

02/03/2026: Continuous Random Variables (Part 1)

CSCI 546: Diffusion Models

Textbook reference: Sec 4.1-4.5

Announcement (Sign-in Sheet)

Please sign the sign-in sheet.

Review Problem Set #5

Concepts for Problem Set #6

Continuous random variables

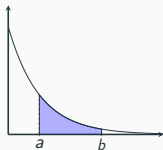
Definition (GS Sec 4.1)

A random variable X is called **continuous** if its distribution function $F(x) = P(X \leq x)$ can be written as

$$F(x) = \int_{-\infty}^x f(u) du$$

for some integrable $f : \mathbb{R} \rightarrow [0, \infty)$.

Remark. If X is a random variable with probability density f below



then

$$P(a \leq X \leq b) = \text{shaded area}$$

Example: Exponential Distribution

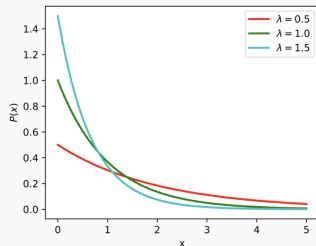
Definition

A continuous random variable whose probability density function is given, for some $\lambda > 0$, by

$$f(x) = \begin{cases} \lambda e^{-\lambda x}, & \text{if } x \geq 0 \\ 0, & \text{if } x < 0 \end{cases}$$

is said to be an **exponential** random variable with parameter λ .

Probability density function



Applications. An exponential distribution often arises as the amount of time until some specific event occurs. For example, the amount of time until...

- ... an earthquake occurs
- ... a new war breaks out
- ... you get a spam telephone call

Definition

We say that a non-negative random variable X is **memoryless** if

$$P(X > s + t \mid X > t) = P(X > s), \quad \text{for all } s, t \geq 0 \quad (1)$$

Memorylessness

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Poll. Who can explain what the above equation is saying?

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Interpretation. If we think of X as being the lifetime of some instrument, Eq. (1) says:

If the instrument is alive at time t , the distribution of the remaining amount of time that it survives is the same as the original lifetime distribution.

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Interpretation. If we think of X as being the lifetime of some instrument, Eq. (1) says:

If the instrument is alive at time t , the distribution of the remaining amount of time that it survives is the same as the original lifetime distribution.

That is, it is as if the instrument does not “remember” that it has already been in use for time t .

Example: Uniform Distribution

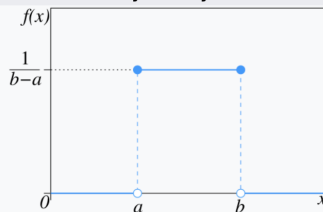
Definition

A continuous random variable X is said to be **uniform** on $[a, b]$ (or, equivalently, on (a, b)) if it has the probability density function

$$f(x) = \begin{cases} \frac{1}{b-a}, & \text{if } a < x < b \\ 0, & \text{otherwise} \end{cases}$$

Continuous uniform

Probability density function



Applications. Although simple, the uniform distribution is surprisingly useful. E.g.:

- Random number generation by a computer. (Non-uniform random numbers are often obtained by transforming uniform samples.)
- Random angles in physics.
- Monte carlo estimates of an integral $\int_a^b f(u) du \approx (b-a) \frac{1}{S} \sum_{s=1}^S f(u^{(s)})$ where $u^{(s)} \sim U[a, b]$.

Random Groups

Aubrey Williams: 7

Austin Barton : 4

Blake Sigmundstad: 3

Diego Moylan: 3

Dillon Shaffer: 2

Ismoiljon Muzaffarov: 5

Jacob Tanner: 8

Josh Stoneback: 1

Joshua Bowen: 2

Joshua Culwell: 6

Laura Banaszewski: 4

Lina Hammel: 1

Logan Racz: 8

Matt Hall: 4

Micah Miller: 2

Mike Kadoshnikov: 1

Owen Cool: 5

Racquel Bowen: 6

Samuel Mocabee: 7

Tatiana Kirillova: 3

Group exercises - Problem Set 6

1. (4.14.5b) Prove that the exponential distribution is memoryless.
(Note: the exponential distribution is the *only* continuous distribution with this property!)
2. (4.5.5a) Let X and Y be independent continuous random variables. Show that

$$\mathbb{E}[g(X)h(Y)] = \mathbb{E}[g(X)]\mathbb{E}[h(Y)]$$

whenever these expectations exist. If X and Y have the exponential distribution with parameter 1, find $\mathbb{E}[e^{\frac{1}{2}(X+Y)}]$.

3. (4.5.4) Let X and Y be independent random variables each having the uniform distribution on $[0, 1]$. Let $U = \min\{X, Y\}$ and $V = \max\{X, Y\}$. Find $\mathbb{E}[U]$ and $\mathbb{E}[V]$, and then calculate $\text{Cov}[U, V]$. Are U and V independent?