PH 4433/6433 HW 5: Due Friday October 9

1 Bifurcation of the damped driven pendulum

Consider a damped, driven pendulum, where the differential equation for θ is given by

$$\frac{d^2\theta}{dt^2} = -\alpha\frac{d\theta}{dt} - (\omega_0^2 + f\cos(\omega t))sin\theta$$

The goal of this problem is to make a bifurcation plot like Fig. 12.12 on page 314 of the textbook. Use Runge-Kutta to integrate the equation. Take the following parameters:

- $\alpha = 0.1$
- $\omega_0 = 1$
- $\omega = 2$
- initial conditions θ=0 and $d\theta/dt$=1

In order to make the bifurcation plot, you will integrate the equation with a range of different forcing amplitudes f (the x axis of the plot). It will be simplest to choose the integration step dt so that the period of the driving force ($2\pi/\omega$) is evenly divided by dt. For each value of f:

- First integrate for 150 driving periods in order to remove any initial transient behavior.
- Integrate for 150 (or more) additional driving periods. Plot $d\theta/dt$ (this is y(2) in the code) when the driving force is zero (this happens every half period).
- Make a plot with the points f,$|d\theta/dt|$.

Note that after each integration you should check if the angle θ is out of the range $-\pi < \theta < \pi$ and add or subtract 2π to bring θ back to this range.

1. Make a plot for the range $0 < f < 2.25$ (like Fig 12.12)
2. Make another plot with more data in just the range where the period doubling takes place.
3. Make a rough estimate of the Feigenbaum constant for the period doubling from the values of f for the first three period doublings.