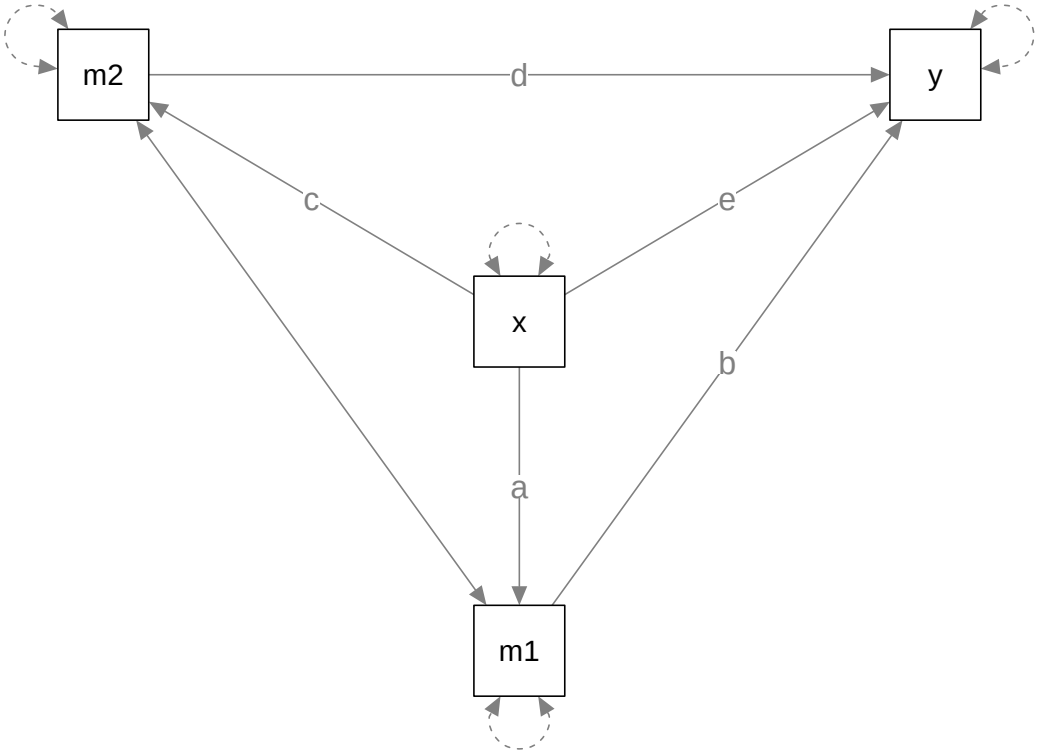
```
## Calculate the indirect and direct effects and their sampling covariance matrix
calEffSizes(model=model1, n=300, Cov=my.cor)
```

```
## $ES
##  Indirect    Direct
## 0.1809524 0.1190476
##
## $VCOV
##                Indirect        Direct
## Indirect  0.0010416478 -0.0004686319
## Direct   -0.0004686319  0.0029289494
```

Two parallel mediators

```
## Model with two specific mediators: x -> m1 -> y and x -> m2 -> y
model2 <- "y ~ e*x + b*m1 + d*m2
           m1 ~ a*x
           m2 ~ c*x
           # m1 and m2 are correlated
           m1 ~~ m2
           # Define indirect and direct effects
           Ind_m1 := a*b
           Ind_m2 := c*d
           Direct := e"

plot(model2, layout="circle")
```




```
## Sample correlation matrix
my.cor <- matrix(c(1, .5, .6, .3,
                   .5, 1, .4, .2,
                   .6, .4, 1, .3,
                   .3, .2, .3, 1),
                 nrow = 4,
                 ncol = 4,
                 dimnames = list(c("y", "m1", "m2", "x"),
                                 c("y", "m1", "m2", "x")))
my.cor
```

```
##      y  m1  m2   x
## y  1.0 0.5 0.6 0.3
## m1 0.5 1.0 0.4 0.2
## m2 0.6 0.4 1.0 0.3
```

```
## x  0.3 0.2 0.3 1.0
```

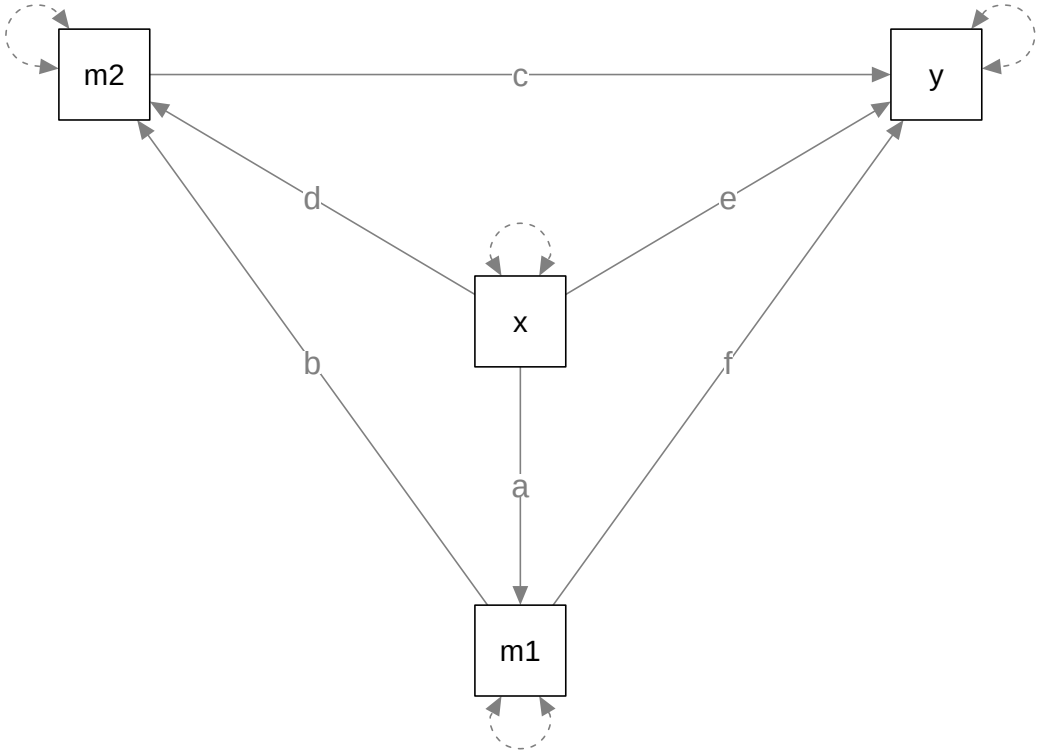
```
## Calculate the indirect and direct effects and their sampling covariance matrix
calEffSizes(model=model2, n=300, Cov=my.cor)
```

```
## $ES
##     Ind_m1     Ind_m2     Direct
## 0.05989446 0.13456464 0.10554090
##
## $VCOV
##                Ind_m1        Ind_m2        Direct
## Ind_m1  0.0003749419  0.0001029453 -0.0000386612
## Ind_m2  0.0001029453  0.0008190651 -0.0001594774
## Direct -0.0000386612 -0.0001594774  0.0020297130
```

Two serial mediators

```
## Model with two intermediate mediators: x -> m1 -> m2 -> y
model3 <- "y ~ e*x + f*m1 + c*m2
           m1 ~ a*x
           m2 ~ b*m1
           m2 ~ d*x
           # Define indirect and direct effects
           Ind_m1m2 := a*b*c
           Ind_m1 := a*f
           Ind_m2 := d*c
           Direct := e"

plot(model3, layout="circle")
```



```
## Calculate the indirect and direct effects and their sampling covariance matrix
calEffSizes(model=model3, n=300, Cov=my.cor)
```

```
## $ES
##   Ind_m1m2     Ind_m1     Ind_m2     Direct
## 0.03177221 0.05989446 0.10279244 0.10554090
##
## $VCOV
##                Ind_m1m2        Ind_m1        Ind_m2        Direct
## Ind_m1m2  1.144605e-04  0.0001405997  1.559037e-05 -0.0000376544
## Ind_m1    1.405997e-04  0.0003749419 -3.765440e-05 -0.0000386612
## Ind_m2    1.559037e-05 -0.0000376544  6.734239e-04 -0.0001218231
## Direct   -3.765440e-05 -0.0000386612 -1.218231e-04  0.0020297130
```

Symbolic calculations

One mediator

```
library(symSEM)

## fn: The effect sizes
## Covfn: Sampling covariance matrix of fn: "b^2*Va+2*b*a*Cba+a^2*Vb"
## Va: Sampling variance of a
## Vb: Sampling variance of b
## Cba: Sampling covariance of a and b
deltamethod(fn="a*b")
```

```
## $fn
##     [,1]
## fn1 "a*b"
##
## $Covfn
##     fn1
## fn1 "b^2*Va+2*b*a*Cba+a^2*Vb"
##
## $vars
## [1] "a" "b"
##
## $Covvars
##   a     b
## a "Va"  "Cba"
## b "Cba" "Vb"
##
## $Jmatrix
##     a   b
## fn1 "b" "a"
```

Two parallel mediators

```
deltamethod(fn=c("a*b", "c*d"))
```

```
## $fn
##     [,1]
```

```
## fn1 "a*b"
## fn2 "c*d"
##
## $Covfn
##     fn1                                 fn2
## fn1 "b^2*Va+2*b*a*Cba+a^2*Vb"           "b*Cca*d+b*Cda*c+a*Ccb*d+a*Cdb*c"
## fn2 "d*Cca*b+d*Ccb*a+c*Cda*b+c*Cdb*a" "d^2*Vc+2*d*c*Cdc+c^2*Vd"
##
## $vars
## [1] "a" "b" "c" "d"
##
## $Covvars
##   a     b     c     d
## a "Va"  "Cba" "Cca" "Cda"
## b "Cba" "Vb"  "Ccb" "Cdb"
## c "Cca" "Ccb" "Vc"  "Cdc"
## d "Cda" "Cdb" "Cdc" "Vd"
##
## $Jmatrix
##     a   b   c   d
## fn1 "b" "a" "0" "0"
## fn2 "0" "0" "d" "c"
```

Two serial mediators

```
deltamethod(fn="a*b*c")
```

```
## $fn
##     [,1]
## fn1 "a*b*c"
##
## $Covfn
##     fn1
## fn1 "b^2*c^2*Va+2*b^2*c*a*Cca+b^2*a^2*Vc+2*b*c^2*a*Cba+2*b*c*a^2*Ccb+c^2*a^2*Vb"
##
## $vars
## [1] "a" "b" "c"
##
## $Covvars
##   a     b     c
## a "Va"  "Cba" "Cca"
## b "Cba" "Vb"  "Ccb"
## c "Cca" "Ccb" "Vc"
##
## $Jmatrix
##     a     b     c
## fn1 "b*c" "a*c" "a*b"
```

```
sessionInfo()
```

```
## R version 4.0.3 (2020-10-10)
## Platform: x86_64-pc-linux-gnu (64-bit)
## Running under: Ubuntu 20.10
##
## Matrix products: default
```

```
## BLAS:   /usr/lib/x86_64-linux-gnu/blas/libblas.so.3.9.0
## LAPACK: /usr/lib/x86_64-linux-gnu/lapack/liblapack.so.3.9.0
##
## locale:
##  [1] LC_CTYPE=en_SG.UTF-8       LC_NUMERIC=C
##  [3] LC_TIME=en_SG.UTF-8        LC_COLLATE=en_SG.UTF-8
##  [5] LC_MONETARY=en_SG.UTF-8    LC_MESSAGES=en_SG.UTF-8
##  [7] LC_PAPER=en_SG.UTF-8       LC_NAME=C
##  [9] LC_ADDRESS=C               LC_TELEPHONE=C
## [11] LC_MEASUREMENT=en_SG.UTF-8 LC_IDENTIFICATION=C
##
## attached base packages:
## [1] stats     graphics  grDevices utils     datasets  methods   base
##
## other attached packages:
## [1] symSEM_0.1    metaSEM_1.2.5 OpenMx_2.18.1
##
## loaded via a namespace (and not attached):
##   [1] minqa_1.2.4         colorspace_1.4-1    rjson_0.2.20
##   [4] ellipsis_0.3.1      rprojroot_1.3-2     htmlTable_1.13.3
##   [7] corpcor_1.6.9       base64enc_0.1-3     rstudioapi_0.11
##  [10] lavaan_0.6-7        mvtnorm_1.1-1       splines_4.0.3
##  [13] mnormt_2.0.2        knitr_1.29          glasso_1.11
##  [16] pkgload_1.0.2       Formula_1.2-4       nloptr_1.2.2.2
##  [19] cluster_2.1.0       png_0.1-7           regsem_1.6.2
##  [22] compiler_4.0.3      backports_1.2.0     assertthat_0.2.1
##  [25] Matrix_1.2-18       acepack_1.4.1       htmltools_0.4.0
##  [28] tools_4.0.3         igraph_1.2.5        coda_0.19-4
##  [31] gtable_0.3.0        glue_1.4.1          reshape2_1.4.4
##  [34] dplyr_1.0.2         Rcpp_1.0.5          carData_3.0-4
##  [37] vctrs_0.3.2         nlme_3.1-149        lisrelToR_0.1.4
##  [40] psych_2.0.7         xfun_0.19           stringr_1.4.0
##  [43] testthat_3.0.0      openxlsx_4.1.5      lme4_1.1-26
##  [46] lifecycle_0.2.0     gtools_3.8.2        statmod_1.4.35
##  [49] XML_3.99-0.3        MASS_7.3-53         scales_1.1.0
##  [52] BDgraph_2.63        Ryacas_1.1.3.1      kutils_1.70
##  [55] parallel_4.0.3      huge_1.3.4.1        RColorBrewer_1.1-2
##  [58] yaml_2.2.1          pbapply_1.4-2       gridExtra_2.3
##  [61] ggplot2_3.3.2       rpart_4.1-15        latticeExtra_0.6-29
##  [64] stringi_1.4.6       desc_1.2.0          sem_3.1-11
##  [67] checkmate_2.0.0     boot_1.3-25         zip_2.0.4
##  [70] truncnorm_1.0-8     rlang_0.4.7         pkgconfig_2.0.3
##  [73] d3Network_0.5.2.1   Rsolnp_1.16         arm_1.11-2
##  [76] evaluate_0.14       lattice_0.20-41     purrr_0.3.4
##  [79] htmlwidgets_1.5.1   tidyselect_1.1.0    plyr_1.8.6
##  [82] magrittr_2.0.1      R6_2.5.0            generics_0.1.0
##  [85] Hmisc_4.4-0         pillar_1.4.4        whisker_0.4
##  [88] foreign_0.8-80      withr_2.2.0         rockchalk_1.8.144
##  [91] survival_3.1-12     semPlot_1.1.2       abind_1.4-5
##  [94] nnet_7.3-14         tibble_3.0.4        crayon_1.3.4
##  [97] fdrtool_1.2.15      ellipse_0.4.2       tmvnsim_1.0-2
## [100] rmarkdown_2.5       jpeg_0.1-8.1        grid_4.0.3
## [103] qgraph_1.6.5        data.table_1.13.0   pbivnorm_0.6.0
## [106] matrixcalc_1.0-3    digest_0.6.25       xtable_1.8-4
```

```
## [109] mi_1.0              stats4_4.0.3        munsell_0.5.0
```