

Project 5

Physics 250 Econophysics

In this project, the goal is to write a computer code that will compute historical volatilities for your 10 time series, and then to compute the correlation coefficient between the volatility and its original time series.

Note that the historical volatility for the SPX is *not* the VIX, although it is similar in some ways.

1. To compute the historical volatility we use the closing prices of the time series as for Project 3, $P(t_i)$ on trading day i . Then the historical volatility $V(t_i)$ on trading day i will be defined by:

$$V(t_i) = \sqrt{\frac{252}{21} \left(\sum_{j=1}^{21} \left(\text{Log} \left(\frac{P(t_{i-j})}{P(t_{i-j-1})} \right) \right)^2 \right)}$$

Make sure you read this formula carefully:

- You compute the ratio of the price on day i to the price on day $i-1$.
- Then compute the natural Log of this ratio.
- Then square it.
- Then repeat a.-c. with the ratio of day $i-1$ to day $i-2$ and add to the term from a.-c.
- Keep repeating this process until you have added up 21 terms over the last 21 trading days (counting today as the 1st day – thus today is $i=1$).
- Now divide by 21 to compute a ratio.
- “Annualize” the result by multiplying by 252 trading days = 1 year.
- Take the square root.

2. The correlation coefficient ρ_{xy} of 2 time series, $x(t), y(t)$, is defined as follows:

$$\sigma_x^2 = \frac{1}{(t_2 - t_1)} \int_{t_1}^{t_2} (x(t) - \langle x \rangle)^2 dt$$

$$\sigma_y^2 = \frac{1}{(t_2 - t_1)} \int_{t_1}^{t_2} (y(t) - \langle y \rangle)^2 dt$$

$$\rho_{xy} = \left(\frac{1}{\sigma_x \sigma_y} \right) \left[\frac{1}{(t_2 - t_1)} \int_{t_1}^{t_2} (x(t) - \langle x \rangle)(y(t) - \langle y \rangle) dt \right]$$

$$\langle x \rangle = \frac{1}{(t_2 - t_1)} \int_{t_1}^{t_2} x(t) dt, \quad \langle y \rangle = \frac{1}{(t_2 - t_1)} \int_{t_1}^{t_2} y(t) dt$$

Compute the correlation coefficient for 1995-today where possible, but in any case, since 2000.

