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Gameplay in HTML5: Homework #3

Vectors

1. Let **u** = [-1, 3], **v** = [7, 2], **w** = [5, 0], **e** = [1, 0], and **f** = [0, 1].
    1. What is u_x? -1
    2. What is v_y? 2
    3. What is w_1? 5 (assuming 1-based)
    4. Compute |**u**|.
       $\sqrt{(-1^2 + 3^2)} = \sqrt{(1 + 9)} = 3.16227766016838$
    5. Compute |**v**|.
       $\sqrt{(7^2 + 2^2)} = \sqrt{(49 + 4)} = 7.28010988928052$
    6. Compute |**w**|.
       $\sqrt{(5^2 + 0^2)} = \sqrt{(25 + 0)} = 5$
    7. Compute |**e**|.
       $\sqrt{(1^2 + 0^2)} = \sqrt{(1 + 0)} = 1$
    8. Compute 3**u**.
       [(3 × -1) , (3 × 3)] = [-3, 9]
    9. Compute 0**v**.
       [(0 × -1), (0 × 3)] = [0, 0]
    10. Compute -3**u**.
        [(-3 × -1) , (-3 × 3)] = [3, -9]
    11. What is -**v**?
        -[7, 2] = [-7, -2]
    12. Compute **w**/2.
        (1/2) [5, 0] = [5 × .5, 0 × .5] = [2.5, 0]
    13. Compute **v**/0.
        NaN (can't divide by zero)
    14. What do you get when you normalize **u**?
        (1 / |**u**|) [-1, 3] =
        [-1 / 3.16227766016838, 3 / 3.16227766016838] =
        [-0.31622776601684, 0.94868329805051]
    15. What do you get when you normalize **w**?
        (1 / |**w**|) [5, 0] =
        [5 / 5, 0 / 5] =
        [1, 0]
    16. Compute **u** + **v**.
        [-1 + 7, 3 + 2] = [6, 5]
    17. Compute **v** + **w**.
        [7 + 5, 2 + 0] = [12, 2]
    18. Compute **v** + **u**.
        [7 + -1, 2 + 3] = [6, 5]
    19. Compute 3(**u** + **v**).
        [(3 × 6), (3 × 5) = [18, 15]

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20. Compute $3\mathbf{u} + 3\mathbf{v}$.
    $3\mathbf{u} = [(3 \times -1) , (3 \times 3)] = [-3, 9]$
    $3\mathbf{v} = [(3 \times 7), (3 \times 2)] = [21, 6]$
    $[-3 + 21, 9 + 6] = [18, 15]$
21. Compute $(2 + 1)\mathbf{u}$.
    $3\mathbf{u} = [(3 \times -1) , (3 \times 3)] = [-3, 9]$
22. Compute $2\mathbf{u} + 1\mathbf{u}$.
    $2\mathbf{u} = [(2 \times -1), (2 \times 3)] = [-2, 6]$
    $1\mathbf{u} = [(1 \times -1), (1 \times 3)] = [-1, 3]$
    $[-2 + -1, 6 + 3] = [-3, 9]$
23. Compute $3\mathbf{e} + 5\mathbf{f}$.
    $3\mathbf{e} = [(3 \times 1), (3 \times 0)] = [3, 0]$
    $5\mathbf{f} = [(5 \times 0), (5 \times 1)] = [0, 5]$
    $[3 + 0, 0 + 5] = [3, 5]$
24. Compute $\mathbf{u} - \mathbf{v}$.
    $[-1, 3] - [7, 2] =$
    $[-1 - 7, 3 - 2] =$
    $[-8, 1]$
25. Compute $\mathbf{v} - \mathbf{u}$.
    $[7, 2] - [-1, 3] =$
    $[7 - -1, 2 - 3] =$
    $[8, -1]$
26. Compute $\mathbf{u} \cdot \mathbf{v}$.
    $[-1, 3] \cdot [7, 2] =$
    $(-1 \times 7) + (3 \times 2) = (-7 + 6) = -1$
27. Compute $\mathbf{v} \cdot \mathbf{w}$.
    $[7, 2] \cdot [5, 0] =$
    $(7 \times 5) + (2 \times 0) = (35 + 2) = 37$
28. Compute $\mathbf{v} \cdot \mathbf{u}$.
    $[7, 2] \cdot [-1, 3] =$
    $(7 \times -1) + (2 \times 3) = (-7 + 6) = -1$
29. Compute $3(\mathbf{u} \cdot \mathbf{v})$.
    $\mathbf{u} \cdot \mathbf{v} = [-1, 3] \cdot [7, 2] = (-1 \times 7) + (3 \times 2) = (-7 + 6) = -1$
    $3 \times -1 = -3$
30. Compute $(3\mathbf{u}) \cdot \mathbf{v}$.
    $3\mathbf{u} = [(3 \times -1) , (3 \times 3)] = [-3, 9]$
    $[-3, 9] \cdot [7, 2] = (-3 \times 7) + (9 \times 2) = (-21 + 18) = -3$
31. Compute $\mathbf{u} \cdot (3\mathbf{v})$.
    $3\mathbf{v} = [(3 \times 7), (3 \times 2)] = [21, 6]$
    $[-1, 3] \cdot [21, 6] = (-1 \times 21) + (3 \times 6) = (-21 + 18) = -3$
32. Compute $\mathbf{u} \cdot \mathbf{u}$. Compare this to $|\mathbf{u}|$.
    $\mathbf{u} \cdot \mathbf{u}$ $[-1, 3] \cdot [-1, 3] = (-1 \times -1) + (3 \times 3) = (1 + 9) = 10$
    $|\mathbf{u}|$ $\sqrt{(-1^2 + 3^2)} = \sqrt{(1 + 9)} = 3.16227766016838$
    $3.16227766016838^2 = 10$
33. Compute $\mathbf{e} \cdot \mathbf{u}$. Compare this to u_x.
    $[1, 0] \cdot [-1, 3] = (1 \times -1) + (0 \times 3) = (-1 + 0) = -1 = u_x$

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1. Compute **f**·**u**. Compare this to u_y.
   [0, 1] · [-1, 3] = (0 × -1) + (1 × 3) = (0 + 3) = 3 = u_y
2. Compute **e**·**v**. Compare this to v_x.
   [1, 0] · [7, 2] = (1 × 7) + (0 × 2) = (7 + 0) = 7 = v_x

3. Compute **f**·**v**. Compare this to v_y.
   [0, 1] · [7, 2] = (0 × 7) + (1 × 2) = (0 + 2) = 2 = v_y

4. Compute the angle between **u** and **v**. What type of angle is it?
   u = [-1, 3], **v** = [7, 2] (vectors)
   [-1, 3] · [7, 2] =
   (-1 × 7) + (3 × 2) = (-7 + 6) = -1 (dot product)
   $\sqrt{(-1^2 + 3^2)} = \sqrt{(1 + 9)}$ = 3.16227766016838 (length u)
   $\sqrt{(7^2 + 2^2)} = \sqrt{(49 + 4)}$ = 7.28010988928052 (length v)
   cos θ = **(u·v) / (|u| · |v|)** (find cos of θ)
   cos θ = (-1) / (3.16227766016838 × 7.28010988928052) =
   -0.04343722427631
   θ = arc cos(-0.04343722427631) = 92.48955292 degrees
   obtuse angle

5. Compute the angle between **v** and **w**. What type of angle is it?
   v = [7, 2], **w** = [5, 0] (vectors)
   [7, 2] · [5, 0] =
   (7 × 5) + (2 × 0) = (35 + 0) = 35 (dot product)
   $\sqrt{(7^2 + 2^2)} = \sqrt{(49 + 4)}$ = 7.28010988928052 (length v)
   $\sqrt{(5^2 + 0^2)} = \sqrt{(25 + 0)}$ = 5 (length w)
   cos θ = (35) / (7.28010988928052 × 5) =
   0.96152395
   θ = arc cos(0.96152395) = 15.94539541
   acute angle

6. Compute the angle between **e** and **w**. What type of angle is it?
   e = [1, 0], **w** = [5, 0] (vectors)
   [1, 0] · [7, 2] =
   (1 × 7) + (0 × 2) = (7 + 0) = 7 (dot product)
   $\sqrt{(1^2 + 0^2)} = \sqrt{(1 + 0)}$ = 1 (length e)
   $\sqrt{(5^2 + 0^2)} = \sqrt{(25 + 0)}$ = 5 (length w)
   cos θ = (7) / (1 × 5) =
   0.99970149
   θ = arc cos(0.99970149) = 1.39999949
   acute angle

7. Compute the angle between **f** and **w**. What type of angle is it?
   f = [0, 1], **w** = [5, 0] (vectors)
   [0, 1] · [5, 0] =
   (0 × 5) + (1 × 0) = (0 + 0) = 0 (dot product)

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$\sqrt{(0^2 + 1^2)} = \sqrt{(0 + 1)} = 1$ (len f)
$\sqrt{(5^2 + 0^2)} = \sqrt{(25 + 0)} = 5$ (len w)
(special case, avoid divide by zero, use acos of dot product)
$\theta = \arccos(0) = 90$
right angle

The “vector.Perp()” function takes a given vector and returns another vector perpendicular to it.