01/15/2026: Events and their probabilities

CSCI 546: Diffusion Models

Textbook reference: CH 1

Opening Discussion ($\approx$ 5 mins)

Find a partner and discuss these questions.

1. What is your early reflection on the class?
    a) goal
    b) hope
    c) fear
    d) like
    e) dislike
2. Reflect on your experience reading the textbook.

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Writing ($\approx$ 5 mins)

Please write your answers to the questions above. I will read them and take them into account as much as possible.

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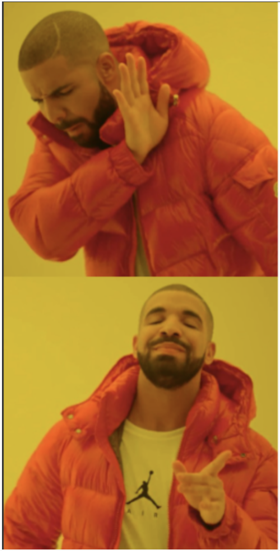
Announcement

Please sign the sign-in sheet.

Thoughts on Chapter 1

Countable Additivity



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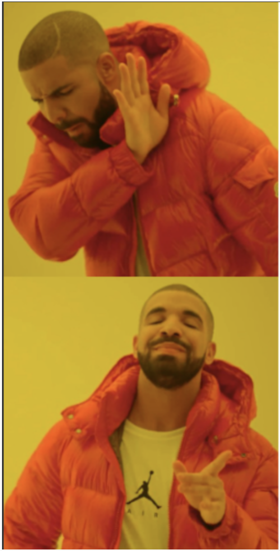
Countable Additivity



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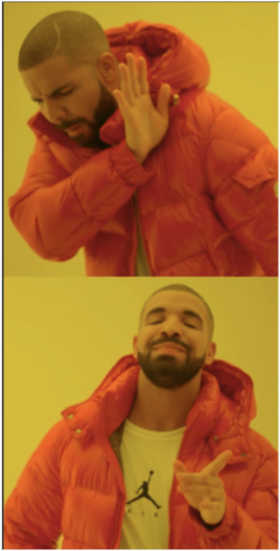
Poll. What does this mean?

Countable Additivity



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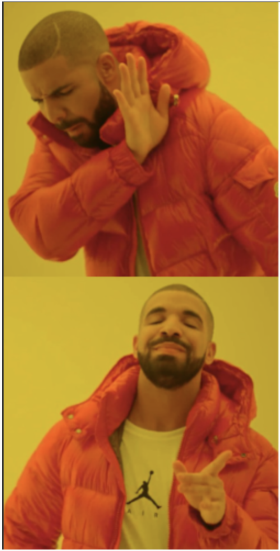
Interpretation. Probability is additive **only** over disjoint unions.

Countable Additivity



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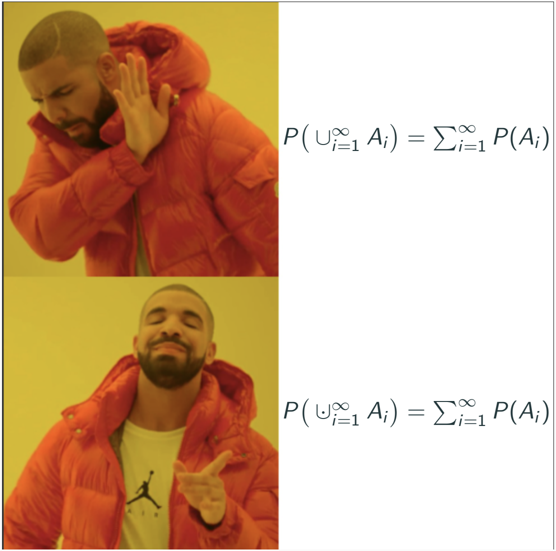
Example (Dice). $P(\texttt{roll 1 or 2}) = P(\texttt{roll 1}) + P(\texttt{roll 2})$.

Countable Additivity



Anti-example. $P(\texttt{male or old}) \neq P(\texttt{male}) + P(\texttt{old})$.

Monty Hall Problem

There are three doors on the set for a game show. Behind one door is a car and behind the other two doors are goats.

You get to pick a door to open. The host of the show then opens one of the other doors and reveals a goat. What should you do if you want to maximize your chance of winning the car: stay with your original door or switch – or would the likelihood of winning be the same either way?

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Try it! `https://www.mathwarehouse.com/monty-hall-simulation-online/`

Group exercises

1. (1.2.1) Let $\{A_i : i \in I\}$ be a collection of sets. Prove De Morgan's Laws:

$$\big(\cup_i A_i \big)^c = \cap_i A_i^c, \qquad \big(\cap_i A_i \big)^c = \cup_i A_i^c$$

2. (1.8.3a) Let $\mathcal{F}$ be a σ-field of subsets of Ω. Show that $\mathcal{F}$ is closed under countable intersections. That is, if $A_1, A_2, \ldots$ are in $\mathcal{F}$, then so is $\cap_i A_i$.
3. (1.2.2) Let A and B belong to some σ-field $\mathcal{F}$. Show that $\mathcal{F}$ contains the sets $A \cap B$, $A \setminus B$, and $A \triangle B$.
4. (1.3.2) A fair coin is tossed repeatedly. Show that, with probability one, a head turns up sooner or later. Show similarly that any finite sequence of heads or tails occurs eventually with probability one.
5. (1.4.5i) **The Monty Hall Problem: goats and cars.** In a game show, you have to choose one of three doors. One conceals a new car, two conceal old goats. You choose, but your chosen door is not opened immediately. Instead, the presenter opens another door, which reveals a goat. He offers you the opportunity to change your choice to a third door (unopened and so far unchosen). Let p be the (conditional) probability that the third door conceals the car. Show that $p = 2/3$, assuming that the presenter is determined to show you a goat (show with a choice of two, he picks one at random).
6. (1.5.9) Two fair dice are rolled. Show that the event that their sum is 7 is independent of the score shown by the first die.
7. (1.7.2) A hand of 13 cards is dealt from a normally shuffled pack of 52 cards. What is the probability that the hand contains exactly one ace given that it contains exactly two kings?