1

1.a

$$\begin{aligned}
F(x) &= P\left(\hat{\theta} < x\right) \\
&= P\left(\max\left(X_1, \ldots, X_n\right)\right) \\
&= P\left(\left(X_1 < x\right) \cup \cdots \cup \left(X_n < x\right)\right) \\
&= P\left(X_1 < x\right) + \cdots + P\left(X_n < x\right) \\
&= \begin{cases} 0 & , \quad x < 0 \\ \left(\frac{x}{\theta}\right)^n & , \quad 0 \le x \le \theta \\ 1 & , \quad \theta < x \end{cases} \\
f(x) &= \frac{d}{dx} F(x) \\
&= \begin{cases} 0 & , \quad x < 0 \\ \frac{n}{\theta}\left(\frac{x}{\theta}\right)^{n-1} & , \quad 0 \le x \le \theta \\ 0 & , \quad \theta < x \end{cases}
\end{aligned}$$

1.b

$$\begin{aligned}
\mathrm{Var}_{F_\theta}\left(\hat{\theta}\right) &= \int_0^\theta x^2 \frac{n}{\theta}\left(\frac{x}{\theta}\right)^{n-1} dx - \left(\int_0^\theta x \frac{n}{\theta}\left(\frac{x}{\theta}\right)^{n-1} dx\right)^2 \\
&= \frac{n}{n+2} \frac{x^{n+2}}{\theta^n}\Bigg|_{x=0}^{x=\theta} - \left(\frac{n}{n+1} \frac{x^{n+1}}{\theta^n}\Bigg|_{x=0}^{x=\theta}\right)^2 \\
&= \frac{n}{n+2}\theta^2 - \left(\frac{n}{n+1}\theta\right)^2 \\
&= \theta^2 \frac{n}{(n+1)^2 (n+2)}
\end{aligned}$$

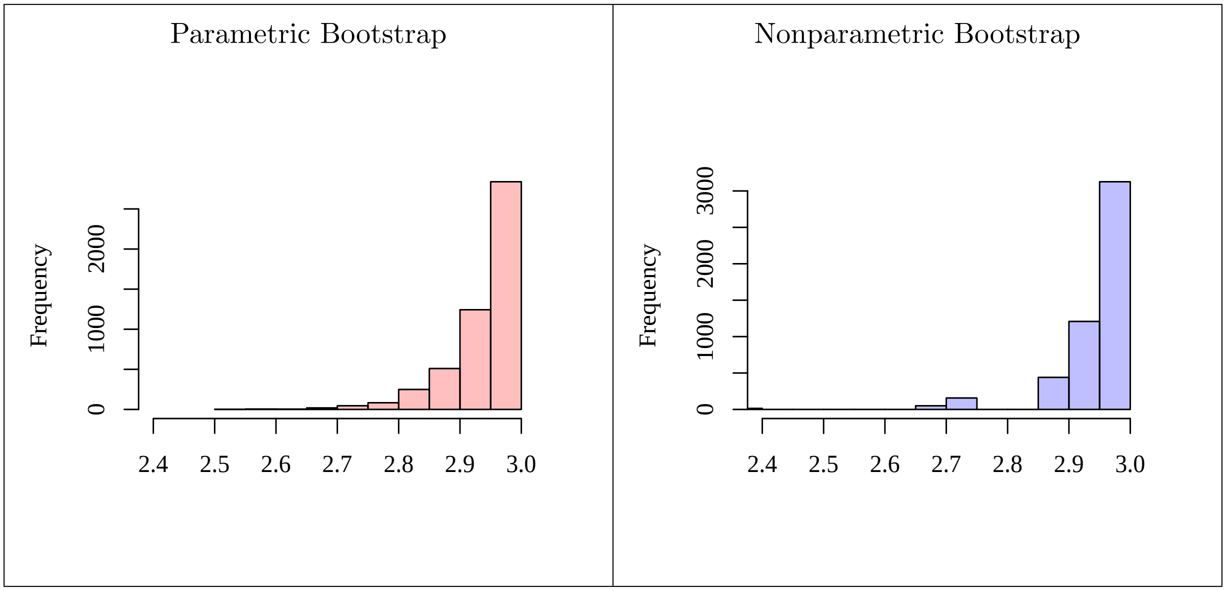
1.c

$$\begin{aligned}
\mathrm{Var}_{F_\theta}\left(\hat{\theta}\right) &= \theta^2 \frac{n}{(n+1)^2 (n+2)} \\
&\approx 0.003327123 \\
\mathrm{Var}_{F_\theta}\left(\hat{\theta}_B\right) &\approx 0.003458246
\end{aligned}$$

1.d

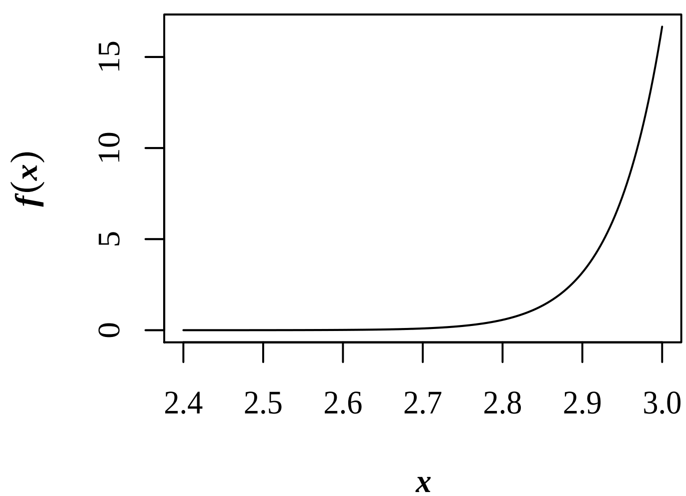
$$\mathrm{Var}_{F_\theta}\left(\hat{\theta}_B\right) \approx 0.004655322$$

1.e



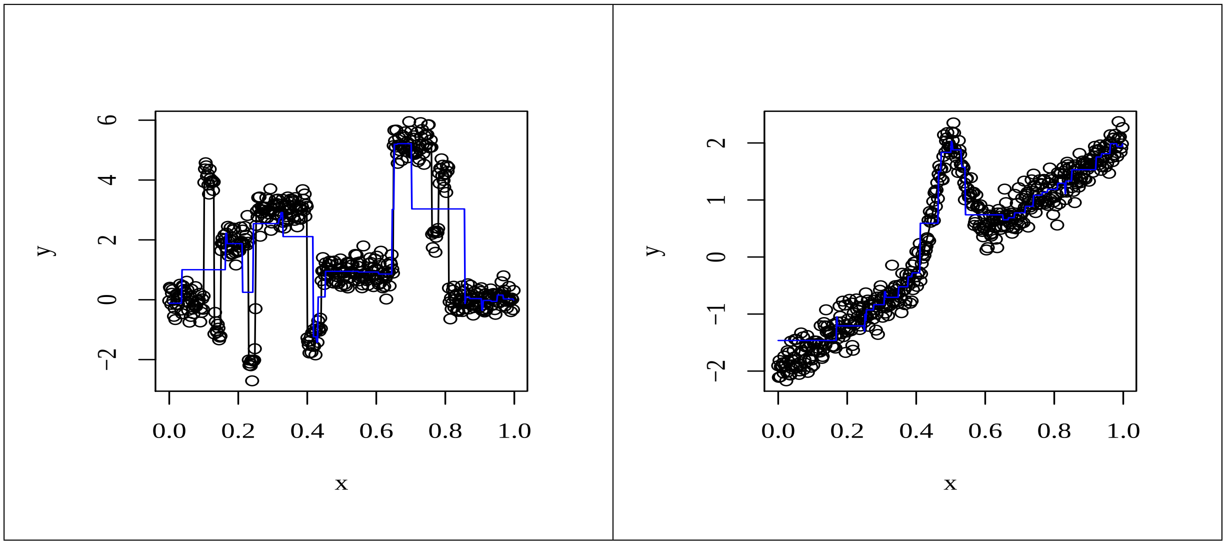
1.f

The distribution seems to matches up very well to the histograms in Part 1.e.

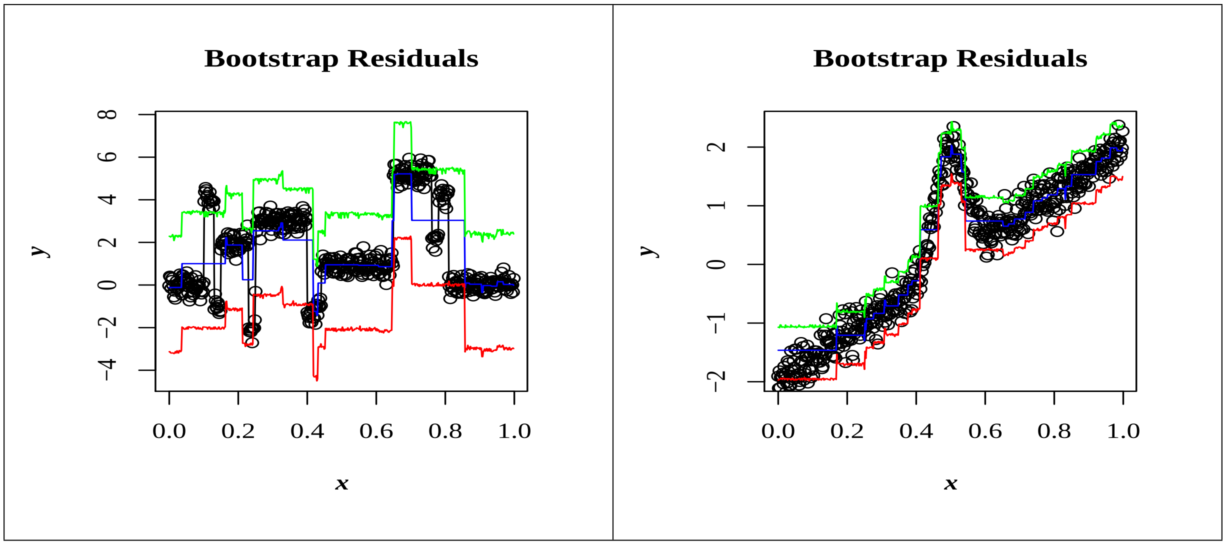


2

2.a



2.b



2.c

A confidence interval for the first jump, for example, could be found by finding B models then finding the mean of the standard deviation of the first jump. Then the 95% confidence interval would be the mean plus or minus two standard deviations.

3

3.a

3.b

3.c