GENERAL ASSEMBLY

INTRO TO DATA SCIENCE

LECTURE 5: THE LINEAR REGRESSION

LAST TIME:

- INTRO TO DATABASES
- WORKING WITH APIS AND JSON
- MYSQL QUERIES

QUESTIONS?

I. INTRODUCTION TO REGRESSION DATA PROBLEMS
II. HOW REGRESSIONS WORK
III. DETERMINING COST

EXERCISES:
IV. IMPLEMENTING THE LINEAR MODEL

I. LINEAR REGRESSION

	continuous	categorical
supervised	???	???
unsupervised	???	???

	continuous	categorical
supervised	regression	classification
unsupervised	dimension reduction	clustering

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The **simple linear regression** model captures a linear relationship between a single input variable x and a response variable y:

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x = input variable (the one we use to train the model)

α = intercept (where the line crosses the y-axis)

β = regression coefficient (the model "parameter")

ε = residual (the prediction error)

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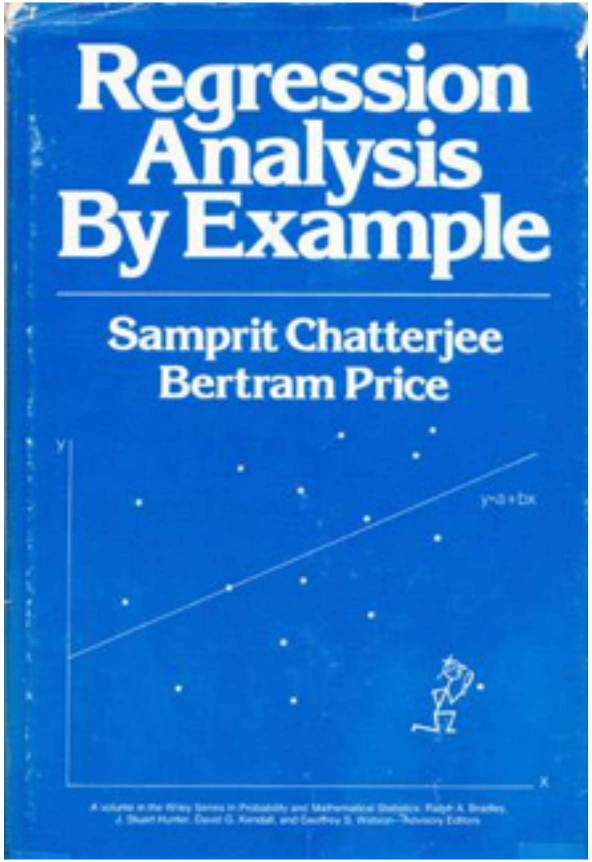
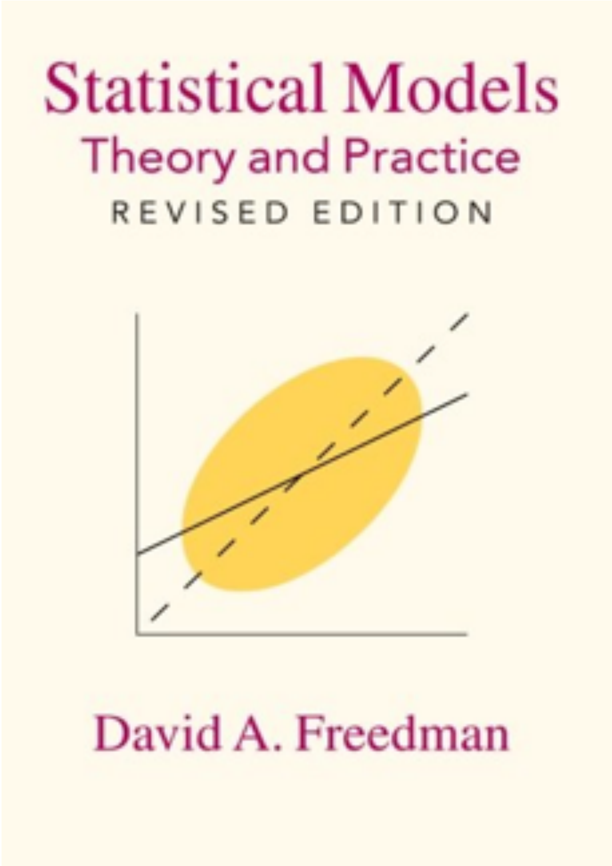
$$y = \alpha + \beta_1 x_1 + \ldots + \beta_n x_n + \varepsilon$$


Regression Analysis By Example
Samprit Chatterjee
Bertram Price


Statistical Models
Theory and Practice
REVISED EDITION
David A. Freedman

Linear regression involves several technical assumptions and is often presented with lots of mathematical formality.

In order for us to gain a deeper understanding of the “magic” behind a regression (and to see why we want a machine to do this!), let’s review the math behind this algorithm!

II: THE MATH WAY

Linear regression is, for the most part, just matrix algebra (the stuff we did already!)

Let’s go over the math by hand so we can understand how we determine the **regression coefficient**.

A linear regression in its simplest form:

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$$y = \alpha + \beta x + \varepsilon$$

but we can assume that our α is either 0 or 1, and ε is zero!

$$y = \beta x$$

So in a more simple form:

$$y = \beta x$$

but we want to solve for β, which means our new equation looks more like this:

$$\beta = (X^T X)^{-1} X^T y$$

So if we had data:

3.385	44.5
0.48	15.5
1.35	8.1
465	423
36.33	119.5

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Response

Input

$$\left(\begin{matrix} 1 & 1 & 1 & 1 & 1 \\ 3.385 & 0.48 & 1.35 & 465 & 36.33 \end{matrix}\begin{matrix} 3.385 & 1 \\ 0.48 & 1 \\ 1.35 & 1 \\ 465 & 1 \\ 36.33 & 1 \end{matrix}\right)^{-1}$$

$$\beta = (X^T X)^{-1} * \dots$$

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 3.385 & 0.48 & 1.35 & 465 & 36.33 \end{pmatrix} \begin{pmatrix} 44.5 \\ 15.5 \\ 8.1 \\ 423 \\ 119.5 \end{pmatrix}$$

... $X^T y$

$$\begin{matrix} 0.2617 & -0.0006 \\ -0.0006 & 0.000006 \end{matrix} \qquad \begin{matrix} 610.6 \\ 201205.4425 \end{matrix}$$

$$\beta = (X^T X)^{-1} X^T y$$

$$\begin{matrix} 37.2 \\ 0.838 \end{matrix} = \begin{matrix} 0.2617 & -0.0006 \\ -0.0006 & 0.000006 \end{matrix} \quad \begin{matrix} 610.6 \\ 201205.4425 \end{matrix}$$

$$\beta = (X^T X)^{-1} X^T y$$

Intercept

$$\begin{bmatrix} 37.2 \\ 0.838 \end{bmatrix} = \begin{bmatrix} 0.2617 & -0.0006 \\ -0.0006 & 0.000006 \end{bmatrix} \begin{bmatrix} 610.6 \\ 201205.4425 \end{bmatrix}$$

β

$$\beta = (X^T X)^{-1} X^T y$$

Q: How did we do compared to a computer?

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```
Call:
lm(formula = brain ~ body, data = head(mammals, 5))

Coefficients:
(Intercept)         body
    37.2009       0.8382
```

A: Not bad!

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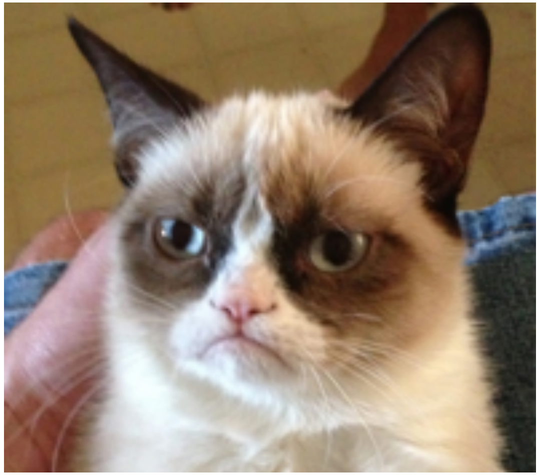

REVIEW: MATRIX ALGEBRA

Review this concept with data that we know has a coefficient of 1 and an intercept of 0:

Input	Response
1	1
2	2
3	3
4	4

III: COST OF LINEAR REGRESSIONS

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In python, we can find this with some quick code:

mean((prediction – actual)^2)

Q: How do measure goodness of fit?

A: In theory, we want to **maximize R^2 (as close to one as possible).**

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If you want to get serious into regression, learn more about the **coefficient of determination**.

REVIEW: COST

1. What values are we looking for when we consider **SSE**? What is the best value we could potentially have?

2. What is the best value we could have for R^2?

3. What's the primary difference between these two values?

EX: LINEAR REGRESSIONS

NEXT TIME:
POLYNOMIAL AND LOGISTIC REGRESSIONS