Homework 2

Due Date: August 13, 2019 by 5:00pm

MAT 022A Linear Algebra
Instructor: Mikhail Gaerlan

Instructions: Write solutions neatly and show all work. Box your solutions, and explain your answers if necessary. Upload a PDF file of your assignment to Gradescope. When uploading to Gradescope, make sure to select which pages correspond to which question.

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1. (Hill 1.6.23) Find all values of a for which the resulting linear system has (a) no solution, (b) a unique solution, and (c) infinitely many solutions.

$$\begin{aligned} x + \ \ y - \qquad\quad\ z &= 2 \\ x + 2y + \qquad\quad\ z &= 3 \\ x + \ \ y + (a^2 - 5)z &= a \end{aligned}$$

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2. (Hill 1.6.29) Solve the linear system with the given augmented matrix.

a. $\left[\begin{array}{cccc:c} 1 & 2 & 3 & 1 & 8 \\ 1 & 3 & 0 & 1 & 7 \\ 1 & 0 & 2 & 1 & 3 \end{array}\right]$

b. $\left[\begin{array}{ccc:c} 1 & -2 & 3 & 4 \\ 2 & -1 & -3 & 5 \\ 3 & 0 & 1 & 2 \\ 3 & -3 & 0 & 7 \end{array}\right]$

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3. (Hill 1.6.T.11) Let $\boldsymbol{u}$ and $\boldsymbol{v}$ be solutions to the homogeneous linear system $A\boldsymbol{x} = \boldsymbol{0}$.

a. Show that $\boldsymbol{u} + \boldsymbol{v}$ is a solution.

b. Show that $\boldsymbol{u} - \boldsymbol{v}$ is a solution.

c. For any scalar r, show that $r\boldsymbol{u}$ is a solution.

d. For any scalars r and s, show that $r\boldsymbol{u} + s\boldsymbol{v}$ is a solution.

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4. (Hill 1.6.T.12) Show that if $\boldsymbol{u}$ and $\boldsymbol{v}$ are solutions to the linear system $A\boldsymbol{x} = \boldsymbol{b}$, then $\boldsymbol{u} - \boldsymbol{v}$ is a solution to the associtaed homogeneous system $A\boldsymbol{x} = 0$.

5. (Strang 1.2.7) Find the angle θ (from its cosine) between these pairs of vectors:

a. $\boldsymbol{v} = \begin{bmatrix} 1 \\ \sqrt{3} \end{bmatrix}$ and $\boldsymbol{w} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

b. $\boldsymbol{v} = \begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix}$ and $\boldsymbol{w} = \begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix}$

c. $\boldsymbol{v} = \begin{bmatrix} 1 \\ \sqrt{3} \end{bmatrix}$ and $\boldsymbol{w} = \begin{bmatrix} -1 \\ \sqrt{3} \end{bmatrix}$

d. $\boldsymbol{v} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$ and $\boldsymbol{w} = \begin{bmatrix} -1 \\ -2 \end{bmatrix}$

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6. (Hill 6.1.10) Show that the space P of all polynomials is a vector space.

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7. (Strang 3.1.1) Suppose $(x_1, x_2) + (y_1, y_2)$ is defined to be $(x_1 + y_2, x_2 + y_1)$. With the usual multiplication $c\boldsymbol{x} = (cx_1, cx_2)$, which of the vector space conditions are not satisfied?

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8. (Hill 4.3.3) Which of the following are linear transformations?

a. $L(x, y, z) = (x + y, 0, 2x - z)$

b. $L\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x^2 - y^2 \\ x^2 + y^2 \end{bmatrix}$

c. $L(x, y) = (x - y, 0, 2x + 3)$

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9. (Strang 8.1.12) Suppose a linear T transforms $(1, 1)$ to $(2, 2)$ and $(2, 0)$ to $(0, 0)$. Find $T(\boldsymbol{v})$.

a. $\boldsymbol{v} = (2, 2)$

b. $\boldsymbol{v} = (3, 1)$

c. $\boldsymbol{v} = (-1, 1)$

d. $\boldsymbol{v} = (a, b)$

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10. (Hill 4.3.T.7) Let $I : \mathbb{R}^n \to \mathbb{R}^n$ be defined by $I(\boldsymbol{u}) = \boldsymbol{u}$, for $\boldsymbol{u}$ in $\mathbb{R}^n$. Show that I is a linear transformation, which is called the **identity operator** on $\mathbb{R}^n$.