Fifth Problem Assignment

EE603 - DSP and its applications

Assigned on: November 6, 2020 Due on: November 20, 2020

Notes:

(1) Copying will be dealt with strictly. Institute disciplinary procedures will be invoked if any form of cheating is detected.

(2) All computer assignments should be solved using Python and submitted as a Colab notebook shared with the instructor and the TAs. The name of the file should be `ROLLNUMBER_HW5.ipynb`

(3) Questions on which it is not mentioned that Python has to be used: you can either solve it using paper and pen, or use Python, according to your choice. Correct answers will be accepted irrespective.

(4) All question responses must be in the same notebook and shared with the TA with edit permissions.

Problem 1

(5 points)

(a) Suppose

$$x[n] = 0, n < 0, n > (N-1).$$

is an N-point sequence having at least one nonzero sample. Is it possible for such a sequence to have a DTFT

$$X(e^{j2\pi k/M}) = 0, k = 0, 1, ..., M-1$$

where M is an integer greater than or equal to N? If your answer is yes, construct an example. If your answer is no, explain your reasoing.

(b) Suppose $M < N$. Repeat Part (a).

Problem 2

(5 points) Suppose $x_1[n]$ is an infinite-length, stable (i.e., absolutely summable) sequence with $z-$ transform given by

$$X_1(z) = \frac{1}{1 - \frac{1}{3}z^{-1}}.$$

Suppose $x_2[n]$ is a finite-length sequence of length N, and the N-point DFT of $x_2[n]$ is

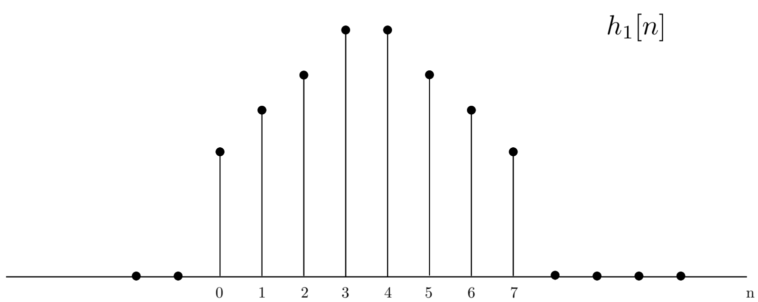
$$X_2[k] = X_1(z)|_{z=e^{j2\pi k/N}}, k = 0, 1, ..., N-1.$$

Determine $x_2[n]$.

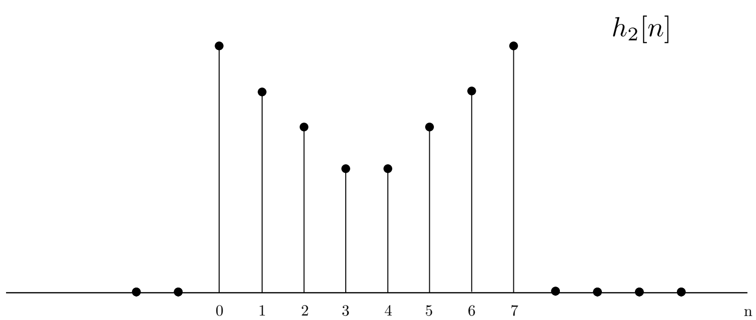
PROBLEM 3

(5 points) Two finite-duration sequences $h_1[n]$ and $h_2[n]$ of length 8 are sketched in the figure. The two sequences are related by a circular shift i.e, $h_1[n] = h_2[((n-m))_8$.

$$h_1[n] = h_2[((n-m))_8]$$



(a)



(b)

(a) Specify whether the magnitudes of the eight point DFTs are equal.

(b) We wish to implement a low pass FIR filter and must be either $h_1[n]$ or $h_2[n]$ as impulse response. Which one of the following statements is correct?

(i) $h_1[n]$ is a better lowpass filter than $h_2[n]$

(ii) $h_2[n]$ is a better lowpass filter than $h_1[n]$

(iii) The two sequences are both about equally good (or bad) as lowpass filters.

Problem 4

(5 points) Let $x_1[n]$ be a sequence obtained by expanding the sequence $x[n] = (\frac{1}{4})^n u[n]$ by a factor of 4 i.e,

$$x_1[n] = \begin{cases} x[n/4], & k = 0, \pm 4, \pm 8, \ldots \\ 0, & \text{otherwise.} \end{cases}$$

Find and sketch a six-point DFT $Q[k]$ that satisfies the two constraints

$$Q[0] = X_1(1), Q[3] = X_1(-1)$$

where $X_1(z)$ represents the z-transform of $x_1[n]$.

Problem 5

(5 points) Two finite-length sequences $x_1[n]$ and $x_2[n]$, which are zero outside the interval $0 \leqslant n \leqslant 99$ are circularly convolved to form a new sequence $y[n]$; i.e.,

$$y[n] = x_1[n] \; (100) \; x_2[n] = \sum_{k=0}^{99} x_1[k] x_2[((n-k))_{100}], \quad 0 \leqslant n \leqslant 99.$$

If, in fact, $x_1[n]$ is non-zero only for $10 \leqslant n \leqslant 39$, determine the set of values of n for which $y[n]$ is guaranteed to be identical to the *linear* convolution of $x_1[n]$ and $x_2[n]$.

Problem 6

(5 points) Consider two finite-length sequences $x[n]$ and $h[n]$ for which $x[n] = 0$ outside the interval $0 \leqslant n \leqslant 49$ and $h[n] = 0$ outside the interval $0 \leqslant n \leqslant 9$.

(a) What is the maximum possible number of non-zero values in the *linear* convolution of $x[n]$ and $h[n]$?

(b) The 50-point *circular* convolution of $x[n]$ and $h[n]$ is

$$x[n] \; (50) \; h[n] = 10, \quad 0 \leqslant n \leqslant 49.$$

The first 5 points of the *linear* convolution of $x[n]$ and $h[n]$ are

$$x[n] * h[n] = 5, \quad 0 \leqslant n \leqslant 4.$$

Determine as many points as possible of the linear convolution of $x[n] * h[n]$.

Problem 7

(5 points) A real values continuous-time segment of a signal $x_c(t)$ is sampled at a rate of 20,000 samples/sec, yielding a 1000-point finite-length discrete-time sequence $x[n]$ that is nonzero in the interval $0 \leqslant n \leqslant 999$. It is known that $x_c(t)$ is also bandlimited such that $X_c(j\Omega) = 0$ for $|\Omega| \geqslant 2\pi(10,000)$; i.e., assume that the sampling operation does not introduce any distortion due to aliasing.

$X[k]$ denotes the 1000-point DFT of $x[n]$. $X[800]$ is known to have the value $X[800] = 1 + j$.

(a) From the information given, can you determine $X[k]$ at any other values of k? If so, state which value(s) of k and what the corresponding value of $X[k]$ is. If not, explain why not.

(b) From the information given, state the value(s) of Ω for which $X_c(j\Omega)$ is known and the corresponding value(s) of $X_c(j\Omega)$.

Problem 8

(5 points)

A continuous-time signal $x_c(t) = \cos(\Omega_0 t)$ is sampled with period T to produce the sequence $x[n] = x_c(nT)$. An N-point rectangular window is applied to $x[n]$ for $0, 1, ..., N-1$, and $X[k]$, for $k = 0, 1, ..., N-1$, is the N-point DFT of resulting sequence.

(a) Assuming that Ω_0, N and k are fixed, how should T be chosen so that $X[k_0]$ and $X[N-k_0]$ are nonzero and $X[k] = 0$ for all other values of k?

(b) Is your answer unique? If not, give another value of T that satisfies the conditions of part (a).

Problem 9

(10 points) Consider the following signal

$$x[n] = \cos(0.2\pi n) + \sin(0.22\pi n) + 0.0005\sin(0.6\pi n);$$

Notice that two frequencies are very close, and the third one is very weak. We consider a windowed version of $x[n]$. For each value of window size N, we wish to observe the DFT and infer what properties of the signal can be obtained.

(a) For window size $N = 64$, plot the absolute value of the DFT of $x[n]$ in log scale versus k when windowed using the rectangular window, Hanning window and Hamming windows. Which windows allow you to see the “frequency split” near 0.2π? Which windows allow you to see the weak 0.6π frequency?

(b) Repeat for $N = 128, 256, 1024, 4096$. What do you observe in each case? Comment on the connection between N and the performance of each window.