08/22/2025: Definition

CSCI 246: Discrete Structures

Textbook reference: Sec 3, Scheinerman

Today's Agenda

- Overview / Q & A ($\approx$ 5 mins)
- Group exercises ($\approx$ 25 mins)
- Discussion ($\approx$ 20 minutes)

Integers

Definition. The set of *integers*, denoted $\mathbb{Z}$, is given by

$$\mathbb{Z} \triangleq \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}.$$

That is, the integers are the positive whole numbers, the negative whole numbers, and zero.

Group exercises

Random group assignments

Aaron Christensen: 3
Aidan Sinclair: 13
Brendan Kelly: 1
Buggy Garza: 6
Cedric Jefferson: 17
Conner Brost: 7
Connor Graville: 3
David Knauert: 1
David Oswald: 4
Elias Martin: 5
Ericson O'Guinn: 8
Erik Halverson: 7
Francis Bush: 9
Garrett Miller: 15
George Cutler: 16
Georgia Franks: 2
Gregor Schmidt: 3

Hakyla Riggs: 6
Izayah Abayomi: 11
Jacob Ketola: 10
Jacob Ruiz: 17
Jaden Hampton: 5
Jeremy Ness: 10
Jonah Day: 1
Karter Gress: 8
Kyle Hoerner: 10
Landry Clarke: 4
Leon BirdHat: 5
Lillian Ziegler: 11
Matthew Rau: 9
Micah Miller: 14
Michael Pitman: 15
Nathan Campbell: 11

Nathan Hooley: 15
Nicholas Rugani: 8
Noah Andersson: 14
Olivia Greuter: 12
Peter Van Vleet: 14
Pierce Dotson: 9
Quinn Carlson: 2
Ridley Christoferson: 16
Riley Smith: 6
Sierra Holleman: 13
Tanner Gramps: 7
Timothy True: 13
Titus Sykes: 12
Trey Randall: 4
William Grant: 16
William Sheldon: 12
Zachary Reller: 2

Group exercises

1. Please determine which of the following are true or false; use Definition 3.2 to explain your answers: (a) $3 \mid 100$, (b) $3 \mid 99$, (c) $-3 \mid 3$, (d) $-5 \mid -5$, (e) $-2 \mid -7$, (f) $0 \mid 4$, (g) $4 \mid 0$, (h) $0 \mid 0$.
2. None of the following is a prime. Explain why they fail to satisfy Definition 3.5. Which of the numbers is composite? (a) 21, (b) 0, (c) π, (d) $\frac{1}{2}$, (e) -2, (f) -1.
3. Define what it means for an integer to be a *perfect square*. For example, the integers 0,1,4,9, and 16 are perfect squares. Your definition should begin:

   An integer x is a *perfect square* provided ...
4. Here is a possible alternative to Definition 3.2: We say that a is *divisible* by b provided $\frac{a}{b}$ is an integer. Explain why this alternative definition is different from Definition 3.2.

   Here, *different* means that the definitions specify *different concepts*. So to answer this question, you should find integers a and b such that a is divisible by b according to one definition, but a is not divisible by b according to the other definition.

Solution to group exercise #1

Definition. Let a and b be integers. We say a is *divisible* by b, written $b \mid a$, provided there is an integer c such that $bc = a$.

Solution.

a. $3 \mid 100$?

Solution to group exercise #1

Definition. Let a and b be integers. We say a is *divisible* by b, written $b \mid a$, provided there is an integer c such that $bc = a$.

Solution.

a. $3 \mid 100$? False. There is no integer c such that $3c = 100$. (Why?

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Definition. Let a and b be integers. We say a is *divisible* by b, written $b \mid a$, provided there is an integer c such that $bc = a$.

Solution.

a. $3 \mid 100$? False. There is no integer c such that $3c = 100$. (Why? Note that there is exactly one number $c = \frac{100}{3} = 33\frac{1}{3}$ that satisfies the equation, but this c is not an integer.)

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b. $3 \mid 99$?

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Solution.

a. $3 \mid 100$? False. There is no integer c such that $3c = 100$. (Why? Note that there is exactly one number $c = \frac{100}{3} = 33\frac{1}{3}$ that satisfies the equation, but this c is not an integer.)

b. $3 \mid 99$? True. There is an integer $c = 33$ such that $3c = 99$.

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b. $3 \mid 99$? True. There is an integer $c = 33$ such that $3c = 99$.
c. $-3 \mid 3$?

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b. $3 \mid 99$? True. There is an integer $c = 33$ such that $3c = 99$.

c. $-3 \mid 3$? True. There is an integer $c = -1$ such that $-3c = 3$.

d. $-5 \mid -5$?

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e. $-2 \mid -7$?

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f. $0 \mid 4$?

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g. $4 \mid 0$? True. There is an integer $c = 0$ such that $4c = 0$.
h. $0 \mid 0$? True. There is an integer c such that $0c = 0$. In fact, there are *infinitely* many integer-valued solutions!

Solution to group exercise #2 (first part)

Problem. None of the following numbers is prime. Explain why they fail to satisfy Definition 3.5.

Definition. An integer p is called *prime* provided that $p > 1$ and the only positive divisors of p are p and 1.

Solution.

a. 21?

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Remark. In the above, I used the shorthand notation $x \notin A$, which means that x is not a member of the set A.

Solution to group exercise #2 (second part)

Problem. None of the following numbers is prime. Which is composite?

Definition. An integer a is called *composite* provided that there is an integer b such that $1 < b < a$ and $b \mid a$.

Solution.

a. 21?

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Solution to group exercise #2 (second part)

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Definition. An integer a is called *composite* provided that there is an integer b such that $1 < b < a$ and $b \mid a$.

Solution.

a. 21? Composite, since $3 \mid 21$, and $1 < 3 < 21$.
b. 0? Not composite, since $0 < 1$.
c. π? Not composite, since $\pi \notin \mathbb{Z}$.
d. $\frac{1}{2}$? Not composite, since $\frac{1}{2} \notin \mathbb{Z}$.
e. -2? Not composite, since $-2 < 1$.
f. -1?

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Remark. Note from the definition that the integer a can be composite only if $a > 1$. We use this fact to answer parts b, e, and f.

Solution to group exercise #3

Problem. Define what it means for an integer to be a *perfect square*. For example, the integers 0,1,4,9, and 16 are perfect squares.

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Solution. An integer x is a *perfect square* provided there is an integer y such that $x = y^2$.

Solution to group exercise #4

Problem. Here is a possible alternative to Definition 3.2: We say that a is *divisible* by b provided $\frac{a}{b}$ is an integer. Explain why this alternative definition is different from Definition 3.2.

Here, *different* means that the definitions specify *different concepts*. So to answer this question, you should find integers a and b such that a is divisible by b according to one definition, but a is not divisible by b according to the other definition.

Solution to group exercise #4

Problem. Here is a possible alternative to Definition 3.2: We say that a is *divisible* by b provided $\frac{a}{b}$ is an integer. Explain why this alternative definition is different from Definition 3.2.

Here, *different* means that the definitions specify *different concepts*. So to answer this question, you should find integers a and b such that a is divisible by b according to one definition, but a is not divisible by b according to the other definition.

Solution. Consider $a = b = 0$. Then, $a \mid b$ according to Definition 3.2 (as we discovered in group exercise #1h). However, $\frac{a}{b}$ is not an integer.