01/29/2026: Discrete Random Variables (Part 2)

CSCI 546: Diffusion Models

Textbook reference: Sec 3.6-3.10

Announcement (Sign-in Sheet)

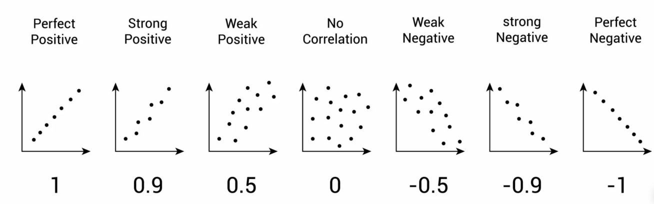
Please sign the sign-in sheet.

Review Problem Set #4

Concepts for Problem Set #5

Correlation Coefficient

The **correlation coefficient** ρ shows the strength and direction of the linear relationship between two random variables X and Y.



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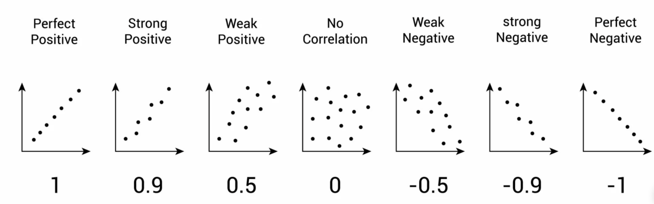

Remark. We have

$$|\rho(X, Y)| \leq 1,$$

with equality if and only if X and Y are linearly related (with probability 1).

Correlation Coefficient

Poll. Can uncorrelated random variables be dependent?

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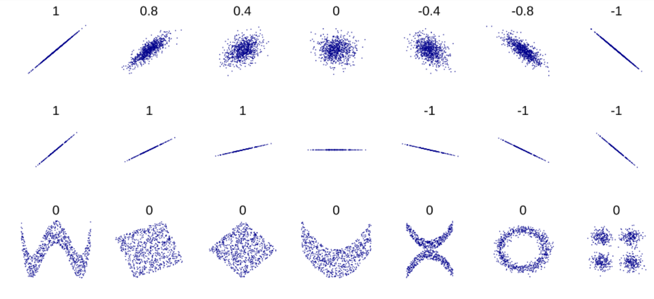
Solution. Yes.



The correlation reflects the noisiness and direction of a linear relationship (top row), but not the slope of that relationship (middle), nor many aspects of nonlinear relationships (bottom).

Random Groups

Aubrey Williams: 1
Austin Barton : 4
Blake Sigmundstad: 7
Diego Moylan: 2
Dillon Shaffer: 1
Ismoiljon Muzaffarov: 3
Jacob Tanner: 6

Josh Stoneback: 1
Joshua Bowen: 5
Joshua Calwell: 8
Laura Banaszewski: 3
Lina Hammel: 8
Logan Racz: 4

Matt Hall: 6
Micah Miller: 7
Mike Kadoshnikov: 5
Owen Cool: 2
Racquel Bowen: 2
Samuel Mocabee: 4
Tatiana Kirillova: 3

Group exercises - Problem Set 5

1. (3.11.16) Let X and Y be independent Bernoulli random variables with parameter $\frac{1}{2}$. Show that $X + Y$ and $|X - Y|$ are dependent though uncorrelated.
2. (Sec 3.6) Use the Cauchy-Schwarz Inequality to show that the correlation coefficient ρ satisfies $|\rho(X, Y)| \leq 1$ with equality if and only if $Y = mX + b$ with probability 1 for some $m, b \in \mathbb{R}$. (Hint: Apply Cauchy-Schwarz to the variables $X - E[X]$ and $Y - E[Y]$.)
3. (3.7.6) Let $X_1, X_2, \ldots$ be identically distributed random variables with mean μ and variance σ^2, and let N be a random variable taking values in the non-negative integers and independent of the X_i. Let $S = X_1 + X_2 + \cdots + X_N$.
    a. Show that $\mathbb{E}[S \mid N] = \mu N$, and deduce that $\mathbb{E}[S] = \mu \mathbb{E}[N]$.
    b. Find $\mathrm{Var}[S]$ using the conditional variance formula $\mathrm{Var}[S] = \mathbb{E}[\mathrm{Var}[S \mid N]] + \mathrm{Var}[\mathbb{E}[S \mid N]]$.