First Problem Assignment

EE603 - DSP and its applications

Assigned on: August 27, 2020 Due on: September 4, 2020

Notes:

(1) Copying will be dealt with strictly. Institute disciplinary procedures will be invoked if any form of cheating is detected.

(2) All computer assignments should be solved using Python and submitted as a Colab notebook shared with the instructor and the TAs. The name of the file should be `ROLLNUMBER_HW1.ipynb`

(3) All question responses must be in the same sheet and shared with the TA with edit permissions.

Problem 1

(10 points) Consider the continuous-time signal $x(t) = \cos(200\pi t)$, where t is measured in seconds. Answer the following questions:

(a) Sketch the continuous-time Fourier transform of this signal, marking all the salient axis values.

(b) What is the minimum sampling rate needed for this signal to be recovered from its samples without any distortion, if uniform sampling is performed?

(c) This signal is sampled at 150 Hz, and then reconstructed using sinc interpolation using the reconstruction pulse $\text{sinc}(150t)$. What are the frequencies present in the reconstructed output?

(d) Continuing with the 150 Hz sampling and reconstruction approach, specify all frequencies f_0 such that, if we sample $\cos 2\pi f_0 t$ at 150 Hz and reconstruct using $\text{sinc}(150t)$, the same output frequencies as in (c) is obtained.

Problem 2

(10 points) An important operation in digital signals is *resampling*. This can be used to convert the sampling rate of a sampled signal without reconstruction. Consider the signal $x[n] = \delta[n]$. Perform the following operations for this signal using `scipy.signal.resample`:

(a) Represent $x[n]$ for $n \in \{-30, -29, ...29, 30\}$. Upsample by a factor of 2, and plot the resulting signal.

(b) Now, upsample by a factor of 10, and plot the resulting signal.

(c) Finally, upsample by a factor of 100, and plot the resulting signal.

Now, repeat the same operations above for the signal $\cos(0.2\pi n)$. What do you observe? Include all the three plots for this signal as well.

Problem 3

(10 points) Let $x(t) = \text{rect}(t)$, where t is measured in seconds. That is, $x(t) = 1$ for $t \in [-0.5, 0.5]$, and 0 otherwise. This signal is sampled at the following rates, and then reconstructed as $\hat{x}(t)$ using a corresponding sinc reconstruction filter:

(a) $f_s = 1$ Hz

(b) $f_s = 10$ Hz

(c) $f_s = 100$ Hz

Plot the reconstructed output $\hat{x}(t)$ for each case above, and compute the sum squared error $\int_{-\infty}^{\infty} |x(t) - \hat{x}(t)|^2 dt$ in each case (you can approximate this using the `numpy.trapz` function). What is your observation? Explain.