Third Problem Assignment

EE603 - DSP and its applications

Assigned on: September 23, 2020 Due on: October 7, 2020

Notes:

(1) Copying will be dealt with strictly. Institute disciplinary procedures will be invoked if any form of cheating is detected.

(2) All computer assignments should be solved using Python and submitted as a Colab notebook shared with the instructor and the TAs. The name of the file should be `ROLLNUMBER_HW3.ipynb`

(3) Questions on which it is not mentioned that Python has to be used: you can either solve it using paper and pen, or use Python, according to your choice. Correct answers will be accepted irrespective.

(4) All question responses must be in the same notebook and shared with the TA with edit permissions.

PROBLEM 1

(10 points)

Let $h[n]$ and $H(z)$ denote the impulse response and system function of stable all-pass LTI system. Let $h_i[n]$ denote the impulse response of the (stable) LTI inverse system. Assume that $h[n]$ is real. Show that $h_i[n] = h[-n]$.

PROBLEM 2

(10 points) System S_1 has real impulse response $h_1[n]$ and real-valued frequency response $H_1(e^{j\omega})$.

(a) Does the impulse response $h_1[n]$ have any symmetry? Explain.

(b) System S_2 is a linear phase system with the same magnitude reponse as system S_1. What is the relationship between $h_2[n]$, the impulse response of the system S_2, and $h_1[n]$?

(c) Can a casual IIR filter have linear phase? Explain. If your answer is yes, provide an example. If not, prove that it is not possible for an IIR filter to have linear phase.

PROBLEM 3

(10 points) An LTI system has generalized linear phase and system function $H(z) = a + bz^{-1} + cz^{-2}$. The impulse response has unit energy, $a \geqslant 0$, and $H(e^{j\pi}) = H(e^{j0}) = 0$.

(a) Determine the impulse response $h[n]$.

(b) Plot $|H(e^{j\omega})|$.

PROBLEM 4

(10 points) A causal LTI has system has the system function

$$H(z) = \frac{(1 - 1.5z^{-1} - z^{-2})(1 + 0.9z^{-1})}{(1 - z^{-1})(1 + 0.7jz^{-1})(1 - 0.7jz^{-1})}$$

(a) Write down the difference equation that is satisfied by the input and output of the system.

(b) Plot the pole-zero diagram and indicate the ROC for the system function.

(c) Sketch $|H(e^{j\omega})|$.

(d) State whether the following are true or false about the system:

- (i) The system is stable.
- (ii) The impulse response approaches a constant for large n.
- (iii) The magnitude of the frequency response has a peak at approximately $\omega = \pm\pi$.
- (iv) The system has a stable and causal inverse.

PROBLEM 5

(10 points) Consider a discrete-time LTI filter whose impulse response $h[n]$ is nonzero only over five consecutive time samples; the filter's frequency response is $H(e^{j\omega})$. Let signals $x[n]$ and $y[n]$ denote the filter's input and output, respectively.

Moreover, you are given the following information about the filter.

(a)

$$\int_{-\pi}^{\pi} H(e^{j\omega})d\omega = 4\pi.$$

(b) There exists a signal $a[n]$ that has a real and even DTFT $A(e^{j\omega})$ given by

$$A(e^{j\omega}) = H(e^{j\omega})e^{j2\omega}.$$

(c)

$$A(e^{j0}) = 8, \quad \text{and} \quad A(e^{j\pi}) = 12.$$

Completely specify the impulse response $h[n]$. That is, specify the value of $h[n]$ for all n for which it is non-zero. Plot $h[n]$ carefully, labelling its salient features.

Problem 6

(10 points) This problem concerns a discrete-time filter with a real-valued impulse response $h[n]$. Determine whether the following statement is true or false:

Statement: If the group delay of the filter is constant for $0 < \omega < \pi$, then the impulse response must have the property that either

$$h[n] = h[M - n]$$

or

$$h[n] = -h[M - n],$$

where M is an integer.

If the statement is true, show why it is true. If it is false, provide a counter example.

Problem 7

(20 points) For this problem, you will use the `rhino.wav` file provided with the homework. You can use the `scipy.io.wavfile.*` functions to help you.

(a) Load the wave file into Python. Plot the samples. What is the sampling frequency?

(b) Plot the frequency content of the waveform versus the frequency (in Hertz) for any one channel and comment on the frequency content.

(c) Add Gaussian noise of variance 0.01 to each channel. You can generate Gaussian noise with variance 0.01 by using `0.01 * randn(N, 2)`, where `N` is the sequence length.

(d) Play the noisy waveform. What do you observe?

(e) Now, let's try filtering the noisy signal with a Butterworth filter. You can create a Butterworth filter using the `scipy.signal.butter` function. Create a sixth order Butterworth low-pass filter with cut-off corresponding to 1 kHz. The documentation will tell you how.

(f) Filter the noisy waveform with the Butterworth filter using the `filter` command. Play the sound again. What do you observe? Why?

Problem 8

(10 points) Look at the course website which has the example plot that shows the group delay when a Gaussian pulse is filtered using a rectangular window. Now, modify the example in the following manner:

(i) Construct an ideal low-pass filter with cut-off $\pi/2$, and truncate it to keep the middle 201 points (i.e., take $\mathrm{sin}(\pi/2n)/\pi n$ for $n \in \{-100, -99, ...99, 100\}$. Use this to filter the Gaussian pulse. Where is the new peak? How much is the delay?

(ii) Repeat for cut-off frequencies $\pi/4$ and $3\pi/4$. Where are the peaks?

(iii) Transform the filter considered in part (a) to a high-pass filter by multiplying the coefficients by $(-1)^n$. Use this to filter the Gaussian pulse. What do you observe?