CHAPTER 2 Parallel Programs

2.1 Introduction

To understand and evaluate design decisions in a parallel machine, we must have an idea of the software that runs on the machine. Understanding program behavior led to some of the most important advances in uniprocessors, including memory hierarchies and instruction set design. It is all the more important in multiprocessors, both because of the increase in degrees of freedom and because of the much greater performance penalties caused by to mismatches between applications and systems.

Understanding parallel software is important for algorithm designers, for programmers, and for architects. As algorithm designers, it helps us focus on designing algorithms that can be run effectively in parallel on real systems. As programmers, it helps us understand the key performance issues and obtain the best performance from a system. And as architects, it helps us understand the workloads we are designing against and their important degrees of freedom. Parallel software and its implications will be the focus of the next three chapters of this book. This chapter describes the process of creating parallel programs in the major programming models. The next chapter focuses on the performance issues that must be addressed in this process, exploring some of the key interactions between parallel applications and architectures. And the following chapter relies on this understanding of software and interactions to develop guidelines for using parallel workloads to evaluate architectural tradeoffs. In addition to architects, the material in these chapters is useful for users of parallel machines as well: the first two chapters for programmers and algorithm designers, and the third for users making decisions about what machines to procure. However, the major focus is on issues that architects should understand before they get into the nuts and bolts of machine design and architectural tradeoffs, so let us look at it from this perspective.

As architects of sequential machines, we generally take programs for granted: The field is mature and there is a large base of programs than can be (or must be) viewed as fixed. We optimize the machine design against the requirements of these programs. Although we recognize that programmers may further optimize their code as caches become larger or floating-point support is improved, we usually evaluate new designs without anticipating such software changes. Compilers may evolve along with the architecture, but the source program is still treated as fixed. In parallel architecture, there is a much stronger and more dynamic interaction between the evolution of machine designs and that of parallel software. Since parallel computing is all about performance, programming tends to be oriented towards taking advantage of what machines provide. Parallelism offers a new degree of freedom—the number of processors—and higher costs for data access and coordination, giving the programmer a wide scope for software optimizations. Even as architects, we therefore need to open up the application “black box”. Understanding the important aspects of the process of creating parallel software, the focus of this chapter, helps us appreciate the role and limitations of the architecture. The deeper look at performance issues in the next chapter will shed greater light on hardware-software tradeoffs.

Even after a problem and a good sequential algorithm for it are determined, there is a substantial process involved in arriving at a parallel program and the execution characteristics that it offers to a multiprocessor architecture. This chapter presents general principles of the parallelization process, and illustrates them with real examples. The chapter begins by introducing four actual problems that serve as case studies throughout the next two chapters. Then, it describes the four major steps in creating a parallel program—using the case studies to illustrate—followed by examples of how a simple parallel program might be written in each of the major programming models. As discussed in Chapter 1, the dominant models from a programming perspective narrow down to three: the data parallel model, a shared address space, and message passing between private address spaces. This chapter illustrates the primitives provided by these models and how they might be used, but is not concerned much with performance. After the performance issues in the parallelization process are understood in the next chapter, the four application case studies will be treated in more detail to create high-performance versions of them.

2.2 Parallel Application Case Studies

We saw in the previous chapter that multiprocessors are used for a wide range of applications—from multiprogramming and commercial computing to so-called “grand challenge” scientific problems—and that the most demanding of these applications for high-end systems tend to be from scientific and engineering computing. Of the four case studies we refer to throughout this chapter and the next, two are from scientific computing, one is from commercial computing and one from computer graphics. Besides being from different application domains, the case studies are chosen to represent a range of important behaviors found in other parallel programs as well.

Of the two scientific applications, one simulates the motion of ocean currents by discretizing the problem on a set of regular grids and solving a system of equations on the grids. This technique of discretizing equations on grids is very common in scientific computing, and leads to a set of very common communication patterns. The second case study represents another major form of scientific computing, in which rather than discretizing the domain on a grid the computational domain is represented as a large number of bodies that interact with one another and move around as a result of these interactions. These so-called N-body problems are common in many areas such as simulating galaxies in astrophysics (our specific case study), simulating proteins and other molecules in chemistry and biology, and simulating electromagnetic interactions. As in many other areas, hierarchical algorithms for solving these problems have become very popular. Hierarchical N-body algorithms, such as the one in our case study, have also been used to solve important problems in computer graphics and some particularly difficult types of equation systems. Unlike the first case study, this one leads to irregular, long-range and unpredictable communication.

The third case study is from computer graphics, a very important consumer of moderate-scale multiprocessors. It traverses a three-dimensional scene and highly irregular and unpredictable ways, and renders it into a two-dimensional image for display. The last case study represents the increasingly important class of commercial applications that analyze the huge volumes of data being produced by our information society to discover useful knowledge, categories and trends. These information processing applications tend to be I/O intensive, so parallelizing the I/O activity effectively is very important. The first three case studies are part of a benchmark suite [SWG92] that is widely used in architectural evaluations in the literature, so there is a wealth of detailed information available about them. They will be used to illustrate architectural tradeoffs in this book as well.

2.2.1 Simulating Ocean Currents

To model the climate of the earth, it is important to understand how the atmosphere interacts with the oceans that occupy three fourths of the earth's surface. This case study simulates the motion of water currents in the ocean. These currents develop and evolve under the influence of several physical forces, including atmospheric effects, wind, and friction with the ocean floor. Near the ocean walls there is additional "vertical" friction as well, which leads to the development of eddy currents. The goal of this particular application case study is to simulate these eddy currents over time, and understand their interactions with the mean ocean flow.

Good models for ocean behavior are complicated: Predicting the state of the ocean at any instant requires the solution of complex systems of equations, which can only be performed numerically by computer. We are, however, interested in the behavior of the currents over time. The actual physical problem is continuous in both space (the ocean basin) and time, but to enable computer simulation we discretize it along both dimensions. To discretize space, we model the ocean basin as a grid of equally spaced points. Every important variable—such as pressure, velocity, and various currents—has a value at each grid point in this discretization. This particular application uses not a three-dimensional grid but a set of two-dimensional, horizontal cross-sections through the ocean basin, each represented by a two-dimensional grid of points (see Figure 2-1). For sim-

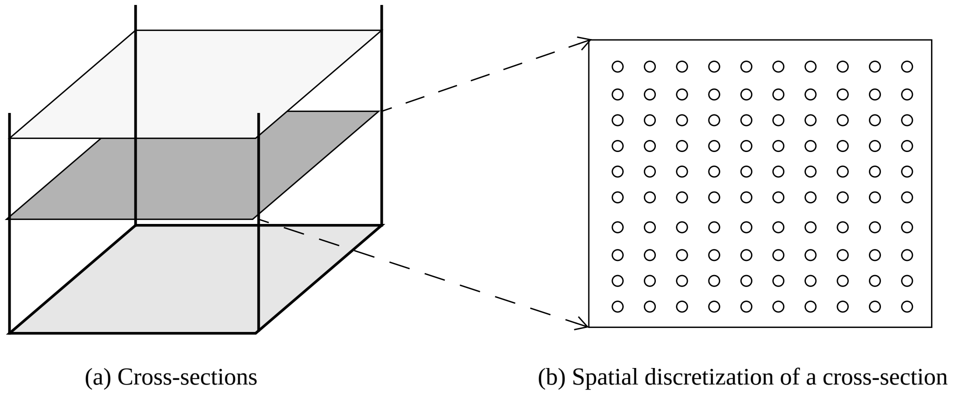


Figure 2-1 Horizontal cross-sections through an ocean basin, and their spatial discretization into regular grids.

plicity, the ocean is modeled as a rectangular basin and the grid points are assumed to be equally spaced. Each variable is therefore represented by a separate two-dimensional array for each cross-section through the ocean. For the time dimension, we discretize time into a series of finite time-steps. The equations of motion are solved at all the grid points in one time-step, the state of the variables is updated as a result, and the equations of motion are solved again for the next time-step, and so on repeatedly.

Every time-step itself consists of several computational phases. Many of these are used to set up values for the different variables at all the grid points using the results from the previous time-step. Then there are phases in which the system of equations governing the ocean circulation are actually solved. All the phases, including the solver, involve sweeping through all points of the relevant arrays and manipulating their values. The solver phases are somewhat more complex, as we shall see when we discuss this case study in more detail in the next chapter.

The more grid points we use in each dimension to represent our fixed-size ocean, the finer the spatial resolution of our discretization and the more accurate our simulation. For an ocean such as the Atlantic, with its roughly 2000km-x-2000km span, using a grid of 100-x-100 points implies a distance of 20km between points in each dimension. This is not a very fine resolution, so we would like to use many more grid points. Similarly, shorter physical intervals between time-steps lead to greater simulation accuracy. For example, to simulate five years of ocean movement updating the state every eight hours we would need about 5500 time-steps.

The computational demands for high accuracy are large, and the need for multiprocessing is clear. Fortunately, the application also naturally affords a lot of concurrency: many of the set-up phases in a time-step are independent of one another and therefore can be done in parallel, and the processing of different grid points in each phase or grid computation can itself be done in parallel. For example, we might assign different parts of each ocean cross-section to different processors, and have the processors perform their parts of each phase of computation (a data-parallel formulation).

2.2.2 Simulating the Evolution of Galaxies

Our second case study is also from scientific computing. It seeks to understand the evolution of stars in a system of galaxies over time. For example, we may want to study what happens when galaxies collide, or how a random collection of stars folds into a defined galactic shape. This problem involves simulating the motion of a number of bodies (here stars) moving under forces exerted on each by all the others, an *n-body* problem. The computation is discretized in space by treating each star as a separate body, or by sampling to use one body to represent many stars. Here again, we discretize the computation in time and simulate the motion of the galaxies for many time-steps. In each time-step, we compute the gravitational forces exerted on each star by all the others and update the position, velocity and other attributes of that star.

Computing the forces among stars is the most expensive part of a time-step. A simple method to compute forces is to calculate pairwise interactions among all stars. This has $O(n^2)$ computational complexity for n stars, and is therefore prohibitive for the millions of stars that we would like to simulate. However, by taking advantage of insights into the force laws, smarter hierarchical algorithms are able to reduce the complexity to $O(n \log n)$. This makes it feasible to simulate problems with millions of stars in reasonable time, but only by using powerful multiprocessors. The basic insight that the hierarchical algorithms use is that since the strength of the gravitational interaction falls off with distance as $G\frac{m_1 m_2}{r^2}$, the influences of stars that are further away are weaker and therefore do not need to be computed as accurately as those of stars that are close by. Thus, if a group of stars is far enough away from a given star, then their effect on the star does not have to be computed individually; as far as that star is concerned, they can be approximated as a single star at their center of mass without much loss in accuracy (Figure 2-2). The further away

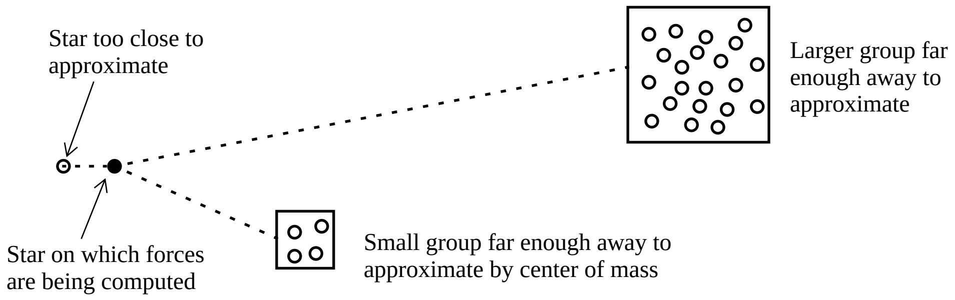

Star too close to approximate
Larger group far enough away to approximate
Star on which forces are being computed
Small group far enough away to approximate by center of mass

Figure 2-2 The insight used by hierarchical methods for n-body problems.

A group of bodies that is far enough away from a given body may be approximated by the center of mass of the group. The further apart the bodies, the larger the group that may be thus approximated.

the stars from a given star, the larger the group that can be thus approximated. In fact, the strength of many physical interactions falls off with distance, so hierarchical methods are becoming increasingly popular in many areas of computing.

The particular hierarchical force-calculation algorithm used in our case study is the Barnes-Hut algorithm. The case study is called Barnes-Hut in the literature, and we shall use this name for it as well. We shall see how the algorithm works in Section 3.6.2. Since galaxies are denser in some regions and sparser in others, the distribution of stars in space is highly irregular. The distribution also changes with time as the galaxy evolves. The nature of the hierarchy implies that stars in denser regions interact with more other stars and centers of mass—and hence have more work associated with them—than stars in sparser regions. There is ample concurrency across stars within a time-step, but given the irregular and dynamically changing nature the challenge is to exploit it efficiently on a parallel architecture.

2.2.3 Visualizing Complex Scenes using Ray Tracing

Our third case study is the visualization of complex scenes in computer graphics. A common technique used to render such scenes into images is ray tracing. The scene is represented as a set of objects in three-dimensional space, and the image being rendered is represented as a two-dimensional array of pixels (picture elements) whose color, opacity and brightness values are to be computed. The pixels taken together represent the image, and the resolution of the image is determined by the distance between pixels in each dimension. The scene is rendered as seen from a specific viewpoint or position of the eye. Rays are shot from that viewpoint through every pixel in the image plane and into the scene. The algorithm traces the paths of these rays—computing their reflection, refraction, and lighting interactions as they strike and reflect off objects—and thus computes values for the color and brightness of the corresponding pixels. There is obvious parallelism across the rays shot through different pixels. This application will be referred to as Raytrace.

2.2.4 Mining Data for Associations

Information processing is rapidly becoming a major marketplace for parallel systems. Businesses are acquiring a lot of data about customers and products, and devoting a lot of computational power to automatically extracting useful information or “knowledge” from these data. Examples from a customer database might include determining the buying patterns of demographic groups or segmenting customers according to relationships in their buying patterns. This process is called data mining. It differs from standard database queries in that its goal is to identify implicit trends and segmentations, rather than simply look up the data requested by a direct, explicit query. For example, finding all customers who have bought cat food in the last week is not data mining; however, segmenting customers according to relationships in their age group, their monthly income, and their preferences in pet food, cars and kitchen utensils is.

A particular, quite directed type of data mining is mining for associations. Here, the goal is to discover relationships (associations) among the information related to different customers and their transactions, and to generate rules for the inference of customer behavior. For example, the database may store for every transaction the list of items purchased in that transaction. The goal of the mining may be to determine associations between sets of commonly purchased items that tend to be purchased together; for example, the conditional probability $P(S_1|S_2)$ that a certain set of items S_1 is found in a transaction given that a different set of items S_2 is found in that transaction, where S_1 and S_2 are sets of items that occur often in transactions.

Consider the problem a little more concretely. We are given a database in which the records correspond to customer purchase transactions, as described above. Each transaction has a transaction identifier and a set of attributes or items, for example the items purchased. The first goal in mining for associations is to examine the database and determine which sets of *k* items, say, are found to occur together in more than a given threshold fraction of the transactions. A set of items (of any size) that occur together in a transaction is called an *itemset*, and an itemset that is found in more than that threshold percentage of transactions is called a *large itemset*. Once the large itemsets of size *k* are found—together with their frequencies of occurrence in the database of transactions—determining the association rules among them is quite easy. The problem we consider therefore focuses on discovering the large itemsets of size *k* and their frequencies.

The data in the database may be in main memory, or more commonly on disk. A simple way to solve the problem is to first determine the large itemsets of size one. From these, a set of candidate itemsets of size two items can be constructed—using the basic insight that if an itemset is large then all its subsets must also be large—and their frequency of occurrence in the transaction database counted. This results in a list of large itemsets of size two. The process is repeated until we obtain the large itemsets of size *k*. There is concurrency in examining large itemsets of size *k-1* to determine candidate itemsets of size *k*, and in counting the number of transactions in the database that contain each of the candidate itemsets.

2.3 The Parallelization Process

The four case studies—Ocean, Barnes-Hut, Raytrace and Data Mining—offer abundant concurrency, and will help illustrate the process of creating effective parallel programs in this chapter and the next. For concreteness, we will assume that the sequential algorithm that we are to make parallel is given to us, perhaps as a description or as a sequential program. In many cases, as in these case studies, the best sequential algorithm for a problem lends itself easily to parallelization; in others, it may not afford enough parallelism and a fundamentally different algorithm may be required. The rich field of parallel algorithm design is outside the scope of this book. However, there is in all cases a significant process of creating a good parallel program that implements the chosen (sequential) algorithm, and we must understand this process in order to program parallel machines effectively and evaluate architectures against parallel programs.

At a high level, the job of parallelization involves identifying the work that can be done in parallel, determining how to distribute the work and perhaps the data among the processing nodes, and managing the necessary data access, communication and synchronization. Note that work includes computation, data access, and input/output activity. The goal is to obtain high performance while keeping programming effort and the resource requirements of the program low. In particular, we would like to obtain good speedup over the best sequential program that solves the same problem. This requires that we ensure a balanced distribution of work among processors, reduce the amount of interprocessor communication which is expensive, and keep the overheads of communication, synchronization and parallelism management low.

The steps in the process of creating a parallel program may be performed either by the programmer or by one of the many layers of system software that intervene between the programmer and the architecture. These layers include the compiler, runtime system, and operating system. In a perfect world, system software would allow users to write programs in the form they find most convenient (for example, as sequential programs in a high-level language or as an even higher-level specification of the problem), and would automatically perform the transformation into efficient parallel programs and executions. While much research is being conducted in parallelizing compiler technology and in programming languages that make this easier for compiler and runtime systems, the goals are very ambitious and have not yet been achieved. In practice today, the vast majority of the process is still the responsibility of the programmer, with perhaps some help from the compiler and runtime system. Regardless of how the responsibility is divided among these parallelizing agents, the issues and tradeoffs are similar and it is important that we understand them. For concreteness, we shall assume for the most part that the programmer has to make all the decisions.

Let us now examine the parallelization process in a more structured way, by looking at the actual steps in it. Each step will address a subset of the issues needed to obtain good performance. These issues will be discussed in detail in the next chapter, and only mentioned briefly here.

2.3.1 Steps in the Process

To understand the steps in creating a parallel program, let us first define three few important concepts: tasks, processes and processors. A *task* is an arbitrarily defined piece of the work done by the program. It is the smallest unit of concurrency that the parallel program can exploit; i.e., an individual task is executed by only one processor, and concurrency is exploited across tasks. In the Ocean application we can think of a single grid point in each phase of computation as being a task, or a row of grid points, or any arbitrary subset of a grid. We could even consider an entire grid computation to be a single task, in which case parallelism is exploited only across independent grid computations. In Barnes-Hut a task may be a particle, in Raytrace a ray or a group of rays, and in Data Mining it may be checking a single transaction for the occurrence of an itemset. What exactly constitutes a task is not prescribed by the underlying sequential program; it is a choice of the parallelizing agent, though it usually matches some natural granularity of work in the sequential program structure. If the amount of work a task performs is small, it is called a *fine-grained* task; otherwise, it is called *coarse-grained*.

A *process* (referred to interchangeably hereafter as a *thread*) is an abstract entity that performs tasks.[1] A parallel program is composed of multiple cooperating processes, each of which performs a subset of the tasks in the program. Tasks are assigned to processes by some *assignment* mechanism. For example, if the computation for each row in a grid in Ocean is viewed as a task, then a simple assignment mechanism may be to give an equal number of adjacent rows to each process, thus dividing the ocean cross section into as many horizontal slices as there are processes. In data mining, the assignment may be determined by which portions of the database are assigned to each process, and by how the itemsets within a candidate list are assigned to processes to look up the database. Processes may need to communicate and synchronize with one another to perform their assigned tasks. Finally, the way processes perform their assigned tasks is by executing them on the physical *processors* in the machine.

It is important to understand the difference between processes and processors from a parallelization perspective. While processors are physical resources, processes provide a convenient way of abstracting or *virtualizing* a multiprocessor: We initially write parallel programs in terms of processes not physical processors; mapping processes to processors is a subsequent step. The number of processes does not *have* to be the same as the number of processors available to the program. If there are more processes, they are multiplexed onto the available processors; if there are fewer processes, then some processors will remain idle.

1. In Chapter 1 we used the correct operating systems definition of a process: an address space and one or more threads of control that share that address space. Thus, processes and threads are distinguished in that definition. To simplify our discussion of parallel programming in this chapter, we do not make this distinction but assume that a process has only one thread of control.

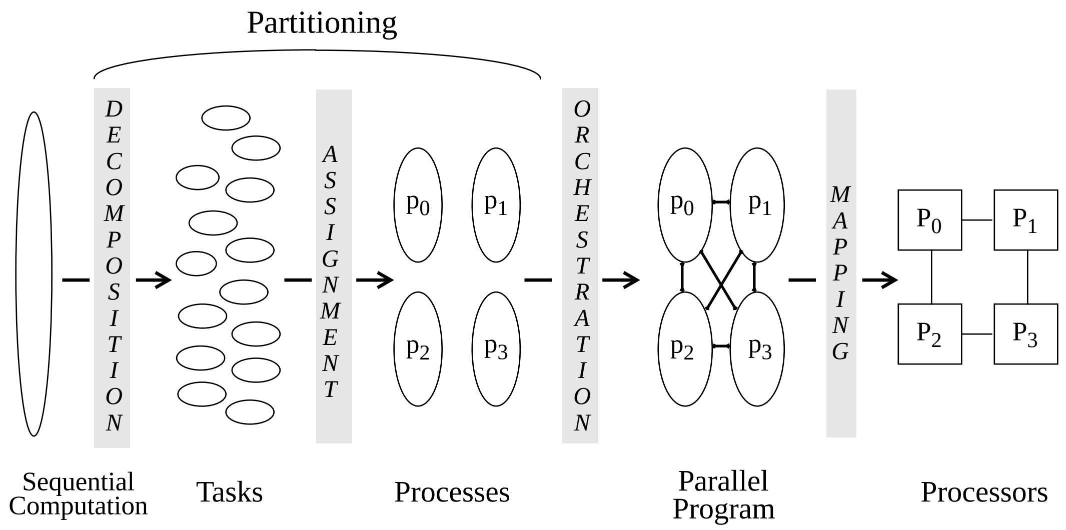


Figure 2-3 Step in parallelization, and the relationships among tasks, processes and processors.

The decomposition and assignment phases are together called partitioning. The orchestration phase coordinates data access, communication and synchronization among processes, and the mapping phase maps them to physical processors.

Given these concepts, the job of creating a parallel program from a sequential one consists of four steps, illustrated in Figure 2-3:

1. **Decomposition** of the computation into tasks,
2. **Assignment** of tasks to processes,
3. **Orchestration** of the necessary data access, communication and synchronization among processes, and
4. **Mapping** or binding of processes to processors.

Together, decomposition and assignment are called partitioning, since they divide the work done by the program among the cooperating processes. Let us examine the steps and their individual goals a little further.

Decomposition

Decomposition means breaking up the computation into a collection of tasks. For example, tracing a single ray in Raytrace may be a task, or performing a particular computation on an individual grid point in Ocean. In general, tasks may become available dynamically as the program executes, and the number of tasks available at a time may vary over the execution of the program. The maximum number of tasks available at a time provides an upper bound on the number of processes (and hence processors) that can be used effectively at that time. Hence, the major goal in decomposition is to *expose enough concurrency* to keep the processes busy at all times, yet not so much that the overhead of managing the tasks becomes substantial compared to the useful work done.

Limited concurrency is the most fundamental limitation on the speedup achievable through parallelism, not just the fundamental concurrency in the underlying problem but also how much of this concurrency is exposed in the decomposition. The impact of available concurrency is codified in one of the few "laws" of parallel computing, called *Amdahl's Law*. If some portions of a program's execution don't have as much concurrency as the number of processors used, then some processors will have to be idle for those portions and speedup will be suboptimal. To see this in its simplest form, consider what happens if a fraction *s* of a program's execution time on a uniprocessor is inherently sequential; that is, it cannot be parallelized. Even if the rest of the program is parallelized to run on a large number of processors in infinitesimal time, this sequential time will remain. The overall execution time of the parallel program will be at least *s*, normalized to a total sequential time of 1, and the speedup limited to 1/s. For example, if s=0.2 (20% of the program's execution is sequential), the maximum speedup available is 1/0.2 or 5 regardless of the number of processors used, even if we ignore all other sources of overhead.

Example 2-1 Consider a simple example program with two phases. In the first phase, a single operation is performed independently on all points of a two-dimensional *n*-by-*n* grid, as in Ocean. In the second, the sum of the n^2 grid point values is computed. If we have *p* processors, we can assign n^2/p points to each processor and complete the first phase in parallel in time n^2/p. In the second phase, each processor can add each of its assigned n^2/p values into a global sum variable. What is the problem with this assignment, and how can we expose more concurrency?

Answer The problem is that the accumulations into the global sum must be done one at a time, or *serialized*, to avoid corrupting the sum value by having two processors try to modify it simultaneously (see mutual exclusion in Section 2.4.5). Thus, the second phase is effectively serial and takes n^2 time regardless of *p*. The total time in parallel is $n^2/p + n^2$, compared to a sequential time of $2n^2$, so the speedup is at most $\frac{2n^2}{\frac{n^2}{p} + n^2}$ or $\frac{2p}{p+1}$, which is at best 2 even if a very large number of processors is used.

We can expose more concurrency by using a little trick. Instead of summing each value directly into the global sum, serializing all the summing, we divide the second phase into two phases. In the new second phase, a process sums its assigned values independently into a private sum. Then, in the third phase, processes sum their private sums into the global sum. The second phase is now fully parallel; the third phase is serialized as before, but there are only *p* operations in it, not *n*. The total parallel time is $n^2/p + n^2/p + p$, and the speedup is at best $p \times \frac{2n^2}{2n^2 + p^2}$. If *n* is large relative to *p*, this speedup limit is almost linear in the number of processors used. Figure 2-4 illustrates the improvement and the impact of limited concurrency.

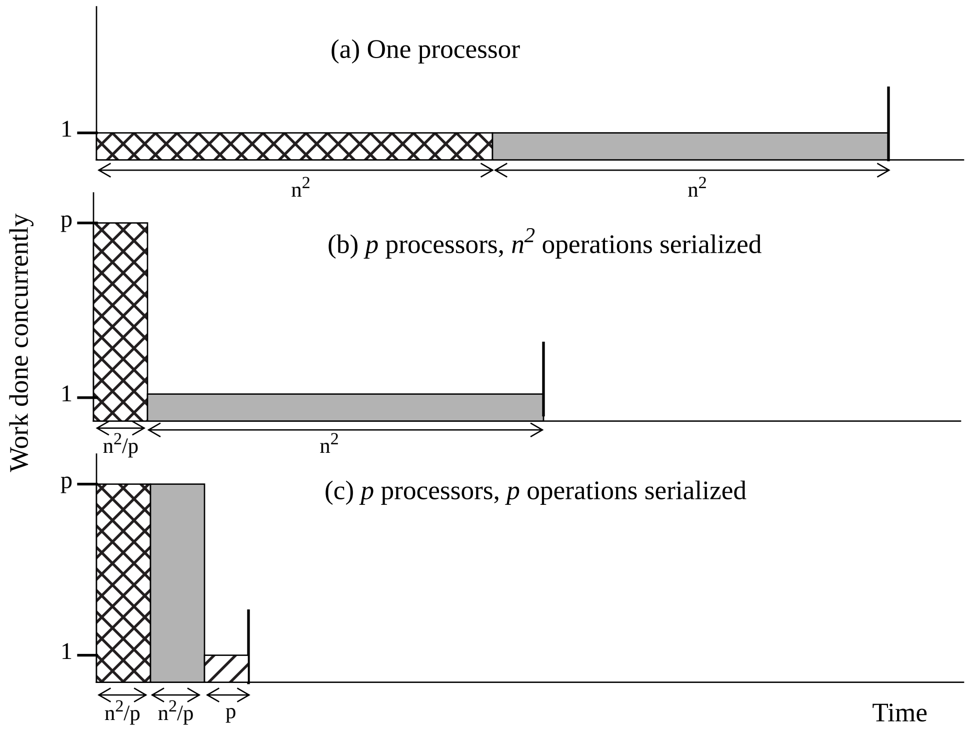


Figure 2-4 Illustration of the impact of limited concurrency.

The x-axis is time, and the y-axis is the amount of work available (exposed by the decomposition) to be done in parallel at a given time. (a) shows the profile for a single processor. (b) shows the original case in the example, which is divided into two phases: one fully concurrent, and one fully serialized. (c) shows the improved version, which is divided into three phases: the first two fully concurrent, and the last fully serialized but with a lot less work in it (*O(p)* rather than *O(n)*).

More generally, given a decomposition and a problem size, we can construct a *concurrency profile* which depicts how many operations (or tasks) are available to be performed concurrently in the application at a given time. The concurrency profile is a function of the problem, the decomposition and the problem size but is independent of the number of processors, effectively assuming that an infinite number of processors is available. It is also independent of the assignment or orchestration. These concurrency profiles may be easy to analyze (as we shall see for matrix factorization in Exercise 2.4) or they may be quite irregular. For example, Figure 2-5 shows a concurrency profile of a parallel event-driven simulation for the synthesis of digital logic systems. The X-axis is time, measured in clock cycles of the circuit being simulated. The Y-axis or amount of concurrency is the number of logic gates in the circuit that are ready to be evaluated at a given time, which is a function of the circuit, the values of its inputs, and time. There is a wide range of unpredictable concurrency across clock cycles, and some cycles with almost no concurrency.

The area under the curve in the concurrency profile is the total amount of work done, i.e. the number of operations or tasks computed, or the “time” taken on a single processor. It’s horizontal extent is a lower bound on the “time” that it would take to run the best parallel program given that decomposition, assuming an infinitely large number of processors. The area divided by the hori-

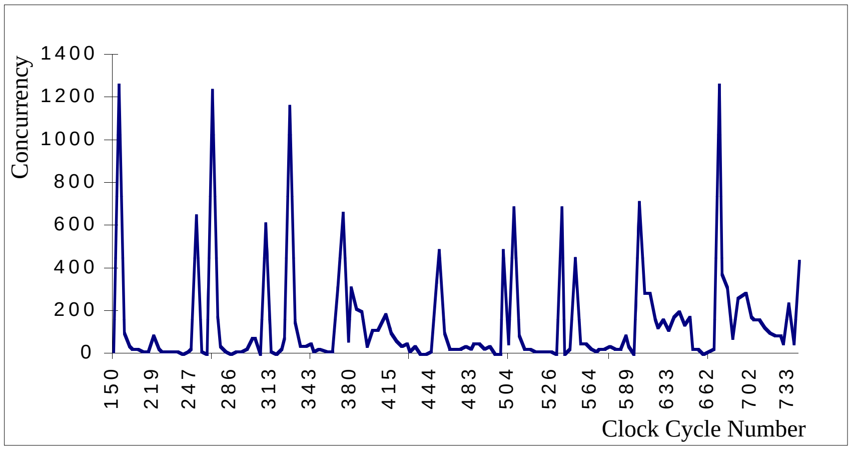


Figure 2-5 Concurrency profile for a distributed-time, discrete-event logic simulator.

The circuit being simulated is a simple MIPS R6000 microprocessor. The y-axis shows the number of logic elements available for evaluation in a given clock cycle.

zontal extent therefore gives us a limit on the achievable speedup with unlimited number of processors, which is thus simply the average concurrency available in the application over time. A rewording of Amdahl's law may therefore be:

$$Speedup \le \frac{AreaUnderConcurencyProfile}{HorizontalExtentofConcurrencyProfile}$$

Thus, if f_k be the number of X-axis points in the concurrency profile that have concurrency k, then we can write Amdahl's Law as:

$$Speedup(p) \le \frac{\sum_{k=1}^{\infty} f_k k}{\sum_{k=1}^{\infty} f_k \left\lceil \frac{k}{p} \right\rceil} . \qquad \textbf{(EQ 2.1)}$$

It is easy to see that if the total work $\sum_{k=1}^{\infty} f_k k$ is normalized to 1 and a fraction s of this is serial, then the speedup with an infinite number of processors is limited by $\frac{1}{s}$, and that with p processors is limited by $\frac{1}{s + \frac{1-s}{p}}$. In fact, Amdahl's law can be applied to any overhead of parallelism, not just limited concurrency, that is not alleviated by using more processors. For now, it quantifies the importance of exposing enough concurrency as a first step in creating a parallel program.

Assignment

Assignment means specifying the mechanism by which tasks will be distributed among processes. For example, which process is responsible for computing forces on which stars in Barnes-Hut, and which process will count occurrences of which itemsets and in which parts of the database in Data Mining? The primary performance goals of assignment are to *balance the workload* among processes, to *reduce interprocess communication*, and to *reduce the runtime overheads of managing the assignment*. Balancing the workload is often referred to as *load balancing*. The workload to be balanced includes computation, input/output and data access or communication, and programs that are not balance these well among processes are said to be load imbalanced.

Achieving these performance goals simultaneously can appear intimidating. However, most programs lend themselves to a fairly structured approach to partitioning (decomposition and assignment). For example, programs are often structured in phases, and candidate tasks for decomposition within a phase are often easily identified as seen in the case studies. The appropriate assignment of tasks is often discernible either by inspection of the code or from a higher-level understanding of the application. And where this is not so, well-known heuristic techniques are often applicable. If the assignment is completely determined at the beginning of the program—or just after reading and analyzing the input—and does not change thereafter, it is called a *static or predetermined assignment*; if the assignment of work to processes is determined at runtime as the program executes—perhaps to react to load imbalances—it is called a *dynamic assignment*. We shall see examples of both. Note that this use of static is a little different than the “compile-time” meaning typically used in computer science. Compile-time assignment that does not change at runtime would indeed be static, but the term is more general here.

Decomposition and assignment are the major algorithmic steps in parallelization. They are *usually* independent of the underlying architecture and programming model, although sometimes the cost and complexity of using certain primitives on a system can impact decomposition and assignment decisions. As architects, we assume that the programs that will run on our machines are reasonably partitioned. There is nothing we can do if a computation is not parallel enough or not balanced across processes, and little we may be able to do if it overwhelms the machine with communication. As programmers, we usually focus on decomposition and assignment first, independent of the programming model or architecture, though in some cases the properties of the latter may cause us to revisit our partitioning strategy.

Orchestration

This is the step where the architecture and programming model play a large role, as well as the programming language itself. To execute their assigned tasks, processes need mechanisms to name and access data, to exchange data (communicate) with other processes, and to synchronize with one another. Orchestration uses the available mechanisms to accomplish these goals correctly and efficiently. The choices made in orchestration are much more dependent on the programming model, and on the efficiencies with which the primitives of the programming model and communication abstraction are supported, than the choices made in the previous steps. Some questions in orchestration include how to organize data structures and schedule tasks to exploit locality, whether to communicate implicitly or explicitly and in small or large messages, and how exactly to organize and express interprocess communication and synchronization. Orchestration also includes scheduling the tasks assigned to a process temporally, i.e. deciding the order in which they are executed. The programming language is important both because this is the step in which the program is actually written and because some of the above tradeoffs in orchestration are influenced strongly by available language mechanisms and their costs.

The major performance goals in orchestration are *reducing the cost of the communication and synchronization* as seen by the processors, *preserving locality of data reference*, *scheduling tasks* so that those on which many other tasks depend are completed early, and *reducing the overheads of parallelism management*. The job of architects is to provide the appropriate primitives with efficiencies that simplify successful orchestration. We shall discuss the major aspects of orchestration further when we see how programs are actually written.

Mapping

The cooperating processes that result from the decomposition, assignment and orchestration steps constitute a full-fledged parallel program on modern systems. The program may control the mapping of processes to processors, but if not the operating system will take care of it, providing a parallel execution. Mapping tends to be fairly specific to the system or programming environment. In the simplest case, the processors in the machine are partitioned into fixed subsets, possibly the entire machine, and only a single program runs at a time in a subset. This is called *space-sharing*. The program can bind or *pin* processes to processors to ensure that they do not migrate during the execution, or can even control exactly which processor a process runs on so as to preserve locality of communication in the network topology. Strict space-sharing schemes, together with some simple mechanisms for time-sharing a subset among multiple applications, have so far been typical of large-scale multiprocessors.

At the other extreme, the operating system may dynamically control which process runs where and when—without allowing the user any control over the mapping—to achieve better resource sharing and utilization. Each processor may use the usual multiprogrammed scheduling criteria to manage processes from the same or from different programs, and processes may be moved around among processors as the scheduler dictates. The operating system may extend the uniprocessor scheduling criteria to include multiprocessor-specific issues. In fact, most modern systems fall somewhere between the above two extremes: The user may ask the system to preserve certain properties, giving the user program some control over the mapping, but the operating system is allowed to change the mapping dynamically for effective resource management.

Mapping and associated resource management issues in multiprogrammed systems are active areas of research. However, our goal here is to understand parallel programming in its basic form, so for simplicity we assume that a single parallel program has complete control over the resources of the machine. We also assume that the number of processes equals the number of processors, and neither changes during the execution of the program. By default, the operating system will place one process on every processor in no particular order. Processes are assumed not to migrate from one processor to another during execution. For this reason, we use the terms “process” and “processor” interchangeably in the rest of the chapter.

2.3.2 Parallelizing Computation versus Data

The view of the parallelization process described above has been centered on computation, or work, rather than on data. It is the computation that is decomposed and assigned. However, due to the communication abstraction or performance considerations, we may be responsible for decomposing and assigning data to processes as well. In fact, in many important classes of problems the decomposition of work and data are so strongly related that they are difficult or even unnecessary to distinguish. Ocean is a good example: Each cross-sectional grid through the ocean is represented as an array, and we can view the parallelization as decomposing the data in each array and assigning parts of it to processes. The process that is assigned a portion of an array will then be responsible for the computation associated with that portion, a so-called *owner computes* arrangement. A similar situation exists in data mining, where we can view the database as being decomposed and assigned; of course, here there is also the question of assigning the item-sets to processes. Several language systems, including the High Performance Fortran standard [KLS+94, HPF93], allow the programmer to specify the decomposition and assignment of data structures; the assignment of computation then follows the assignment of data in an owner computes manner. However, the distinction between computation and data is stronger in many other applications, including the Barnes-Hut and Raytrace case studies as we shall see. Since the *computation-centric* view is more general, we shall retain this view and consider data management to be part of the orchestration step.

2.3.3 Goals of the Parallelization Process

As stated previously, the major goal of using a parallel machine is to improve performance by obtaining speedup over the best uniprocessor execution. Each of the steps in creating a parallel program has a role to play in achieving that overall goal, and each has its own performance goals.

Table 2-1 Steps in the Parallelization Process and Their Goals

Step	Architecture-dependent?	Major Performance Goals
Decomposition	Mostly no	Expose enough concurrency, but not too much
Assignment	Mostly no	Balance workload Reduce communication volume
Orchestration	Yes	Reduce non-inherent communication via data locality (see next chapter) Reduce cost of comm/synch as seen by processor Reduce serialization of shared resources Schedule tasks to satisfy dependences early
Mapping	Yes	Put related processes on the same processor if necessary Exploit locality in network topology

These are summarized in Table 2-1, and we shall discuss them in more detail in the next chapter.

Creating an effective parallel program requires evaluating cost as well as performance. In addition to the dollar cost of the machine itself, we must consider the resource requirements of the program on the architecture and the effort it takes to develop a satisfactory program. While costs and their impact are often more difficult to quantify than performance, they are very important and we must not lose sight of them; in fact, we may need to compromise performance to reduce them. As algorithm designers, we should favor high-performance solutions that keep the resource requirements of the algorithm small and that don't require inordinate programming effort. As architects, we should try to design high-performance systems that, in addition to being low-cost, reduce programming effort and facilitate resource-efficient algorithms. For example, an architecture that delivers gradually improving performance with increased programming effort may be preferable to one that is capable of ultimately delivering better performance but only with inordinate programming effort.

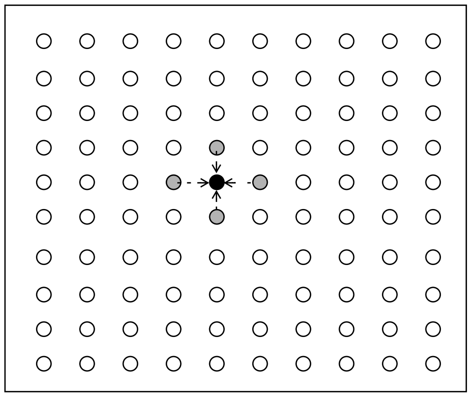
Having understood the basic process and its goals, let us apply it to a simple but detailed example and see what the resulting parallel programs look like in the three major modern programming models introduced in Chapter 1: shared address space, message passing, and data parallel. We shall focus here on illustrating programs and programming primitives, not so much on performance.

2.4 Parallelization of an Example Program

The four case studies introduced at the beginning of the chapter all lead to parallel programs that are too complex and too long to serve as useful sample programs. Instead, this section presents a simplified version of a piece or *kernel* of Ocean: its equation solver. It uses the equation solver to dig deeper and illustrate how to implement a parallel program using the three programming models. Except for the data parallel version, which necessarily uses a high-level data parallel language, the parallel programs are not written in an aesthetically pleasing language that relies on software layers to hide the orchestration and communication abstraction from the programmer. Rather, they are written in C or Pascal-like pseudocode augmented with simple extensions for parallelism, thus exposing the basic communication and synchronization primitives that a shared address space or message passing communication abstraction must provide. Standard sequential languages augmented with primitives for parallelism also reflect the state of most real parallel programming today.

2.4.1 A Simple Example: The Equation Solver Kernel

The equation solver kernel solves a simple partial differential equation on a grid, using what is referred to as a finite differencing method. It operates on a regular, two-dimensional grid or array of *(n+2)*-by-*(n+2)* elements, such as a single horizontal cross-section of the ocean basin in Ocean. The border rows and columns of the grid contain boundary values that do not change, while the interior *n*-by-*n* points are updated by the solver starting from their initial values. The computation proceeds over a number of sweeps. In each sweep, it operates on all the elements of the grid, for each element replacing its value with a weighted average of itself and its four nearest neighbor elements (above, below, left and right, see Figure 2-6). The updates are done in-place in the grid, so a point sees the new values of the points above and to the left of it, and the old values of the points below it and to its right. This form of update is called the Gauss-Seidel method. During each sweep the kernel also computes the average difference of an updated element from its previous value. If this average difference over all elements is smaller than a predefined “tolerance” parameter, the solution is said to have converged and the solver exits at the end of the sweep. Otherwise, it performs another sweep and tests for convergence again. The sequential pseudocode is shown in Figure 2-7. Let us now go through the steps to convert this simple equation solver to a parallel program for each programming model. The decomposition and assignment are essentially the same for all three models, so these steps are examined them in a general context. Once we enter the orchestration phase, the discussion will be organized explicitly by programming model.



Expression for updating each interior point:

A[i,j] = 0.2 * (A[i,j] + A[i,j-1] + A[i-1,j] +
A[i,j+1] + A[i+1,j])

Figure 2-6 Nearest-neighbor update of a grid point in the simple equation solver.

The black point is A[i,j] in the two-dimensional array that represents the grid, and is updated using itself and the four shaded points that are its nearest neighbors according to the equation at the right of the figure.

2.4.2 Decomposition

For programs that are structured in successive loops or loop nests, a simple way to identify concurrency is to start from the loop structure itself. We examine the individual loops or loop nests in the program one at a time, see if their iterations can be performed in parallel, and determine whether this exposes enough concurrency. We can then look for concurrency across loops or take a different approach if necessary. Let us follow this program structure based approach in Figure 2-7.

Each iteration of the outermost loop, beginning at line 15, sweeps through the entire grid. These iterations clearly are not independent, since data that are modified in one iteration are accessed in the next. Consider the loop nest in lines 17-24, and ignore the lines containing `diff`. Look at the inner loop first (the *j* loop starting on line 18). Each iteration of this loop reads the grid point (A[i,j-1]) that was written in the previous iteration. The iterations are therefore sequentially dependent, and we call this a *sequential loop*. The outer loop of this nest is also sequential, since the elements in row *i-1* were written in the previous (*i-1th*) iteration of this loop. So this simple analysis of existing loops and their dependences uncovers no concurrency in this case.

In general, an alternative to relying on program structure to find concurrency is to go back to the fundamental dependences in the underlying algorithms used, regardless of program or loop structure. In the equation solver, we might look at the fundamental data dependences at the granularity of individual grid points. Since the computation proceeds from left to right and top to bottom in the grid, computing a particular grid point in the sequential program uses the updated values of the grid points directly above and to the left. This dependence pattern is shown in Figure 2-8. The result is that the elements along a given anti-diagonal (south-west to north-east) have no dependences among them and can be computed in parallel, while the points in the next anti-diagonal depend on some points in the previous one. From this diagram, we can observe that of the $O(n^2)$

```
1. int n;                                  /* size of matrix: n-by-n elements*/
2. float **A, diff = 0;

3. main()
4. begin
5.   read(n) ;                             /* read input parameter: matrix size*/
6.   A ← malloc (a 2-d array of size n+2 by n+2 doubles);
7.   initialize(A);                        /* initialize the matrix A somehow */
8.   Solve (A);                            /* call the routine to solve equation*/
9. end main

10. procedure Solve (A)                    /* solve the equation system */
11.   float **A;                           /* A is an n+2 by n+2 array*/
12. begin
13.   int i, j, done = 0;
14.   float diff = 0, temp;
15.   while (!done) do                    /* outermost loop over sweeps */
16.        diff = 0;                      /* initialize maximum difference to 0 */
17.        for i ← 1 to n do              /* sweep over non-border points of grid */
18.             for j ← 1 to n do
19.                  temp = A[i,j];          /* save old value of element */
20.                  A[i,j] ← 0.2 * (A[i,j] + A[i,j-1] + A[i-1,j] +
21.                       A[i,j+1] + A[i+1,j]);      /*compute average */
22.                  diff += abs(A[i,j] - temp);
23.             end for
24.        end for
25.        if (diff/(n*n) < TOL) then done = 1;
26.   end while
27.end procedure
```

Figure 2-7 Pseudocode describing the sequential equation solver kernel.

The main body of work to be done in each iteration is in the nested for loop in lines 17 to 23. This is what we would like to parallelize.

work involved in each sweep, there a sequential dependence proportional to *n* along the diagonal and inherent concurrency proportional to *n*.

Suppose we decide to decompose the work into individual grid points, so updating a single grid point is a task. There are several ways to exploit the concurrency this exposes. Let us examine a few. First, we can leave the loop structure of the program as it is, and insert point-to-point synchronization to ensure that a grid point has been produced in the current sweep before it is used by the points to the right of or below it. Thus, different loop nests and even different sweeps might be in progress simultaneously on different elements, as long as the element-level dependences are not violated. But the overhead of this synchronization at grid-point level may be too

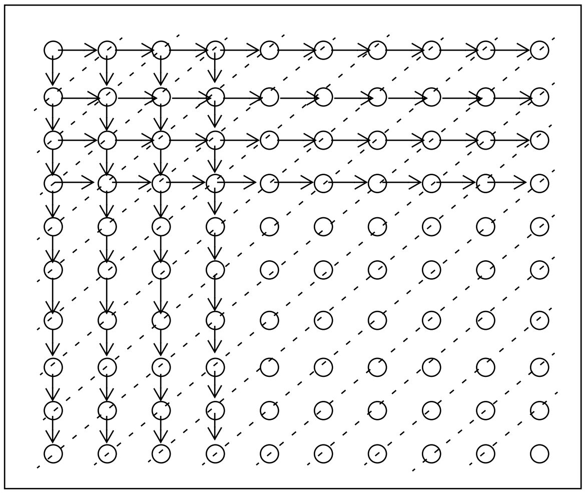


Figure 2-8 Dependences and concurrency in the Gauss-Seidel equation solver computation.

The horizontal and vertical lines with arrows indicate dependences, while the anti-diagonal, dashed lines connect points with no dependences among them and that can be computed in parallel.

high. Second, we can change the loop structure, and have the first for loop (line 17) be over anti-diagonals and the inner for loop be over elements within an anti-diagonal. The inner loop can now be executed completely in parallel, with global synchronization between iterations of the outer for loop to preserve dependences conservatively across anti-diagonals. Communication will be orchestrated very differently in the two cases, particularly if communication is explicit. However, this approach also has problems. Global synchronization is still very frequent: once per anti-diagonal. Also, the number of iterations in the parallel (inner) loop changes with successive outer loop iterations, causing load imbalances among processors especially in the shorter anti-diagonals. Because of the frequency of synchronization, the load imbalances, and the programming complexity, neither of these approaches is used much on modern architectures.

The third and most common approach is based on exploiting knowledge of the problem beyond the sequential program itself. The order in which the grid points are updated in the sequential program (left to right and top to bottom) is not fundamental to the Gauss-Seidel solution method; it is simply one possible ordering that is convenient to program sequentially. Since the Gauss-Seidel method is not an exact solution method (unlike Gaussian elimination) but rather iterates until convergence, we can update the grid points in a different order as long as we use updated values for grid points frequently enough.[1] One such ordering that is used often for parallel versions is called *red-black* ordering. The idea here is to separate the grid points into alternating red points and black points as on a checkerboard (Figure 2-9), so that no red point is adjacent to a black point or vice versa. Since each point reads only its four nearest neighbors, it is clear that to

1. Even if we don't use updated values from the current while loop iteration for any grid points, and we always use the values as they were at the end of the previous while loop iteration, the system will still converge, only much slower. This is called Jacobi rather than Gauss-Seidel iteration.

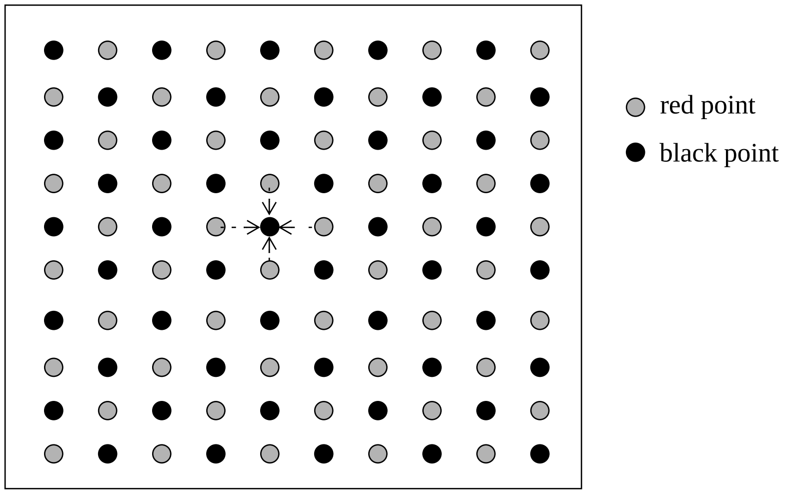


Figure 2-9 Red-black ordering for the equation solver.

The sweep over the grid is broken up into two sub-sweeps: The first computes all the red points, and the second all the black points. SInce red points depend only on black points and vice versa, there are no dependences within a sub-sweep.

compute a red point we do not need the updated value of any other red point, but only the updated values of the above and left black points (in a standard sweep), and vice versa. We can therefore divide a grid sweep into two phases: first computing all red points and then computing all black points. Within each phase there are no dependences among grid points, so we can compute all $\frac{n^2}{2}$ red points in parallel, then synchronize globally, and then compute all $\frac{n^2}{2}$ black points in parallel. Global synchronization is conservative and can be replaced by point-to-point synchronization at the level of grid points—since not all black points need to wait for all red points to be computed—but it is convenient.

The red-black ordering is different from our original sequential ordering, and can therefore both converge in fewer or more sweeps as well as produce different final values for the grid points (though still within the convergence tolerance). Note that the black points will see the updated values of all their (red) neighbors in the current sweep, not just the ones to the left and above. Whether the new order is sequentially better or worse than the old depends on the problem. The red-black order also has the advantage that the values produced and convergence properties are independent of the number of processors used, since there are no dependences within a phase. If the sequential program itself uses a red-black ordering then parallelism does not change the properties at all.

The solver used in Ocean in fact uses a red-black ordering, as we shall see later. However, red-black ordering produces a longer kernel of code than is appropriate for illustration here. Let us therefore examine a simpler but still common asynchronous method that does not separate points into red and black. Global synchronization is used between grid sweeps as above, and the loop structure for a sweep is not changed from the top-to-bottom, left-to-right order. Instead, within a sweep a process simply updates the values of all its assigned grid points, accessing its nearest neighbors whether or not they have been updated in the current sweep by their assigned processes or not. That is, it ignores dependences among grid points within a sweep. When only a single process is used, this defaults to the original sequential ordering. When multiple processes are used, the ordering is unpredictable; it depends on the assignment of points to processes, the number of processes used, and how quickly different processes execute at runtime. The execution is no longer deterministic, and the number of sweeps required to converge may depend on the number of processors used; however, for most reasonable assignments the number of sweeps will not vary much.

If we choose a decomposition into individual inner loop iterations (grid points), we can express the program by writing lines 15 to 26 of Figure 2-7 as shown in Figure 2-10. All that we have

```
15.  while (!done) do                                   /* a sequential loop*/
16.       diff = 0;
17.       for_all i ← 1 to n do                         /* a parallel loop nest */
18.            for_all j ← 1 to n do
19.                 temp = A[i,j];
20.                 A[i,j] ← 0.2 * (A[i,j] + A[i,j-1] + A[i-1,j] +
21.                      A[i,j+1] + A[i+1,j]);
22.                 diff += abs(A[i,j] - temp);
23.            end for_all
24.       end for_all
25.       if (diff/(n*n) < TOL) then done = 1;
26.  end while
```

Figure 2-10 Parallel equation solver kernel with decomposition into elements and no explicit assignment.

Since both for loops are made parallel by using for_all instead of for, the decomposition is into individual grid elements. Other than this change, the code is the same as the sequential code.

done is replace the keyword `for` in the parallel loops with `for_all`. A `for_all` loop simply tells the underlying software/hardware system that all iterations of the loop can be executed in parallel without dependences, but says nothing about assignment. A loop nest with both nesting levels being `for_all` means that all iterations in the loop nest ($n*n$ or n^2 here) can be executed in parallel. The system can orchestrate the parallelism in any way it chooses; the program does not take a position on this. All it assumes is that there is an implicit global synchronization after a `for_all` loop nest.

In fact, we can decompose the computation into not just individual iterations as above but any aggregated groups of iterations we desire. Notice that decomposing the computation corresponds very closely to decomposing the grid itself. Suppose now that we wanted to decompose into rows of elements instead, so that the work for an entire row is an indivisible task which must be assigned to the same process. We could express this by making the inner loop on line 18 a sequential loop, changing its `for_all` back to a `for`, but leaving the loop over rows on line 17 as a parallel `for_all` loop. The parallelism, or *degree of concurrency*, exploited under this decomposition is reduced from n^2 inherent in the problem to n: Instead of n^2 independent tasks of duration one unit each, we now have n independent tasks of duration n units each. If each task executed on a different processor, we would have approximately $2n$ communication operations (accesses to data computed by other tasks) for *n* points.

Let us proceed past decomposition and see how we might assign rows to processes explicitly, using a row-based decomposition.

2.4.3 Assignment

The simplest option is a static (predetermined) assignment in which each processor is responsible for a contiguous block of rows, as shown in Figure 2-11. Row i is assigned to process $\left\lfloor \frac{i}{P} \right\rfloor$. Alternative static assignments to this so-called *block* assignment are also possible, such as a *cyclic* assignment in which rows are interleaved among processes (process i is assigned rows i, $i + P$, and so on). We might also consider a dynamic assignment, where each process repeatedly grabs the next available (not yet computed) row after it finishes with a row task, so it is not predetermined which process computes which rows. For now, we will work with the static block assignment. This simple partitioning of the problem exhibits good load balance across processes as long as the number of rows is divisible by the number of processes, since the work per row is uniform. Observe that the static assignments have further reduced the parallelism or degree of concurrency, from n to p, and the block assignment has reduced the communication required by assigning adjacent rows to the same processor. The communication to computation ratio is now only $O\left(\frac{n}{p}\right)$.

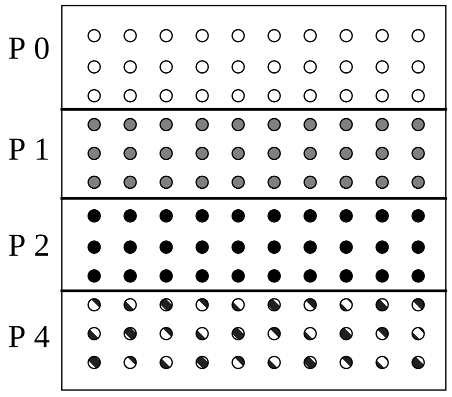


Figure 2-11 A simple assignment for the parallel equation solver.

Each of the four processors is assigned a contiguous, equal number of rows of the grid. In each sweep, it will perform the work needed to update the elements of its assigned rows.

Given this decomposition and assignment, we are ready to dig into the orchestration phase. This requires that we pin down the programming model. We begin with a high level, data parallel model. Then, we examine the two major programming models that this and other models might compile down to: a shared address space and explicit message passing.

2.4.4 Orchestration under the Data Parallel Model

The data parallel model is convenient for the equation solver kernel, since it is natural to view the computation as a single thread of control performing global transformations on a large array data structure, just as we have done above [Hil85,HiS86]. Computation and data are quite interchangeable, a simple decomposition and assignment of the data leads to good load balance across processes, and the appropriate assignments are very regular in shape and can be described by simple primitives. Pseudocode for the data-parallel equation solver is shown in Figure 2-12. We assume that global declarations (outside any procedure) describe shared data, and all other data are private to a process. Dynamically allocated shared data, such as the array `A`, are allocated with a `G_MALLOC` (global `malloc`) call rather than a regular `malloc`. Other than this, the main differences from the sequential program are the use of an `DECOMP` statement, the use of `for_all` loops instead of `for` loops, the use of a private `mydiff` variable per process, and the use of a `REDUCE` statement. We have already seen that for_all loops specify that the iterations can be performed in parallel. The `DECOMP` statement has a two-fold purpose. First, it specifies the assignment of the iterations to processes (DECOMP is in this sense an unfortunate choice of word). Here, it is a [`BLOCK`, `*`, `nprocs`] assignment, which means that the first dimension (rows) is partitioned into contiguous pieces among the nprocs processes, and the second dimension is not partitioned at all. Specifying [`CYCLIC`, `*`, `nprocs`] would have implied a cyclic or interleaved partitioning of rows among nprocs processes, specifying [`BLOCK`, `BLOCK`, `nprocs`] would have implied a subblock decomposition, and specifying [`*`, `CYCLIC`, `nprocs`] would have implied an interleaved partitioning of columns. The second and related purpose of `DECOMP` is that it also specifies how the grid data should be distributed among memories on a distributed-memory machine (this is restricted to be the same as the assignment in most current data-parallel languages, following the owner computes rule, which works well in this example).

The `mydiff` variable is used to allow each process to first independently compute the sum of the difference values for its assigned grid points. Then, the `REDUCE` statement directs the system to add all their partial `mydiff` values together into the shared `diff` variable. The reduction operation may be implemented in a library in a manner best suited to the underlying architecture.

While the data parallel programming model is well-suited to specifying partitioning and data distribution for regular computations on large arrays of data, such as the equation solver kernel or the Ocean application, the desirable properties do not hold true for more irregular applications, particularly those in which the distribution of work among tasks or the communication pattern changes unpredictably with time. For example, think of the stars in Barnes-Hut or the rays in Raytrace. Let us look at the more flexible, lower-level programming models in which processes have their own individual threads of control and communicate with each other when they please.

2.4.5 Orchestration under the Shared Address Space Model

In a shared address space we can simply declare the matrix *A* as a single shared array—as we did in the data parallel model—and processes can reference the parts of it they need using loads and stores with exactly the same array indices as in a sequential program. Communication will be generated implicitly as necessary. With explicit parallel processes, we now need mechanisms to create the processes, coordinate them through synchronization, and control the assignment of

```
int n, nprocs;                    /* grid size (n+2-by-n+2) and number of processes*/
float **A, diff = 0;

main()
begin
  read(n); read(nprocs);;          /* read input grid size and number of processes*/
  A ← G_MALLOC (a 2-d array of size n+2 by n+2 doubles);
  initialize(A);                       /* initialize the matrix A somehow */
  Solve (A);                           /* call the routine to solve equation*/
end main

procedure Solve(A)                     /* solve the equation system */
  float **A;                           /* A is an n+2 by n+2 array*/
begin
  int i, j, done = 0;
  float mydiff = 0, temp;
  DECOMP A[BLOCK,*];
  while (!done) do                   /* outermost loop over sweeps */
       mydiff = 0;                   /* initialize maximum difference to 0 */
       for_all i ← 1 to n do         /* sweep over non-border points of grid */
            for_all j ← 1 to n do
                 temp = A[i,j];           /* save old value of element */
                 A[i,j] ← 0.2 * (A[i,j] + A[i,j-1] + A[i-1,j] +
                      A[i,j+1] + A[i+1,j]);        /*compute average */
                 mydiff += abs(A[i,j] - temp);
            end for_all
       end for_all
       REDUCE (mydiff, diff, ADD);
       if (diff/(n*n) < TOL) then done = 1;
  end while
end procedure
```

Figure 2-12 Pseudocode describing the data-parallel equation solver.

Differences from the sequential code are shown in italicized bold font. The decomposition is still into individual elements, as indicated by the nested for_all loop. The assignment, indicated by the unfortunately labelled DECOMP statement, is into blocks of contiguous rows (the first or column dimension is partitioned into blocks, and the second or row dimension is not partitioned). The REDUCE statement sums the locally computed mydiffs into a global diff value. The while loop is still serial.

work to processes. The primitives we use are typical of low-level programming environments such as *parmacs* [LO+87], and are summarized in Table 2-2.

Table 2-2 Key Shared Address Space Primitives

Name	**Syntax**	**Function**
CREATE	`CREATE(p,proc,args)`	Create `p` processes that start executing at procedure `proc` with arguments `args`.
G_MALLOC	`G_MALLOC(size)`	Allocate shared data of `size` bytes
LOCK	`LOCK(name)`	Acquire mutually exclusive access
UNLOCK	`UNLOCK(name)`	Release mutually exclusive access
BARRIER	`BARRIER(name, number)`	Global synchronization among `number` processes: None gets past BARRIER until `number` have arrived
WAIT_FOR_END	`WAIT_FOR_END(number)`	Wait for `number` processes to terminate
wait for flag	`while (!flag); or` `WAIT(flag)`	Wait for `flag` to be set (spin or block); for point-to-point event synchronization.
set flag	`flag = 1; or` `SIGNAL(flag)`	Set `flag`; wakes up process spinning or blocked on `flag`, if any

Pseudocode for the parallel equation solver in a shared address space is shown in Figure 2-13. The special primitives for parallelism are shown in italicized bold font. They are typically implemented as library calls or macros, each of which expands to a number of instructions that accomplish its goal. Although the code for the Solve procedure is remarkably similar to the sequential version, let's go through it one step at a time.

A single process is first started up by the operating system to execute the program, starting from the procedure called main. Let's call it the *main* process. It reads the input, which specifies the size of the grid *A* (recall that input `n` denotes an (`n+2`) -by-(`n+2`) grid of which `n`-by-`n` points are updated by the solver). It then allocates the grid *A* as a two-dimensional array in the shared address space using the `G_MALLOC` call, and initializes the grid. The `G_MALLOC` call is similar to a usual `malloc` call—used in the C programming language to allocate data dynamically in the program's heap storage—and it returns a pointer to the allocated data. However, it allocates the data in a shared region of the heap, so they can be accessed and modified by any process. For data that are not dynamically allocated on the heap, different systems make different assumptions about what is shared and what is private to a process. Let us assume that all "global" data in the sequential C programming sense—i.e. data declared outside any procedure, such as `nprocs` and `n` in Figure 2-13—are shared. Data on a procedure's stack (such as `mymin`, `mymax`, `mydiff`, `temp`, `i` and `j`) are private to a process that executes the procedure, as are data allocated with a regular `malloc` call (and data that are explicitly declared to be private, not used in this program).

Having allocated data and initialized the grid, the program is ready to start solving the system. It therefore creates (`nprocs-1`) "worker" processes, which begin executing at the procedure called Solve. The main process then also calls the Solve procedure, so that all `nprocs` processes

```
1. int n, nprocs;             /* matrix dimension and number of processors to be used */
2a. float **A, diff;          /*A is global (shared) array representing the grid */
                              /* diff is global (shared) maximum difference in current sweep */
2b. LOCKDEC(diff_lock);       /* declaration of lock to enforce mutual exclusion */
2c. BARDEC (bar1);            /* barrier declaration for global synchronization between sweeps*/

3. main()
4. begin
5.    read(n);   read(nprocs);     /* read input matrix size and number of processes*/
6.    A ← G_MALLOC (a two-dimensional array of size n+2 by n+2 doubles);
7.    initialize(A);                          /* initialize A in an unspecified way*/
8a.   CREATE (nprocs-1, Solve, A);
8     Solve(A);                    /* main process becomes a worker too*/
8b.   WAIT_FOR_END;                /* wait for all child processes created to terminate */
9. end main

10. procedure Solve(A)
11. float **A;             /* A is entire n+2-by-n+2 shared array, as in the sequential program */
12. begin
13.   int i,j, pid, done = 0;
14.   float temp, mydiff = 0;                /* private variables/
14a.  int mymin ← 1 + (pid * n/nprocs);        /* assume that n is exactly divisible by  */
14b.  int mymax ← mymin + n/nprocs - 1;        /* nprocs for simplicity here*/

15. while (!done) do                         /* outer loop over all diagonal elements */
16.   mydiff = diff = 0;                     /* set global diff to 0 (okay for all to do it) */
17.   for i ← mymin to mymax do              /* for each of my rows */
18.         for j ← 1 to n do                /* for all elements in that row */
19.               temp = A[i,j];
20.               A[i,j] = 0.2 * (A[i,j] + A[i,j-1] + A[i-1,j] +
21.                     A[i,j+1] + A[i+1,j]);
22.               mydiff += abs(A[i,j] - temp);
23.         endfor
24.   endfor
25a.  LOCK(diff_lock);                 /* update global diff if necessary */
25b.        diff += mydiff;
25c.  UNLOCK(diff_lock);
25d.  BARRIER(bar1, nprocs);                 /* ensure all have got here before checking if done*/

25e.  if (diff/(n*n) < TOL) then done = 1; /* check convergence; all get same answer*/
25f.  BARRIER(bar1, nprocs);           /* see Exercise c */
26. endwhile
27.end procedure
```

Figure 2-13 Pseudocode describing the parallel equation solver in a shared address space.

Line numbers followed by a letter denote lines that were not present in the sequential version. The numbers are chosen to match the line or control structure in the sequential code with which the new lines are most closely related. The design of the data structures does not have to change from the sequential program. Processes are created with the CREATE call, and the main process waits for them to terminate at the end of the program with the WAIT_FOR_END call. The decomposition is into rows, since the inner loop is unmodified, and the outer loop specifies the assignment of rows to processes. Barriers are used to separate sweeps (and to separate the convergence test from further modification of the global diff variable), and locks are used to provide mutually exclusive access to the global diff variable.

enter the procedure in parallel as equal partners. All created processes execute the same code image until they exit from the program and terminate. This does not mean that they proceed in lock-step or even execute the same instructions, since in general they may follow different control paths through the code. That is, we use a structured, *single-program-multiple-data* style of programming. Control over the assignment of work to processes—and which data they access—is maintained by a few private variables that acquire different values for different processes (e.g. `mymin` and `mymax`), and by simple manipulations of loop control variables. For example, we assume that every process upon creation obtains a unique process identifier (`pid`) between 0 and `nprocs`-1 in its private address space, and in lines 12-13 uses this pid to determine which rows are assigned to it. Processes synchronize through calls to synchronization primitives. We shall discuss these primitives shortly

We assume for simplicity that the total number of rows `n` is an integer multiple of the number of processes `nprocs`, so that every process is assigned the same number of rows. Each process calculates the indices of the first and last rows of its assigned block in the private variables `mymin` and `mymax`. It then proceeds to the actual solution loop.

The outermost while loop (line 21) is still over successive grid sweeps. Although the iterations of this loop proceed sequentially, each iteration or sweep is itself executed in parallel by all processes. The decision of whether to execute the next iteration of this loop is taken separately by each process (by setting the `done` variable and computing the `while (!done)` condition) even though each will make the same decision: The redundant work performed here is very small compared to the cost of communicating a completion flag or the diff value.

The code that performs the actual updates (lines 19-22) is essentially identical to that in the sequential program. Other than the bounds in the loop control statements, used for assignment, the only difference is that each process maintains its own private variable `mydiff`. This private variable keeps track of the total difference between new and old values for only its assigned grid points. It is accumulated it once into the shared `diff` variable at the end of the sweep, rather than adding directly into the shared variable for every grid point. In addition to the serialization and concurrency reason discussed in Section 2.3.1, all processes repeatedly modifying and reading the same shared variable causes a lot of expensive communication, so we do not want to do this once per grid point.

The interesting aspect of the rest of the program (line 23 onward) is synchronization, both mutual exclusion and event synchronization, so the rest of the discussion will focus on it. First, the accumulations into the shared variable by different processes have to be mutually exclusive. To see why, consider the sequence of instructions that a processor executes to add its `mydiff` variable (maintained say in register `r2`) into the shared `diff` variable; i.e. to execute the source statement `diff += mydiff`:

```
load the value of diff into register r1
add the register r2 to register r1
store the value of register r1 into diff
```

Suppose the value in the variable `diff` is 0 to begin with, and the value of `mydiff` in each process is 1. After two processes have executed this code we would expect the value in `diff` to be

2. However, it may turn out to be 1 instead if the processes happen to execute their operations in the interleaved order shown below.

P1	P2	
`r1 ← diff`		*{P1 gets 0 in its r1}*
	`r1 ← diff`	*{P2 also gets 0}*
`r1 ← r1+r2`		*{P1 sets its r1 to 1}*
	`r1 ← r1+r2`	*{P2 sets its r1 to 1}*
`diff ← r1`		*{P1 sets cell_cost to 1}*
	`diff ← r1`	*{P2 also sets cell_cost to 1}*

This is not what we intended. The problem is that a process (here P2) may be able to read the value of the logically shared `diff` between the time that another process (P1) reads it and writes it back. To prohibit this interleaving of operations, we would like the sets of operations from different processes to execute *atomically* (mutually exclusively) with respect to one another. The set of operations we want to execute atomically or mutually exclusively is called a *critical section.*: Once a process starts to execute the first of its three instructions above (its critical section), no other process can execute any of the instructions in its corresponding critical section until the former process has completed the last instruction in the critical section. The `LOCK`-`UNLOCK` pair around line 25b achieves mutual exclusion.

A lock, such as `cell_lock`, can be viewed as a shared token that confers an exclusive right. Acquiring the lock through the `LOCK` primitive gives a process the right to execute the critical section. The process that holds the lock frees it when it has completed the critical section by issuing an `UNLOCK` command. At this point, the lock is either free for another process to acquire or be granted, depending on the implementation. The `LOCK` and `UNLOCK` primitives must be implemented in a way that guarantees mutual exclusion. Our `LOCK` primitive takes as argument the name of the lock being used. Associating names with locks allows us to use different locks to protect unrelated critical sections, reducing contention and serialization.

Locks are expensive, and even a given lock can cause contention and serialization if multiple processes try to access it at the same time. This is another reason to use a private `mydiff` variable to reduce accesses to the shared variable. This technique is used in many operations called *reductions* (implemented by the `REDUCE` operation in the data parallel program). A reduction is a situation in which many processes (all, in a global reduction) perform associative operations (such as addition, taking the maximum, etc.) on the same logically shared data. Associativity implies that the order of the operations does not matter. Floating point operations, such as the ones here, are strictly speaking not associative, since how rounding errors accumulate depends on the order of operations. However, the effects are small and we usually ignore them, especially in iterative calculations that are anyway approximations.

Once a process has added its `mydiff` into the global `diff`, it waits until all processes have done so and the value contained in `diff` is indeed the total difference over all grid points. This requires global event synchronization implemented here with a `BARRIER`. A barrier operation takes as an argument the name of the barrier and the number of processes involved in the synchronization, and is issued by all those processes. Its semantics are as follows. When a process calls the barrier, it registers the fact that it has reached that point in the program. The process is not allowed to proceed past the barrier call until the specified number of processes participating in the barrier have issued the barrier operation. That is, the semantics of `BARRIER(name,p)` are: wait until `p` processes get here and only then proceed.

Barriers are often used to separate distinct phases of computation in a program. For example, in the Barnes-Hut galaxy simulation we use a barrier between updating the positions of the stars at the end of one time-step and using them to compute forces at the beginning of the next one, and in data mining we may use a barrier between counting occurrences of candidate itemsets and using the resulting large itemsets to generate the next list of candidates. Since they implement all-to-all event synchronization, barriers are usually a conservative way of preserving dependences; usually, not all operations (or processes) after the barrier actually need to wait for all operations before the barrier. More specific, point-to-point or group event synchronization would enable some processes to get past their synchronization event earlier; however, from a programming viewpoint it is often more convenient to use a single barrier than to orchestrate the actual dependences through point-to-point synchronization.

When point-to-point synchronization is needed, one way to orchestrate it in a shared address space is with wait and signal operations on semaphores, with which we are familiar from operating systems. A more common way is by using normal shared variables as synchronization *flags*, as shown in Figure 2-14. Since *P1* simply spins around in a tight while loop for something to

```
                P1                                    P2
                                                       A = 1;
a: while (flag is 0) do nothing;                      b: flag = 1;
                print A;
```

Figure 2-14 Point-to-point event synchronization using flags.

Suppose we want to ensure that a process P1 does not get past a certain point (say a) in the program until some other process P2 has already reached another point (say b). Assume that the variable flag (and A) was initialized to 0 before the processes arrive at this scenario. If P1 gets to statement a after P2 has already executed statement b, P1 will simply pass point a. If, on the other hand, P2 has not yet executed b, then P1 will remain in the "idle" while loop until P2 reaches b and sets flag to 1, at which point P1 will exit the idle loop and proceed. If we assume that the writes by P2 are seen by P1 in the order in which they are performed, then this synchronization will ensure that P1 prints the value 1 for A.

happen, keeping the processor busy during this time, we call this *spin-waiting* or *busy-waiting*. Recall that in the case of a semaphore the waiting process does not spin and consume processor resources but rather blocks (suspends) itself, and is awoken when the other process signals the semaphore.

In event synchronization among subsets of processes, or *group event synchronization*, one or more processes may act as producers and one or more as consumers. Group event synchronization can be orchestrated either using ordinary shared variables as flags or by using barriers among subsets of processes.

Once it is past the barrier, a process reads the value of `diff` and examines whether the average difference over all grid points (`diff/(n*n)`) is less than the error tolerance used to determine convergence. If so, it sets the done flag to exit from the while loop; if not, it goes on to perform another sweep.

Finally, the WAIT_FOR_END called by the main process at the end of the program (line 11) is a particular form of all-to-one synchronization. Through it, the main process waits for all the worker processes it created to terminate. The other processes do not call WAIT_FOR_END, but implicitly participate in the synchronization by terminating when they exit the Solve procedure that was their entry point into the program.

In summary, for this simple equation solver the parallel program in a shared address space is not too different in structure from the sequential program. The major differences are in the control flow—which are implemented by changing the bounds on some loops—in creating processes and in the use of simple and generic synchronization primitives. The body of the computational loop is unchanged, as are the major data structures and references to them. Given a strategy for decomposition, assignment and synchronization, inserting the necessary primitives and making the necessary modifications is quite mechanical in this example. Changes to decomposition and assignment are also easy to incorporate. While many simple programs have this property in a shared address space we will see later that more substantial changes are needed as we seek to obtain higher and higher parallel performance, and also as we address more complex parallel programs.

Example 2-2 How would the code for the shared address space parallel version of the equation solver change if we retained the same decomposition into rows but changed to a cyclic or interleaved assignment of rows to processes?

Answer Figure 2-15 shows the relevant pseudocode. All we have changed in the code is the

```
17.  for i ← pid+1 to n by nprocs do   /* for my interleaved set of rows*/
18.       for j ← 1 to n do             /* for all elements in that row */
19.            temp = A[i,j];
20.            A[i,j] ¨ 0.2 * (A[i,j] + A[i,j-1] + A[i-1,j] +
21.                 A[i,j+1] + A[i+1,j]);
22.            mydiff += abs(A[i,j] - temp);
23.       endfor
24.  endfor
```

Figure 2-15 Cyclic assignment of row-based solver in a shared address space.

All that changes in the code from the block assignment of rows in Figure 2-13 is first for statement in line 17. The data structures or accesses to them do not have to be changed.

control arithmetic in line 17. The same global data structure is used with the same indexing, and all the rest of the parallel program stays exactly the same.

2.4.6 Orchestration under the Message Passing Model

We now examine a possible implementation of the parallel solver using explicit message passing between private address spaces, employing the same decomposition and assignment as before. Since we no longer have a shared address space, we cannot simply declare the matrix *A* to be shared and have processes reference parts of it as they would in a sequential program. Rather, the logical data structure *A* must be represented by smaller data structures, which are allocated among the private address spaces of the cooperating processes in accordance with the assignment of work. The process that is assigned a block of rows allocates those rows as a sub-grid in its local address space.

The message-passing program shown in Figure 2-16 is structurally very similar to the shared address space program in Figure 2-13 (more complex programs will reveal further differences in the next chapter, see Section 3.7). Here too, a main process is started by the operating system when the program executable is invoked, and this main process creates (`nprocs-1`) other processes to collaborate with it. We assume that every created process acquires a process identifier pid between 0 and `nprocs-1`, and that the `CREATE` call allows us to communicate the program's input parameters (`n` and `nprocs`) to the address space of each process.[1] The outermost

1. An alternative organization is to use what is called a "hostless" model, in which there is no single main process. The number of processes to be used is specified to the system when the program is invoked. The system then starts up that many processes and distributes the code to the relevant processing nodes. There is no need for a `CREATE` primitive in the program itself, every process reads the program inputs (`n` and `nprocs`) separately, though processes still acquire unique user-level `pid`'s.

```
1.   int   pid, n, nprocs   ;        /* process id, matrix dimension and number of processors to be used */
2. float **myA;
3.   main ()
4.   begin
5.       read (n);   read  (nprocs);          /* read input matrix size and number of processes*/
8a.      CREATE (nprocs-1 processes that start at procedure Solve);
8b.   Solve();                              /* main process becomes a worker too*/
8c.      WAIT_FOR_END;                      /* wait for all child processes created to terminate */
9.   end main

10.   procedure   Solve()
11.   begin
13.      int   i,j, pid, nÕ = n/nprocs, done = 0;
14.      float   temp, tempdiff, mydiff = 0;         /* private variables */
6.    myA  ¬  malloc(a 2-d array of size [n/nprocs + 2] by n+2);     /* my assigned rows of A */
7.    initialize(myA);                           /* initialize my rows of A, in an unspeciÞed way*/

15. while  (!done)   do
16.    mydiff = 0;                               /* set local diff to 0 */
16a. if (pid != 0) then          SEND(&myA[1,0],n*sizeof(float),pid-1,ROW);
16b.   if (pid = nprocs-1) then          SEND(&myA[nÕ,0],n*sizeof(float),pid+1,ROW);
16c.   if (pid != 0) then          RECEIVE(&myA[0,0],n*sizeof(float),pid-1,ROW);
16d.   if (pid != nprocs-1) then          RECEIVE(&myA[nÕ+1,0],n*sizeof(float),pid+1,ROW);
              /* border rows of neighbors have now been copied into myA[0,*] and myA[nÕ+1,*] */
17.      for  i   ¬  1   to  nÕ   do                /* for each of my rows */
18.            for  j   ¬  1   to  n   do           /* for all elements in that row */
19.                 temp = myA[i,j];
20.                 myA[i,j] ¬ 0.2 * (myA[i,j] + myA[i,j-1] + myA[i-1,j] +
21.                       myA[i,j+1] + myA[i+1,j]);
22.                 mydiff += abs(myA[i,j] - temp);
23.            endfor
24.      endfor
         /* communicate local diff values and obtain determine if done; can be replaced by reduction and broadcast */
25a.     if   (pid != 0)    then                    /* process 0 holds global total diff*/
25b.           SEND(mydiff,sizeof(float),0,DIFF);
25c.           RECEIVE(mydiff,sizeof(float),0,DONE);
25d.     else
25e.           for  i   ¬  1   to  nprocs-1    do       /* for each of my rows */
25f.                 RECEIVE(tempdiff,sizeof(float),*,DONE);
25g.                mydiff += tempdiff;                /* accumulate into total */
25h.           endfor
25i.           for  i   ¬  1   to  nprocs-1    do       /* for each of my rows */
25j.                 SEND(done,sizeof(int),i,DONE);
25k.           endfor
25l.     endif
26.            if  (mydiff/(n*n) < TOL)      then  done = 1;
27.   endwhile
28.   end procedure
```

Figure 2-16 Pseudocode describing parallel equation solver with explicit message-passing.

Now the meaning of the data structures and the indexing of them changes in going to the parallel code. Each process has its own myA data structure that represents its assigned part of the grid, and myA[i,j] referenced by different processes refers to different parts of the logical overall grid. The communication is all contained in lines 16 a-d and 25 a-f. No locks or barriers are needed, since the synchronization is implicit in the send-receive pairs. Several extra lines of code are added to orchestrate the communication with simple sends and receives.

loop of the `Solve` routine (line 15) still iterates over grid sweeps until convergence. In every iteration a process performs the computation for its assigned rows and communicates as necessary. The major differences are in the data structures used to represent the logically shared matrix *A* and in how interprocess communication is implemented. We shall focus on these differences.

Instead of representing the matrix to be factored as a single global (`n+2`)-by-(`n+2`) array *A*, each process in the message-passing program allocates an array called `myA` of size (`nprocs/n + 2`)-by-(`nprocs/n + 2`) in its private address space. This array represents its assigned `nprocs/n` rows of the logically shared matrix *A*, plus two rows at the edges to hold the boundary data from its neighboring partitions. The boundary rows from its neighbors must be communicated to it explicitly and copied into these extra or *ghost* rows. Ghost rows are used because without them the communicated data would have to received into separate one-dimensional arrays with different names created specially for this purpose, which would complicate the referencing of the data when they are read in the inner loop (lines 20-21). Communicated data have to be copied into the receiver's private address space anyway, so programming is made easier by extending the existing data structure rather than allocating new ones.

Recall from Chapter 1 that both communication and synchronization in a message passing program are based on two primitives: `SEND` and `RECEIVE`. The program event that initiates data transfer is the `SEND` operation, unlike in a shared address space where data transfer is usually initiated by the consumer or receiver using a load instruction. When a message arrives at the destination processor, it is either kept in the network queue or temporarily stored in a system buffer until a process running on the destination processor posts a `RECEIVE` for it. With a `RECEIVE`, a process reads an incoming message from the network or system buffer into a designated portion of the private (application) address space. A `RECEIVE` does not in itself cause any data to be transferred across the network.

Our simple SEND and RECEIVE primitives assume that the data being transferred are in a contiguous region of the virtual address space. The arguments in our simple `SEND` call are the start address—in the sending processor's private address space—of the data to be sent, the size of the message in bytes, the pid of the destination process—which we must be able to name now, unlike in a shared address space—and an optional tag or type associated with the message for matching at the receiver. The arguments to the `RECEIVE` call are a local address at which to place the received data, the size of the message, the sender's process id, and the optional message tag or type. The sender's id and the tag, if present, are used to perform a match with the messages that have arrived and are in the system buffer, to see which one corresponds to the receive. Either or both of these fields may be wild-cards, in which case they will match a message from any source process or with any tag, respectively. `SEND` and `RECEIVE` primitives are usually implemented in a library on a specific architecture, just like `BARRIER` and `LOCK` in a shared address space. A full set of message passing primitives commonly used in real programs is part of a standard called the Message Passing Interface (MPI), described at different levels of detail in [Pac96, MPI93, GLS94]. A significant extension is transfers of non-contiguous regions of memory, either with regular stride—such as every tenth word in between addresses *a* and *b*, or four words every sixth word—or by using index arrays to specify unstructured addresses from which to *gather* data on the sending side or to which to *scatter* data on the receiving side. Another is a large degree of flexibility in specifying tags to match messages and in the potential complexity of a match. For example, processes may be divided into groups that communicate certain types of messages only to each other, and collective communication operations may be provided as described below.

Semantically, the simplest forms of SEND and RECEIVE we can use in our program are the so called *synchronous* forms. A synchronous SEND returns control to the calling process only when it is clear that the corresponding RECEIVE has been performed. A synchronous RECEIVE returns control when the data have been received into the destination buffer. Using synchronous messages, our implementation of the communication in lines 16a-d is actually deadlocked. All processors do their SEND first and stall until the corresponding receive is performed, so none will ever get to actually perform their RECEIVE! In general, synchronous message passing can easily deadlock on pairwise exchanges of data if we are not careful. One way to avoid this problem is to have every alternate processor do its SENDs first followed by its RECEIVEs, and the others start do their RECEIVEs first followed by their SENDs. The alternative is to use different semantic flavors of send and receive, as we will see shortly.

The communication is done all at once at the beginning of each iteration, rather than element-by-element as needed in a shared address space. It could be done element by element, but the overhead of send and receive operations is usually too large to make this approach perform reasonably. As a result, unlike in the shared address space version there is no computational asynchrony in the message-passing program: Even though one process updates its boundary rows while its neighbor is computing in the same sweep, the neighbor is guaranteed not see the updates in the current sweep since they are not in its address space. A process therefore sees the values in its neighbors' boundary rows as they were at the end of the previous sweep, which may cause more sweeps to be needed for convergence as per our earlier discussion (red-black ordering would have been particularly useful here).

Once a process has received its neighbors' boundary rows into its ghost rows, it can update its assigned points using code almost exactly like that in the sequential and shared address space programs. Although we use a different name (myA) for a process's local array than the A used in the sequential and shared address space programs, this is just to distinguish it from the logically shared entire grid *A* which is here only conceptual; we could just as well have used the name A). The loop bounds are different, extending from 1 to `nprocs/n` (substituted by `n'` in the code) rather than 0 to `n-1` as in the sequential program or `mymin` to `mymax` in the shared address space program. In fact, the indices used to reference `myA` are local indices, which are different than the global indices that would be used if the entire logically shared grid `A` could be referenced as a single shared array. That is, the `myA[1,j]` reference by different processes refers to different rows of the logically shared grid `A`. The use of local index spaces can be somewhat more tricky in cases where a global index must also be used explicitly, as seen in Exercise 2.4.

Synchronization, including the accumulation of private `mydiff` variables into a logically shared `diff` variable and the evaluation of the done condition, is performed very differently here than in a shared address space. Given our simple synchronous sends and receives which block the issuing process until they complete, the send-receive match encapsulates a synchronization event, and no special operations (like locks and barriers) or additional variables are needed to orchestrate mutual exclusion or event synchronization. Consider mutual exclusion. The logically shared `diff` variable must be allocated in some process's private address space (here process 0). The identity of this process must be known to all the others. Every process sends its `mydiff` value to process 0, which receives them all and adds them to the logically shared global `diff`. Since only it can manipulate this logically shared variable mutual exclusion and serialization are natural and no locks are needed. In fact, process 0 can simply use its own `mydiff` variable as the global `diff`.

Now consider the global event synchronization for determining the done condition. Once it has received the `mydiff` values from all the other processes and accumulated them, it sends the accumulated value to all the other processes, which are waiting for it with receive calls. There is no need for a barrier since the completion of the receive implies that all processes's `mydiff`s have been accumulated since process 0 has sent out the result. The processes then compute the done condition to determine whether or not to proceed with another sweep. We could instead have had process 0 alone compute the done condition and then send the `done` variable, not the `diff` value, to the others, saving the redundant calculation of the done condition. We could, of course, implement lock and barrier calls using messages if that is more convenient for programming, although that may lead to request-reply communication and more round-trip messages. More complex send-receive semantics than the synchronous ones we have used here may require additional synchronization beyond the messages themselves, as we shall see.

Notice that the code for the accumulation and done condition evaluation communication has expanded to several lines when using only point-to-point SENDs and RECEIVEs as communication operations. In practice, programming environments would provide library functions like REDUCE (accumulate values from private variables in multiple processes to a single variable in a given process) and BROADCAST (send from one process to all processes) to the programmer which the application processes could use directly to simplify the codein these stylized situations. Using these, lines 25a through 25b in Figure 2-16 can be replaced by the two lines in Figure 2-17. The system may provide special support to improve the performance of these and other *collective communication* operations (such as *multicast* from one to several or even several to several processes, or all-to-all communication in which every process transfers data to every other process), for example by reducing the software overhead at the sender to that of a single message, or they may be built on top of the usual point to point send and receive in user-level libraries for programming convenience only.

```
     /* communicate local diff values and determine if done, using reduction and broadcast */
25b. REDUCE(0,mydiff,sizeof(float),ADD);
25c. if (pid == 0) then
25i.      if (mydiff/(n*n) < TOL) then done = 1;
26.  endif
25k. BROADCAST(0,done,sizeof(int),DONE);
```

Figure 2-17 Accumulation and convergence determination in the solver using REDUCE and BROADCAST instead of SEND and RECEIVE.

The first argument to the REDUCE call is the destination process. All but this process will do a send to this process in the implementation of REDUCE, while this process will do a receive. The next argument is the private variable to be reduced from (in all other processes than the destination) and to (in the destination process), and the third argument is the size of this variable. The last argument is the function to be performed on the variables in the reduction. Similarly, the first argument of the BROADCAST call is the sender; this process does a send and all others do a receive. The second argument is the variable to be broadcast and received into, and the third is its size. The final argument is the optional message type.

Finally, we said earlier that send and receive operations come in different semantic flavors, which we could use to solve our deadlock problem. Let us examine this a little further. The main axis along which these flavors differ is their completion semantics; i.e. when they return control to the user process that issued the send or receive. These semantics affect when the data structures or buffers they use can be reused without compromising correctness. There are two major kinds of send/receive—*synchronous* and *asynchronous*. Within the class of asynchronous messages, there are two major kinds: *blocking* and *non-blocking*. Let us examine these and see how they might be used in our program.

Synchronous sends and receives are what we assumed above, since they have the simplest semantics for a programmer. A synchronous send returns control to the calling process only when the corresponding synchronous receive at the destination end has completed successfully and returned an acknowledgment to the sender. Until the acknowledgment is received, the sending process is suspended and cannot execute any code that follows the send. Receipt of the acknowledgment implies that the receiver has retrieved the entire message from the system buffer into application space. Thus, the completion of the send guarantees (barring hardware errors) that the message has been successfully received, and that all associated data structures and buffers can be reused.

A *blocking asynchronous* send returns control to the calling process when the message has been taken from the sending application's source data structure and is therefore in the care of the system. This means that when control is returned, the sending process can modify the source data structure without affecting that message. Compared to a synchronous send, this allows the sending process to resume sooner, but the return of control does not guarantee that the message will actually be delivered to the appropriate process. Obtaining such a guarantee would require additional handshaking. A blocking asynchronous receive is similar in that it returns control to the calling process only when the data it is receiving have been successfully removed from the system buffer and placed at the designated application address. Once it returns, the application can immediately use the data in the specified application buffer. Unlike a synchronous receive, a blocking receive does not send an acknowledgment to the sender.

The *nonblocking* asynchronous send and receive allow the greatest overlap between computation and message passing by returning control most quickly to the calling process. A nonblocking send returns control immediately. A nonblocking receive returns control after simply posting the intent to receive; the actual receipt of the message and placement into a specified application data structure or buffer is performed asynchronously at an undetermined time by the system, on the basis of the posted receive. In both the nonblocking send and receive, however, the return of control does not imply anything about the state of the message or the application data structures it uses, so it is the user's responsibility to determine that state when necessary. The state can be determined through separate calls to primitives that probe (query) the state. Nonblocking messages are thus typically used in a two-phase manner: first the send/receive operation itself, and then the probes. The probes—which must be provided by the message-passing library—might either block until the desired state is observed, or might return control immediately and simply report what state was observed.

Which kind of send/receive semantics we choose depends on how the program uses its data structures, and on how much we want to optimize performance over ease of programming and portability to systems with other semantics. The semantics mostly affects event synchronization, since mutual exclusion falls out naturally from having only private address spaces. In the equation solver example, using asynchronous sends would avoid the deadlock problem since processes would proceed past the send and to the receive. However, if we used nonblocking asynchronous receives, we would have to use a probe before actually using the data specified in the receive.

Note that a blocking send/receive is equivalent to a nonblocking send/receive followed immediately by a blocking probe.

Table 2-3 Some Basic Message Passing Primitives.

Name	**Syntax**	**Function**
`CREATE`	`CREATE(procedure)`	Create process that starts at `procedure`
`BARRIER`	`BARRIER(name, number)`	Global synchronization among `number` processes: None gets past BARRIER until `number` have arrived
`WAIT_FOR_END`	`WAIT_FOR_END(number)`	Wait for `number` processes to terminate
`SEND`	`SEND(src_addr, size, dest, tag)`	Send `size` bytes starting at `src_addr` to the `dest` process, with `tag` identifier
`RECEIVE`	`RECEIVE(buffer_addr, size, src, tag)`	Receive a message with the `tag` identifier from the src process, and put `size` bytes of it into buffer starting at `buffer_addr`
`SEND_PROBE`	`SEND_PROBE(tag, dest)`	Check if message with identifier tag has been sent to process dest (only for asynchronous message passing, and meaning depends on semantics discussed above)
`RECV_PROBE`	`RECV_PROBE(tag, src)`	Check if message with identifier tag has been received from process src (only for asynchronous message passing, and meaning depends on semantics)

We leave it as an exercise to transform the message passing version to use a cyclic assignment, as was done for the shared address space version in Example 2-2. The point to observe in that case is that while the two message-passing versions will look syntactically similar, the meaning of the `myA` data structure will be completely different. In one case it is a section of the global array and in the other is a set of widely separated rows. Only by careful inspection of the data structures and communication patterns can one determine how a given message-passing version corresponds to the original sequential program.

2.5 Concluding Remarks

Starting from a sequential application, the process of parallelizing the application is quite structured: We decompose the work into tasks, assign the tasks to processes, orchestrate data access, communication and synchronization among processes, and optionally map processes to processors. For many applications, including the simple equation solver used in this chapter, the initial decomposition and assignment are similar or identical regardless of whether a shared address space or message passing programming model is used. The differences are in orchestration, particularly in the way data structures are organized and accessed and the way communication and synchronization are performed. A shared address space allows us to use the same major data structures as in a sequential program: Communication is implicit through data accesses, and the decomposition of data is not required for correctness. In the message passing case we must synthesize the logically shared data structure from per-process private data structures: Communication is explicit, explicit decomposition of data among private address spaces (processes) is necessary, and processes must be able to name one another to communicate. On the other hand, while a shared address space program requires additional synchronization primitives separate from the loads and stores used for implicit communication, synchronization is bundled into the explicit send and receive communication in many forms of message passing. As we examine the parallelization of more complex parallel applications such as the four case studies introduced in this chapter, we will understand the implications of these differences for ease of programming and for performance.

The parallel versions of the simple equation solver that we described here were purely to illustrate programming primitives. While we did not use versions that clearly will perform terribly (e.g. we reduced communication by using a block rather than cyclic assignment of rows, and we reduced both communication and synchronization dramatically by first accumulating into local `mydiff`s and only then into a global `diff`), they can use improvement. We shall see how in the next chapter, as we turn our attention to the performance issues in parallel programming and how positions taken on these issues affect the workload presented to the architecture.

2.6 References

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2.7 Exercises

2.1 Short Answer Questions:

a. Describe two examples where the a good parallel algorithm must be based on a serial algorithm that is different than the best serial algorithm, since the latter does not afford enough concurrency.

b. Which of the case study applications that we have described (Ocean, Barnes-Hut, Raytrace) do you think are amenable to decomposing data rather than computation and using an owner computes rule in parallelization? What do you think would be the problem(s) with using a strict data distribution and owner computes rule in the others?

2.2 <2.4.5>There are two dominant models for how parent and children processes relate to each other in a shared address space. In the heavyweight UNIX process fork model, when a process creates another, the child gets a private *copy* of the parent's image: that is, if the parent had allocated a variable x, then the child also finds a variable x in its address space which is initialized to the value that the parent had for x when it created the child; however, any modifications that either process makes subsequently are to its own copy of x and are not visible to the other process. In the lightweight threads model, the child process or thread gets a pointer to the parent's image, so that it and the parent now see the same storage location for x. All data that any process or thread allocates are shared in this model, except those that are on a procedure's stack.

a. Consider the problem of a process having to reference its process identifier procid in various parts of a program, in different routines in a call chain from the routine at which the process begins execution. How would you implement this in the first case? In the second? Do you need private data per process, or could you do this with all data being globally shared?

b. A program written in the former (fork) model may rely on the fact that a child process gets its own private copies of the parents' data structures. What changes would you make to port the program to the latter (threads) model for data structures that are (i) only read by processes after the creation of the child, (ii) are both read and written? What performance issues might arise in designing the data structures (you will be better equipped to answer this part of the question after reading the next few chapters)?

2.3 Synchronization.

a. The classic bounded buffer problem provides an example of point-to-point event synchronization. Two processes communicate through a finite buffer. One process—the producer—adds data items to a buffer when it is not full, and another—the consumer—reads data items from the buffer when it is not empty. If the consumer finds the buffer empty, it must wait till the producer inserts an item. When the producer is ready to insert an item, it checks to see if the buffer is full, in which case it must wait till the consumer removes something from the buffer. If the buffer is empty when the producer tries to add an item, depending on the implementation the consumer may be waiting for notification, so the producer may need to notify the consumer. Can you implement a bounded buffer with only point-to-point event synchronization, or do you need mutual exclusion as well. Design an implementation, including pseudocode.

b. Why wouldn't we use spinning for interprocess synchronization in uniprocessor operating systems? What do you think are the tradeoffs between blocking and spinning on a multiprocessor?

c. In the shared address space parallel equation solver (Figure 2-13 on page 114), why do we need the second barrier at the end of a while loop iteration (line 25f)? Can you eliminate it without inserting any other synchronization, but perhaps modifying when certain operations are performed? Think about all possible scenarios.

2.4 Do LU factorization as an exercise. Describe, give a simple contiguous decomposition and assignment, draw concurrency profile, estimate speedup assuming no communication overheads, and ask to implement in a shared address space and in message passing. Modify the message-passing pseudocode to implement an interleaved assignment. Use both synchronous and asynchronous (blocking) sends and receives. Then interleaving in both directions, do the same.

2.5 More synchronization. Suppose that a system supporting a shared address space did not support barriers but only semaphores. Event synchronization would have to be constructed through semaphores or ordinary flags. To coordinate P1 indicating to P2 that it has reached point a (so that P2 can proceed past point b where it was waiting) using semaphores, *P1* performs a *wait* (also called *P* or *down*) operation on a semaphore when it reaches point *a*, and *P2* performs a *signal* (or *V* or *up*) operation on the same semaphore when it reaches point *b*. If *P1* gets to *a* before *P2* gets to *b*, *P1* suspends itself and is awoken by *P2*'s signal operation.

a. How might you orchestrate the synchronization in LU factorization with (i) flags and (ii) semaphores replacing the barriers. Could you use point-to-point or group event synchronization instead of global event synchronization?

b. Answer the same for the equation solver example.

2.6 Other than the above "broadcast" approach, LU factorization can also be parallelized in a form that is more aggressive in exploiting the available concurrency. We call this form the *pipelined* form of parallelization, since an element is computed from the producing process to a consumer in pipelined form via other consumers, which use the element as they communicate it in the pipeline.

a. Write shared-address-space pseudocode, at a similar level of detail as Figure 2-13 on page 114, for a version that implements pipelined parallelism at the granularity of individual elements (as described briefly in Section 2.4.2). Show all synchronization necessary. Do you need barriers?

b. Write message-passing pseudocode at the level of detail of Figure 2-16 on page 120 for the above pipelined case. Assume that the only communication primitives you have are synchronous and asynchronous (blocking and nonblocking) sends and receives. Which versions of send and receive would you use, and why wouldn't you choose the others?

c. Discuss the tradeoffs (programming difficulty and likely performance differences) in programming the shared address space and message-passing versions.

d. Discuss the tradeoffs in programming the loop-based versus pipelined parallelism

2.7 Multicast (sending a message from one process to a named list of other processes) is a useful mechanism for communicating among subsets of processes.

a. How would you implement the message-passing, interleaved assignment version of LU factorization with multicast rather than broadcast? Write pseudocode, and compare the programming ease of the two versions.

b. Which do you think will perform better and why?

c. What other “group communication” primitives other than multicast do you think might be useful for a message passing system to support? Give examples of computations in which they might be used.