PH4433/6433 HW #7 Due Wed. 10/28

1 Linear least-squares fit

Consider the following set of (x, y) data. A copy will be placed on MyCourses:

```
0.15  3.85
0.24  3.82
0.43  3.27
0.47  4.20
0.64  4.12
0.68  3.38
0.73  3.91
0.77  4.26
0.79  3.79
0.88  3.76
0.97  3.83
1.03  4.36
1.18  3.70
1.19  3.82
1.54  4.72
1.76  4.62
1.84  3.98
1.61  3.97
1.91  4.12
1.96  4.74
```

1. Plot the data in xmgrace and fit to a straight line $y = a + bx$. What are the parameters a and b and their estimated errors?

2. Write a program to solve for a and b using LAPACK and the method discussed in class. Assume an error bar of σ_i=0.5. Compare your results for a, b, and the errors to xmgrace.

2 Fitting an ellipse

You have the following (noisy) measurements of the points of an object in an elliptical orbit in the (x, y) plane:

```
0.6728 0.0589
0.3380 0.4093
0.2510 0.3559
-0.0684 0.5449
-0.4329 0.3657
-0.6921 0.0252
-0.3681 -0.2020
0.0019 -0.3769
```

```
0.0825 -0.3508
0.5294 -0.2918
```

The following equation defines an ellipse

$$a_1 x^2 + a_2 y^2 + a_3 xy = 1. \tag{1}$$

a_1, a_2, and a_3 are parameters you are to determine.

1. Using Eq. 1 and the 10 points (x_i, y_i), write a system of linear equations of the form $\mathbf{A} \cdot \mathbf{a} = \mathbf{b}$, where $\mathbf{a}$ is the vector of unknown parameters a_i. $\mathbf{A}$ is not a square matrix.

2. By multiplying both sides by $\mathbf{A^T}$, convert this equation to a matrix equation of the form we considered in class, $\boldsymbol{\alpha} \cdot \boldsymbol{a} = \boldsymbol{\beta}$, where $\boldsymbol{\alpha}$ is a square matrix.

3. Solve for $\boldsymbol{a}$ (LAPACK DGESV).

4. Plot the data and the fitted function. You can plot the ellipse by using polar coordinates:

$$\begin{aligned} x &= \rho \cos\theta \\ y &= \rho \sin\theta \\ \frac{1}{\rho^2} &= a_1 \cos^2\theta + a_2 \sin^2\theta + a_3 \cos\theta \sin\theta \end{aligned}$$