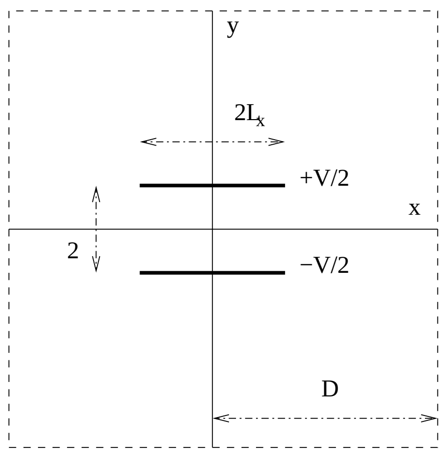
PH 4433/6433 #9 due Mon. 11/23: A strip capacitor

In introductory physics the capacitance of a parallel plate capacitor is derived as

$$C = \frac{Q}{V} = \frac{\epsilon_0 A}{d},$$

where A is the plate area and d the spacing between the plates. For the case of a long strip capacitor for which the plate width is L_x, the capacitance per unit length is $c = \epsilon_0 L_x/d$. These formulas assume that the electric field is only nonzero between the plates and uniform. In reality, the fringing electric field at the edge changes C, with the deviation from the simple formula becoming worse as $b = d/L_x$ increases. In this assignment you will calculate the actual capacitance for a strip capacitor. Here is a cross-section of the capacitor, whose long dimension is along z:



Set $\epsilon_0 = 1$. The top plate has potential $\phi = +V/2$, the bottom plate $\Phi = -V/2$. Note that with this geometry $\Phi = 0$ everywhere in the xz plane. The plate spacing is 2 (in whatever units) and the plate length $2L_x$. Far from the capacitor, $\Phi = 0$; assume this is true on the dashed box width dimensions $2D \times 2D$ as shown.

To calculate c you need to solve Laplace's equation, $\nabla^2\Phi = 0$ in two dimensions, the xy plane. To do this use the Gauss-Seidel method as we covered in class. Some hints:

- Define $\Phi(x, y)$ as a 2D array, $u(N, N)$. You will need to take N in the 100's to get good results.
- You can save a lot of time if you use symmetry and just solve *half* of the capacitor, i.e. only for $y > 0$.
- Boundary conditions: $u = 0$ along the x axis and the dotted boundary. Make the capacitor plate a single line of points with $u = 0.5$ (this corresponds to taking $V = 1$).
- Gauss-Seidel (or SOR) relaxation is very simple. The only tricky part is to be sure the boundary and plate points are *not* updated; they are fixed boundary values.

- Choosing a better starting guess for $u(x, y)$ will speed up the process quite a bit. A good reasonable guess is the potential assuming no fringing effects are present.
- Calculating Q: to determine the capacitance you need to calculate the charge on the plates. The charge density is proportional to the perpendicular component of the electric field, which can be determined from $-\nabla\Phi$. The total charge can also be determined using Gauss' law (see discussion in class).

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1. Write the program to solve for the potential $\Phi(x, y)$. Make contour plots of $\Phi(x, y)$ (use Mathematica) for b=0.1, 0.5, and 1.0.
2. Calculate and plot the charge density on the upper and lower surfaces of the plate for each b. How does the charge density in each case compare to its value in the simple formula?
3. Calculate the total charge using Gauss' law and the capacitance per unit length c for each b. Find the ratio c/c_0, where c_0 is the capacitance per unit length in the simple formula.

   Try different numbers of grid points and D to determine roughly how accurate you think the results are.