The Output Resistance of a Power Supply

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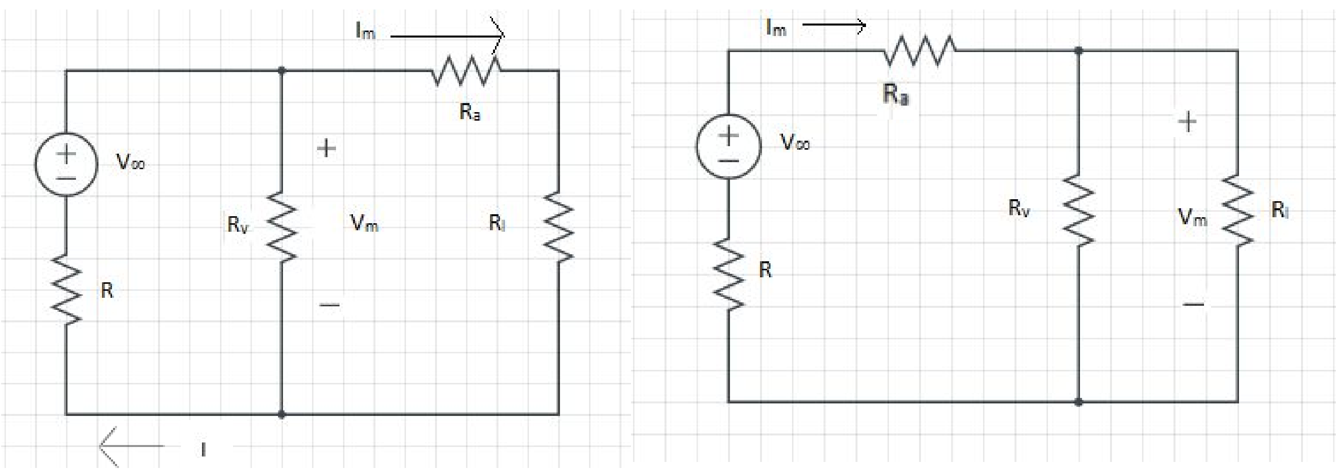
In this lab, we examined a simple circuit consisting of a single external resistor wired in series with a ammeter and a power supply and in parallel with a voltmeter. We used the values collected in these recordings to ultimately calculate the output resistance of the power source, given by the governing equation: $V = V_\infty - RI$, with V being the terminal voltage across the power supply, V_∞ being the electromotive force - the voltage potential with no load attached, R being the output resistance of the power source, and I being the current across the power supply.

When considering the possible wiring configurations for the power source, voltmeter, ammeter, and resistor in our experiment, there existed two possible options. The two options should theoretically output the same values of voltage and current had the conditions been ideal, however, the multimeters used are not ideal - they have internal resistances. A real ammeter can be modeled as the current across the examined resistor when a resistor with a very low resistance is connected in series,[1] whereas a real voltmeter can be modeled as the voltage across the examined resistor when a resistor with a very large resistance is connected in parallel.[2] As a result, the different wiring configurations have slightly different readings which can be explained by the diagram and formulas below,

Option 1 Option 2



with I_m being the measured current, V_m being the measured voltage, V_∞ being the emf (electromotive force), R_a being the resistance of the ammeter (~0 ohms), R_V being the resistance of the voltmeter (~∞ ohms) and R being the output resistance of the power source. I is the current in the left loop of Option 1.

[1] http://www.allaboutcircuits.com/textbook/direct-current/chpt-8/ammeter-impact-measured-circuit/
[2] http://www.allaboutcircuits.com/textbook/direct-current/chpt-8/voltmeter-impact-measured-circuit/

Option 1:

$V_m = I_m(R_l + R_a)$ (Kirchoff's Voltage Law, right loop)

$V_m = V_\infty - IR$ (Kirchoff's Voltage Law, left loop. Note that I is not the measured I)

$V_\infty - IR = I_m(R_l + R_a)$

$I_m = (V_\infty - IR)/(R_l + R_a)$

$I = (I_m(R_l + R_a) - V_\infty)/(-R)$

$V_m = V_\infty - IR$

$V_m = V_\infty - (I_m(R_l + R_a) - V_\infty)(R)/(-R)$

$V_m = V_\infty + (I_m(R_l + R_a) - V_\infty)$

$V_m = I_m(R_l + R_a)$

If we assume that $R_a \sim 0$,

$I_m = (V_\infty - IR)/(R_l)$

$V_m = I_m R_l$

Option 2:

$V_\infty = I_m R_a + V_m + I_m R$ (Kirchoff's Voltage Law, left loop)

$V_\infty = I_m(R_a + R) + V_m$

If we assume that $R_a \sim 0$,

$V_\infty = I_m(R) + V_m$, which is the governing equation proposed.

In relation to R_l,

$((1/R_l)/(1/R_l + 1/R_v)) * (I_m) * (R_l) = V_m$ (using current division)

$(1/(1/R_l + 1/R_v)) = V_m/I_m$

$(R_l R_v)/(R_l + R_v) = V_m/I_m$

$(R_l R_v) = V_m * (R_l + R_v)/I_m$

$(R_l)(R_v - (V_m/I_m)) = V_m * (R_v)/I_m$

$(R_l) = (V_m * (R_v)/I_m)/(R_v - (V_m/I_m))$

If we take the limit as R_v goes to infinity:

$R_l = V_m/I_m$

Therefore, in option 1, the reading of the voltmeter will be $I_m(R_l + R_a)$ and the reading of the ammeter will be $(V_\infty - IR)/(R_l + R_a)$ In option 2, the relation between the voltmeter and the ammeter will be given by the equation $V_\infty = I_m(R_a + R) + V_m$. Because option 2 better suits the governing equation when the assumption $R_a \sim 0$ is made, we will use it in our experiment.

Option 1:

The measured formula for R_l as calculated above (before assumptions) is:

$V_m = I_m(R_l + R_a)$

$R_l = V_m/I_m - R_a$

Using the Addition Formula and Division Formula for Error Propagation provided from the course materials, For $X = A + B$, $\sigma_x = \sqrt{(\sigma_A)^2 + (\sigma_B)^2}$ and for $X = A/B$, $\sigma_x = X\sqrt{(\sigma_A/A)^2 + (\sigma_B/B)^2}$

The uncertainty of R_l can be calculated using the following formula:

$$\sigma_{R_l} = \sqrt{(R_l + R_a)^2((\sigma_{V_m}/V_m)^2 + (\sigma_{I_m}/I_m)^2) + (\sigma_{R_a})^2}$$

If we assume that $R_a \sim 0$, and therefore $\sigma_{Ra} \sim 0$, the formula simplifies to

$$\sigma_{R_l} = R_l\sqrt{(\sigma_{V_m}/V_m)^2 + (\sigma_{I_m}/I_m)^2} = (V_m/I_m)\sqrt{(\sigma_{V_m}/V_m)^2 + (\sigma_{I_m}/I_m)^2}$$

With I_m being the measured current, V_m being the measured voltage, R_a being the resistance of the ammeter, σ_R being the propagated uncertainty of R_l, σ_V being the propagated uncertainty of V_m, σ_I being the propagated uncertainty of I_m and σ_{Ra} being the propagated uncertainty of R_a.

Option 2:

The measured formula for R_l as calculated above (before assumptions) is:

$R_l = (V_m * (R_v)/I_m)/(R_v - (V_m/I_m)) = (V_m/I_m)/(1 - (V_m/I_mR_v))$

Using the Addition Formula and Division Formula for Error Propagation provided from the course materials,

For $X = A + B$, $\sigma_x = \sqrt{(\sigma_A)^2 + (\sigma_B)^2}$ and for $X = A/B$, $\sigma_x = X\sqrt{(\sigma_A/A)^2 + (\sigma_B/B)^2}$

The uncertainty of R_l can be calculated using the following formula:

$$\sigma_{R_l} = R_l\sqrt{(V_m/I_m)^2((\sigma_{V_m}/V_m)^2 + (\sigma_{I_m}/I_m)^2) + (V_m/I_mR_v)^2((\sigma_{V_m}/V_m)^2 + (\sigma_{I_m}/I_m)^2 + (\sigma_{Rv}/R_v)^2)}$$

If we take the limit as R_v goes to infinity, the formula simplifies to

$$\sigma_{R_l} = R_l\sqrt{(\sigma_{V_m}/V_m)^2 + (\sigma_{I_m}/I_m)^2} = (V_m/I_m)\sqrt{(\sigma_{V_m}/V_m)^2 + (\sigma_{I_m}/I_m)^2}$$

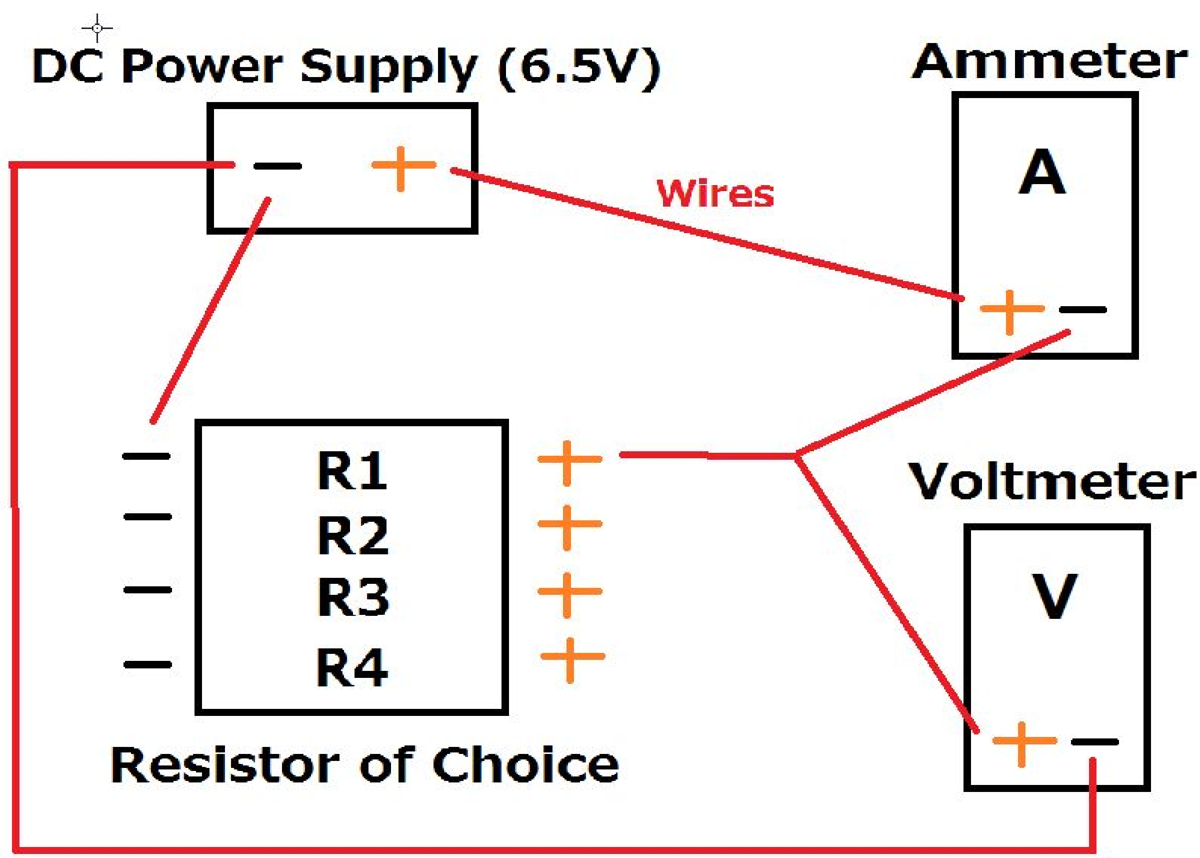
With I_m being the measured current, V_m being the measured voltage, R_a being the resistance of the ammeter, σ_R being the propagated uncertainty of R_l, σ_V being the propagated uncertainty of V_m, σ_I being the propagated uncertainty of I_m and σ_{Rv} being the propagated uncertainty of R_v.

In the given apparatus, there were seven possible resistors we could use, with values of $27*10^1 \pm 10\%$, $68*10^1 \pm 5\%$, $82*10^1 \pm 5\%$, $82*10^2 \pm 5\%$, $33*10^3 \pm 5\%$, $82*10^3 \pm 5\%$, $32*10^5 \pm 1\%$ and we were to choose 4 of them for use in our measurements. There were several factors that played into our resistor choice.

Firstly, we wanted to avoid resistors that would require changing the knobs of the multimeter, because these would influence the internal shunt resistors and would alter their effect on our measured quantities. In particular, we were wary of the ammeter because, according to the provided equation, current would be the more volatile variable (small internal resistance does not create a large difference between terminal voltage). In other words, we wanted to choose resistors that would keep the current of the circuit in similar orders of magnitude. By keeping similar orders of magnitude in current, we realized that this would also help us in graphing (as current would be the independent variable in the given linear governing equation $V = V_\infty - RI$. Having large gaps between measured data points corresponding to order of magnitude differences in the independent variable creates uncertainty in the graph behavior between these points and nullifies the percentage influence of absolute uncertainty on larger variables.

Secondly, we wanted to choose resistors that would allow us to take the highest precision readings using the most number of significant digits. We knew that the data we were collecting for this experiment was that of current and voltage, and we knew that the digital multimeter we were using had a limited range of values it could measure. We did not want to use a resistor that would exceed the range of measuring values, or would only be cutoff at a single significant digit when reading the value. In particular, we focussed on the ammeter, as current was the more volatile variable. For example, using the highest value resistor ($32*10^5$ ohms) on the provided 6.5 V power supply would create a current of about 0.006 mA, a value that can only be displayed as one significant digit on the ammeter. This is highly undesirable.

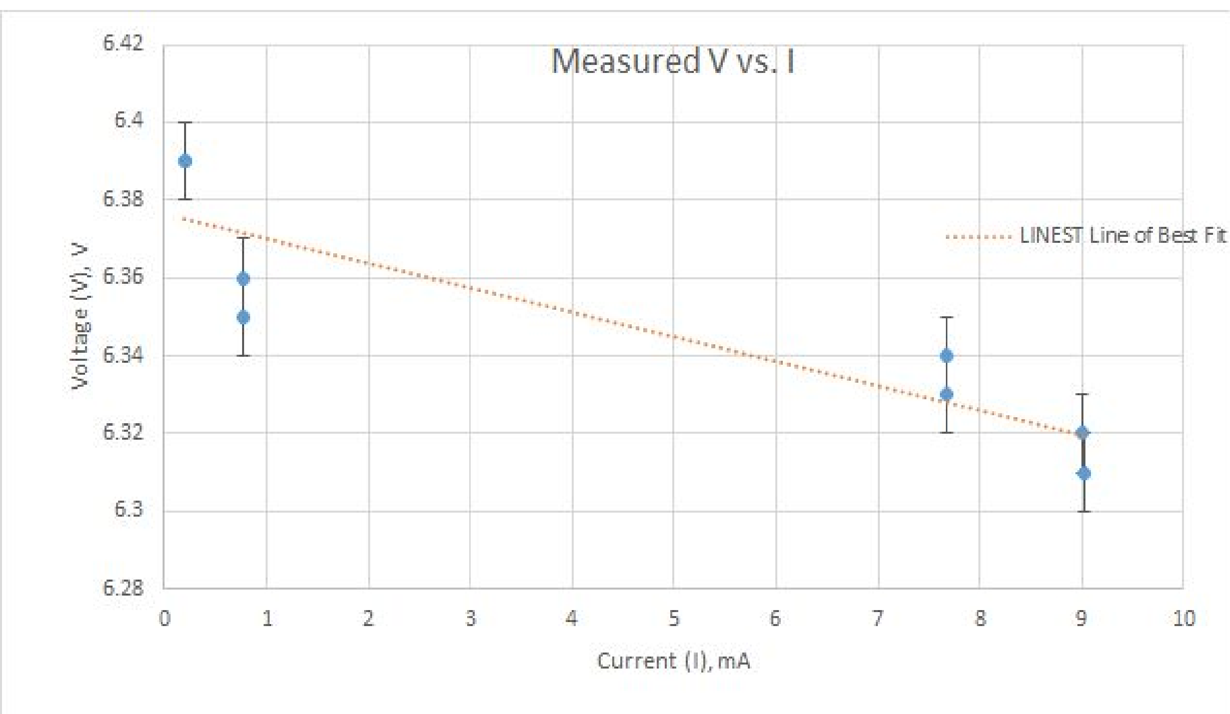
With these two factors in mind, we chose resistors of resistance $68*10^1 \pm 5\%$, $82*10^1 \pm 5\%$, $82*10^2 \pm 5\%$, $33*10^3 \pm 5\%$. These resistors were chosen because 1) we would only have to turn the ammeter knob once, and 2) they avoid cutoff of significant digits on the ammeter. The disadvantage of this choice is that it spans two orders of magnitude (0.1~10) which negatively impacts graphing but we determined this to be inevitable. With these chosen resistors, we proceeded to take recreate the setup to take recordings of the terminal voltage V and the current I using the setup described by option 2.



The collected data for the selected resistors are shown below. Two measurements for each resistor were taken.

R_l (Ohms)	σ_R (Ohms)	I_m (mA)	σ_I (mA)	V_m (V)	σ_V (V)
680	34	9.03	0.01	6.31	0.01
680	34	9.01	0.01	6.32	0.01
820	41	7.67	0.01	6.34	0.01
820	41	7.68	0.01	6.33	0.01
8200	410	0.764	0.001	6.35	0.01
8200	410	0.764	0.001	6.36	0.01
33000	1600	0.197	0.001	6.39	0.01
33000	1600	0.198	0.001	6.39	0.01

In the graph below, V_m vs I_m is plotted with Excel with the LINEST array function applied to show the output resistance of the power source, R. We took advantage of the linearity of the governing equation $V_m = V_\infty - RI_m$, with V_m being the dependent variable, I_m being the independent variable, and -R being the slope (in kOhms). The error bars for the x values are present. However, the reading error for the x values are too small, and no noticeable error bars can be seen on the graph.



The slope of the line of best fit is -0.006 ± 0.001 kOhms with a y-intercept of 6.376 ± 0.008 V.

These uncertainties were calculated using the equations for the uncertainty of slope and the uncertainty of the y-intercept, given by

$$\Delta = N\sum x_i^2 - (\sum x_i)^2$$

$$s_{y,x}^2 = (1/N-2)\sum(y_i - (b + mx_i))^2$$

$$s_m = \sqrt{Ns_{y,x}^2/\Delta}$$ (the uncertainty of slope)

$$s_b = \sqrt{s_{y,x}^2 \sum x_i^2/\Delta}$$ (the uncertainty of the y-intercept)

These formulas were found in the uncertainty notes posted online under the course materials, and were programmed in Python and used to calculate the uncertainties in our experiment. These uncertainties were corroborated against the uncertainty values given by LINEST. Therefore, the resistance of the DC power supply is the negative value of the calculated slope, equivalent to 6 ± 1 ohm.

As can be seen on the graph, the line fails to cross all the error bars. This indicates that this might not be a correct fit, and to alleviate this, more readings must be taken. One possibility that might have caused this was the changing of the ammeter dial. While taking measurements, we noted that for the same configuration of resistor, multimeters, and voltage source, when the ammeter dial was switched, the voltage changed quite significantly. The switching of dials is associated with a change in the ammeter's internal resistance. Therefore, it is possible that this could have played a part the apparent fitting error. To test this hypothesis, we would have to redo the experiment, but would choose resistor values that kept the ammeter dial constant (by keeping current in the same order of magnitude).

Appendix A

```
import math
N = 8
data = [[9.03,6.31],
        [9.01,6.32],
        [7.67,6.34],
        [7.68,6.33],
        [0.764,6.35],
        [0.764,6.36],
        [0.197,6.39],
        [0.198,6.39]]
m = -0.00631
b = 6.376

def slope_uncertainty(N, data, m, b):
    x_vals = []
    y_vals = []
    for i in range(len(data)):
        x_vals.append(data[i][0])
        y_vals.append(data[i][1])
    x_vals_sum = sum(x_vals)
    delta = 0
    sum_sqr = 0
    for i in range(len(x_vals)):
        sum_sqr += x_vals[i]**2
    delta = sum_sqr*N - (x_vals_sum)**2

    sxysqr = 0
    for i in range(len(x_vals)):
        sxysqr += (y_vals[i] - (b+m*x_vals[i]))**2
    sxysqr /= (N-2)

    std_dev_m = math.sqrt(N*sxysqr/delta)
    std_dev_b = math.sqrt(sxysqr*sum_sqr/delta)
    return [std_dev_m, std_dev_b]

print(slope_uncertainty(N, data, m, b))
```