

# Homework 1

Due Date: August 8, 2019 by 5:00pm

MAT 022A Linear Algebra

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**Instructions:** Write solutions neatly and show all work. Box your solutions, and explain your answers if necessary. Upload a PDF file of your assignment to Gradescope. When uploading to Gradescope, make sure to select which pages correspond to which question.

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1. (Hill 1.3.22) Consider the following linear system:

$$3x - y + 2z = 4$$

$$2x + y = 2$$

$$y + 3z = 7$$

$$4x - z = 4$$

- a. Find the coefficient matrix.
  - b. Write the linear system in matrix form.
  - c. Find the augmented matrix.
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2. (Hill 1.3.21) Write the linear system with augmented matrix

$$\left[ \begin{array}{cccc|c} -2 & -1 & 0 & 4 & 5 \\ -3 & 2 & 7 & 8 & 3 \\ 1 & 0 & 0 & 2 & 4 \\ 3 & 0 & 1 & 3 & 6 \end{array} \right].$$

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3. (Hill 1.3.11) Let

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 2 & -1 \\ -3 & 4 \end{bmatrix}.$$

Show that  $AB \neq BA$ .

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4. (Strang PS 2.3.16) Write these ancient problems in a 2 by 2 matrix form  $A\mathbf{x} = \mathbf{b}$  and solve them:
- a.  $X$  is twice as old as  $Y$  and their ages add to 33.
  - b.  $(x, y) = (2, 5)$  and  $(3, 7)$  lie on the line  $y = mx + c$ . Find  $m$  and  $c$ .

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5. (Strang PS 2.3.17) The parabola  $y = a + bx + cx^2$  goes through the points  $(x, y) = (1, 4)$  and  $(2, 8)$  and  $(3, 14)$ . Find and solve a matrix equation for the unknowns  $(a, b, c)$ .
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6. (Hill 1.7.7) Find the inverses of the given matrices, if possible.

a.  $\begin{bmatrix} 1 & 3 \\ -2 & 6 \end{bmatrix}$

b.  $\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & -1 & 2 \\ 1 & -1 & 2 & 1 \\ 1 & 3 & 3 & 2 \end{bmatrix}$

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7. (Hill 1.8.7) Find an LU-factorization of the coefficient matrix of the given linear system  $A\mathbf{x} = \mathbf{b}$ . Solve the linear system using a forward substitution followed by a back substitution.

$$A = \begin{bmatrix} 4 & 2 & 3 \\ 2 & 0 & 5 \\ 1 & 2 & 1 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 1 \\ -1 \\ -3 \end{bmatrix}$$