Linear Algebra:

Theory, Intuition, Code

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0.1 Front matter

This page contains some important details about the book that basically no one reads but somehow is always in the first page.

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This book was written and formatted in LaTeX by Mike X Cohen. (Mike X Cohen is the friendlier and more approachable persona of Professor Michael X Cohen; Michael deals with the legal and business aspects while Mike gets to have fun writing.)

Book edition 1.

0.2 Dedication

If you're reading this, then the book is dedicated to you. I wrote this book for *you*. Now turn the page and start learning math!

0.3 Forward

The past is immutable and the present is fleeting. Forward is the only direction.

Contents

Chapter 1

Introduction to this book

1.1 What is linear algebra and why learn it?

FACT: Linear algebra is SUUUUPER IMPORTANT!!

Linear algebra is the branch of mathematics concerned with vectors and matrices, their linear combinations, and operations acting upon them. Linear algebra has a long history in pure mathematics, in part because it provides a compact notation that is powerful and general enough to be used in geometry, calculus, differential equations, physics, economics, and many other areas.

But the importance and application of linear algebra is quickly increasing in modern applications. Many areas of science, technology, finance, and medicine are moving towards large-scale data collection and analysis. Data are often stored in matrices, and operations on those data—ranging from statistics to filtering to machine learning to computer graphics to compression—are typically implemented via linear algebra operations. Indeed, linear algebra has arguably exceeded statistics and time series analysis as the most important branch of mathematics in which to gain proficiency for data-focused areas of science and industry.

Human civilization is moving towards increasing digitization, quantitative methods, and data. Therefore, knowledge of foundational topics such as linear algebra are increasingly important. One may (indeed: *should*) question the appropriateness and utility of the trend towards "big data" and the over-reliance on algorithms to make decisions for us, but it is inarguable that familiarity with matrix analysis, statistics, and multivariate methods have become crucial skills for any data-related job in academia and in industry.

1.2 About this book

The purpose of this book is to teach you how to think about and work with matrices, with an eye towards applications in machine learning, multivariate statistics, time series, and image processing.

If you are interested in data science, quantitative biology, statistics, or machine learning and artificial intelligence, then this book is for you. If you don't have a strong background in mathematics, then don't be concerned: You need only high-school math and a bit of dedication to learn linear algebra from this book.

This book is written with the self-studying reader in mind. Many people do not realize how important linear algebra is until after university, or they do not meet the requirements of university-level linear algebra courses (typically, calculus). Linear algebra textbooks are often used as a compendium to a lecture-based course embedded in a traditional university math program, and therefore can be a challenge to use as an independent resource. I hope that this book is a self-contained resource that works well inside or outside of a formal course.

Many extant textbooks are theory-oriented, with a strong focus on abstract concepts as opposed to practical implementations. You might have encountered such books: They avoid showing numerical examples in the interest of generalizations; important proofs are left "as an exercise for the reader"; mathematical statements are simply plopped onto the page without discussion of relevance, importance, or application; and there is no mention of whether or how operations can be implemented in computers.

I do not write these as criticisms—abstract linear algebra is a beautiful topic, and infinite-dimensional vector spaces are great. But for those interested in using linear algebra (and mathematics more generally) as a tool for understanding data, statistics, deep learning, etc., then abstract treatments of linear algebra may seem like a frustrating waste of time. My goal here is to present applied linear algebra in an approachable and comprehensible way, with little focus on abstract concepts that lack a clear link to applications.

Ebook version The ebook version is identical to the physical version of this book, in terms of the text, formulas, visualizations, and code. However, the formatting is necesarily quite different. The book was designed to be a *physical* book; and thus, margins, fonts, text and figure placements, and code blocks are optimized for pages, not for ereaders.

Therefore, I recommend getting the physical copy of the book if you have the choice. If you get the ebook version, then please accept my apologies for any ugly or annoying formatting issues. If you have difficulties reading the code, please download it from github.com/mikexcohen/LinAlgBook.

Equations This is a math book, so you won't be surprised to see equations. But math is more than just equations: In my view, the purpose of math is to understand concepts; equations are one way to present those concepts, but words, pictures, and code are also important. Let me outline the balance:

1. Equations provide rigor and formalism, but they rarely provide intuition.
2. Descriptions, analogies, visualizations, and code provide intuition but often lack sufficient rigor.

This balance guides my writing: Equations are pointless if they lack descriptions and visualizations, but words and pictures without equations can be incomplete or misinterpreted.

So yes, there is a respectable number of equations here. There are three levels of *hierarchy* in the equations throughout this book. Some equations are simple or reminders of previously discussed equations; these are lowest on the totem pole are are presented in-line with text like this: $x(yz) = (xy)z$.

More important equations are given on their own lines. The number in parentheses to the right will allow me to refer back to that equation later in the text (the number left of the decimal point is the chapter, and the number to the right is the equation number).

$$\sigma = x(yz) = (xy)z \tag{1.1}$$

And the most important equations—the ones you should really make sure to understand and be comfortable using and reproducing—are presented in their own box with a title:

Something important!

$$\sigma = x(yz) = (xy)z \tag{1.2}$$

Algebraic and geometric perspectives on matrices Many concepts in linear algebra can be formulated using both geometric and algebraic (analytic) methods. This "dualism" promotes comprehension and I try to utilize it often. The geometric perspective provides visual intuitions, although it is usually limited to 2D or 3D. The algebraic perspective facilitates rigorous proofs and computational methods, and is easily extended to N-D. When working on problems in $\mathbb{R}^2$ or $\mathbb{R}^3$, I recommend sketching the problem on paper or using a computer graphing program.

Just keep in mind that *not every* concept in linear algebra has both a geometric and an algebraic concept. The dualism is useful in many cases, but it's not a fundamental fact that necessarily applies to all linear algebra concepts.

1.3 Prerequisites

The obvious. Dare I write it? You need to be motivated to learn linear algebra. Linear algebra isn't so difficult, but it's also not so easy. An intention to learn applied linear algebra—and a willingness to expend mental energy towards that goal—is the single most important prerequisite. Everything below is minor in comparison.

High-school math. You need to be comfortable with arithmetic and basic algebra. Can you solve for x in $4x^2 = 9$? Then you have enough algebra knowledge to continue. Other concepts in geometry, trigonometry, and complex numbers ($a\ ib, e^{i\theta}$) will be introduced as the need arises.

Calculus. Simply put: none. I strongly object to calculus being taught before linear algebra. No offense to calculus, of course; it's a rich, beautiful, and incredibly important subject. But linear algebra can be learned without any calculus, whereas many topics in calculus involve some linear algebra. Furthermore, many modern applications of linear algebra invoke no calculus concepts. Hence, linear algebra should be taught assuming no calculus background.

Vectors, matrices and <insert fancy-sounding linear algebra term here>. If this book is any good, then you don't need to know anything about linear algebra before reading it. That said, some familiarity with matrices and matrix operations will be beneficial.

Programming. Before computers, advanced concepts in mathematics could be understood only by brilliant mathematicians with great artistic skills and a talent for being able to visualize equations. Computers changed that. Now, a reasonably good mathematics student with some perseverance and moderate computer skills can implement and visualize equations and other mathematical concepts. The computer deals with the arithmetic and low-level graphics, so you can worry about the concepts and intuition.

This doesn't mean you should forgo solving problems by hand; it is only through laboriously solving lots and lots of problems on paper that you will internalize a deep and flexible understanding of linear algebra. However, only simple (often, integer) matrices are feasible to work through by hand; computer simulations and plotting will allow you to understand an equation visually, when staring at a bunch of letters and Greek characters might give you nothing but a sense of dread. So, if you really want to learn modern, applied linear algebra, it's helpful to have some coding proficiency in a language that interacts with a visualization engine.

I provide code for all concepts and problems in this book in both MATLAB and Python. I find MATLAB to be more comfortable for implementing linear algebra concepts. If you don't have access to MATLAB, you can use Octave, which is a free cross-platform software that emulates nearly all MATLAB functionality. But the popularity of Python is undeniable, and you should use whichever program you (1) feel more comfortable using or (2) anticipate working with in the future. Feel free to use any other coding language you like, but it is your responsibility to translate the code into your preferred language.

I have tried to keep the code as simple as possible, so you need only minimal coding experience to understand it. On the other hand, this is not an intro-coding book, and I assume some basic coding familiarity. If you understand variables, for-loops, functions, and basic plotting, then you know enough to work with the book code.

To be clear: You do not *need* any coding to work through the book. The code provides additional material that I believe will help solidify the concepts as well as adapt to specific applications. But you can successfully and completely learn from this book without looking at a single line of code.

1.4 Practice, exercises and code challenges

Math is not a spectator sport. If you simply read this book without solving any problems, then sure, you'll learn something and I hope you enjoy it. But to really understand linear algebra, you need to solve problems.

Some math textbooks have a seemingly uncountable number of exercises. My strategy here is to have a manageable number of exercises, with the expectation that you can solve *all* of them.

There is a hierarchy of problems to solve in this book:

Practice problems are a few problems at the end of chapter subsections. They are designed to be easy, the answers are given immediately below the problems, and are simply a way for you to confirm that you get the basic idea of that section. If you can't solve the practice problems, go back and re-read that subsection.

Exercises are found at the end of each chapter and focus on drilling and practicing the important concepts. The answers (yes, *all* of them; not just the odd-numbered) follow the exercises, and in many cases you can also check your own answer by solving the problem on a computer (using MATLAB or Python). Keep in mind that these exercises are designed to be solved by hand, and you will learn more by solving them by hand than by computer.

Code challenges are more involved, require some effort and creativity, and can only be solved on a computer. These are opportunities for you to explore concepts, visualizations, and parameter spaces in ways that are difficult or impossible to do by hand. I provide my solutions to all code challenges, but keep in mind that there are many correct solutions; the point is for you to explore and understand linear algebra using code, not to reproduce my code.

If you are more interested in concepts than in computer implementation, feel free to skip the coding challenges.

1.5 Online and other resources

Although I have tried to write this book to be a self-contained one-stop-shop for all of your linear algebra needs, it is naive to think that everyone will find it to be the perfect resource that I intend it to be. Everyone learns differently and everyone has a different way of understanding and visualizing mathematical concepts.

If you struggle to understand something, don't jump to the conclusion that you aren't smart enough; a simpler possibility is that the explanation I find intuitive is not the explanation that you find intuitive. I try to give several explanations of the same concept, in hopes that you'll find traction with at least one of them.

Therefore, you shouldn't hesitate to search the Internet or other textbooks if you need different or alternative explanations, or if you want additional exercises to work through.

This book is based on an online course that I created. The book and the course are similar but not entirely redundant. You don't need to enroll in the online course to follow along with this book (or the other way around). I appreciate that some people prefer to learn from online video lectures while others prefer to learn from textbooks. I am trying to cater to both kinds of learners.

You can find a list of all my online courses at sincxpress.com.