(Whoops, I didn't read the "must be typed" requirement until I already completed the assignment; I will type the next one.)

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STA 243 Lee

Assignment 1

1. (a) $L(\theta) = \prod_{i=1}^{n} f(x_i|\theta) = \prod_{i=1}^{n} [\pi(1+(x_i-\theta)^2)]^{-1} = \frac{1}{\pi^n} \prod_{i=1}^{n} [1+(\theta-x_i)^2]^{-1}$

$$\ell(\theta) = \log(L(\theta)) = \log\left(\frac{1}{\pi^n} \prod_{i=1}^{n} [1+(\theta-x_i)^2]^{-1}\right) = \boxed{-n\log\pi - \sum_{i=1}^{n} \log[1+(\theta-x_i)^2]}$$

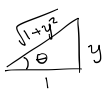
$$\ell'(\theta) = -\sum_{i=1}^{n} \frac{2(\theta-x_i)}{1+(\theta-x_i)^2} = \boxed{-2\sum_{i=1}^{n} \frac{\theta-x_i}{1+(\theta-x_i)^2}}$$

$$\ell''(\theta) = -2\sum_{i=1}^{n} \frac{1+(\theta-x_i)^2 - (\theta-x_i)[2(\theta-x_i)]}{[1+(\theta-x_i)^2]^2} = \boxed{-2\sum_{i=1}^{n} \frac{1-(\theta-x_i)^2}{[1+(\theta-x_i)^2]^2}}$$

(b) $I(\theta) = E_\theta[\ell'(\theta)\ell'(\theta)^T] = -E_\theta[\ell''(\theta)] = n I_i(\theta)$

$$= n\int_{\mathbb{R}} \frac{-2[1-(\theta-x)^2]}{[1+(\theta-x)^2]^2} \frac{1}{\pi(1+(\theta-x)^2)} dx$$

$$= n\int_{\mathbb{R}} \frac{-2}{\pi} \frac{1-(\theta-x)^2}{[1+(\theta-x)^2]^3} dx = \frac{2n}{\pi}\int_{\mathbb{R}} \frac{1-y^2}{(1+y^2)^3} dy = \frac{2n}{\pi}\left[\int_{\mathbb{R}} \frac{dy}{(1+y^2)^3} - \int_{\mathbb{R}} \frac{y^2 dy}{(1+y^2)^3}\right]$$



$\cos\theta = \frac{1}{\sqrt{1+y^2}}$

$\tan\theta = y$

$\sec^2\theta\, d\theta = dy$

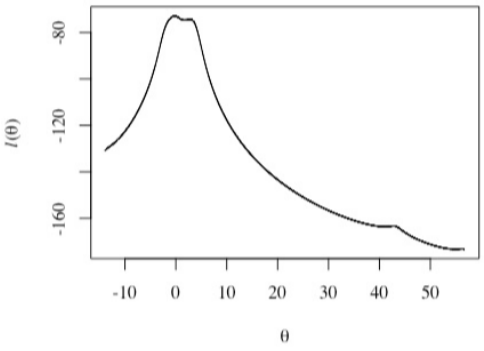
$\lim_{y\to\pm\infty} \frac{1}{\sqrt{1+y^2}} = 0_\pm = \cos\theta \Rightarrow \theta = \pm\frac{\pi}{2}$

$$= \frac{2n}{\pi}\left[\int_{-\pi/2}^{\pi/2} \cos^6\theta \sec^2\theta\, d\theta - \int_{-\pi/2}^{\pi/2} \cos^6\theta \tan^2\theta \sec^2\theta\, d\theta\right]$$

$$= \frac{2n}{\pi}\left[\int_{-\pi/2}^{\pi/2} \cos^4\theta\, d\theta - \int_{-\pi/2}^{\pi/2} \cos^2\theta \sin^2\theta\, d\theta\right]$$

$$= \frac{2n}{\pi}\left[\frac{3\pi}{8} - \frac{\pi}{8}\right] = \frac{2n}{\pi}\left(\frac{\pi}{4}\right) = \boxed{\frac{n}{2}}$$

(c)



(d)

```
[1] "Newton-Raphson"
[1] "theta = -1.00677e+61, L(theta) = 0.00000e+00"
[1] "theta = -1.92287e-01, L(theta) = 2.15309e-32"
[1] "theta = -1.92287e-01, L(theta) = 2.15309e-32"
[1] "theta = 1.71359e+00, L(theta) = 3.83164e-33"
[1] "theta = 2.81747e+00, L(theta) = 5.07764e-33"
[1] "theta = -2.37127e+60, L(theta) = 0.00000e+00"
[1] "theta = 4.10408e+01, L(theta) = 8.83219e-72"
[1] "theta = -7.01996e+60, L(theta) = 0.00000e+00"
[1] "theta = 4.27954e+01, L(theta) = 1.18608e-71"
```

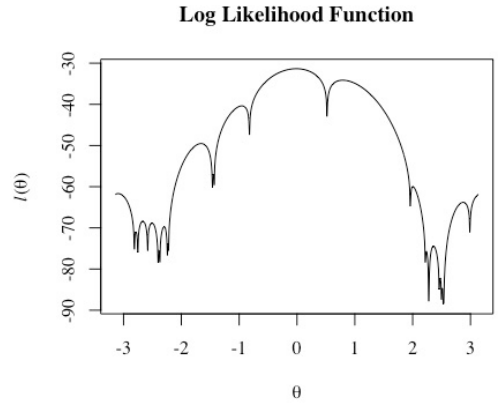
The results from various starting points are very different with some not converging.

(e)

```
[1] "Fisher-Scoring + Newton-Raphson"
[1] "theta = -1.92287e-01, L(theta) = 2.15309e-32"
[1] "theta = -1.92287e-01, L(theta) = 2.15309e-32"
[1] "theta = -1.92287e-01, L(theta) = 2.15309e-32"
[1] "theta = -1.92287e-01, L(theta) = 2.15309e-32"
[1] "theta = 2.81747e+00, L(theta) = 5.07764e-33"
[1] "theta = 2.81747e+00, L(theta) = 5.07764e-33"
[1] "theta = 2.81747e+00, L(theta) = 5.07764e-33"
[1] "theta = 2.81747e+00, L(theta) = 5.07764e-33"
[1] "theta = 5.48766e+01, L(theta) = 5.26954e-76"
```

The Fisher-Scoring method is clearly more stable with all the starting points converging to a reasonable result.

2. (a)



(b) $E[X] = \int_0^{2\pi} \frac{x}{2\pi}[1-\cos(x-\theta)]\, dx$

$$= \frac{1}{2\pi}\int_0^{2\pi}[x - x\cos(x-\theta)]\, dx$$

$$= \frac{1}{2\pi}\left[\frac{x^2}{2}\Big|_0^{2\pi} - x\sin(x-\theta)\Big|_0^{2\pi} - \cos(x-\theta)\Big|_0^{2\pi}\right]$$

$$= \frac{1}{2\pi}\left[2\pi^2 - 2\pi\sin(2\pi-\theta) - \cancel{\cos(2\pi-\theta)} + \cancel{\cos(-\theta)}\right]$$

$$= \pi + \sin(\theta)$$

$$\hat{\theta} = \hat{\pi} + \sin(\hat{\theta}) \Rightarrow \boxed{\hat{\theta} = \pi}$$

2. (c) $\ell(\theta) = -n\log 2\pi + \sum_{i=1}^{n} \log(1-\cos(x_i-\theta))$

$$\ell'(\theta) = \sum_{i=1}^{n} \frac{-\sin(x_i-\theta)}{1-\cos(x_i-\theta)}$$

$$\ell''(\theta) = \sum_{i=1}^{n} \frac{(1-\cos(x_i-\theta))(\cos(x_i-\theta)) + \sin(x_i-\theta)(-\sin(x_i-\theta))}{(1-\cos(x_i-\theta))^2}$$

$$= \sum_{i=1}^{n} \frac{\cos(x_i-\theta)-1}{(1-\cos(x_i-\theta))^2} = \sum_{i=1}^{n} \frac{1}{\cos(x_i-\theta)-1}$$

$\theta \approx 3.19009$, $\boxed{MLE \approx 1.62924 \times 10^{-27}}$

(d) $\theta_0 = -2.7$, $\boxed{\theta \approx -2.66670}$, $\boxed{MLE \approx 1.97065 \times 10^{-30}}$

$\theta_0 = 2.7$, $\boxed{\theta \approx 2.87309}$, $\boxed{MLE \approx 1.94786 \times 10^{-28}}$

(e)

$a \le \theta_0 \le b$	θ	MLE
[1] "a = -3.14159,	b = -2.82743,	theta = -3.09309,	MLE = 1.62924e-27"
[1] "a = -2.79602,	b = -2.76460,	theta = -2.78617,	MLE = 1.47761e-31"
[1] "a = -2.73319,	b = -2.60752,	theta = -2.66670,	MLE = 1.97065e-30"
[1] "a = -2.57611,	b = -2.41903,	theta = -2.50761,	MLE = 1.27098e-30"
[1] "a = -2.38761,	b = -2.38761,	theta = -2.38820,	MLE = 2.00092e-33"
[1] "a = -2.35619,	b = -2.26195,	theta = -2.29726,	MLE = 5.26310e-31"
[1] "a = -2.23053,	b = -2.23053,	theta = -2.23217,	MLE = 8.71100e-33"
[1] "a = -2.19911,	b = -1.47655,	theta = -1.65828,	MLE = 3.11476e-22"
[1] "a = -1.44513,	b = -1.44513,	theta = -1.44748,	MLE = 2.23526e-25"
[1] "a = -1.41372,	b = -0.84823,	theta = -0.95334,	MLE = 2.90971e-18"
[1] "a = -0.81681,	b = 0.50265,	theta = -0.01197,	MLE = 2.44313e-14"
[1] "a = 0.53407,	b = 1.94779,	theta = 0.79060,	MLE = 1.48799e-15"
[1] "a = 1.97920,	b = 2.19911,	theta = 2.00364,	MLE = 8.78580e-27"
[1] "a = 2.23053,	b = 2.26195,	theta = 2.23622,	MLE = 1.42122e-33"
[1] "a = 2.29336,	b = 2.45044,	theta = 2.36072,	MLE = 4.60706e-33"
[1] "a = 2.48186,	b = 2.48186,	theta = 2.47537,	MLE = 2.11847e-36"
[1] "a = 2.51327,	b = 2.51327,	theta = 2.51359,	MLE = 2.16205e-37"
[1] "a = 2.54469,	b = 2.98451,	theta = 2.87309,	MLE = 1.94786e-28"
[1] "a = 3.01593,	b = 3.14159,	theta = 3.19009,	MLE = 1.62924e-27"

3. (a) $\beta_0 = \frac{1}{\theta_1} \Rightarrow \theta_1 = \frac{1}{\beta_0}$

$\beta_1 = \frac{\theta_2}{\theta_1} \Rightarrow \theta_2 = \theta_1\beta_1 = \frac{\beta_1}{\beta_0}$

	θ_1	θ_2
y_1	205.28	0.06752
y_2	187.16	0.03098

(c)

	θ_1	θ_2
y_1	163.55	-136.01
y_2	196.50	357.30

(??)

(b) $$\frac{\partial RSS}{\partial \theta_1} = -2\sum_{i=1}^{n}\left(y_i - \frac{\theta_1 x_i}{x_i+\theta_2}\right)\left(\frac{x_i}{x_i+\theta_2}\right) = -2\sum_{i=1}^{n}\left[\frac{x_i y_i}{x_i+\theta_2} - \frac{\theta_1 x_i^2}{(x_i+\theta_2)^2}\right]$$

$$\frac{\partial RSS}{\partial \theta_2} = 2\sum_{i=1}^{n}\left(y_i - \frac{\theta_1 x_i}{x_i+\theta_2}\right)\left(\frac{\theta_1 x_i}{(x_i+\theta_2)^2}\right)$$

$$\frac{\partial^2 RSS}{\partial \theta_1^2} = 2\sum_{i=1}^{n}\left(\frac{x_i}{x_i+\theta_2}\right)^2$$

$$\frac{\partial^2 RSS}{\partial \theta_2^2} = 2\sum_{i=1}^{n}\left[\frac{\theta_1^2 x_i^2}{(x_i+\theta_2)^4} - \left(\frac{2\theta_1 x_i}{(x_i+\theta_2)^3}\right)\left(y_i - \frac{\theta_1 x_i}{x_i+\theta_2}\right)\right]$$

$$\frac{\partial^2 RSS}{\partial \theta_1 \partial \theta_2} = -2\sum_{i=1}^{n}\left[\frac{-x_i y_i}{(x_i+\theta_2)^2} + \frac{2\theta_1 x_i^2}{(x_i+\theta_2)^3}\right]$$

	θ_1	θ_2
y_1	213.17	0.07679
y_2	212.12	0.05278

(d) $f(\underset{\sim}{\theta}) = \frac{\theta_1 x}{x+\theta_2}$

$$f_i'(\underset{\sim}{\theta}) = \left(\frac{x_i}{x_i+\theta_2}, \frac{-\theta_1 x_i}{(x_i+\theta_2)^2}\right)$$

	θ_1	θ_2
y_1	213.17	0.07679
y_2	212.12	0.05278

```
par(family = 'serif')
setwd("/Users/mikhailgaerlan/Box Sync/Education/UC Davis/2016-2017 Spring/STA 243 Computational
Statistics/Assignments/Assignment 1")

#=========
#    1
#=========
#----------
#   1(c)
#----------
sample =
c(-13.87,-2.53,-2.44,-2.40,-1.75,-1.34,-1.05,-0.23,-0.07,0.27,1.77,2.76,3.29,3.47,3.71,3.80,4.24,4.53,43.21,56.75)

n = length(sample)
thetas = seq(min(sample),max(sample),0.01)
loglikelihood = thetas*0
for (x in sample)
  loglikelihood = loglikelihood - log(1+(thetas-x)^2)
loglikelihood = loglikelihood - n*log(pi)

plot(loglikelihood ~ thetas,type='l',xlab=expression(theta),ylab=expression(paste(italic("l"),"
(",theta,")")),main="Log Likelihood Function")

#----------
#   1(d)
#----------
starting = c(-11,-1,0,1.4,4.1,4.8,7,8,38)

print("1d")
maxiter = 200
err = 10^(-5)
for (theta0 in starting){
  theta = theta0
  for (t in 1:maxiter){
    oldtheta = theta
    lptheta = 0
    for (x in sample){
      lptheta = lptheta - 2*(theta-x)/(1+(theta-x)^2)
    }
    lpptheta = 0
    for (x in sample){
      lpptheta = lpptheta - 2*(1-(theta-x)^2)/(1+(theta-x)^2)^2
    }
    h = -lptheta/lpptheta
    theta = theta + h
    experr = abs(theta - oldtheta)
    if (experr < err) {
      break
    }
  }
  biglтheta = 1
  for (x in sample){
    biglтheta = biglтheta * 1/(pi*(1+(x-theta)^2))
  }
  print(sprintf("theta = %12.5e, L(theta) = %12.5e",theta,biglтheta))
}

#----------
#   1(e)
#----------
print("1e")
for (theta0 in starting){
  theta = theta0

  #Fisher-Scoring
  maxiter = 100
  err = 10^(-2)
  for (t in 1:maxiter){
    oldtheta = theta
    lptheta = 0
    for (x in sample){
      lptheta = lptheta - 2*(theta-x)/(1+(theta-x)^2)
    }
    h = 2*lptheta/n
    theta = theta + h
    experr = abs(theta - oldtheta)
    if (experr < err) {
```

```
      break
    }
  }

  #Newton-Raphson
  maxiter = 200
  err = 10^(-5)
  for (t in 1:maxiter){
    oldtheta = theta
    lptheta = 0
    for (x in sample){
      lptheta = lptheta - 2*(theta-x)/(1+(theta-x)^2)
    }
    lpptheta = 0
    for (x in sample){
      lpptheta = lpptheta - 2*(1-(theta-x)^2)/(1+(theta-x)^2)^2
    }
    h = -lptheta/lpptheta
    theta = theta + h
    experr = abs(theta - oldtheta)
    if (experr < err) {
      break
    }
  }
  biglтheta = 1
  for (x in sample){
    biglтheta = biglтheta * 1/(pi*(1+(x-theta)^2))
  }
  print(sprintf("theta = %12.5e, L(theta) = %12.5e",theta,biglтheta))
}

rm(list=ls())
#==========
#    2
#==========
#----------
#   2(a)
#----------
sample = c(0.52,1.96,2.22,2.28,2.28,2.46,2.50,2.53,2.54,2.99,3.47,3.53,3.70,3.88,3.91,4.04,4.06,4.82,4.85,5.46)
thetas = seq(-pi,pi,0.01)
loglikelihood = thetas*0
for (x in sample){
  loglikelihood = loglikelihood + log((1-cos(x-thetas))/(2*pi))
}
plot(loglikelihood ~ thetas,type='l',xlab=expression(theta),ylab=expression(paste(italic("l"),"
(",theta,")")),main="Log Likelihood Function")

#----------
#   2(c)
#----------
print("2c")
maxiter = 200
err = 10^(-5)
theta = pi
for (t in 1:maxiter){
  oldtheta = theta
  lptheta = 0
  for (x in sample){
    lptheta = lptheta - sin(x-theta)/(1-cos(x-theta))
  }
  lpptheta = 0
  for (x in sample){
    lpptheta = lpptheta +  1/(cos(x-theta)-1)
  }
  h = -lptheta/lpptheta
  theta = theta + h
  experr = abs(theta - oldtheta)
  if (experr < err) {
    break
  }
}
biglтheta = 1
for (x in sample){
  biglтheta = biglтheta * ((1-cos(x-theta))/(2*pi))
}
print(sprintf("theta = %12.5e, L(theta) = %12.5e",theta,biglтheta))

#----------
```

```
#   2(d)
#----------
starting = c(-2.7,2.7)

print("2d")
maxiter = 200
err = 10^(-5)
for (theta0 in starting){
  theta = theta0
  for (t in 1:maxiter){
    oldtheta = theta
    lptheta = 0
    for (x in sample){
      lptheta = lptheta - sin(x-theta)/(1-cos(x-theta))
    }
    lpptheta = 0
    for (x in sample){
      lpptheta = lpptheta +  1/(cos(x-theta)-1)
    }
    h = -lptheta/lpptheta
    theta = theta + h
    experr = abs(theta - oldtheta)
    if (experr < err) {
      break
    }
  }
  biglтheta = 1
  for (x in sample){
    biglтheta = biglтheta * ((1-cos(x-theta))/(2*pi))
  }
  print(sprintf("theta = %12.5e, L(theta) = %12.5e",theta,biglтheta))
}
#----------
#   2(e)
#----------
starting = seq(-pi,pi,pi/100)
ending = starting*0
biglтheta = 0*ending+1

print("2e")
maxiter = 300
err = 10^(-5)
for (i in 1:length(starting)){
  ending[i] = starting[i]
  for (t in 1:maxiter){
    oldtheta = ending[i]
    lptheta = 0
    for (x in sample){
      lptheta = lptheta - sin(x-ending[i])/(1-cos(x-ending[i]))
    }
    lpptheta = 0
    for (x in sample){
      lpptheta = lpptheta +  1/(cos(x-ending[i])-1)
    }
    h = -lptheta/lpptheta
    ending[i] = ending[i] + h
    experr = abs(ending[i] - oldtheta)
    if (experr < err) {
      break
    }
  }
  for (x in sample){
    biglтheta[i] = biglтheta[i] * ((1-cos(x-ending[i]))/(2*pi))
  }
#  print(sprintf("i = %3d theta = %8.5f, L(theta) = %12.5e",i,ending[i],biglтheta[i]))
}

i = 1
while (i < length(starting)){
  for (j in i:length(starting)){
    if (sprintf("%8.5f",ending[j]) != sprintf("%8.5f",ending[j+1])){
      print(sprintf("a = %8.5f, b = %8.5f, theta = %8.5f, MLE =
%12.5e",starting[i],starting[j],ending[i],biglтheta[i]))
      i = j
      break
    }
  }
  i = i+1
```

```
}

rm(list=ls())
#==========
#    3
#==========
#----------
#   3(a)
#----------
print("3a")
x = c(0.02,0.06,0.11,0.22,0.56,1.10)
y1 = c(47,97,123,152,191,200)

xm = 1/x
ym = 1/y1

model = lm(formula=ym~xm)
theta1 = c(1/coef(model)[1],coef(model)[2]/coef(model)[1])
print(sprintf("theta_1 = %8.5f, theta_2 = %8.5f",theta1[1],theta1[2]))
xtheo = seq(min(x),max(x),0.01)
ytheo = theta1[1]*xtheo/(xtheo+theta1[2])
plot(x,y1,xlab="substrate concentration",ylab="velocity")
lines(xtheo,ytheo)

y2 = c(76,107,139,159,201,207)

xm = 1/x
ym = 1/y2

model = lm(formula=ym~xm)
theta2 = c(1/coef(model)[1],coef(model)[2]/coef(model)[1])
print(sprintf("theta_1 = %8.5f, theta_2 = %8.5f",theta2[1],theta2[2]))
xtheo = seq(min(x),max(x),0.01)
ytheo = theta2[1]*xtheo/(xtheo+theta2[2])
plot(x,y2,xlab="substrate concentration",ylab="velocity")
lines(xtheo,ytheo)

#----------
#   3(b)
#----------
print("3b")
theta = theta1
maxiter = 400
err = 10^(-5)
for (t in 1:maxiter){
  dRSSdtheta1 = 0
  for (i in 1:length(x)){
    dRSSdtheta1 = dRSSdtheta1 - 2*(y1[i]-theta[1]*x[i]/(x[i]+theta[2]))*(x[i]/(x[i]+theta[2]))
  }
  dRSSdtheta2 = 0
  for (i in 1:length(x)){
    dRSSdtheta2 = dRSSdtheta2 + 2*(y1[i]-theta[1]*x[i]/(x[i]+theta[2]))*(theta[1]*x[i]/(x[i]+theta[2])^2)
  }
  d2RSSdtheta12 = 0
  for (i in 1:length(x)){
    d2RSSdtheta12 = d2RSSdtheta12 + 2*(x[i]/(x[i]+theta[2]))^2
  }
  d2RSSdtheta22 = 0
  for (i in 1:length(x)){
    d2RSSdtheta22 = d2RSSdtheta22 + 2*((theta[1]^2*x[i]^2)/(x[i]+theta[2])^4-
(2*theta[1]*x[i]/(x[i]+theta[2])^3)*(y1[i]-theta[1]*x[i]/(x[i]+theta[2])))
  }
  d2RSSdtheta1dtheta2 = 0
  for (i in 1:length(x)){
    d2RSSdtheta1dtheta2 = d2RSSdtheta1dtheta2 - 2*(-
x[i]*y1[i]/(x[i]+theta[2])^2+2*theta[1]*x[i]^2/(x[i]+theta[2])^3)
  }
  gradRSS = c(dRSSdtheta1,dRSSdtheta2)
  hessRSS = matrix(c(d2RSSdtheta12,d2RSSdtheta1dtheta2,d2RSSdtheta1dtheta2,d2RSSdtheta22),nrow=2,ncol=2)
  theta = theta - solve(hessRSS) %*% gradRSS
}
print(sprintf("theta_1 = %8.5f, theta_2 = %8.5f",theta[1],theta[2]))
plot(x,y1,xlab="substrate concentration",ylab="velocity")
xtheo = seq(min(x),max(x),0.01)
ytheo = theta[1]*xtheo/(xtheo+theta[2])
lines(xtheo,ytheo)

theta = theta2
```

```
maxiter = 400
err = 10^(-5)
for (t in 1:maxiter){
  dRSSdtheta1 = 0
  for (i in 1:length(x)){
    dRSSdtheta1 = dRSSdtheta1 - 2*(y2[i]-theta[1]*x[i]/(x[i]+theta[2]))*(x[i]/(x[i]+theta[2]))
  }
  dRSSdtheta2 = 0
  for (i in 1:length(x)){
    dRSSdtheta2 = dRSSdtheta2 + 2*(y2[i]-theta[1]*x[i]/(x[i]+theta[2]))*(theta[1]*x[i]/(x[i]+theta[2])^2)
  }
  d2RSSdtheta12 = 0
  for (i in 1:length(x)){
    d2RSSdtheta12 = d2RSSdtheta12 + 2*(x[i]/(x[i]+theta[2]))^2
  }
  d2RSSdtheta22 = 0
  for (i in 1:length(x)){
    d2RSSdtheta22 = d2RSSdtheta22 + 2*((theta[1]^2*x[i]^2)/(x[i]+theta[2])^4-
(2*theta[1]*x[i]/(x[i]+theta[2])^3)*(y2[i]-theta[1]*x[i]/(x[i]+theta[2])))
  }
  d2RSSdtheta1dtheta2 = 0
  for (i in 1:length(x)){
    d2RSSdtheta1dtheta2 = d2RSSdtheta1dtheta2 - 2*(-
x[i]*y2[i]/(x[i]+theta[2])^2+2*theta[1]*x[i]^2/(x[i]+theta[2])^3)
  }
  gradRSS = c(dRSSdtheta1,dRSSdtheta2)
  hessRSS = matrix(c(d2RSSdtheta12,d2RSSdtheta1dtheta2,d2RSSdtheta1dtheta2,d2RSSdtheta22),nrow=2,ncol=2)
  theta = theta - solve(hessRSS) %*% gradRSS
}
print(sprintf("theta_1 = %8.5f, theta_2 = %8.5f",theta[1],theta[2]))
plot(x,y2,xlab="substrate concentration",ylab="velocity")
xtheo = seq(min(x),max(x),0.01)
ytheo = theta[1]*xtheo/(xtheo+theta[2])
lines(xtheo,ytheo)

#----------
#   3(c)
#----------
print("3c")
theta = theta1
maxiter = 600
err = 10^(-5)
for (t in 1:maxiter){
  dRSSdtheta1 = 0
  for (i in 1:length(x)){
    dRSSdtheta1 = dRSSdtheta1 + 2*(y1[i]-theta[1]*x[i]/(x[i]+theta[2]))*(x[i]/(x[i]+theta[2]))
  }
  dRSSdtheta2 = 0
  for (i in 1:length(x)){
    dRSSdtheta2 = dRSSdtheta2 - 2*(y1[i]-theta[1]*x[i]/(x[i]+theta[2]))*(theta[1]*x[i]/(x[i]+theta[2])^2)
  }
  gradRSS = c(dRSSdtheta1,dRSSdtheta2)
  alphat = 0.01
  theta = theta + alphat*gradRSS
}
print(sprintf("theta_1 = %8.5f, theta_2 = %8.5f",theta[1],theta[2]))
plot(x,y1,xlab="substrate concentration",ylab="velocity")
xtheo = seq(min(x),max(x),0.01)
ytheo = theta[1]*xtheo/(xtheo+theta[2])
lines(xtheo,ytheo)

theta = theta2
maxiter = 400
err = 10^(-5)
for (t in 1:maxiter){
  dRSSdtheta1 = 0
  for (i in 1:length(x)){
    dRSSdtheta1 = dRSSdtheta1 - 2*(y2[i]-theta[1]*x[i]/(x[i]+theta[2]))*(x[i]/(x[i]+theta[2]))
  }
  dRSSdtheta2 = 0
  for (i in 1:length(x)){
    dRSSdtheta2 = dRSSdtheta2 + 2*(y2[i]-theta[1]*x[i]/(x[i]+theta[2]))*(theta[1]*x[i]/(x[i]+theta[2])^2)
  }
  gradRSS = c(dRSSdtheta1,dRSSdtheta2)
  alphat = alphat = 0.01
  theta = theta - alphat*gradRSS
}
print(sprintf("theta_1 = %8.5f, theta_2 = %8.5f",theta[1],theta[2]))
```

```
plot(x,y2,xlab="substrate concentration",ylab="velocity")
xtheo = seq(min(x),max(x),0.01)
ytheo = theta[1]*xtheo/(xtheo+theta[2])
lines(xtheo,ytheo)

#----------
#   3(d)
#----------
print("3d")
theta = theta1
maxiter = 400
err = 10^(-5)
for (t in 1:maxiter){
  amat = matrix(data=0,nrow=length(x),ncol=2)
  for (i in 1:length(x)){
    amat[i,] = c(x[i]/(x[i]+theta[2]),-theta[1]*x[i]/(x[i]+theta[2])^2)
  }
  zvec = x*0
  for (i in 1:length(x)){
    zvec[i] = y1[i]-theta[1]*x[i]/(x[i]+theta[2])
  }
  theta = theta + solve(t(amat) %*% amat) %*% (t(amat) %*% zvec)
}
print(sprintf("theta_1 = %8.5f, theta_2 = %8.5f",theta[1],theta[2]))
plot(x,y1,xlab="substrate concentration",ylab="velocity")
xtheo = seq(min(x),max(x),0.01)
ytheo = theta[1]*xtheo/(xtheo+theta[2])
lines(xtheo,ytheo)

theta = theta2
maxiter = 400
err = 10^(-5)
for (t in 1:maxiter){
  amat = matrix(data=0,nrow=length(x),ncol=2)
  for (i in 1:length(x)){
    amat[i,] = c(x[i]/(x[i]+theta[2]),-theta[1]*x[i]/(x[i]+theta[2])^2)
  }
  zvec = x*0
  for (i in 1:length(x)){
    zvec[i] = y2[i]-theta[1]*x[i]/(x[i]+theta[2])
  }
  theta = theta + solve(t(amat) %*% amat) %*% (t(amat) %*% zvec)
}
print(sprintf("theta_1 = %8.5f, theta_2 = %8.5f",theta[1],theta[2]))
plot(x,y2,xlab="substrate concentration",ylab="velocity")
xtheo = seq(min(x),max(x),0.01)
ytheo = theta[1]*xtheo/(xtheo+theta[2])
lines(xtheo,ytheo)
```