Second Problem Assignment

EE603 - DSP and its applications

Assigned on: September 4, 2020 Due on: September 18, 2020

Notes:

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(1) Copying will be dealt with strictly. Institute disciplinary procedures will be invoked if any form of cheating is detected.

(2) All computer assignments should be solved using Python and submitted as a Colab notebook shared with the instructor and the TAs. The name of the file should be `ROLLNUMBER_HW2.ipynb`

(3) Questions on which it is not mentioned that Python has to be used: you can either solve it using paper and pen, or use Python, according to your choice. Correct answers will be accepted irrespective.

(4) All question responses must be in the same notebook and shared with the TA with edit permissions.

Problem 1

(10 points) Suppose that $h[n]$ is the impulse response of a discrete-time LTI system.

(i) What are the necessary and sufficient conditions on $h[n]$ for the system to be stable?

(ii) Consider the system $h[n] = (-1)^n u[n]$, where $u[n]$ is the discrete-time step sequence. Prove that this system is stable, or, alternately, provide a counter-example to show that this system is not stable.

Problem 2

(10 points) Consider a system described by the LCCDE

$$y[n] = x[n] + ay[n-1]$$

where a is a real number.

(i) Suppose that the system has an initial condition $y[-1] = b$. What is the output for $x[n] = 0$?

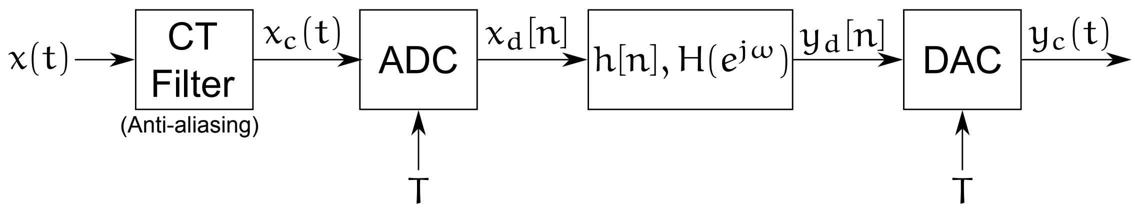
(ii) What is the impulse response of this system?

(iii) What is the solution for the input $x[n] = c\delta[n]$?

(iv) What are the conditions on a, b and c for the system to be a stable LTI system?

Problem 3

(10 points) Consider the system shown in the figure below:



The frequency response of the discrete-time LTI system between the ADC and DAC is given by

$$H_d(e^{j\omega}) = e^{-j\omega/3}, \quad |\omega| < \pi/3$$

(i) What is the effective continuous-time frequency response $H(f)$ of the overall system?

(ii) Which of the following is the most accurate statement?

- (a) $y_c(t) = \frac{d}{dt}x_c(t)$.
- (b) $y_c(t) = x_c(t - \frac{T}{3})$.
- (c) $y_c(t) = \frac{d}{dt}x_c(t - 3T)$.
- (d) $y_c(t) = x_c(t - \frac{1}{3})$.

(iii) Express $y_d[n]$ in terms of $y_c(t)$.

(iv) Determine the impulse response $h[n]$ of the discrete-time LTI system.

Problem 4

(10 points) The DTFT pair

$$a^n u[n] \Leftrightarrow \frac{1}{1 - ae^{-j\omega}}, \quad |a| < 1$$

is given.

(a) Using the above equation, determine the DTFT, $X(e^{j\omega})$, of the sequence

$$x[n] = -b^n u[-n-1] = \begin{cases} -b^n, & n \leqslant -1 \\ 0, & \geqslant 0. \end{cases}$$

What restriction on b is necessary for the DTFT of $x[n]$ to exist?

(b) Determine the sequence $y[n]$ whose DTFT is

$$Y(e^{j\omega}) = \frac{2e^{-j\omega}}{1+2e^{-j\omega}}.$$

Problem 5

(10 points) Consider the LTI system with frequency response

$$H(e^{j\omega}) = e^{-j[(\omega/2)+(\pi/4)]}, \qquad -\pi \leqslant \omega < \pi$$

Determine $y[n]$, the output of the system, if the input to the system is

$$x[n] = \cos\left(\frac{15\pi n}{4} - \frac{\pi}{3}\right)$$

Problem 6

(10 points) **Computer assignment**: In this exercise, you will approximate a low-pass filter and perform some operations on it.

(i) Construct an approximate low-pass filter $h[n]$ with cut-off frequency 0.2π by approximating an ideal low-pass filter using 101 coefficients. In NumPy, you can do it using: `import numpy as np n = np.arange(-50, 51) myfilter = np.sinc(0.2 * n);`

(ii) What is the gain of this filter at dc in dB? Hint: you can use `scipy.signal.freqz` and plot the result to figure this out.

(iii) How can you fix the gain for dc to be 0 dB? Provide the code for it here.

(iv) Construct the filter $(-1)^n h[n]$. Provide the magnitude and phase response plot of this filter. What type of filter is this?

(v) Now, construct the filter $\cos(0.5\pi n)h[n]$. Provide the magnitude and phase response plot of this filter. What type of filter is this?

(vi) Finally, construct the filter $\sin(0.1\pi n)h[n]/\pi n$. Provide the magnitude and phase response plot of this filter. What type of filter is this?

Problem 7

(10 points) This is much like the previous problem. Consider an ideal low-pass filter with impulse response $h_{lp}[n]$ and frequency response

$$H_{lp}(e^{j\omega}) = \begin{cases} 1, & |\omega| < 0.2\pi \\ 0, & 0.2\pi \leqslant |\omega| \leqslant \pi \end{cases}$$

(a) A new filter is designed by the equation $h_1[n] = (-1)^n h_{lp}[n] = e^{j\pi n} h_{lp}[n]$. Determine an equation for the frequency response of $H_1(e^{j\omega})$, and plot the equation for $|\omega| < \pi$. What kind of filter is this?

(b) A new filter is designed by the equation $h_2[n] = 2h_{lp}[n]\cos(0.5\pi n)$. Determine an equation for the frequency response of $H_2(e^{j\omega})$, and plot the equation for $|\omega| < \pi$. What kind of filter is this?

(c) A new filter is designed by the equation

$$h_3[n] = \frac{\sin(0.1\pi n)}{\pi n} h_{lp}[n]$$

Determine an equation for the frequency response of $H_3(e^{j\omega})$, and plot the equation for $|\omega| < \pi$. What kind of filter is this?

Problem 8

(10 points) Consider the following system for which the input $x[n]$ and output $y[n]$ satisfy the difference equation

$$y[n] - \frac{1}{2}y[n-1] = x[n]$$

and for which $y[-1]$ is constrained to be zero for every point. Determine whether or not the system is stable. If you conclude that system is stable, show your reasoning. If you conclude that the system is not stable, give an example of a bounded input that results in an unbounded output.

Problem 9

(10 points) A linear time-invariant system has frequency response

$$H(e^{j\omega}) = \begin{cases} e^{-j\omega 3}, |\omega| < \frac{2\pi}{16}\left(\frac{3}{2}\right) \\ 0, \frac{2\pi}{16}\left(\frac{3}{2}\right) \leqslant |\omega| \leqslant \pi \end{cases}$$

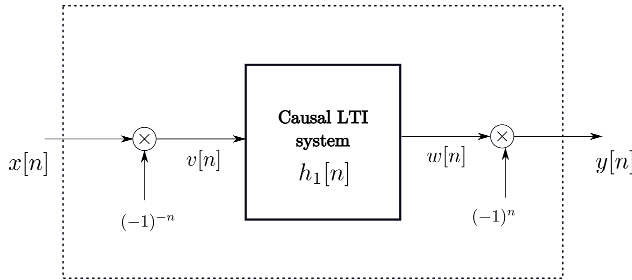
The input to the system is a periodic unit-impulse train with period $N = 16$ i.e

$$x[n] = \sum_{k=-\infty}^{\infty} \delta[n + 16k]$$

Find the output of the system.

Problem 10

(10 points) The overall system in the dotted box in the figure below can be shown to be linear and time invariant.



(a) Determine an expression for $H(e^{j\omega})$, the frequency response of the overall system from the input $x[n]$ to the output $y[n]$, in terms of $H_1(e^{j\omega})$, the frequency response of the internal LTI system. Remember that $(-1)^n = e^{j\omega n}$.

(b) Plot $H(e^{j\omega})$ for the case when the frequency response of the internal LTI system is

$$H_1(e^{j\omega}) = \begin{cases} 1, & |\omega| < \omega_c \\ 0, & \omega_c \leqslant |\omega| \leqslant \pi \end{cases}$$

Problem 11

(10 points) A commonly used numerical operation called the first backward difference is defined as

$$y[n] = \nabla(x[n]) = x[n] - x[n-1]$$

where $x[n]$ is the input and $y[n]$ is the output of the first-backward-difference system.

(a) Show that this system is linear and time invariant.

(b) Find the impulse response of the system.

(c) Find and sketch the frequency response (magnitude and phase)

(d) Show that if

$$x[n] = f[n] * g[n]$$

$$\nabla(x[n]) = \nabla(f[n]) * g[n] = f[n] * \nabla(g[n])$$

where $*$ denotes discrete convolution.

(e) Find the impulse response of a system that could be cascaded with the first-difference system to recover the input; i.e find $h_1[n]$ where

$$h_1[n] * \nabla(x[n]) = x[n]$$

Problem 12

(30 points) **Use Python!**
Consider the following continuous-time signal:

$$x(t) = \cos(2\pi t) + 2\sin(12\pi t)$$

The above signal is sampled with $f_s = 20$ Hz. You will work with this signal for values `n = np.arange(-100, 101)` for all the questions below.

(a) Plot $x[n]$.

(b) Plot the frequency and phase response of $x[n]$ (just use `scipy.signal.freqz`).

(c) Create a discrete-time low-pass filter with unity gain and cut-off frequency corresponding to 2 Hz by approximating an ideal low-pass filter, just like in Question 6. Plot the frequency and phase response.

(d) Create a discrete-time band-pass filter with pass-band between 5 Hz and 9 Hz by modulating the filter from the previous section. (Hint: multiply by $\cos\omega_0 n$ for appropriate ω_0. Plot the frequency response using `freqz`).

(e) Filter the $x[n]$ with the band-pass filter and plot the response vs. n, as well as the `freqz` output. What do you observe? Why?