Домашняя работа
Задача №1

а)
(1)
$$\begin{vmatrix} 1 & a & ba \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix} \overset{?}{=} 2 \cdot \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} (2)$$

(1)
$$\begin{vmatrix} 1 & a & ba \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix} = 1 \cdot b \cdot ab + 1 \cdot a \cdot ca + 1 \cdot ba \cdot c -$$
$$- (1 \cdot b \cdot ba + 1 \cdot a \cdot ab + 1 \cdot ca \cdot c) =$$

$$= ab^2 + a^2c + abc - ab^2 - a^2b - ac^2 =$$

$$= abc + a^2c - a^2b - ac^2$$

(2)
$$2 \cdot \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = 2(1 \cdot b \cdot c^2 + 1 \cdot a \cdot ca + 1 \cdot ba \cdot c -$$
$$(1 \cdot b \cdot a^2 + 1 \cdot a \cdot c^2 + 1 \cdot b^2 \cdot c)) =$$

$$= 2(bc^2 + a^2c + abc - a^2b - ac^2 - b^2c) =$$

$$= 2bc^2 + 2a^2c + 2abc - 2a^2b - 2ac^2 - 2b^2c$$

(1) - (2) $\overset{?}{=} 0$

$$abc + a^2c - a^2b - ac^2 + 2a^2b + 2ac^2 + 2b^2c - 2bc^2 - 2a^2c - 2abc =$$

$$= -abc - a^2c + a^2b + ac^2 + 2b^2c - 2bc^2 \neq 0$$

б)
$$\begin{vmatrix} a_1+b_1x & a_1x+b_1 & c_1 \\ a_2+b_2x & a_2x+b_2 & c_2 \\ a_3+b_3x & a_3x+b_3 & c_3 \end{vmatrix} \equiv$$

$$= (a_1+b_1x)(a_2x+b_2)\cdot c_3 + (a_3+b_3x)(a_1x+b_1)\cdot c_2 +$$
$$+ (a_2+b_2x)(a_3x+b_3)\cdot c_1 -$$
$$- (a_3+b_3x)(a_2x+b_2)\cdot c_1 - (a_2+b_2x)(a_1x+b_1)\cdot c_3 -$$
$$- (a_1+b_1x)(a_3x+b_3)\cdot c_2 =$$

$$= (a_1a_2x + a_1b_2 + a_2b_1x^2 + b_1b_2x)\cdot c_3 +$$
$$+ (a_1a_3x + a_3b_1 + a_1b_3x^2 + b_1b_3x)\cdot c_2 +$$
$$+ (a_2a_3x + a_2b_3 + a_3b_2x^2 + b_2b_3x)\cdot c_1 -$$
$$- (a_2a_3x + a_3b_2 + a_2b_3x^2 + b_2b_3x)\cdot c_1 -$$
$$- (a_1a_2x + a_2b_1 + a_1b_2x^2 + b_1b_2x)\cdot c_3 -$$
$$- (a_1a_3x + a_1b_3 + a_3b_1x^2 + b_1b_3x)\cdot c_2 =$$
$$= a_1a_2c_3x + a_1b_2c_3 + a_2b_1c_3x^2 + b_1b_2c_3x +$$
$$+ a_1a_3c_2x + a_3b_1c_2 + a_1b_3c_2x^2 + b_1b_3c_2x +$$
$$+ a_2a_3c_1x + a_2b_3c_1 + a_3b_2c_1x^2 + b_2b_3c_1x -$$
$$- a_2a_3c_1x - a_3b_2c_1 - a_2b_3c_1x^2 - b_2b_3c_1x -$$
$$- a_1a_2c_3x - a_2b_1c_3 - a_1b_2c_3x^2 - b_1b_2c_3x -$$
$$- a_1a_3c_2x - a_1b_3c_2 - a_3b_1c_2x^2 - b_1b_3c_2x =$$
$$= x(a_1a_2c_3 + b_1b_2c_3 + a_1a_3c_2 + b_1b_3c_2 + a_2a_3c_1 + b_2b_3c_1 +$$
$$- a_2a_3c_1 - b_2b_3c_1 - a_1a_2c_3 - b_1b_2c_3 - a_1a_3c_2 - b_1b_3c_2) +$$
$$+ (1-x^2)(a_1b_2c_3) + (1-x^2)(a_3b_1c_2) + (1-x^2)(a_2b_3c_1) +$$
$$+ (x^2-1)(a_3b_2c_1) + (x^2-1)(a_2b_1c_3) + (x^2-1)(a_1b_3c_2) =$$

$$= x\cdot 0 + (1-x^2)(a_1b_2c_3 + a_3b_1c_2 + a_2b_3c_1 - a_3b_2c_1 - a_2b_1c_3 -$$
$$- a_1b_2c_2). \quad (*)$$

$$(1-x^2) \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} =$$

$$= (a_1 b_2 c_3 + a_3 b_1 c_2 + a_2 b_3 c_1 - a_3 b_2 c_1 - a_2 b_1 c_3$$
$$- a_1 b_3 c_2)(1-x^2) \quad (**)$$

Получается: $(*) = (**)$ \qquad верно.

Задача №2

$c_1 c_3 -$

а)
$$\begin{vmatrix} (5-x) & 6 & -3 \\ -1 & -x & 1 \\ 1 & 2 & (1-x) \end{vmatrix} = (5-x)\cdot(-x)\cdot(1-x) + 1\cdot6\cdot1 + $$
$$+ 2\cdot(-1)\cdot(-3) - 1(-x)\cdot(-3) - $$
$$- (-1)\cdot6\cdot(1-x) - (5-x)(\cdot1)(2) = 0$$

$(x^2-5x)(1-x) + 6 + 6 - 3x + 6 - 6x - 10 + 2x = 0$

$(x^2 - x^3 - 5x + 5x^2) + 18 - 9x - 10 + 2x = 0$

$-x^3 + 6x^2 - 5x - 7x + 8 = 0$

$x^3 - 6x^2 + 12x - 8 = 0$

$x = 2$

$(x-2)(x^2-4x+4) = 0$

$(x-2)(x-2)^2 = 0$

$(x-2)^3 = 0$

$x = 2$

$$\begin{array}{r} -x^3 - 6x^2 + 12x - 8 \\ \underline{x^3 - 2x^2} \\ -4x^2 + 12x \\ \underline{-\,4x^2 + 8x} \\ -4x - 8 \\ \underline{-\,4x - 8} \\ 0 \end{array} \bigg| \begin{array}{l} x - 2 \\ \hline x^2 - 4x + 4 \end{array}$$

Ответ: $x = 2$

б)

$$\begin{vmatrix} (x-2) & 3 & -1 \\ -2 & (x-2) & -2 \\ -1 & -3 & (x-2) \end{vmatrix} = 0$$

$(x-2)^3 + (-1)\cdot 3 \cdot(-2) + (-2)\cdot(-1)\cdot(-3) - (-1)\cdot(x-2)\cdot(-1) -$

$- (-2)\cdot 3 \cdot(x-2) - (x-2)\cdot 1 \cdot(-2)\cdot(-3) = 0$

$(x-2)^3 + 6 - 6 - x + 2 + 6x - 12 - 6x + 12 = 0$

$(x-2)^3 - (x+2) = 0$

$(x-2)\big((x-2)^2 - 1\big) = 0$

$(x-2)(x-3)(x-1) = 0$

Ответ: $\left[\begin{array}{l} x = 2 \\ x = 3 \\ x = 1 \end{array}\right.$

Задача № 3

a)
$$\begin{vmatrix} 3 & 1 & -1 & 2 \\ -5 & 1 & 3 & -4 \\ 2 & 0 & 1 & -1 \\ 1 & -5 & 3 & -3 \end{vmatrix} =$$

$$= (-1)^{1+2} \cdot 1 \begin{vmatrix} -5 & 3 & -4 \\ 2 & 1 & -1 \\ 1 & 3 & -3 \end{vmatrix} + (-1)^{2+2} \cdot 1 \begin{vmatrix} 3 & -1 & 2 \\ 2 & 1 & -1 \\ 1 & 3 & -3 \end{vmatrix} +$$

$$+ (-1)^{2+3} \cdot 0 \begin{vmatrix} 3 & -1 & 2 \\ -5 & 3 & -4 \\ 1 & 3 & -3 \end{vmatrix} + (-1)^{2+4} \cdot (-5) \cdot \begin{vmatrix} 3 & -1 & 2 \\ -5 & 3 & -4 \\ 2 & 1 & -1 \end{vmatrix} =$$

$$= -(-5 \cdot 1 (-3) + (1 \cdot 3) \cdot (-1) + 2 \cdot (-4) \cdot 3 - 1 \cdot 1 \cdot (-4) - 2 \cdot (3)(-3) -$$
$$- (-5) \cdot (-1) \cdot 3) +$$
$$+ (3 \cdot 1 \cdot (-3) + 1 \cdot (-1) \cdot (-1) + 2 \cdot 2 \cdot 3 - 1 \cdot 1 \cdot 2 - 2 \cdot (-1) \cdot (-3) - 3 \cdot (-1) \cdot 3) +$$
$$+ (-5)(3 \cdot 3 \cdot (-1) + 2 \cdot (-1) \cdot (-4) + (-5) \cdot 2 \cdot 1 - 2 \cdot 3 \cdot 2 - (-5)(-1) \cdot (-1) -$$
$$- 3 \cdot (-4) \cdot 1) =$$
$$= -(15 - 3 - 24 + 4 + 18 - 15) + (-9 + 1 + 12 - 2 - 6 + 9) +$$
$$+ (-5)(9 + 8 - 10 - 12 + 5 + 12) =$$
$$= -(-5) + (5) + (-5)(-6) = 10 + 30 = \boxed{40}$$

в)

$$\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{vmatrix} 2 & -1 & 0 & 3 \\ -1 & 3 & 0 & -5 \\ 4 & -2 & 2 & -2 \\ 5 & 1 & 0 & 1 \end{vmatrix} = (-1)^{3+1} \cdot 0 \begin{vmatrix} -1 & 3 & -5 \\ 4 & -2 & -2 \\ 5 & 1 & 1 \end{vmatrix} +$$

$$+ (-1)^{2+3} \cdot 0 \cdot \begin{vmatrix} -1 & 3 & -5 \\ 4 & -2 & -2 \\ 5 & 1 & 1 \end{vmatrix} + (-1)^{3+3} \cdot 2 \begin{vmatrix} 2 & -1 & 3 \\ -1 & 3 & -5 \\ 5 & 1 & 1 \end{vmatrix}$$

$$= 2 \begin{vmatrix} 2 & -1 & 3 \\ -1 & 3 & -5 \\ 5 & 1 & 1 \end{vmatrix} =$$

$$= 2 (2 \cdot 3 \cdot 1 + 5 \cdot (-1) \cdot (-5) + (-1) \cdot 3 \cdot 1 - 5 \cdot 3 \cdot 3 - (-1)(-1) \cdot 1$$

$$- 2 \cdot (-5) \cdot 1) = 2 (6 + 25 - 3 - 45 - 1 + 10) =$$

$$= 2 (31 - 48 + 9) = 2(31 - 39) = -16$$

Ответ: -16

Задача № 4

а) $\begin{vmatrix} 2 & 1 & 1 & 1 & 1 \\ 1 & 3 & 1 & 1 & 1 \\ 1 & 1 & 4 & 1 & 1 \\ 1 & 1 & 1 & 5 & 1 \\ 1 & 1 & 1 & 1 & 6 \end{vmatrix} = - \begin{vmatrix} 1 & 1 & 1 & 1 & 6 \\ 1 & 3 & 1 & 1 & 1 \\ 1 & 1 & 4 & 1 & 1 \\ 1 & 1 & 1 & 5 & 1 \\ 2 & 1 & 1 & 1 & 1 \end{vmatrix} =$

$= \begin{vmatrix} 1 & 1 & 1 & 1 & 6 \\ 1 & 1 & 1 & 5 & 1 \\ 1 & 1 & 4 & 1 & 1 \\ 1 & 3 & 1 & 1 & 1 \\ 2 & 1 & 1 & 1 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 & 1 & 6 \\ 1 & 1 & 1 & 5 & 1 \\ 1 & 1 & 4 & 1 & 1 \\ 1 & 3 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & -5 \end{vmatrix} =$

$= \begin{vmatrix} 1 & 1 & 1 & 1 & 6 \\ 1 & 1 & 1 & 5 & 1 \\ 1 & 1 & 4 & 1 & 1 \\ 0 & 2 & 0 & 0 & -5 \\ 1 & 0 & 0 & 0 & -5 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 & 1 & 6 \\ 0 & 0 & 0 & 4 & -5 \\ 0 & 0 & 3 & 0 & -5 \\ 0 & 2 & 0 & 0 & -5 \\ 1 & 0 & 0 & 0 & -5 \end{vmatrix} =$

$= \begin{vmatrix} 1 & 1 & 1 & 1 & 6 \\ 0 & 0 & 0 & 4 & -5 \\ 0 & 0 & 3 & 0 & -5 \\ 0 & 2 & 0 & 0 & -5 \\ 0 & -1 & -1 & -1 & -11 \end{vmatrix} = - \begin{vmatrix} 1 & 1 & 1 & 1 & 6 \\ 0 & 2 & 0 & 0 & -5 \\ 0 & 0 & 3 & 0 & -5 \\ 0 & 0 & 0 & 4 & -5 \\ 0 & -1 & -1 & -1 & -11 \end{vmatrix} \Big) +\tfrac{1}{2}$

$=$

$-\left(\frac{22+5}{2}\right)$

$= - \begin{vmatrix} 1 & 1 & 1 & 1 & 6 \\ 0 & 2 & 0 & 0 & -5 \\ 0 & 0 & 3 & 0 & -5 \\ 0 & 0 & 0 & 4 & -5 \\ 0 & 0 & -1 & -1 & (-11-2,5) \end{vmatrix} \Big) +\tfrac{1}{3} = - \begin{vmatrix} 1 & 1 & 1 & 1 & 6 \\ 0 & 2 & 0 & 0 & -5 \\ 0 & 0 & 3 & 0 & -5 \\ 0 & 0 & 0 & 4 & -5 \\ 0 & 0 & 0 & -1 & \left(\frac{27}{2}-\frac{5}{3}\right) \end{vmatrix} \Big) +4 = - \begin{vmatrix} 1 & 1 & 1 & 1 & 6 \\ 0 & 2 & 0 & 0 & -5 \\ 0 & 0 & 3 & 0 & -5 \\ 0 & 0 & 0 & 4 & -5 \\ 0 & 0 & 0 & 0 & \left(-\frac{27}{2}\cdot\frac{5}{3}-\frac{5}{2}\right) \end{vmatrix} \cdot \frac{27}{2}$

$= -(1\cdot2\cdot3\cdot4)\cdot\left(-\frac{86}{6}-\frac{5}{4}\right) = 1\cdot2\cdot3\cdot4\cdot\frac{197}{12} = \frac{24\cdot197}{12} = 197\cdot2 = \underline{394}$

(left margin, partial)

$\begin{vmatrix} -1 & 3 \\ 3 & -5 \\ 1 & 1 \end{vmatrix} =$

$(-1)\cdot1 -$

$$\begin{vmatrix} 7 \\ -1 \\ 1 \\ 3 \\ 1 \end{vmatrix} =$$

$$\left.\begin{matrix} 8 \\ 1 \\ 2 \\ 3 \\ 1 \end{matrix}\right\} =$$

$=$

$=$

б) $\begin{vmatrix} 5 & 13 & 21 & 2 & 10 \\ 4 & 10 & 16 & 7 & 3 \\ 3 & 7 & 11 & 7 & 5 \\ 2 & 3 & 5 & -7 & 4 \\ 1 & 2 & 3 & 2 & 1 \end{vmatrix} = \begin{vmatrix} 5 & 4 & 3 & 2 & 1 \\ 13 & 10 & 7 & 3 & 2 \\ 21 & 16 & 11 & 5 & 3 \\ 2 & 7 & 7 & -7 & 2 \\ 10 & 3 & 5 & 4 & 1 \end{vmatrix}^{T} =$

$= \begin{vmatrix} 5 & 4 & 3 & 2 & 1 \\ 13 & 10 & 7 & 3 & 2 \\ 21 & 16 & 11 & 5 & 3 \\ 2 & 7 & 7 & -7 & 2 \\ 10 & 3 & 5 & 4 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 7 & 10 & 13 \\ 3 & 5 & 11 & 16 & 21 \\ 2 & -7 & 7 & 7 & 2 \\ 1 & 4 & 5 & 3 & 10 \end{vmatrix} \begin{smallmatrix} \cdot -2 \\ \cdot -3 \\ \cdot -2 \\ \cdot -1 \end{smallmatrix} =$

$= \begin{vmatrix} 1 & 2 & 3 & 4 & 5 \\ 0 & -1 & 1 & 2 & 3 \\ 0 & -1 & 2 & 4 & 6 \\ 0 & -11 & 1 & -1 & -8 \\ 0 & 2 & 2 & -1 & 5 \end{vmatrix} = \begin{vmatrix} 1 & 2 & 3 & 4 & 5 \\ 0 & -1 & 1 & 2 & 3 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & -11 & 1 & -1 & -8 \\ 0 & 2 & 2 & -1 & 5 \end{vmatrix} \cdot 11 =$

$= \begin{vmatrix} 1 & 2 & 3 & 4 & 5 \\ 0 & -11 & 11 & 22 & 33 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & -11 & 1 & -1 & -8 \\ 0 & 2 & 2 & -1 & 5 \end{vmatrix} = \begin{vmatrix} 1 & 2 & 3 & 4 & 5 \\ 0 & -1 & 1 & 2 & 3 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & -10 & -23 & -4 \\ 0 & 0 & 4 & 3 & 11 \end{vmatrix} \begin{smallmatrix} \cdot 2 \\ \cdot 10 \\ \cdot -4 \end{smallmatrix} =$

$= \begin{vmatrix} 1 & 2 & 3 & 4 & 5 \\ 0 & -1 & 1 & 2 & 3 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 3 & -11 \\ 0 & 0 & 0 & -5 & -1 \end{vmatrix} = \begin{vmatrix} 1 & 2 & 3 & 4 & 5 \\ 0 & -1 & 1 & 2 & 3 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 3 & -11 \\ 0 & 0 & 0 & -2 & -1 \end{vmatrix} =$

$$= \begin{vmatrix} 1 & 2 & 3 & 4 & 5 \\ 0 & -1 & 1 & 2 & 3 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 3 & -11 \\ 0 & 0 & 0 & 0 & -\frac{22}{3} - \frac{3}{3} \end{vmatrix} = \begin{vmatrix} 1 & 2 & 3 & 4 & 5 \\ 0 & -1 & 1 & 2 & 3 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 3 & -11 \\ 0 & 0 & 0 & 0 & -\frac{25}{3} \end{vmatrix} =$$

$$= 1 \cdot (-1) \cdot 1 \cdot 3 \cdot \left(-\frac{25}{3}\right) = 25$$

Ответ: 25.