

Homework 1 Key

Solutions by:
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1. a. $\begin{bmatrix} 3 & -1 & 2 \\ 2 & 1 & 0 \\ 0 & 1 & 3 \\ 4 & 0 & -1 \end{bmatrix}$

b. $\begin{bmatrix} 3 & -1 & 2 \\ 2 & 1 & 0 \\ 0 & 1 & 3 \\ 4 & 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \\ 7 \\ 4 \end{bmatrix}$

c. $\begin{bmatrix} 3 & -1 & 2 & | & 4 \\ 2 & 1 & 0 & | & 2 \\ 0 & 1 & 3 & | & 7 \\ 4 & 0 & -1 & | & 4 \end{bmatrix}$

2.
$$\begin{aligned} -2x - y + 4w &= 5 \\ -3x + 2y + 7z + 8w &= 3 \\ x + 2w &= 4 \\ 3x + z + 3w &= 6 \end{aligned}$$

3. $AB = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ -3 & 4 \end{bmatrix} = \begin{bmatrix} 1(2) + 2(-3) & 1(-1) + 2(4) \\ 3(2) + 2(-3) & 3(-1) + 2(4) \end{bmatrix} = \begin{bmatrix} -4 & 7 \\ 0 & 5 \end{bmatrix}$

$$BA = \begin{bmatrix} 2 & -1 \\ -3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} 2(1) - 1(3) & 2(2) - 1(2) \\ -3(1) + 4(3) & -3(2) + 4(2) \end{bmatrix} = \begin{bmatrix} -1 & 2 \\ 9 & 2 \end{bmatrix}$$

$$AB = \begin{bmatrix} -4 & 7 \\ 0 & 5 \end{bmatrix} \neq \begin{bmatrix} -1 & 2 \\ 9 & 2 \end{bmatrix} = BA$$

4. a. $X = 2Y$ \sim $X - 2Y = 0$ \sim $\begin{bmatrix} 1 & -2 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} 0 \\ 33 \end{bmatrix}$

$$X + Y = 33$$

$$X + Y = 33$$

$$2Y + Y = 33$$

$$X = 2(11)$$

$$X = 11$$

$$3Y = 33$$

$$= 22$$

$$Y = 22$$

$$Y = 11$$

$$\begin{aligned} b. \quad 5 &= 2m + c \\ 7 &= 3m + c \end{aligned} \sim \begin{bmatrix} 2 & 1 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} m \\ c \end{bmatrix} = \begin{bmatrix} 5 \\ 7 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 & | & 5 \\ 3 & 1 & | & 7 \end{bmatrix} \xrightarrow{2R_2 - 3R_1 \rightarrow R_2} \begin{bmatrix} 2 & 1 & | & 5 \\ 0 & -1 & | & -1 \end{bmatrix} \xrightarrow{-R_2 \rightarrow R_2} \begin{bmatrix} 2 & 1 & | & 5 \\ 0 & 1 & | & 1 \end{bmatrix}$$

$$\xrightarrow{R_1 - R_2 \rightarrow R_1} \begin{bmatrix} 2 & 0 & | & 4 \\ 0 & 1 & | & 1 \end{bmatrix} \xrightarrow{\frac{1}{2}R_1 \rightarrow R_1} \begin{bmatrix} 1 & 0 & | & 2 \\ 0 & 1 & | & 1 \end{bmatrix} \quad \begin{array}{l} m=2 \\ c=1 \end{array}$$

$$\begin{aligned} 5. \quad 4 &= a + b + c \\ 8 &= a + 2b + 4c \\ 14 &= a + 3b + 9c \end{aligned} \sim \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 3 & 9 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 4 \\ 8 \\ 14 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 & | & 4 \\ 1 & 2 & 4 & | & 8 \\ 1 & 3 & 9 & | & 14 \end{bmatrix} \xrightarrow{R_2 - R_1 \rightarrow R_2} \begin{bmatrix} 1 & 1 & 1 & | & 4 \\ 0 & 1 & 3 & | & 4 \\ 0 & 2 & 8 & | & 10 \end{bmatrix} \xrightarrow{R_1 - R_2 \rightarrow R_1} \begin{bmatrix} 1 & 0 & -2 & | & 0 \\ 0 & 1 & 3 & | & 4 \\ 0 & 2 & 8 & | & 10 \end{bmatrix}$$

$$\xrightarrow{R_3 - 2R_2 \rightarrow R_3} \begin{bmatrix} 1 & 0 & -2 & | & 0 \\ 0 & 1 & 3 & | & 4 \\ 0 & 0 & 2 & | & 2 \end{bmatrix} \xrightarrow{\begin{array}{l} R_1 + R_3 \rightarrow R_1 \\ R_2 - \frac{3}{2}R_3 \rightarrow R_2 \\ \frac{1}{2}R_3 \rightarrow R_3 \end{array}} \begin{bmatrix} 1 & 0 & 0 & | & 2 \\ 0 & 1 & 0 & | & 1 \\ 0 & 0 & 1 & | & 1 \end{bmatrix} \Rightarrow \begin{array}{l} a=2 \\ b=1 \\ c=1 \end{array}$$

$$6.a. \quad \begin{bmatrix} 1 & 3 & | & 1 & 0 \\ -2 & 6 & | & 0 & 1 \end{bmatrix} \xrightarrow{R_2 + 2R_1 \rightarrow R_2} \begin{bmatrix} 1 & 3 & | & 1 & 0 \\ 0 & 12 & | & 2 & 1 \end{bmatrix} \xrightarrow{R_1 - \frac{1}{4}R_2 \rightarrow R_1} \begin{bmatrix} 1 & 0 & | & \frac{1}{2} & -\frac{1}{4} \\ 0 & 1 & | & \frac{1}{6} & \frac{1}{12} \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 \\ -2 & 6 \end{bmatrix}^{-1} = \begin{bmatrix} \frac{1}{2} & -\frac{1}{4} \\ \frac{1}{6} & \frac{1}{12} \end{bmatrix}$$

$$\begin{aligned}
 & 6. b. \left[\begin{array}{cccc|cccc} 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 2 & -1 & 2 & 0 & 1 & 0 & 0 \\ 1 & -1 & 2 & 1 & 0 & 0 & 1 & 0 \\ 1 & 3 & 3 & 2 & 0 & 0 & 0 & 1 \end{array} \right] \begin{array}{l} R_2 - R_1 \rightarrow R_2 \\ R_3 - R_1 \rightarrow R_3 \\ \sim \\ R_4 - R_1 \rightarrow R_4 \end{array} \left[\begin{array}{cccc|cccc} 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & -2 & 1 & -1 & 1 & 0 & 0 \\ 0 & -2 & 1 & 0 & -1 & 0 & 1 & 0 \\ 0 & 2 & 2 & 1 & -1 & 0 & 0 & 1 \end{array} \right] \\
 & \begin{array}{l} R_1 - R_2 \rightarrow R_1 \\ R_3 + 2R_2 \rightarrow R_3 \\ \sim \\ R_4 - 2R_2 \rightarrow R_4 \end{array} \left[\begin{array}{cccc|cccc} 1 & 0 & 3 & 0 & 2 & -1 & 0 & 0 \\ 0 & 1 & -2 & 1 & -1 & 1 & 0 & 0 \\ 0 & 0 & -3 & 2 & -3 & 2 & 1 & 0 \\ 0 & 0 & 6 & -1 & 1 & -2 & 0 & 1 \end{array} \right] \begin{array}{l} R_1 + R_3 \rightarrow R_1 \\ R_2 - \frac{2}{3}R_3 \rightarrow R_2 \\ \sim \\ -\frac{1}{3}R_3 \rightarrow R_3 \\ R_4 + 2R_3 \rightarrow R_4 \end{array} \\
 & \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 2 & -1 & 1 & 1 & 0 \\ 0 & 1 & 0 & -\frac{1}{3} & 1 & -\frac{1}{3} & -\frac{2}{3} & 0 \\ 0 & 0 & 1 & -\frac{2}{3} & 1 & -\frac{2}{3} & -\frac{1}{3} & 0 \\ 0 & 0 & 0 & 3 & -5 & 2 & 2 & 1 \end{array} \right] \begin{array}{l} R_1 - \frac{2}{3}R_4 \rightarrow R_1 \\ R_2 + \frac{1}{9}R_4 \rightarrow R_2 \\ \sim \\ R_3 + \frac{2}{9}R_4 \rightarrow R_3 \\ \frac{1}{3}R_4 \rightarrow R_4 \end{array} \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & \frac{7}{3} & \frac{1}{3} & -\frac{1}{3} & -\frac{2}{3} \\ 0 & 1 & 0 & 0 & \frac{4}{9} & -\frac{1}{9} & -\frac{4}{9} & \frac{1}{9} \\ 0 & 0 & 1 & 0 & -\frac{1}{9} & -\frac{2}{9} & \frac{1}{9} & \frac{2}{9} \\ 0 & 0 & 0 & 1 & -\frac{5}{3} & \frac{2}{3} & \frac{2}{3} & \frac{1}{3} \end{array} \right] \\
 & \left[\begin{array}{cccc|cccc} 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 2 & -1 & 2 & 0 & 1 & 0 & 0 \\ 1 & -1 & 2 & 1 & 0 & 0 & 1 & 0 \\ 1 & 3 & 3 & 2 & 0 & 0 & 0 & 1 \end{array} \right]^{-1} = \boxed{\begin{bmatrix} \frac{7}{3} & \frac{1}{3} & -\frac{1}{3} & -\frac{2}{3} \\ \frac{4}{9} & -\frac{1}{9} & -\frac{4}{9} & \frac{1}{9} \\ -\frac{1}{9} & -\frac{2}{9} & \frac{1}{9} & \frac{2}{9} \\ -\frac{5}{3} & \frac{2}{3} & \frac{2}{3} & \frac{1}{3} \end{bmatrix}}
 \end{aligned}$$

$$\begin{aligned}
 & 7. \left[\begin{array}{ccc} 4 & 2 & 3 \\ 2 & 0 & 5 \\ 1 & 2 & 1 \end{array} \right] \begin{array}{l} R_2 - \frac{1}{2}R_1 \rightarrow R_2 \\ R_3 - \frac{1}{4}R_1 \rightarrow R_3 \end{array} \left[\begin{array}{ccc} 1 & 0 & 0 \\ \frac{1}{2} & 1 & 0 \\ \frac{1}{4} & 0 & 1 \end{array} \right] \left[\begin{array}{ccc} 4 & 2 & 3 \\ 0 & -1 & \frac{7}{2} \\ 0 & \frac{3}{2} & \frac{1}{4} \end{array} \right] \\
 & R_3 + \frac{3}{2}R_2 \rightarrow R_3 \quad \boxed{\left[\begin{array}{ccc} 1 & 0 & 0 \\ \frac{1}{2} & 1 & 0 \\ \frac{1}{4} & -\frac{3}{2} & 1 \end{array} \right] \left[\begin{array}{ccc} 4 & 2 & 3 \\ 0 & -1 & \frac{7}{2} \\ 0 & 0 & \frac{11}{2} \end{array} \right]}
 \end{aligned}$$

$$\text{Forward } (L\vec{y} = \vec{b}) \quad x' = 1 \quad \frac{1}{2}(1) + y' = -1 \quad \frac{1}{4}(1) - \frac{3}{2}(-\frac{3}{2}) + z' = -3$$

$$y' = -\frac{3}{2}$$

$$z' = -\frac{11}{2}$$

$$\text{Back } (U\vec{x} = \vec{y}) \quad \frac{11}{2}z = -\frac{11}{2} \quad -y + \frac{7}{2}(-1) = -\frac{3}{2} \quad 4x + 2(-2) + 3(-1) = 1$$

$$z = -1$$

$$y = -2$$

$$x = 2$$

$$\boxed{\vec{x} = \begin{bmatrix} 2 \\ -2 \\ -1 \end{bmatrix}}$$