Mathematical Model

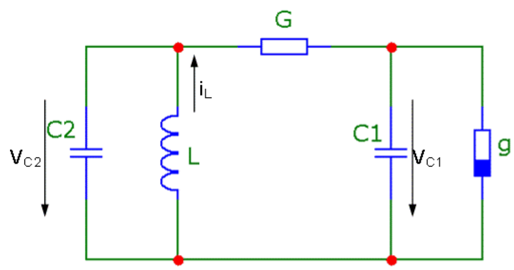

Figure 1: Chua circuit.

The system of equations for the circuit (as shown in the photo) can be obtained using Kirchhoff's first law and the expression for the voltage across the inductor:

$$\begin{cases} C_1 \frac{dv_{C_1}}{dt} = G\big(v_{C_2} - v_{C_1}\big) \; - \; g\big(v_{C_1}\big), \\ C_2 \frac{dv_{C_2}}{dt} = G\big(v_{C_1} - v_{C_2}\big) \; + \; i_L, \\ L \frac{di_L}{dt} = -\, v_{C_2}. \end{cases}$$

Here, v_{C_1} and v_{C_2} are the voltages across the capacitors C_1 and C_2, respectively, and i_L is the current through the inductor L. The function

$$g(v_{C_1}) \;=\; G_b\, v_{C_1} \;+\; \tfrac{1}{2}\,(G_a - G_b)\,\big(\,|v_{C_1} + E| \;-\; |v_{C_1} - E|\big)$$

is a piecewise-linear function describing the nonlinear element (Chua's diode). The slopes in its different linear segments are determined by the parameters G_a and G_b, and the breakpoints by $\pm E$.

To simplify analysis, one often introduces dimensionless variables:

$$\begin{cases} \frac{dx}{d\tau} = \alpha\,\big(y \;-\; x \;-\; h(x)\big), \\ \frac{dy}{d\tau} = x \;-\; y \;+\; z, \\ \frac{dz}{d\tau} = -\,\beta\, y, \end{cases}$$

where

$$h(x) \;=\; m_1\, x \;+\; \tfrac{1}{2}\,(m_0 - m_1)\,\big(\,|x+1| \;-\; |x-1|\big).$$

In these equations, α and β are dimensionless parameters, and m_0, m_1 correspond to the slopes of the piecewise-linear characteristic of the nonlinear element.

Additionally, the relationships among these parameters and variables can be expressed as:

$$m_0 = \frac{G_a}{G'}, \quad m_1 = \frac{G_b}{G'}, \quad \alpha = \frac{C_2}{C_1}, \quad \beta = \frac{C_2}{L\,G^2}, \quad \tau = \frac{t\,G}{C_2}, \quad x = \frac{v_{C_1}}{E}, \quad y = \frac{v_{C_2}}{E}, \quad z = \frac{i_L}{E\,G}.$$