

What Is Digital

Cinematography begins with light.

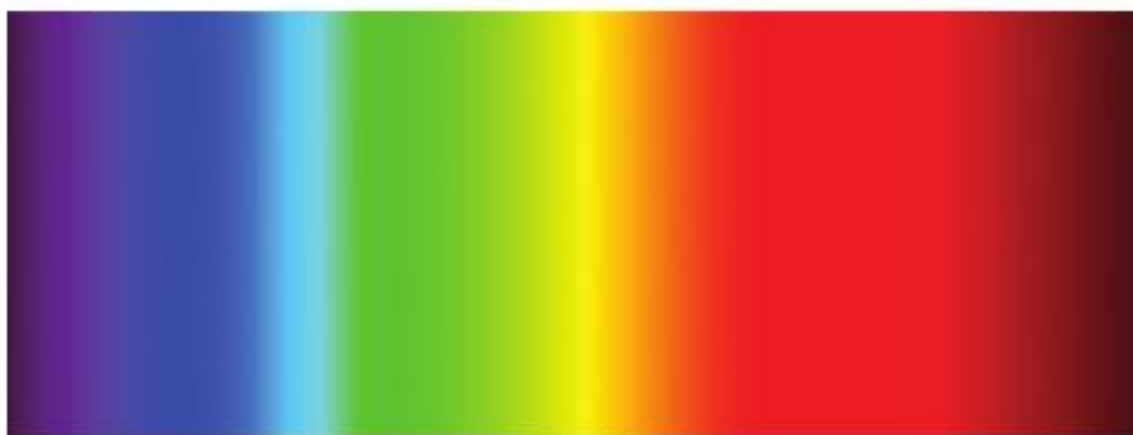


Figure 1.1 The spectrum of visible color.

Cinematography is the art of manipulating, capturing and recording motion pictures on a medium such as film, or in the case of digital cinematography, on an image sensor such as a charge-coupled device (CCD) or complementary metal oxide semiconductor (CMOS) chip set.



Figure 1.2 Charlie Chaplin at his hand-cranked Bell and Howell model 2709 camera.

In order to understand how digital photography works, it is important to understand how visible light gets separated by narrow band color filters into the three primary colors (red, green and blue) that we use to reproduce images.

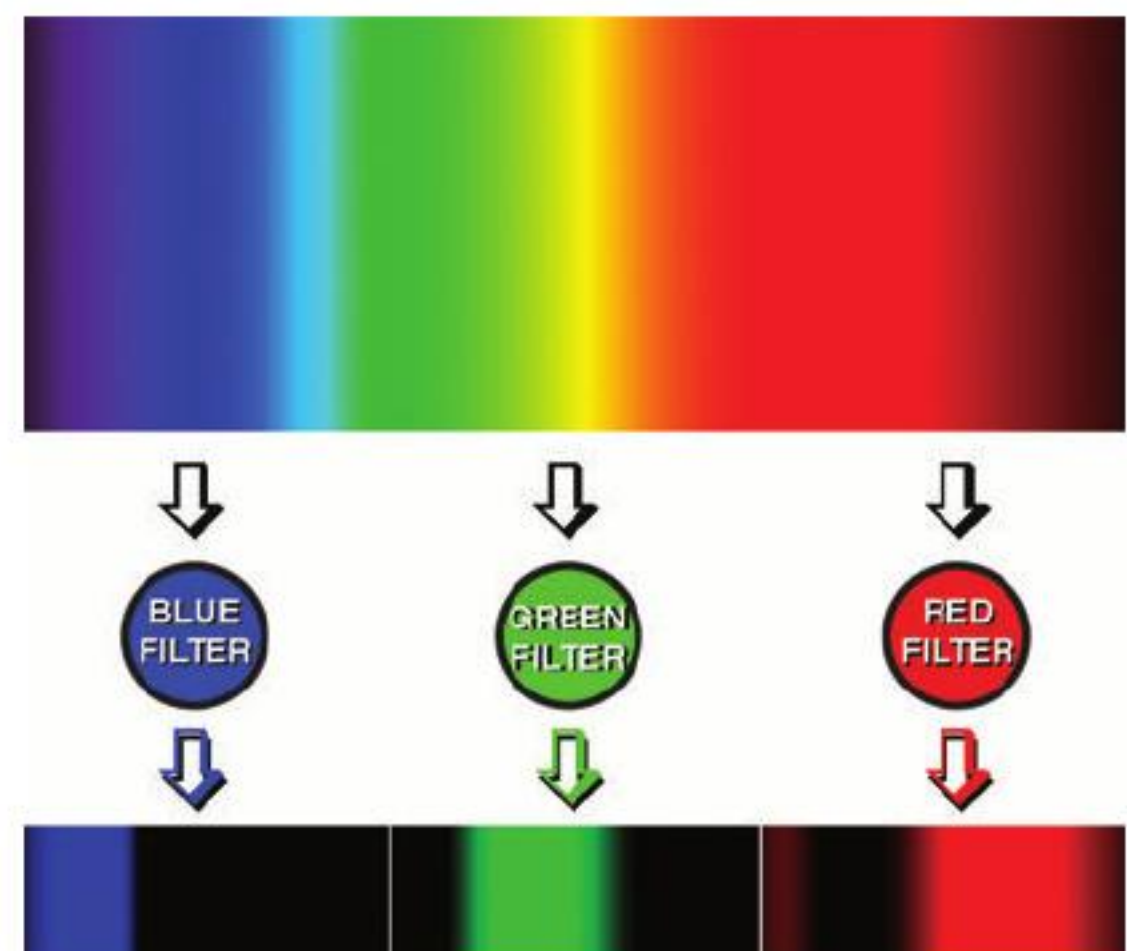


Figure 1.3 Visible light divided into red, green, and blue (RGB) components.

In a modern 3-chip CCD camera, light of various color wavelengths is directed to three individual 1920×1080 photosite monochrome red-only, green-only, and blue-only sensors by a system of filters and prisms.

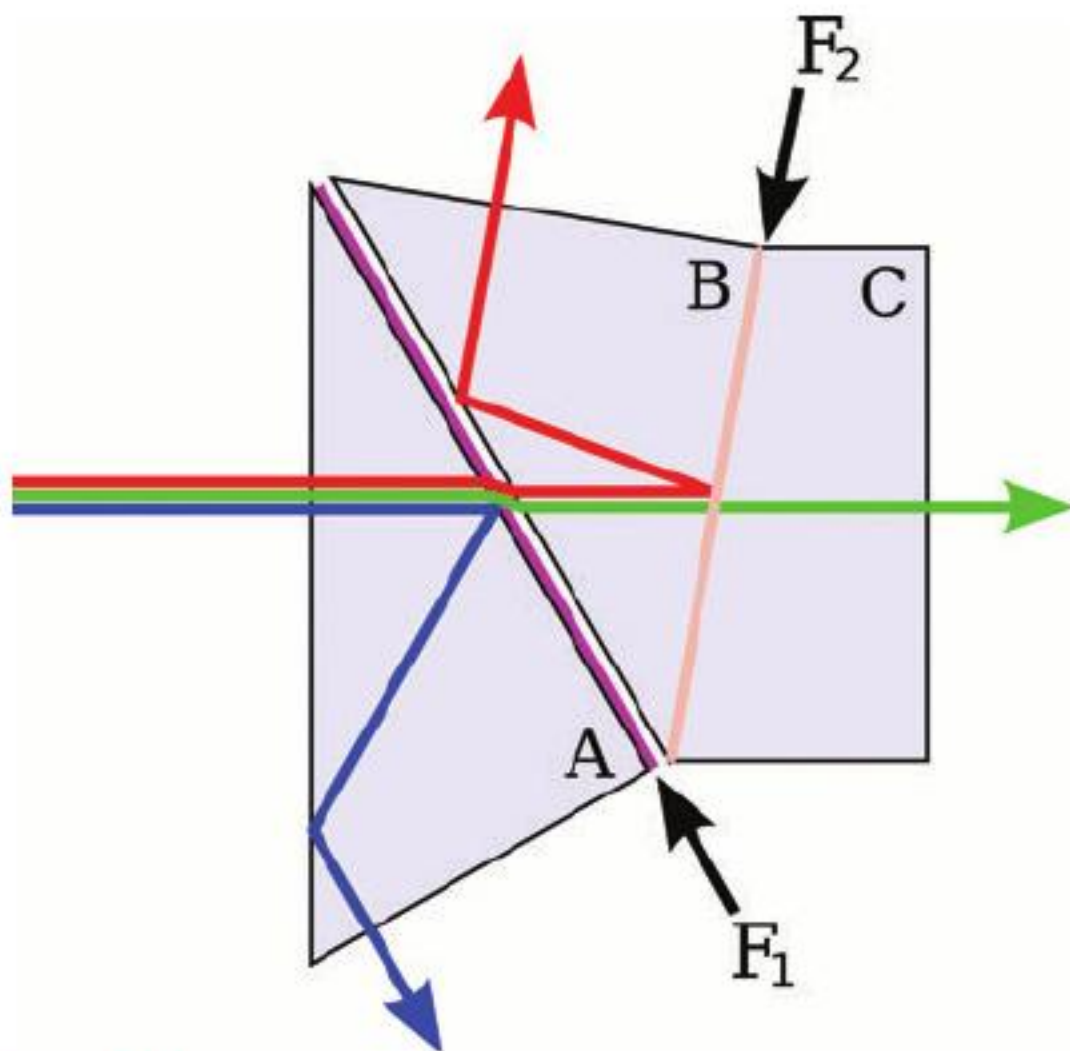


Figure 1.4 How an image is separated into red, green and blue components. A blue sensor (A), a red sensor (B), and a green sensor (C) collect light directed to them by dichroic prism surfaces F1 and F2.

Each of these three sensors collects photons from all its photosites to create three individual pictures of the scene: red only, green only, and blue only.

Three-chip cameras are very efficient collectors of light from the scene they record. Relatively speaking, not much of the light from the scene is wasted, because the photosites are said to be co-sited; they have the ability to sample light in all three color wavelengths from the same apparent place by using a semitransparent dichroic prism system.

Every photosite functions like a microscopic light meter; more photons entering and collecting in a light well generate a higher voltage out, whereas fewer photons entering generates a lower voltage out.

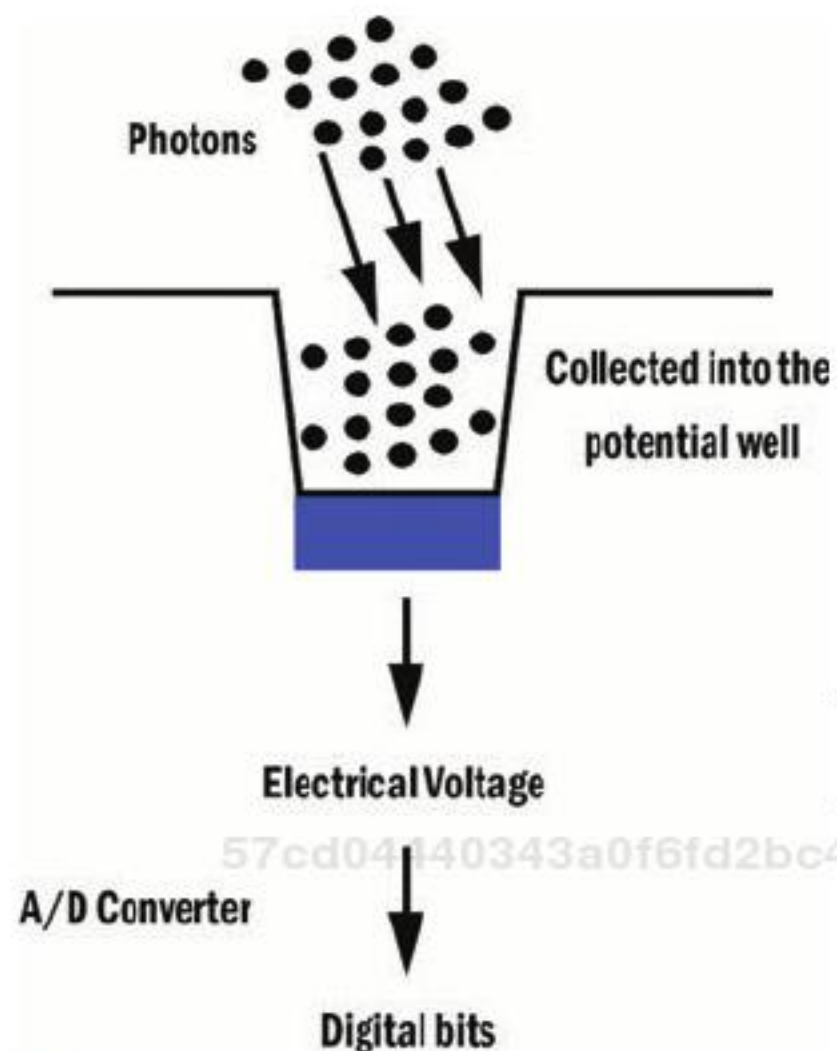


Figure 1.5 Photosites are photon-collection buckets that turn light into voltages.

Thousands of these photon-collecting buckets work like microscopic light meters, giving light readings on a pixel-for-pixel, frame-for-frame basis.



Figure 1.6 Photosites turn light into voltages.

For every frame, on a frame-by-frame basis, each of the three color sensors generates and measures the individual voltage from each discrete photosite commensurate with the number of photons that arrived at that photosite.

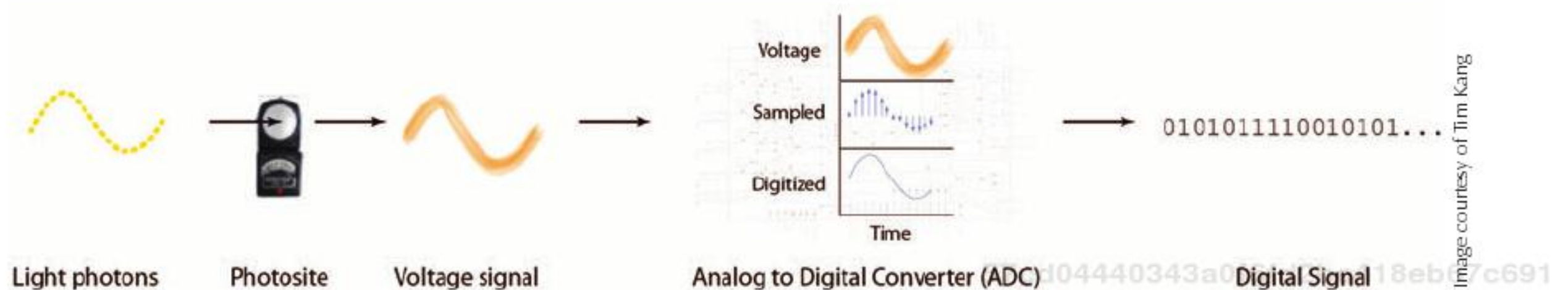


Figure 1.7 The process of analog to digital.

Those voltages are sampled at very a high frequency and converted to digital code values in an analog to digital (A-to-D) sampling processor.

In single-chip (monoplanar) sensor cameras, light is directed to a grid of adjacent individual photosites that are optically filtered by microscopic red, green, and blue (RGB) filters at each site. Each photosite captures light from only one of the primary colors while rejecting light from the other two primary colors.

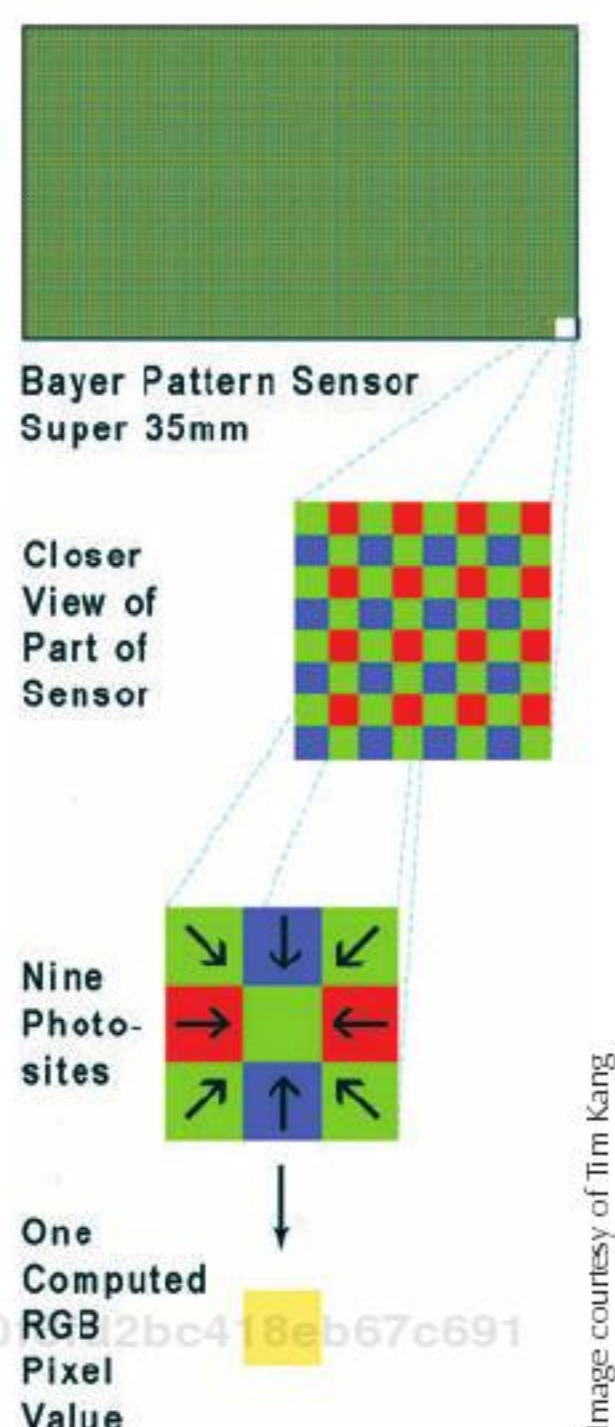


Figure 1.8 Bayer pattern color filter array.

Much of the light (and, therefore, color information) arriving at such a sensor is discarded, rejected by the color filtration scheme, and RGB pixels must be created by combining samples from adjacent, non-co-sited photosites.

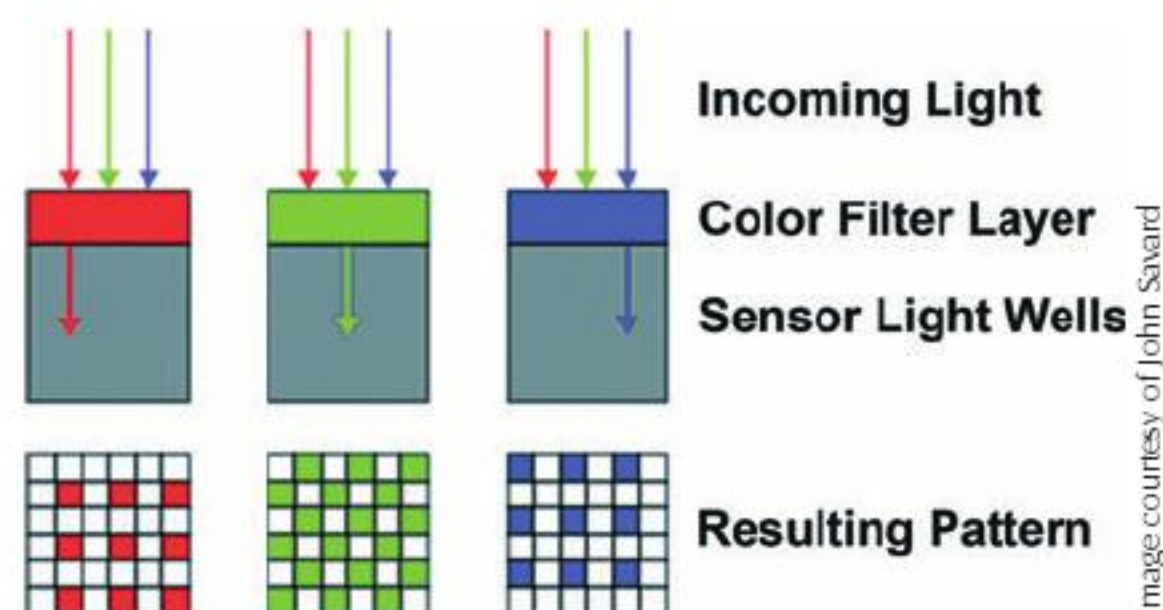


Figure 1.9 Filtered red, green, and blue light landing on non-co-sited photosites.

Those photosites are arranged in one of numerous possible patterns according to the dictates of the hardware manufacturer (see Figure 1.10). The light falling on such an array of photosites is largely wasted. A green photosite can only collect the green light that falls on it; red and blue light are rejected. A red photosite can only collect the red light that falls on it; green and blue are rejected. A blue photosite can only collect the blue light that falls on it, rejecting red and green light. The inefficiency of such systems can be fairly easily intuited.

What Are Pixels?

The word *pixel* is a contraction of *pix* ("picture") and *el* (for "element").

A pixel is the smallest addressable full-color (RGB) element in a digital imaging device. The address of a pixel corresponds to its physical coordinates on a sensor or screen.

Pixels are full-color samples of an original image. More pixels provide a more accurate presentation of the original image. The color and tonal intensity of a pixel are variable. In digital motion picture cinematography systems, a color is typically represented by three component intensities of red, green, and blue.

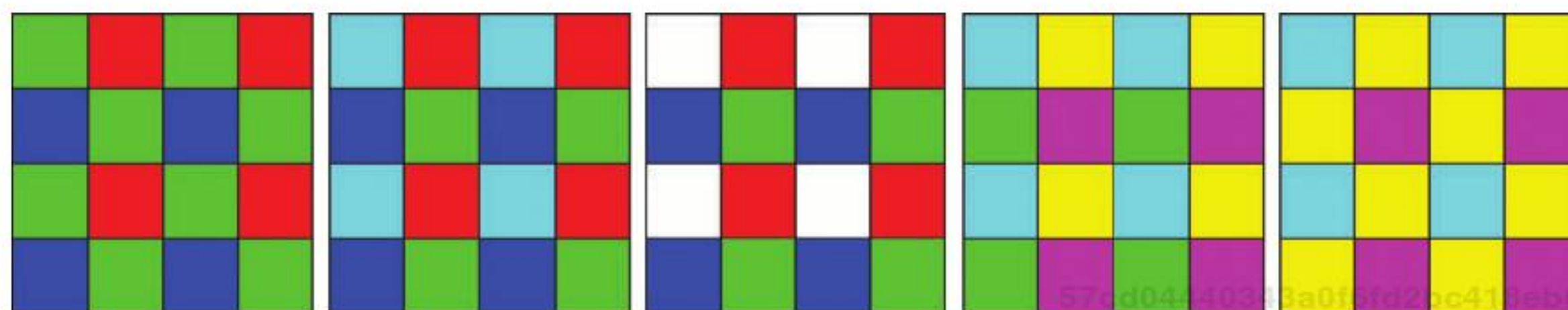


Figure 1.10 A variety of color filter patterns.



Figure 1.11 Only pixels contain RGB (red, green, and blue) information.

Photosites Are *Not* Pixels!

This is one of the most important distinctions we can make when talking about digital cinema cameras!

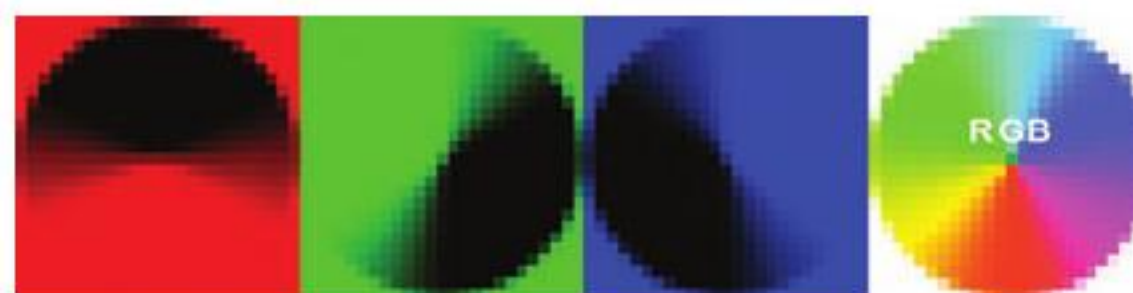


Figure 1.12 Red, green, and blue photosites combine to create full color RGB pixels.

Photosites (or sensels as they are referred to in camera sensor design) can only carry information about one color. A photosite can only be red *or* green *or* blue.

Photosites must be combined to make pixels. Pixels carry tricolor RGB information.

Analog to Digital

Quantization is the process of converting continuously varying analog voltages into a series of numerical values called samples. A sample is a numerical code value that represents a waveform's amplitude at a specific moment in time. Because digital samples are numbers, they can be stored in a computer's memory and saved to a computer hard drive as data.

**Signal
(bits)**

57cd04440343a0f6fd2bc418eb67c691
ebruary

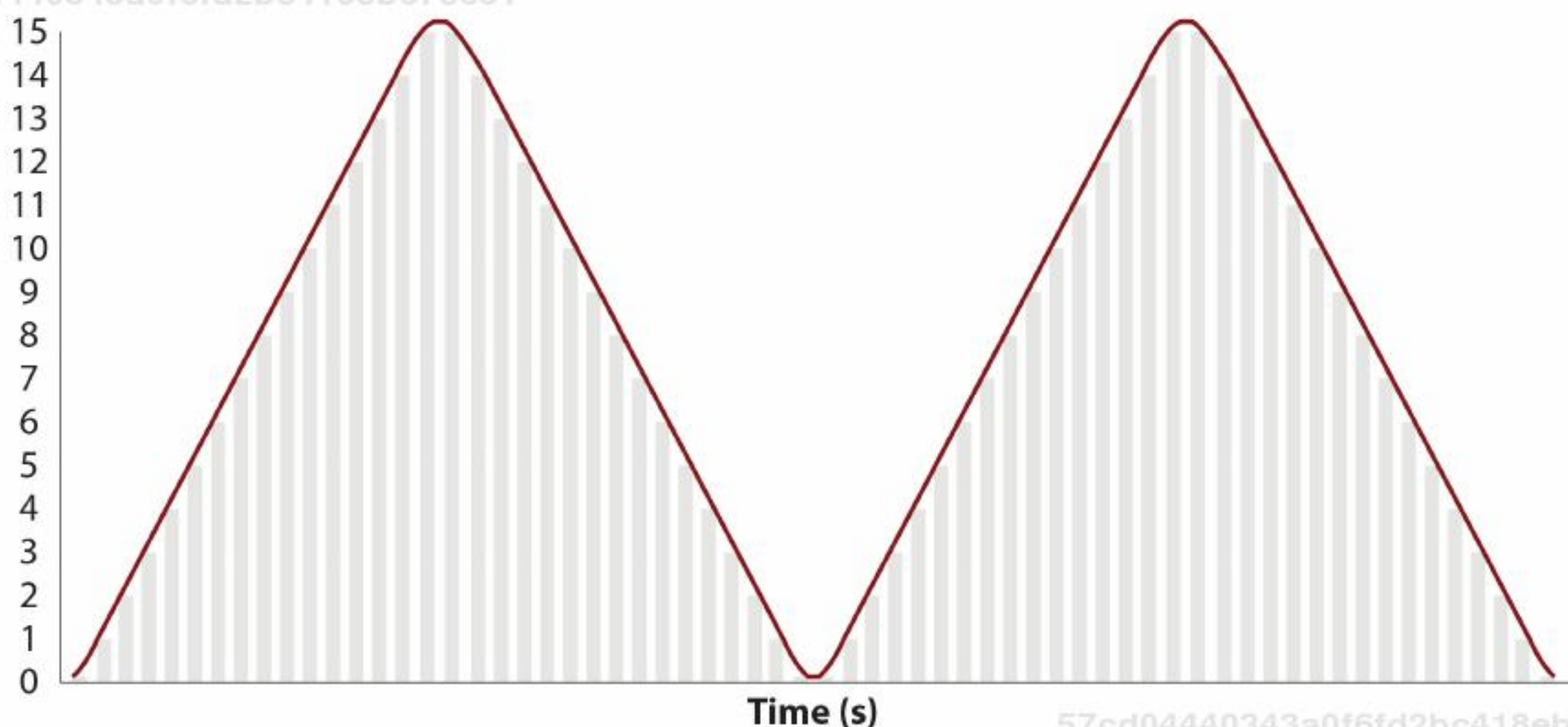


Image courtesy of Tim Kang

Figure 1.13 Voltages are quantized over time to create digital code values.

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ebruary

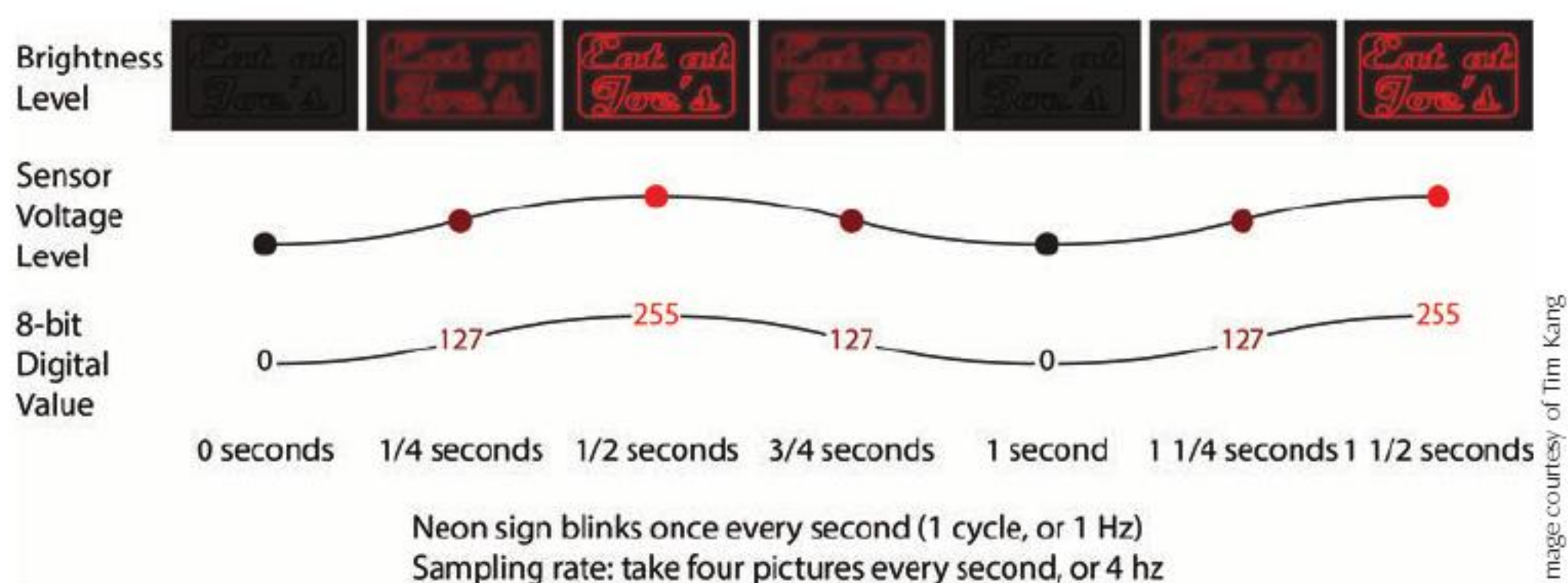


Figure 1.14 Sampling over time.

Sampling precision is the maximum number of digits (bits) each sample can represent. The sampling precision determines how accurately the amplitudes in the original waveform can be recreated.

Let's draw a simplified example of how waves are formed and quantified in the real world. Suppose we have a neon sign—"Eat at Joe's"—that blinks on and off with a frequency one cycle per second and we sample that picture at 4 times per second (see Figure 1.14).

The sensor reacts by generating voltages proportional to the number of photons arriving from the sign. The voltages from sensors are then processed by an analog-to-digital converter (ADC), a device that periodically samples the input voltage at a specified number of times per second and then creates a numeric code value from that voltage.

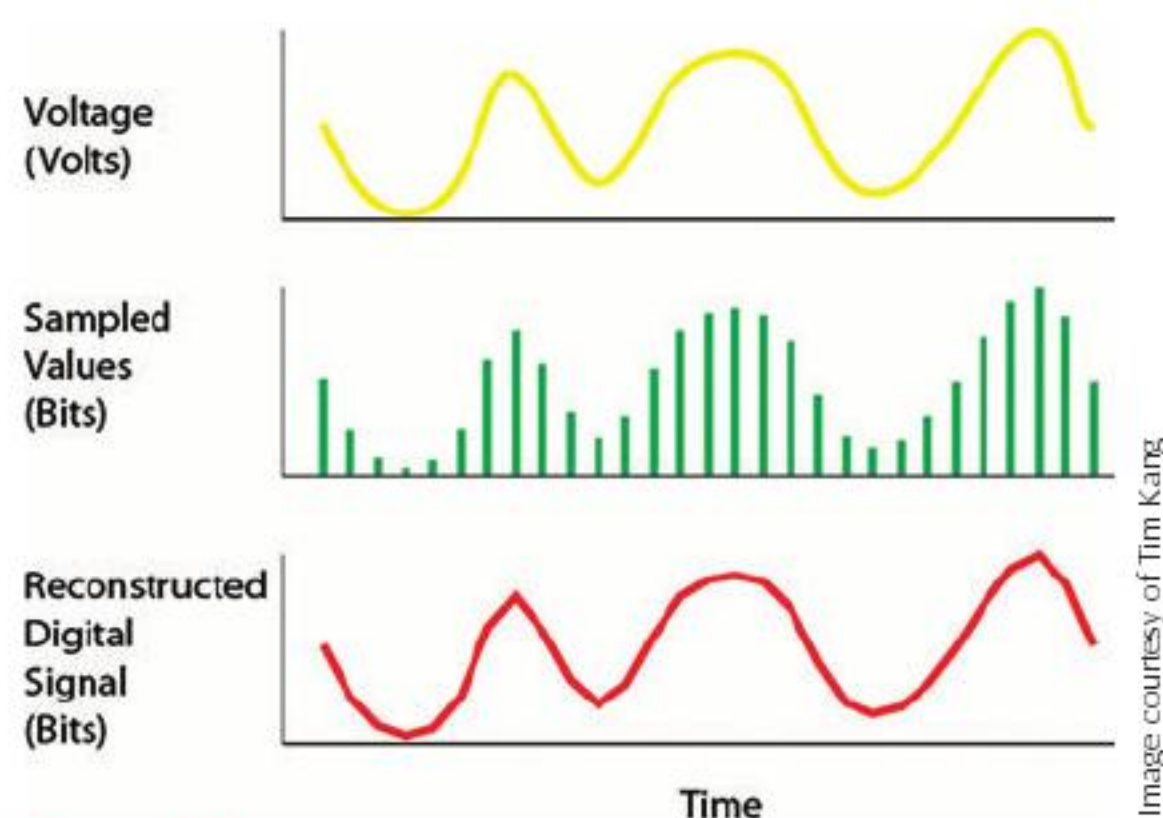


Figure 1.15 Voltage sampled to bits to reconstruct a signal.

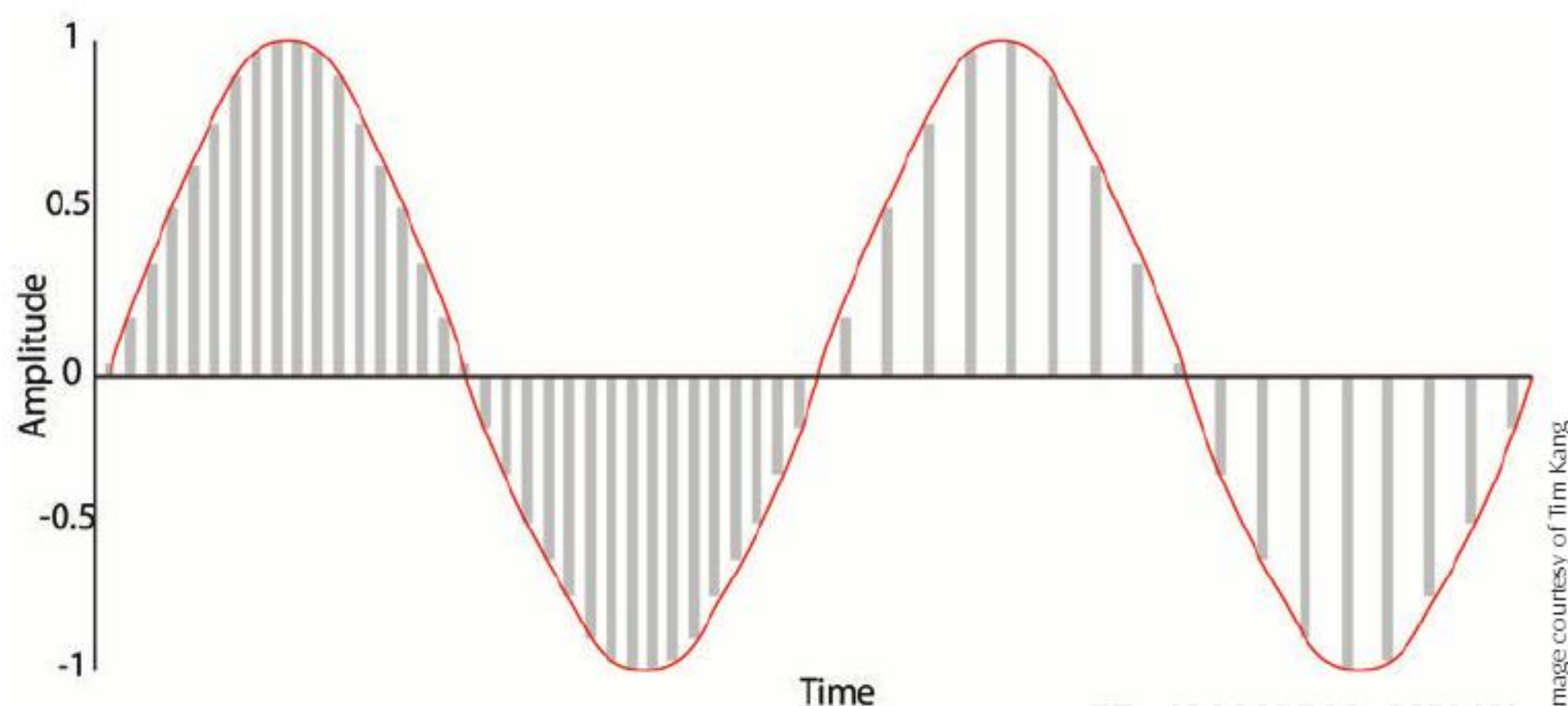
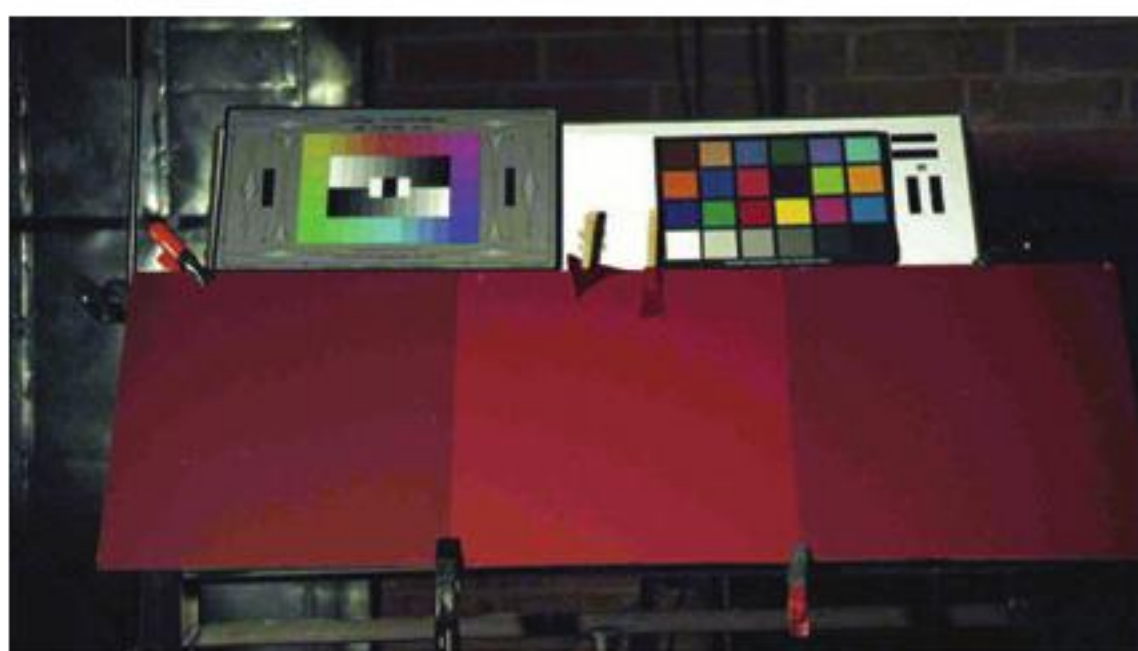


Figure 1.16 Higher and lower sampling frequencies.

In this illustration, a 1-Hz wave is sampled at two different sampling rates and precisions. The first full cycle depicted in blue is sampled at a rate of 50 Hz, and the second cycle depicted in red is sampled at 25 Hz. As we can see in Figure 1.16, the left samples are more accurate in both time and amplitude than the right samples.

How Many Crayons in the Box?

Bit depth refers to the number of digital bits used to store the gray scale or color information of each pixel as a digital representation of the analog world. The higher the bit depth, the more shades of gray or range of colors in an image, and the bigger the file size for that image. “Higher color bit depth gives you more shades of color—it’s like having more crayons in your coloring box,” as Scott Billups says.¹



57cd01448243a0f6fd2bc418eb67c691
ebrary

Figure 1.17 Low color bit depth results in color aliasing, which looks like distinct separate bands of red shades across the image instead of a subtle graduation of color.



Figure 1.18 Fewer colors to choose from reduces the accuracy with which subtle graduations in colors can be reproduced.



Image courtesy of Tim Kang

Figure 1.19 More colors to choose from means that subtler graduations of color can be achieved.

Higher color bit depth (more gradations of brightness value) and higher sampling frequency (more samples per second) will yield pictures that have better fidelity to the original scene. The more times per second that a waveform is sampled, the more faithful the digital recording is to changes in the original analog phenomenon. Sampling resolution (bit depth) is the maximum number of digits (bits) each sample’s components can represent. The sampling resolution determines how accurately the amplitudes in the original analog waveform can be re-created.

An 8-bit sampling resolution means that the continuous values of the input signal will be quantized to 2 to the 8th power, or 256 code values—in other words, 256 shades of red, 256 shades of green, and 256 shades of blue. When the red, green, and blue color palettes are multiplied to define the entire color palette, $256 \times 256 \times 256$, the result is 16,777,216—defined as 8-bit color in the digital realm.

The current de facto workflow in the motion picture industry is Cineon 10-bit log, which employs 1,024 code values per channel of red, green, and blue to digitally encode red, green, and blue film densities. This encoding uses code value 95 for black, code value 445 for Cineon Digital LAD, approximate code value 685 for white, and code value 1023 for peak white (the most extreme highlights). Cineon encoding provides roughly 90 code values per stop of light.

The broadcast video de facto standard SMPTE 292M implementation of 10-bit color constrains the range of 1,024 total bits per channel used to code value 64 for reference black to code value 940 for reference white, and code values 64 to 960 for

chrominance. We will delve further into these issues later in this text.

Other encodings are available for both motion picture and video work, but these two encodings have historically accounted for the majority of content created until very recently.



Figure 1.20 With even more colors to choose from we can make more beautiful pictures.

Source: Detail from True Color Series—"Girl 1"

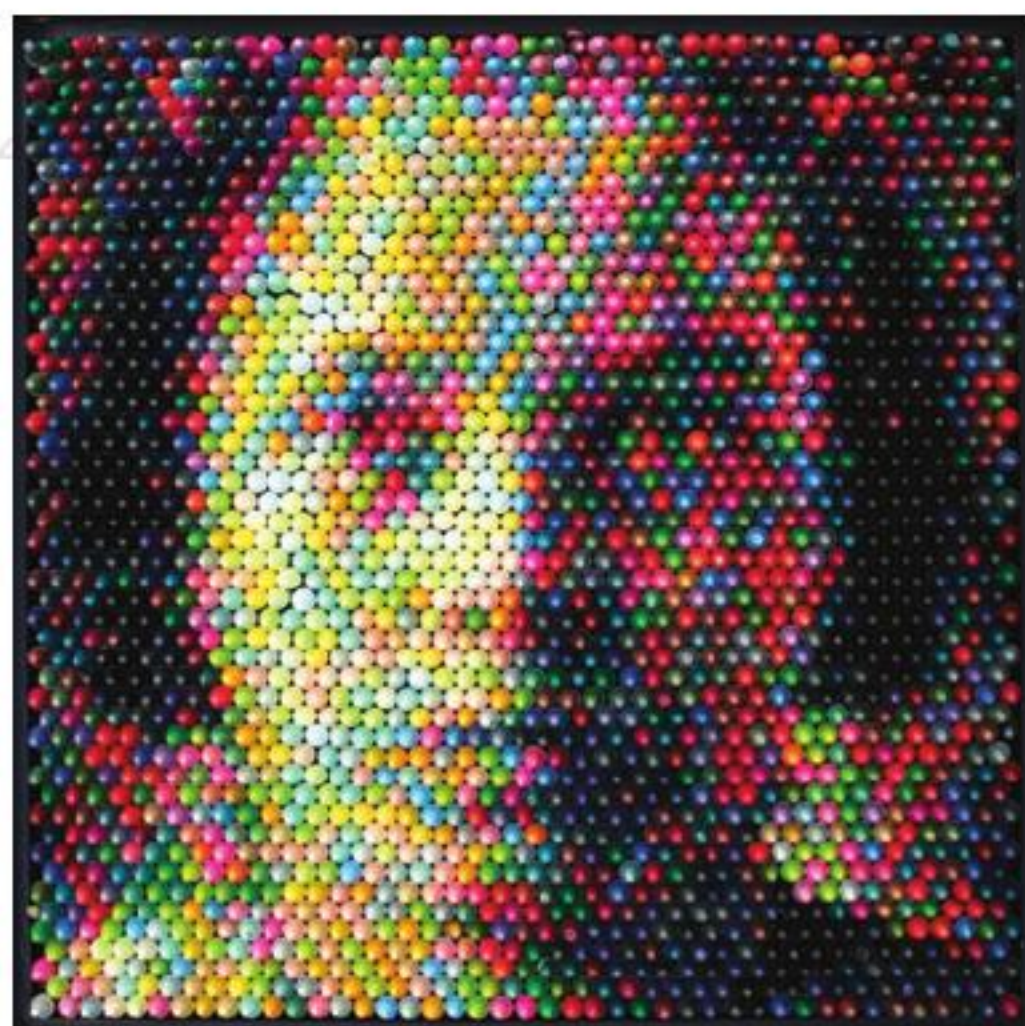


Figure 1.21 More colors to choose from means prettier pictures.

Source: "Girl 1" Crayon Art, courtesy of Artist Christian Faur, www.christianfaur.com/.

A 16-bit sampling quantizes to 2 to the 16th power, or 65,536, code values (65,536 shades of red, 65,536 shades of green, and 65,536 shades of blue), which provides much more accuracy and subtlety of color shading. This is especially important when working with wide-gamut color spaces where most of the more common colors are located relatively close together or in digital intermediate where a large number of digital transform algorithms such as in digital intermediate work are used consecutively.

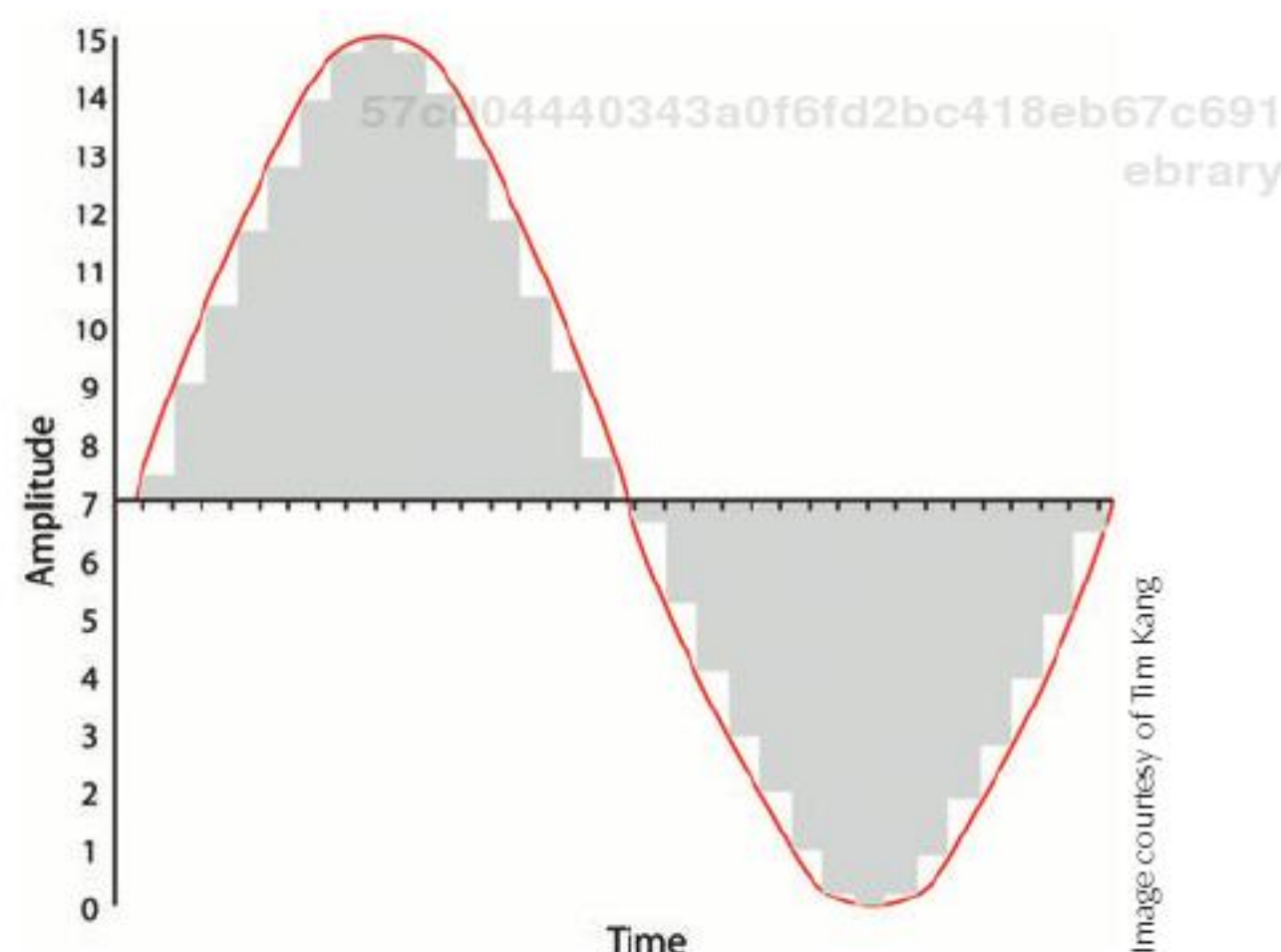


Figure 1.22 Colors are numerically sampled.

The lower the sampling precision, the more likely the quantized sample will differ from the actual value in the original analog wave. The digital recording is only an approximation of the original continuous wave form, but it has several advantages over an analog recording of that waveform. No recording noise is introduced except by the sampling process itself, digital numbers are easily manipulated and transmitted, and digital copies of the waveform are as good as the original recording.

There is much effort being given to creating a 16-bit RGB color space file format as the eventual future of digital cinema color space, and several research and development efforts are underway to implement 16-bit floating-point file formats for image acquisition, color correction, and ultimately display purposes. The Academy of Motion Picture Arts and Sciences "ACES" project is an attempt to create a ubiquitous 16-bit motion picture file format

that will serve our industry all the way from acquisition through archive. These efforts are covered more deeply in later chapters of this book.

The Issues of Encoding Luminance²

Now that we understand how analog waveforms are sampled for reconstruction from numerical values, let's explore how best to apply digital technique to images that human beings will view.

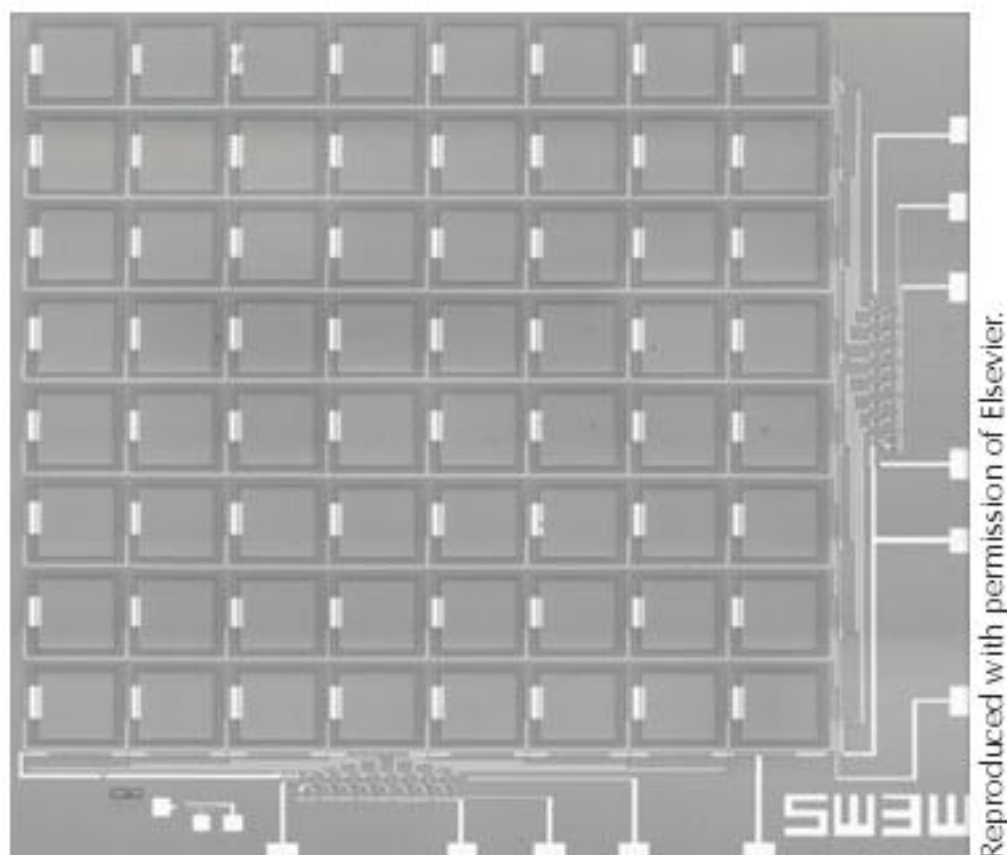


Figure 1.23 Electron microscope view of photosites on a sensor.

In a digital image, each pixel is represented as the color of a sample point on a rectangular grid. Each sample represents the color across some nearby area.

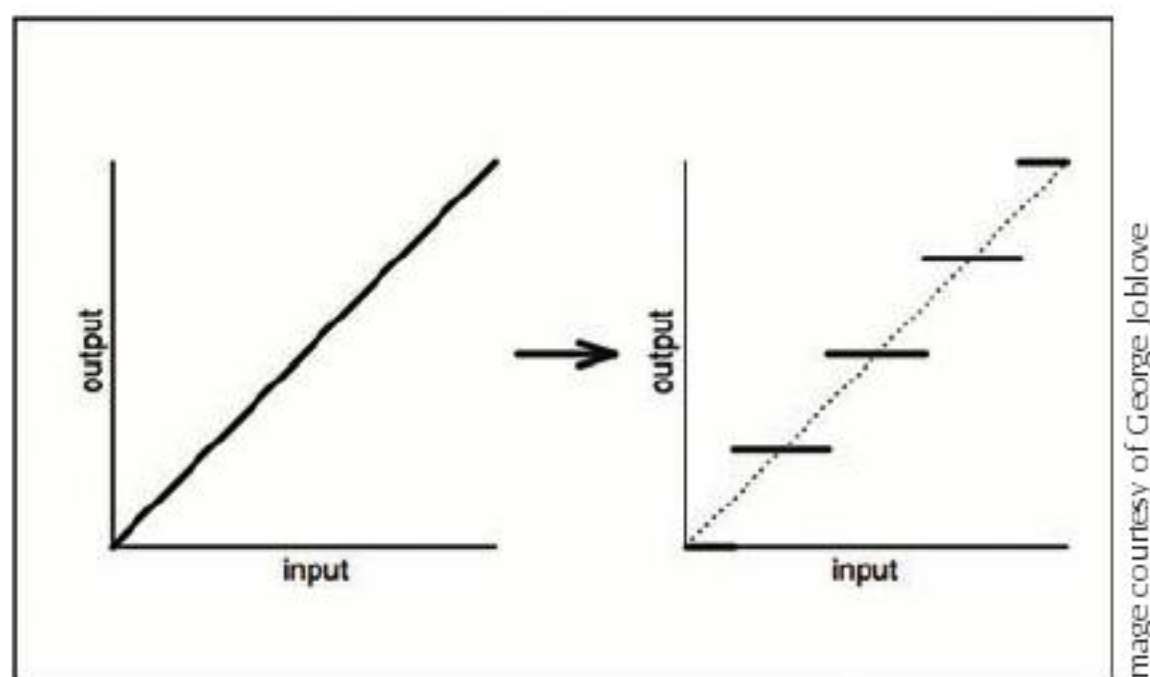


Figure 1.24 Quantization creates single numerical values for sampled values.

In a digital image, each pixel value is limited to a fixed number of discrete values.

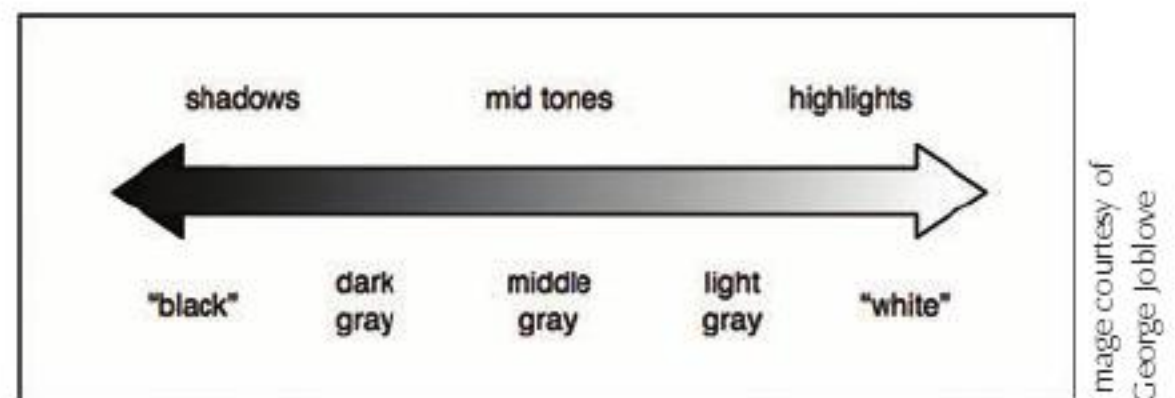


Figure 1.25 The scene has a tonal range.

The range of grays, or luminance levels.

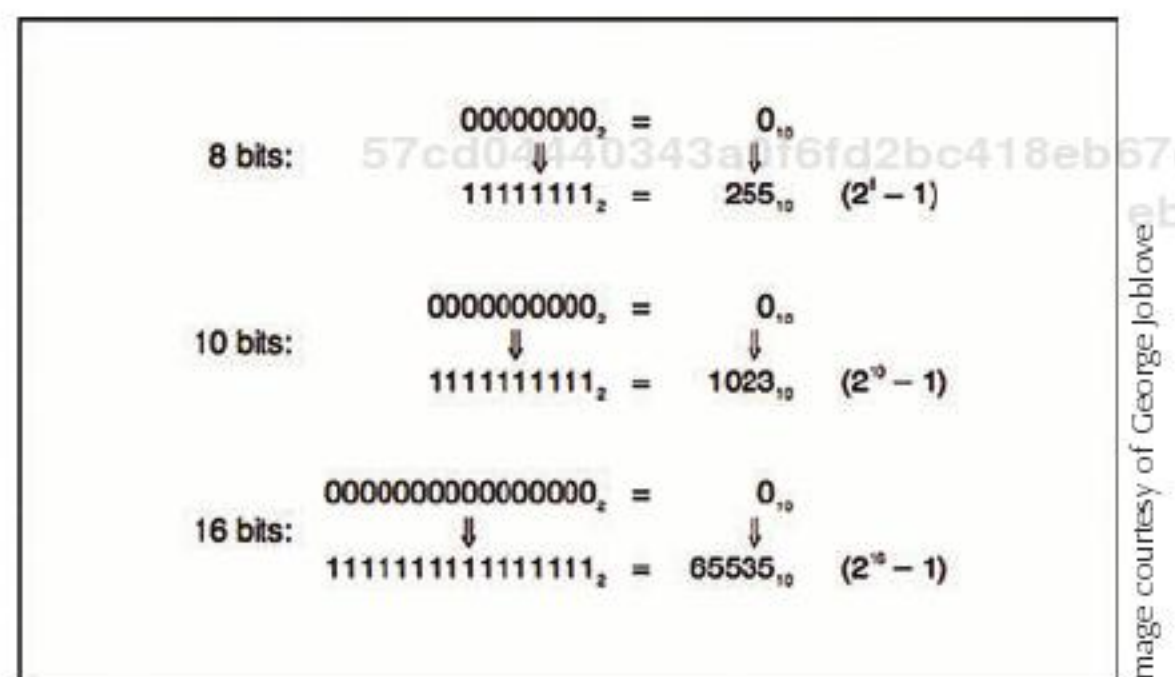


Figure 1.26 Bit depth is a function of the number of code values.

In digital images, pixel values are frequently represented as integers. The number of bits used determines the number of possible discrete code values available.

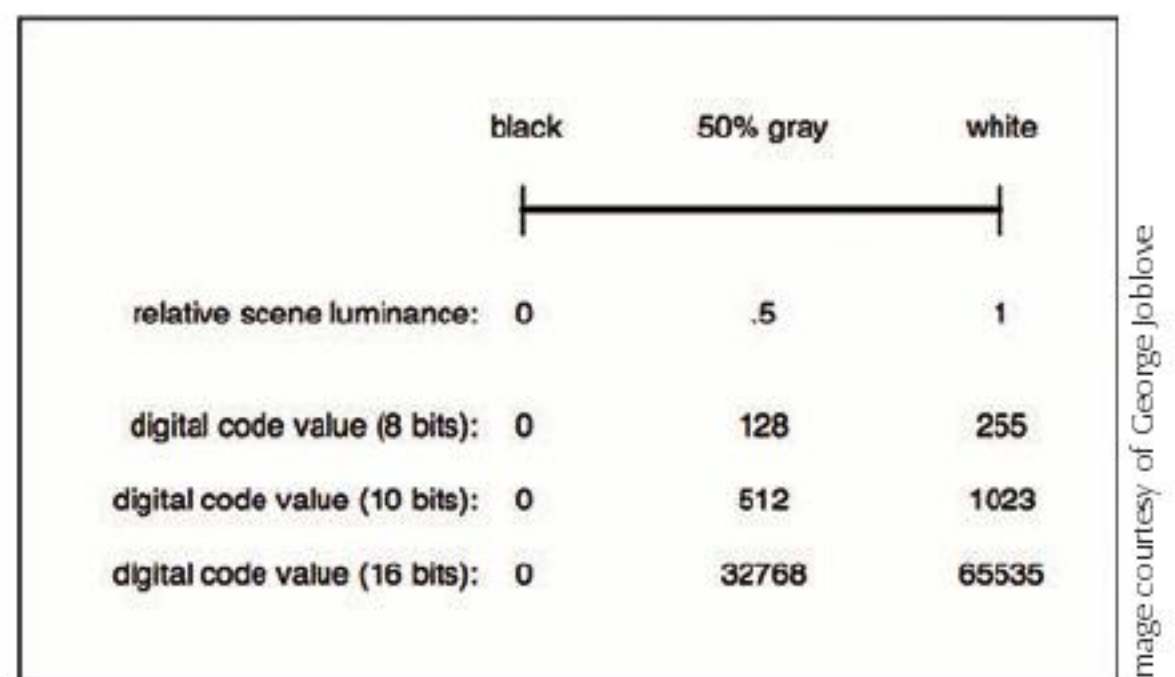


Figure 1.27 A linear coding of scene luminance and its resulting distributions of code values for three different bit depths.

Note: A simple coding function: the code value is directly proportional to the scene luminance, with the maximum code value representing a 100% reflector white. (Note that current generation digital cinema cameras can record luminances far above that of a 100% reflector.) The middle code value results in middle gray.

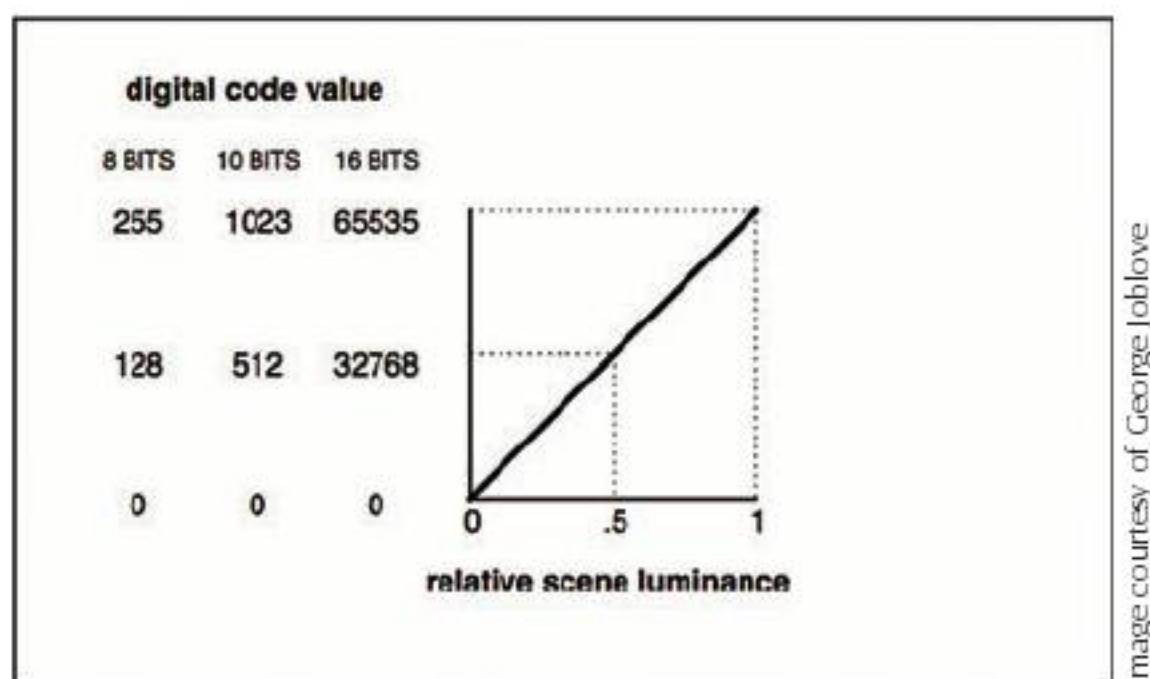


Figure 1.28 Graphic plot of a linear coding of scene luminance.

Note: Code values plotted as a function of scene luminance, in this case the entire scene luminance from an arbitrary range of 0 to 1 is plotted over linear encodings. What happens if the relative scene luminance at some pixel is greater than 1?

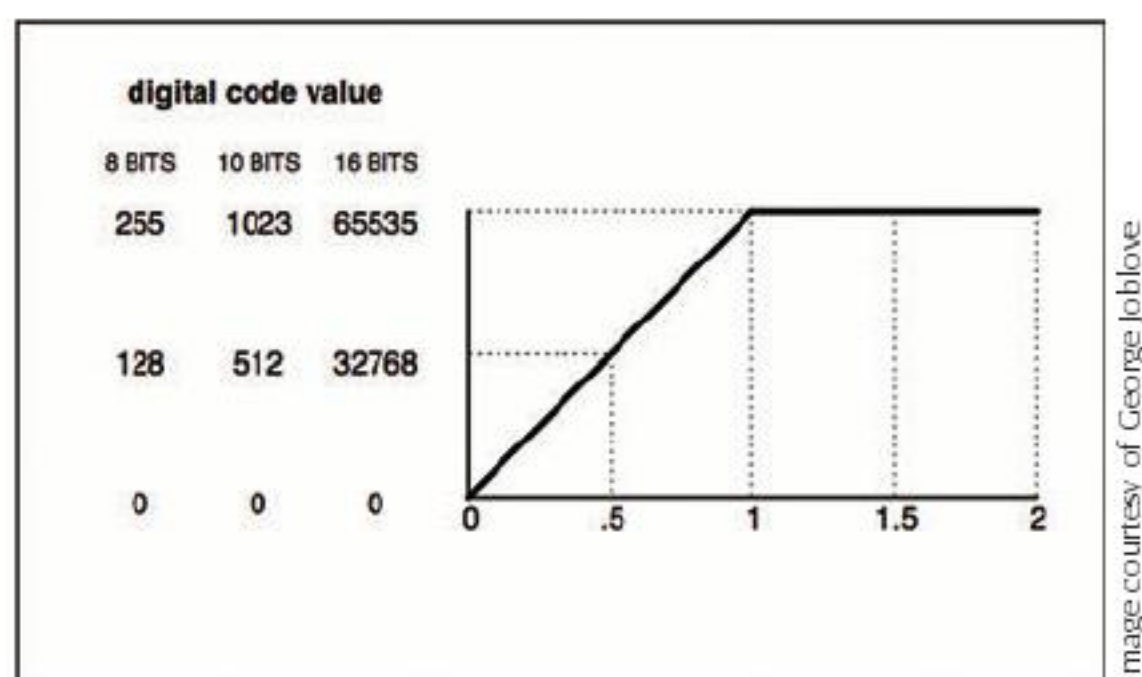


Figure 1.29 Highlight clipping results when scene luminance exceeds the maximum encoding value.

Note: The digital representation of luminance is “clipped” to the maximum code value. Any visual information in the image above this level—“highlight detail”—is lost.

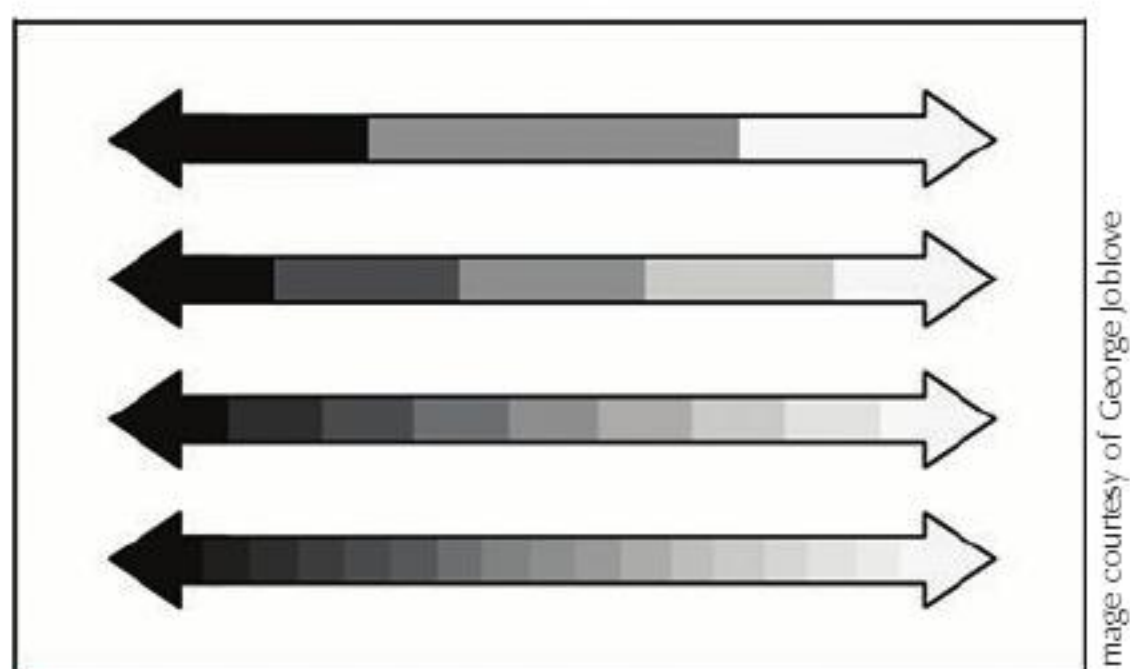


Figure 1.30 Luminance discrimination.

How many code values (discrete digital luminance steps) must there be to insure that the quantizing is never visible in an image? The sensitivity of the human visual system to contrast is limited. If two gray levels are close enough in luminance, the difference is indistinguishable to human vision.

The Just Noticeable Difference (JND) and Square Root Integral (SQRI)

It is important to understand that there has been enormous technical research into how many shades of color that humans can distinguish in an image. That threshold is generally described by the concept of the “just noticeable difference,” or JND. This concept is important because if we employ an encoding scheme with fewer code values than “just noticeable differences,” we can perceive the different shades that make up subtle gradations of color as distinct bands of color. This banding is often referred to as “color aliasing,” which significantly detracts from faithful reproduction of original images.

Researcher Peter Barten has developed and refined a number of equations for understanding human visual capabilities. He defines a metric for describing perceived image quality that is based on the square root of the normalized modulation of the picture, this metric he calls the SQRI, or square root integral. He observes experiments where comparable image contrast was varied in steps of one “just noticeable difference” (or JND); the minimum amount of contrast change that is detectable more than 50% of the time in test subjects, and his tests show a very close correlation to the results of real world testing. Detecting “just noticeable differences” depends on many factors—resolution, luminance, refresh rate and others—and the number of “JNDs” per stop of light varies dependent on color and luminance, but humans are quite sensitive to these differences in contrast.

For a scholarly look at human vision and our ability to distinguish subtle differences in contrast please read *Contrast Sensitivity of the Human Eye and Its Effects on Image Quality* by Peter G. J. Barten.

Practical testing of human sensitivity to “just noticeable differences” in contrast conducted recently determined the minimum quantizing level for the threshold of quantizing visibility in digital cinema to be 12 bits as a minimum.



Figure 1.31 12 bits of color per channel is the minimum required to avoid perception of “just noticeable differences” in quantized images.

The Problem With Linear Encoding

Digital cameras are basically tricolor light meters. For a given quantity of light, they report a voltage value. For twice that amount of light, they report twice the voltage. This presents us with a mathematical coding problem when we try to reproduce images from those reported code values.

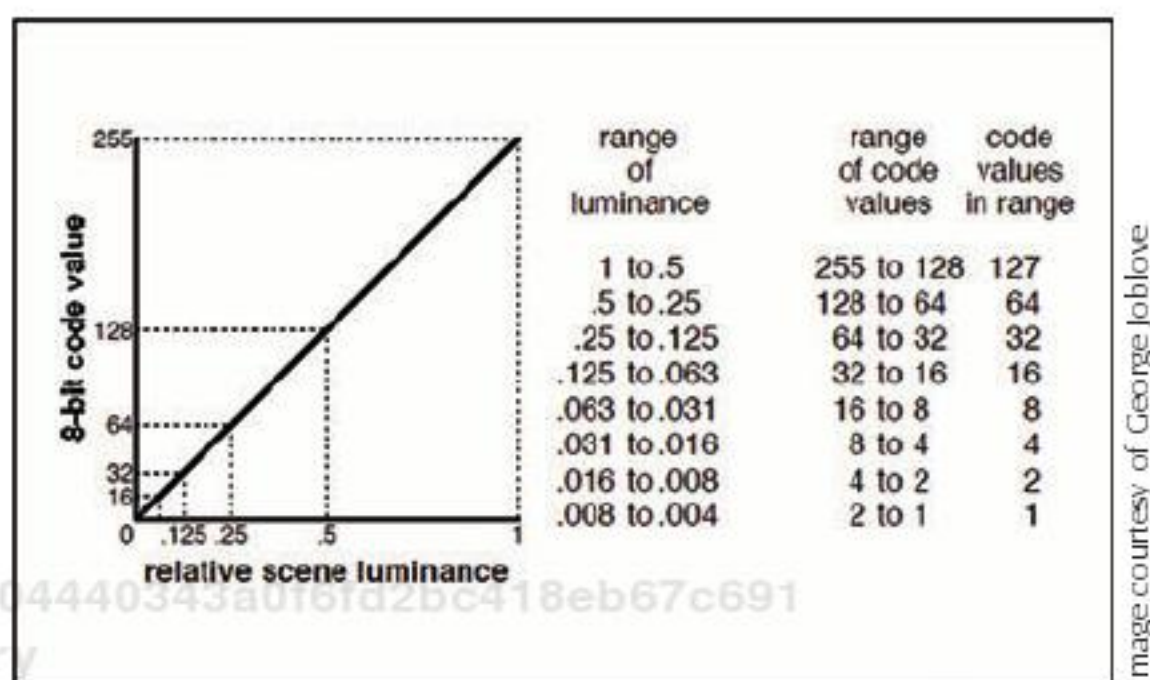


Figure 1.32 8-bit linear coding.

The range in relative scene luminance from 1 down to .5 covers a 1-stop (factor-of-2) difference. If we employ an 8-bit linear coding scheme, this range is represented by the code values 255 to 128, so there are 127 discrete steps. From .5 luminance down to .25 luminance is also 1 stop, but here there are only 64 code values (from 128 down to 64). From .25 luminance down to .125 luminance is also 1 stop, but here there are only 32 code values (from 64 down to 32). From .125 luminance down to .0625 is also 1 stop, but here there are only 16 code values (from 32 down to 16). We only have enough numerical bit values to represent the first stop of luminance, and almost none

to represent the toe of the exposure. The lower the luminance, the fewer the number of code values available to represent it.

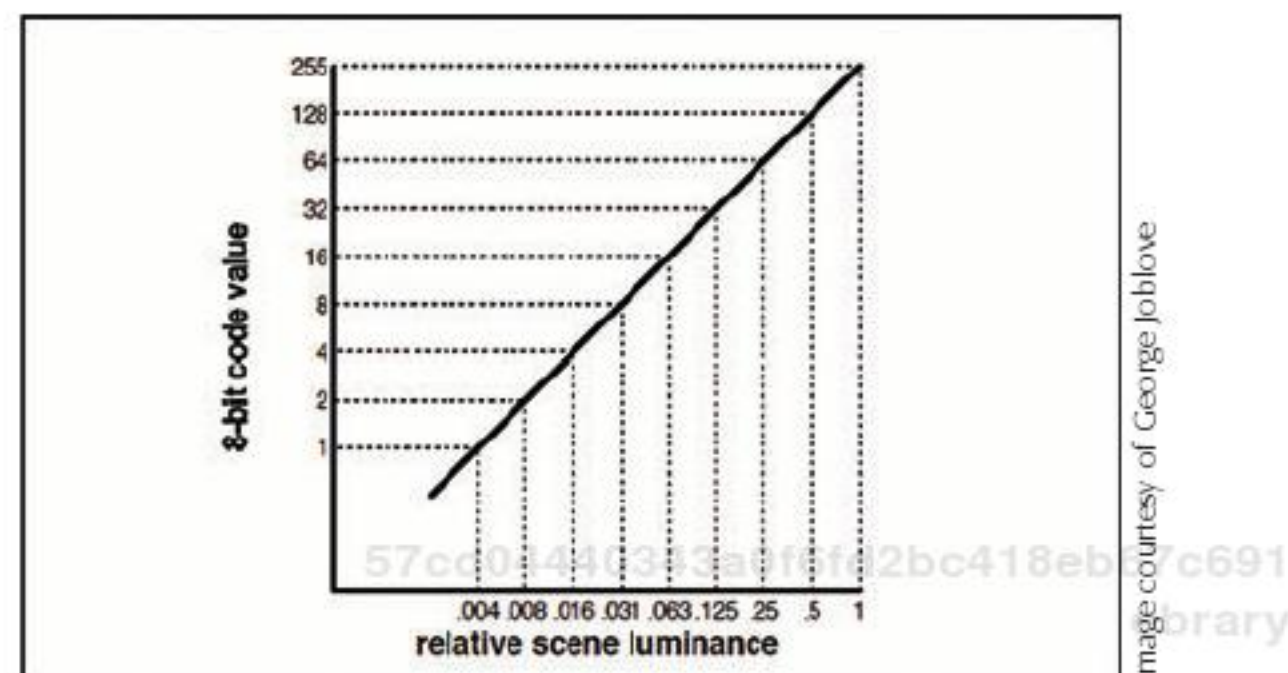


Figure 1.33 8-bit linear coding (logarithmic plot).

The same coding function is shown here plotted on logarithmic scales (in which equal steps along either axis represent equal ratios rather than equal differences). Here each marked step is a factor of 2 (1 stop). The problem at the lower end of the scale is evident. For example, the luminance range from 6% down to 3% is represented by only 8 code steps. This lack of precision causes contouring in an image’s shadow details.

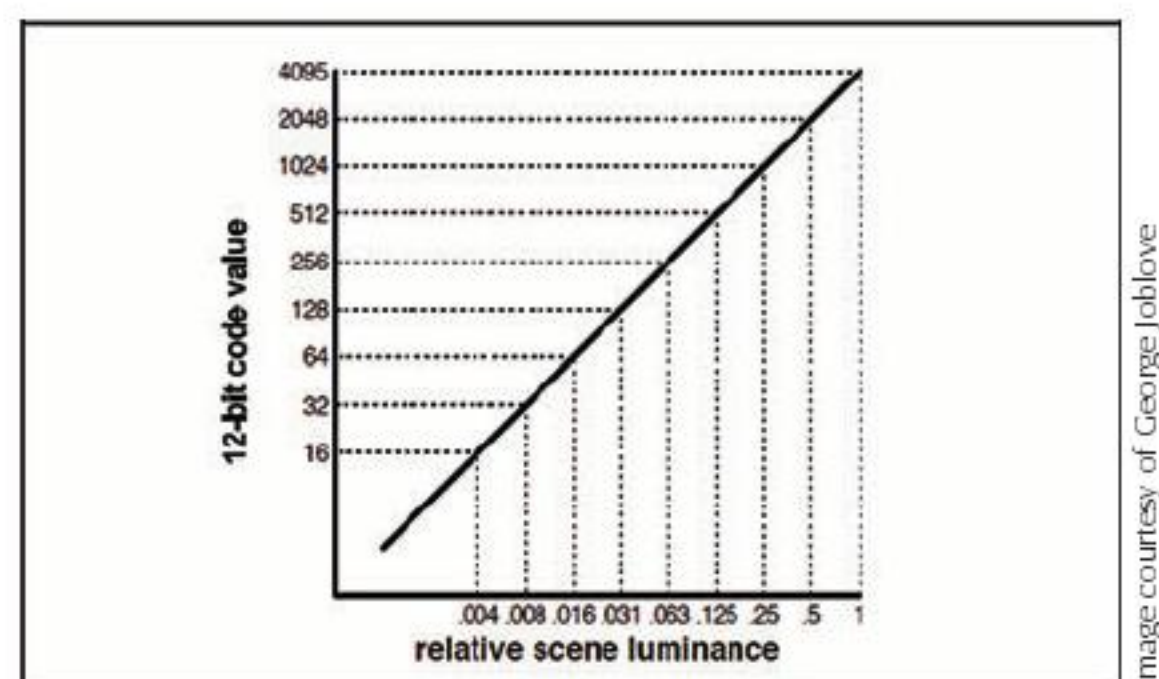


Figure 1.34 12-bit linear coding (logarithmic plot).

Coding with 12 bits instead of 8 provides 16 times as many steps. The luminance range from 6% down to 3% is now represented by a range of 128 code values, enough to ensure that 1-stop differences are below the threshold of perception. This may be sufficient, but

doesn't leave much room for brightness adjustment that may need to be done.

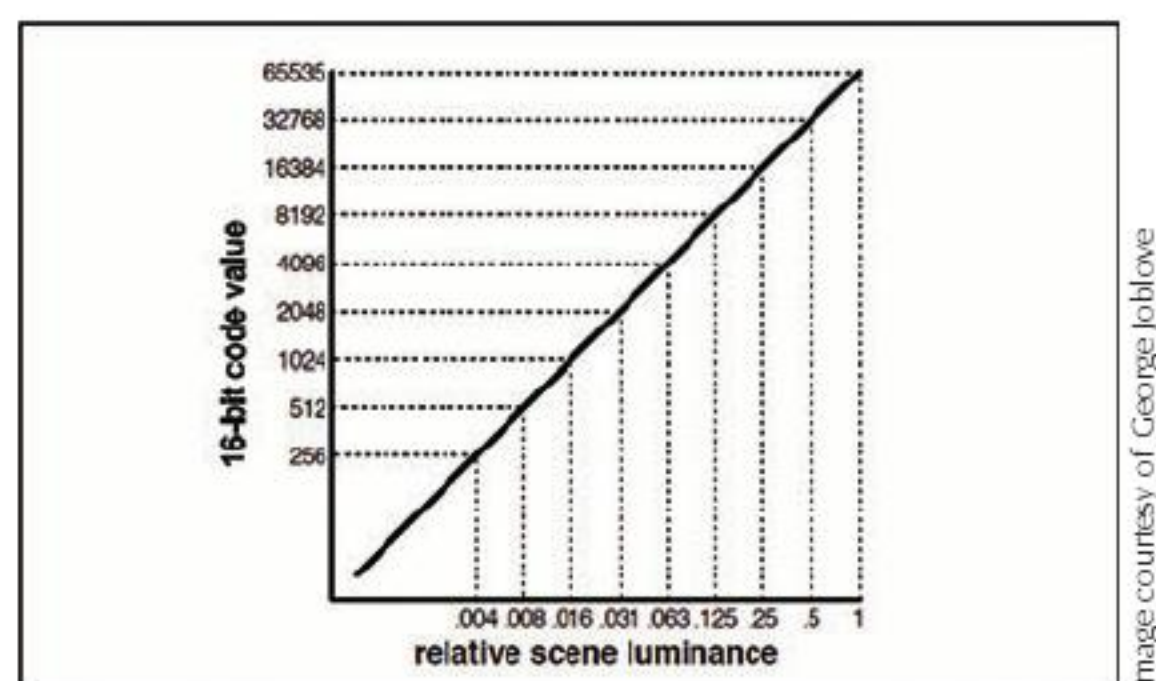


Figure 1.35 16-bit linear coding (logarithmic plot).

For linear coding of scene luminance, 16 bits provide enough code values to avoid visible quantizing through a range of scene luminance of at least 9 stops. But this requires double the storage of an 8-bit coding, and most of the 65536 code values are wasted: For example, across the luminance range from white (65535) to 50% gray (32768), only 70 steps would suffice, but 32,767 are used.

The concern for having enough bits to properly encode both shadow detail and blacks while also preserving highlight details dictates the way we allocate encoding bits all throughout any practical color encoding scheme.

Gamma

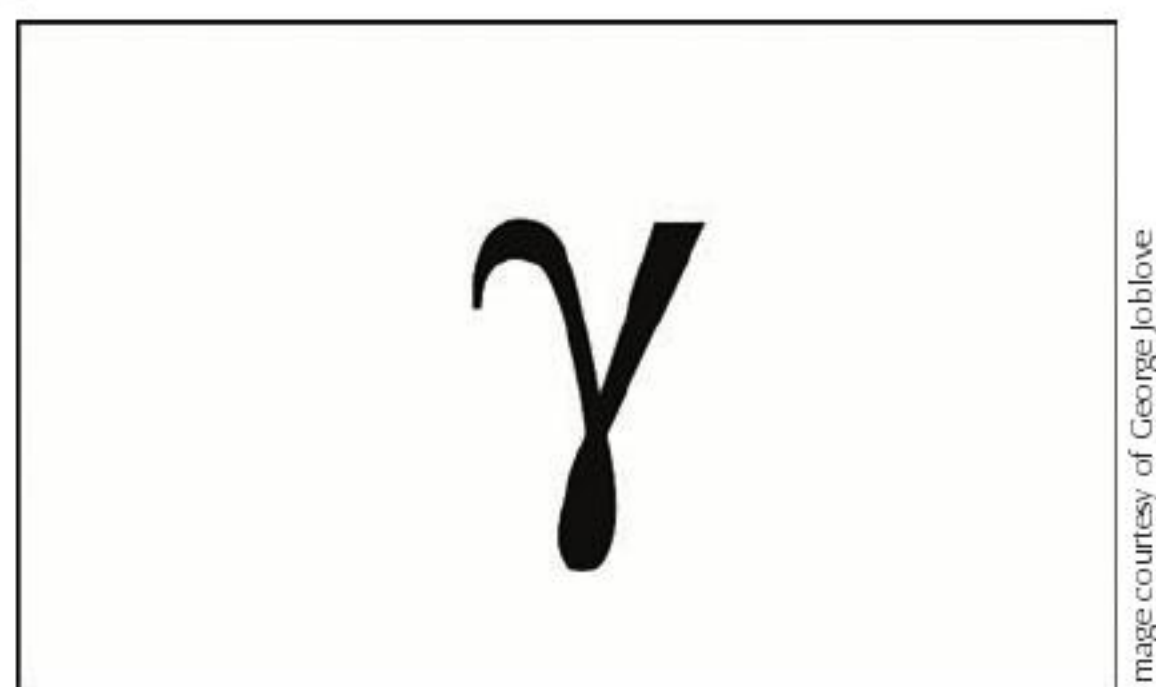


Figure 1.36 Gamma.

The Greek letter gamma is used to denote the numerical characteristic of images which must be

multiplied for proper reproduction. It is perceived as contrast, or a change in contrast. Gamma of 1 (unity gamma) means no change, values greater than 1 increase contrast, and values less than 1 (fractions) decrease contrast.

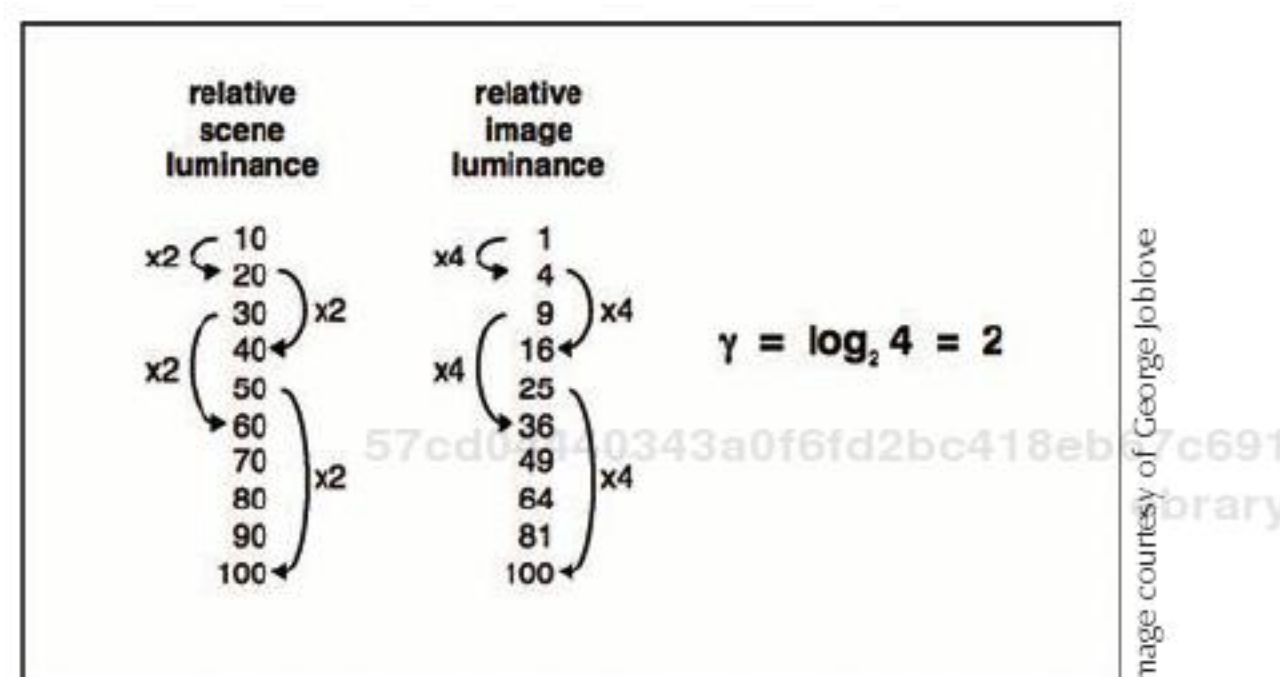


Figure 1.37 In this arbitrary example, gamma = 2.

If some ratio of luminance in an image represents a fixed ratio of luminance in the original scene throughout the range of luminance levels, then the image has a gamma, and it is the log of the ratio in the image to the base of the ratio in the scene. In this example, any change in scene luminance by a factor of 2 results in a change in luminance in the image by a factor of 4, so the gamma of the image is 2.

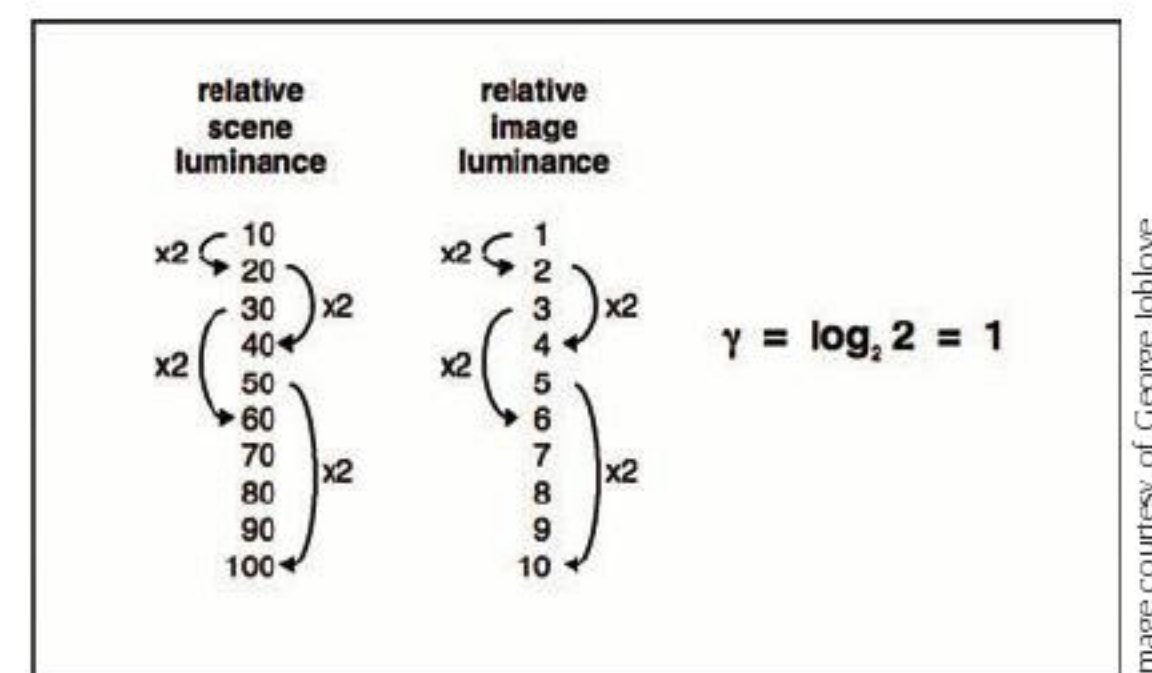


Figure 1.38 An example of gamma = 1 (linear).

A linear digital image by definition has a gamma of 1, since multiplying the luminance in the original scene by some factor results in an increase in the digital value by the same factor ($\log_x x = 1$ for all x).

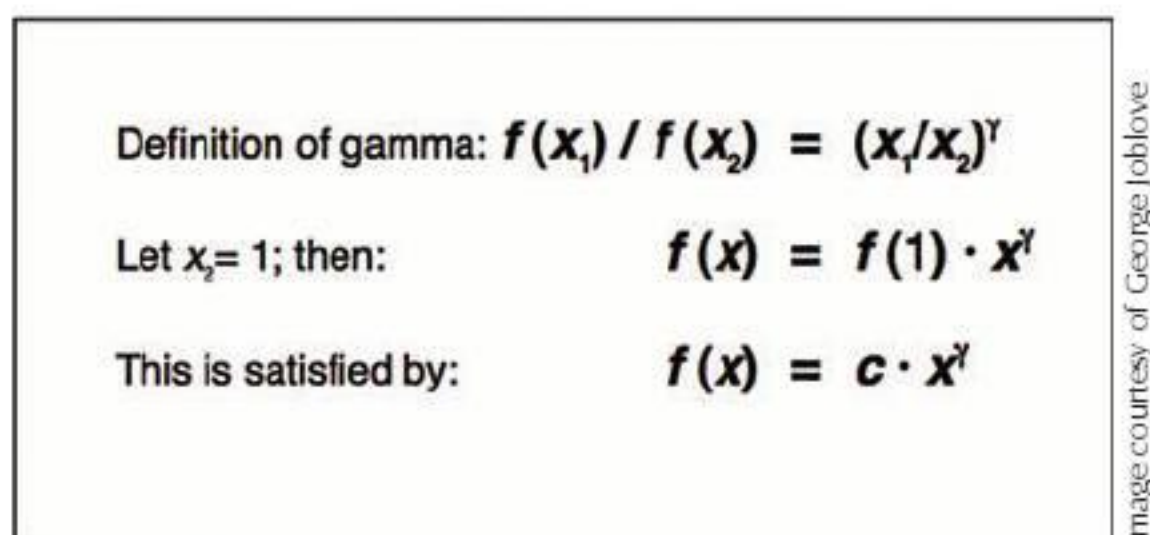


Figure 1.39 The gamma function.

X_i is a luminance level in the scene, $f(x_i)$ is the corresponding luminance in the image, and γ is gamma. c is a constant. Because $(x^a)^b = x^{(ab)}$, multiple changes in gamma are equivalent to a single change that is the product of the individual values. For example, applying a gamma of 2 and then a gamma of 3 is the same as applying a gamma of 6 $((x^2)^3 = x^6)$.

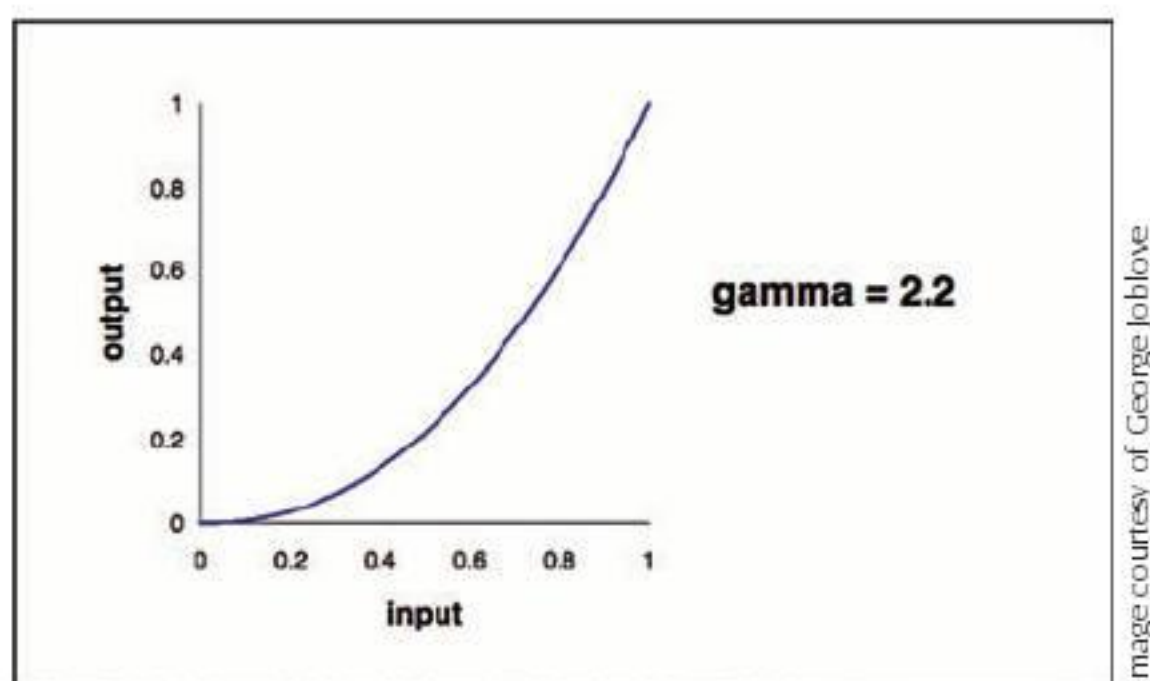


Figure 1.40 Graph of gamma = 2.2 output = input^{2.2}.

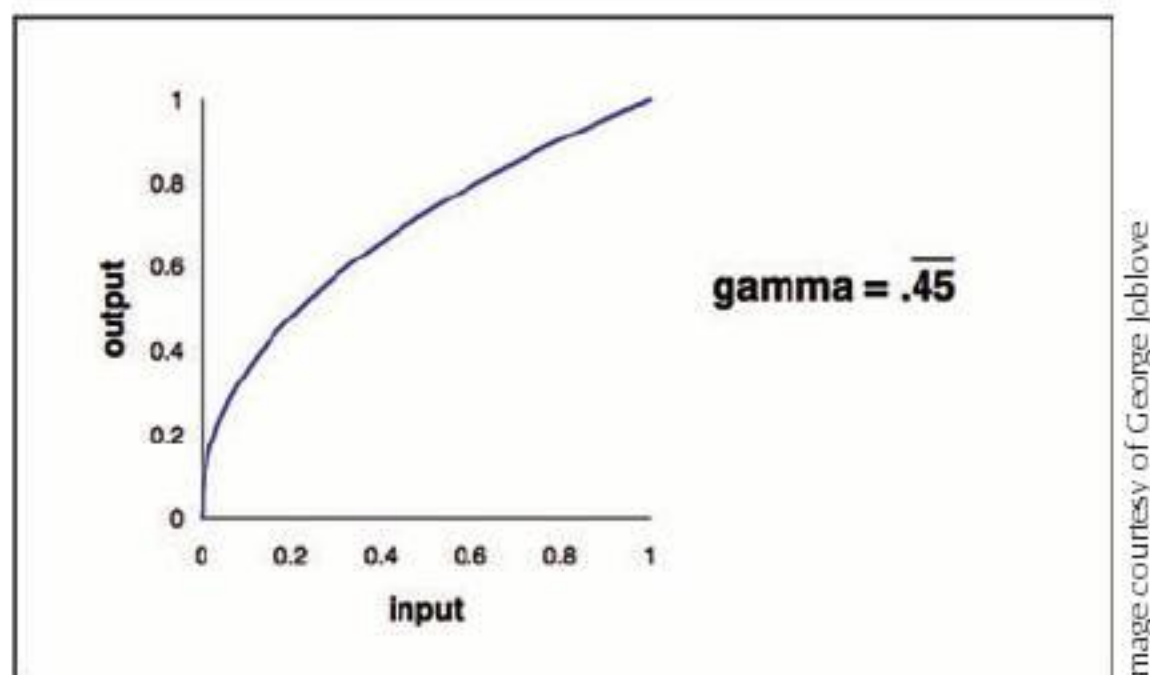


Figure 1.41 Graph of gamma = 1/2.2 = .45 output = input^{.45}.

All video images (analog and digital) have this gamma.

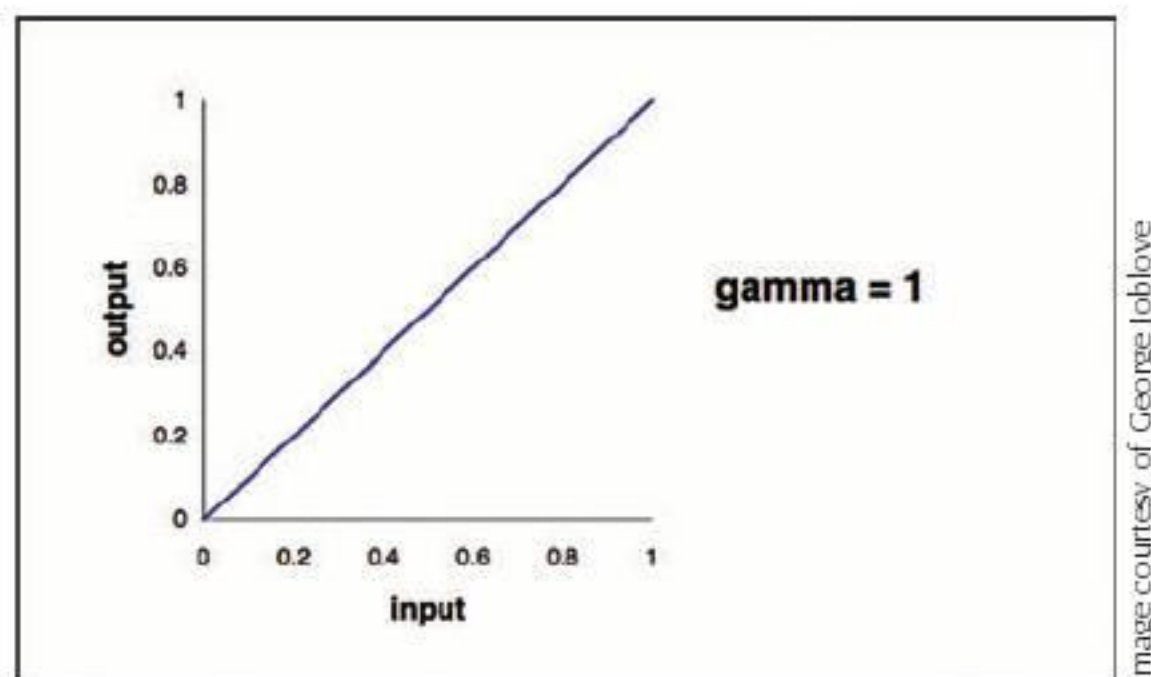


Figure 1.42 Graph of gamma = 1 (linear) output = input¹.

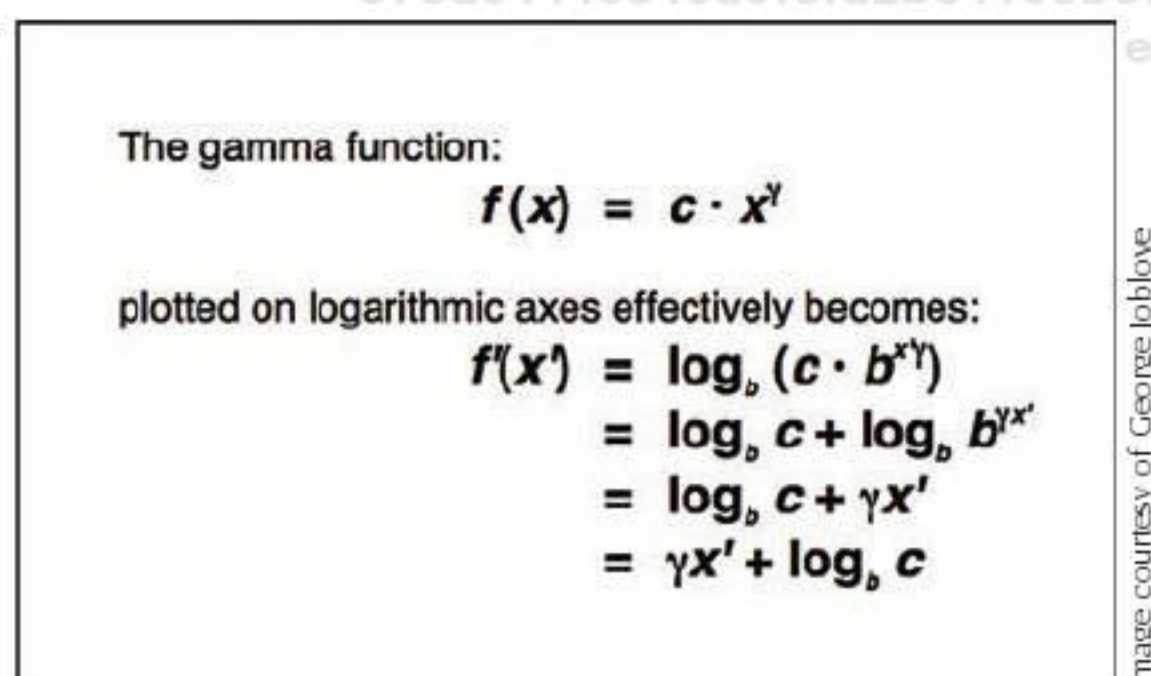


Figure 1.43 The gamma function as plotted logarithmically.

Since gamma is a power function, its graph on linear axes is curved for any value of γ other than 1. When plotted on logarithmic axes, however, the function effectively becomes f' shown here. (b is the logarithm base used to scale the axes.) The last form of the function is recognizable as that of a straight line, where γ is the slope and $\log_b c$ is the intercept at $x' = 0$.

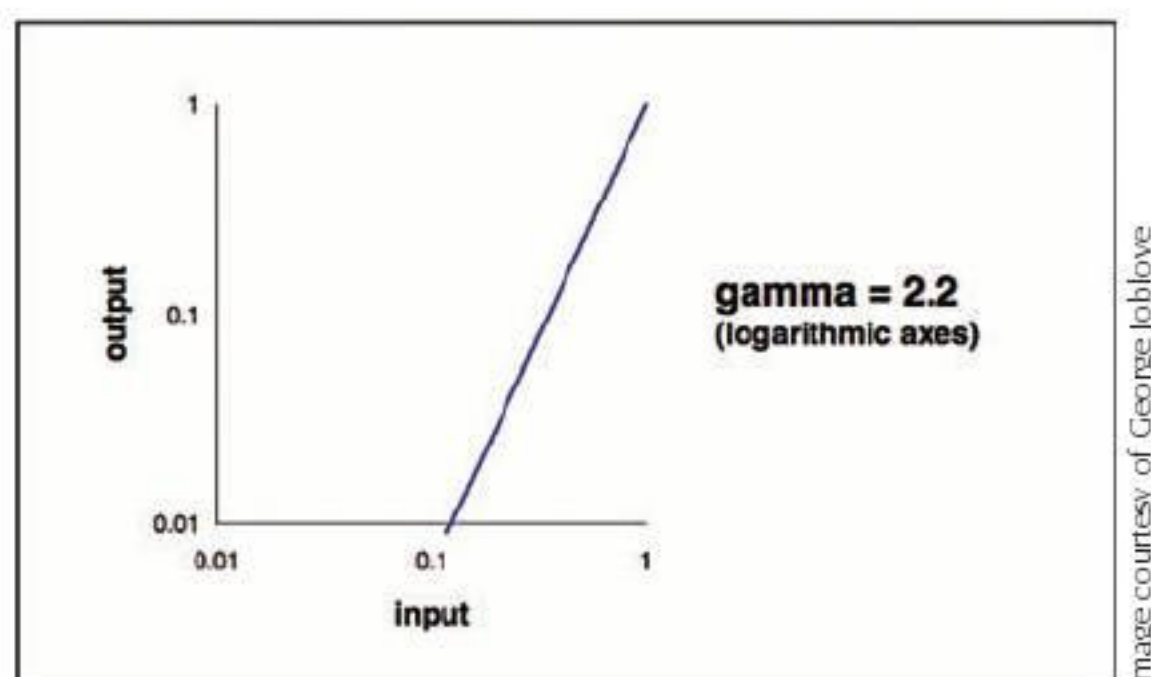


Figure 1.44 Graph of gamma = 2.2 (logarithmic axes) output = input^{2.2}.

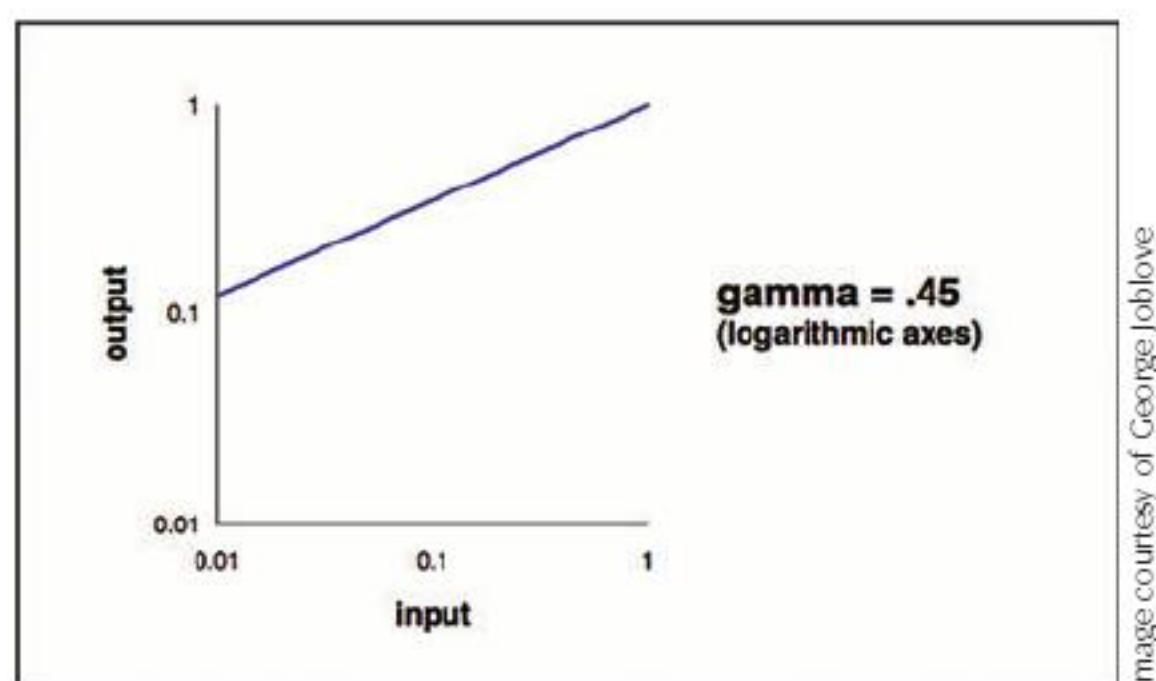


Figure 1.45 Graph of $\gamma = 1/2.2 = .45$ (logarithmic axes) $\text{output} = \text{input}^{.45}$.

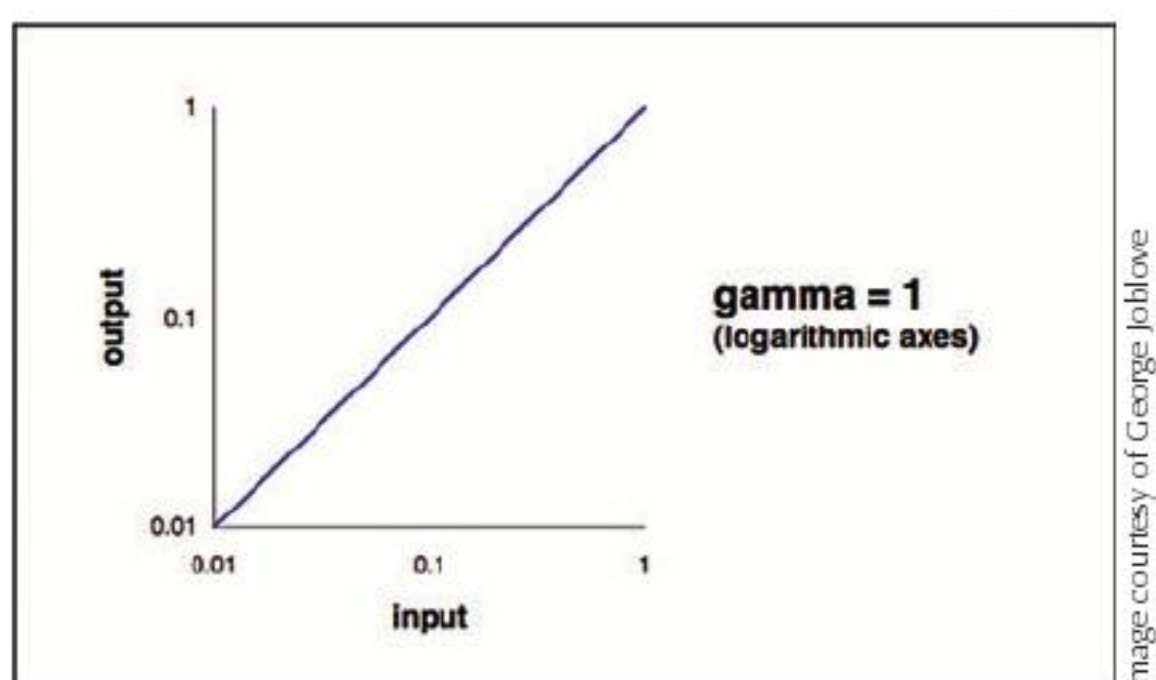


Figure 1.46 Graph of $\gamma = 1$ (linear) (logarithmic axes) $\text{output} = \text{input}^1$.

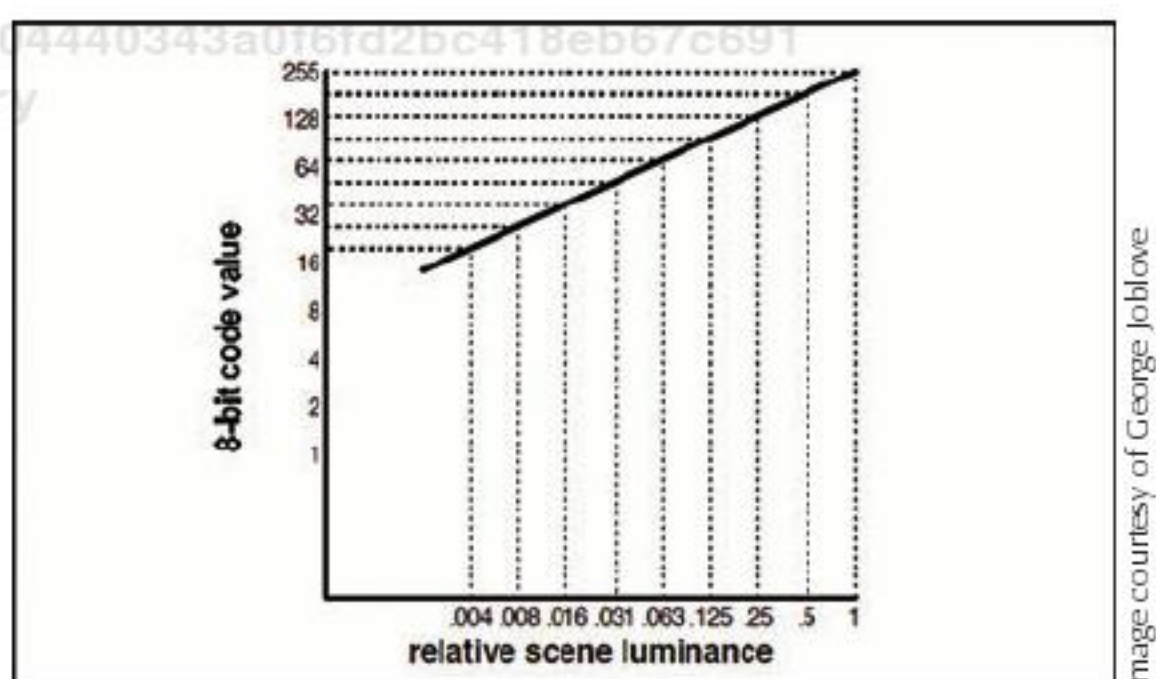


Figure 1.47 8-bit $\gamma = 1/2.2$ coding (logarithmic plot).

By boosting the code values in a manner that produces a greater increase for the lower luminance values, the 256 code values are effectively distributed more efficiently with respect to human perception. Proceeding down from white in 1-stop increments,

the corresponding code values are 255, 186, 136, 99, 72, 53, 39, 28, 21. So the range in luminance from 6% to 3% is now coded by the values from 72 to 53, or 19 steps.

range of luminance	8-bit linear		8-bit $\gamma = 1/2.2$	
	range of code values	code values in range	range of code values	code values in range
1 to .5	255 to 128	127	255 to 186	69
.5 to .25	128 to 64	64	186 to 136	50
.25 to .125	64 to 32	32	136 to 99	37
.125 to .063	32 to 16	16	99 to 72	27
.063 to .031	16 to 8	8	72 to 53	19
.031 to .016	8 to 4	4	53 to 39	14
.016 to .008	4 to 2	2	39 to 28	11
.008 to .004	2 to 1	1	28 to 21	7

Figure 1.48 Comparison between 8-bit linear and 8-bit $\gamma = 1/2.2$ coding.

The better distribution of code values with the gamma coding can clearly be seen in this table. In fact, below about the .002 luminance level (not shown here) the 8-bit gamma coding provides more code values than 12-bit linear coding.

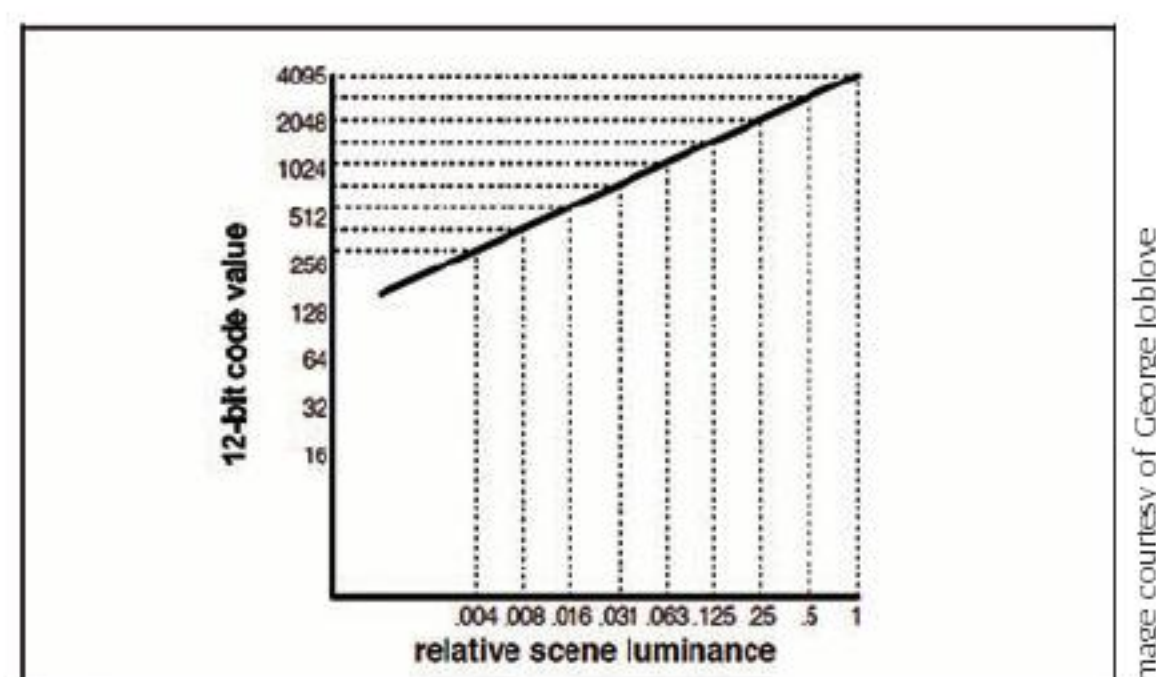


Figure 1.49 12-bit $\gamma = 1/2.2$ coding (logarithmic plot).

Coding with the same gamma but with 12 bits instead of 8 provides 16 times as many steps. The luminance range from 6% down to 3% is now represented by a range of 314 steps, and the range from .8% down to .4% is represented by a range of 122 steps. This is more than adequate to avoid visible quantizing. Below about the .002 luminance level the 12-bit gamma coding provides more code values than 16-bit linear coding.

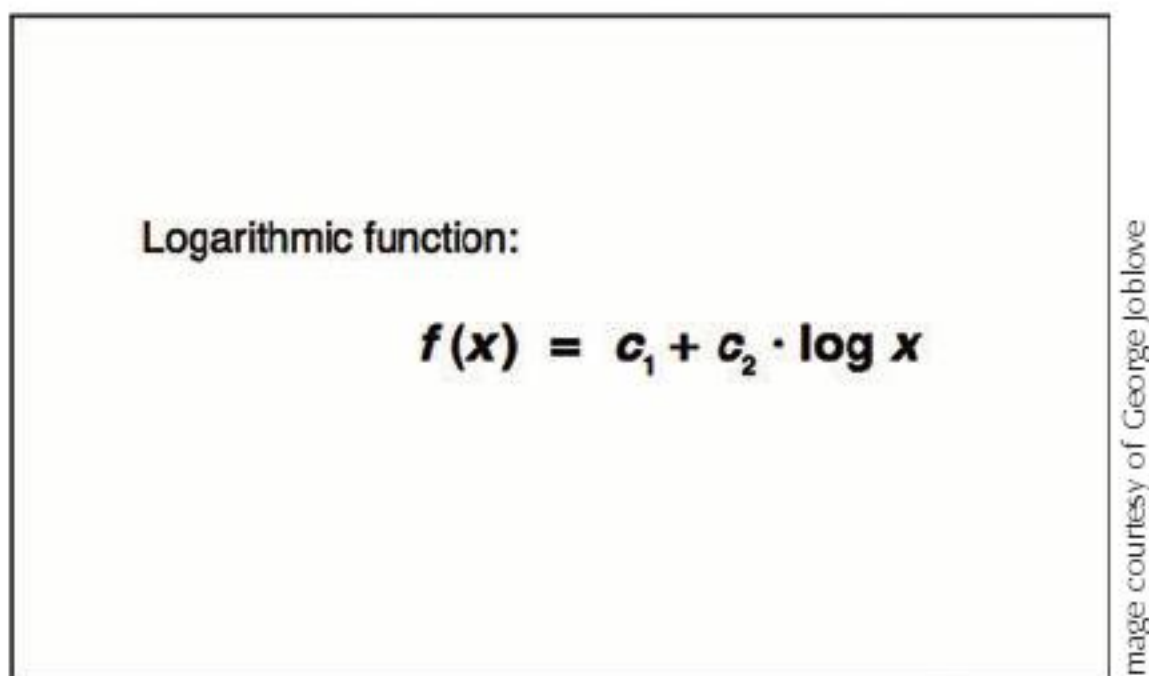


Figure 1.50 The logarithmic function x is the luminance; c_1 and c_2 are scaling constants.

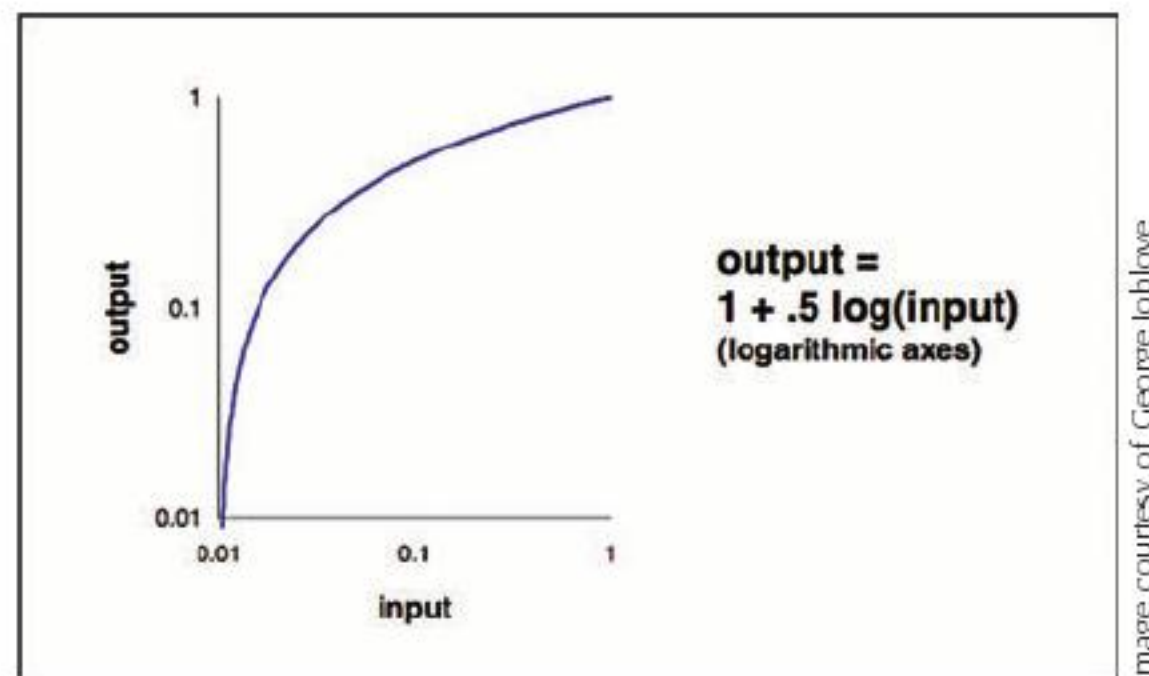


Figure 1.53 Graph of scaled log function (logarithmic axes).

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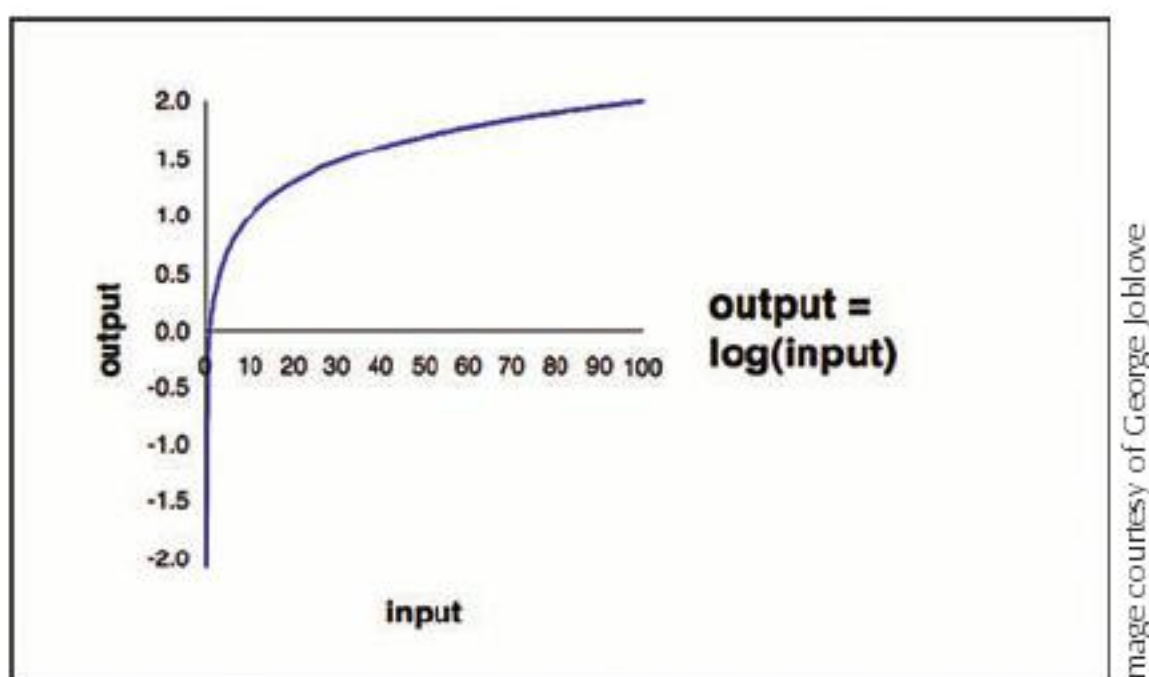


Figure 1.51 Graph of log function.

bit depth:	8	10	10
number of code values:	256	1024	1024
dynamic range, stops:	8	8	16
code values per stop:	32	128	64
relative scene luminance:	1.	255	1023
	.5	223	895
	.25	191	767
	.125	159	639
	.0625	128	512
	.03125	96	384
	.01563	64	256
	.00781	32	128
	.00391	0	0
	.00195		
	...		
	.000031		
	.000015		

Image courtesy of George Joblove

Figure 1.54 Log coding of scene luminance.

Note that since logarithms are defined only for positive numbers, the log of zero is undefined.

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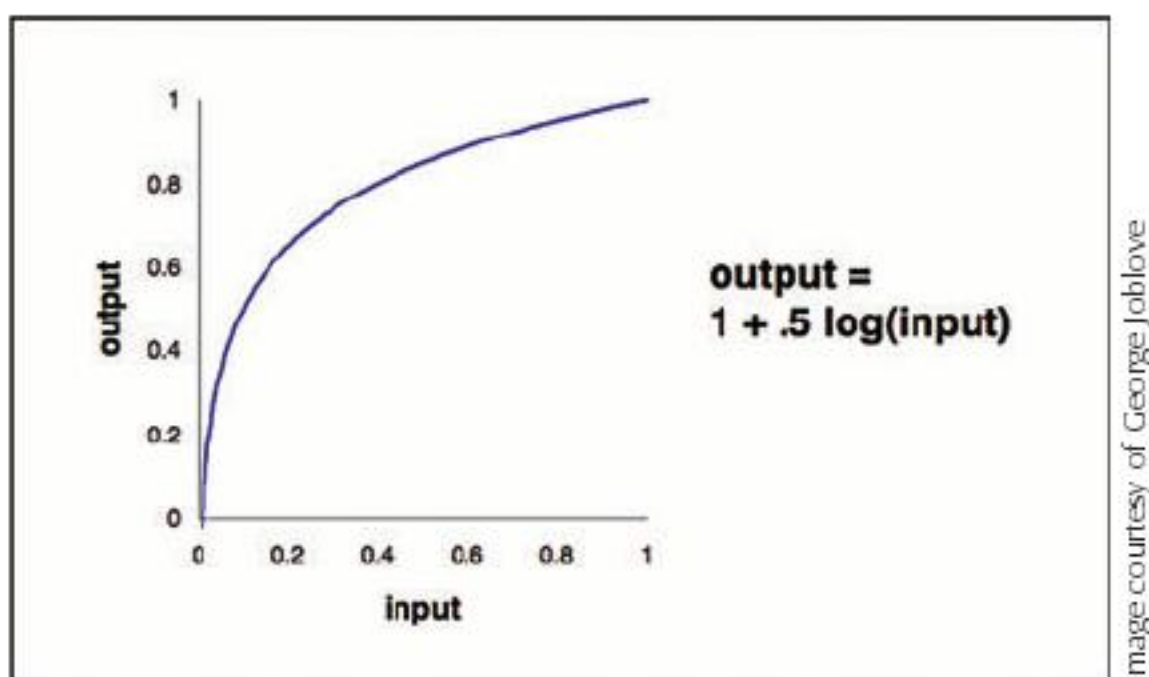


Figure 1.52 Graph of scaled log function.

Note that this function is negative for input values less than .01.

Shown in Figure 1.54 are three sample schemes (one with 8 bits, two with 10 bits) for representing luminance logarithmically. As with gamma coding, the available code values are distributed more evenly with respect to human contrast discrimination. In fact, log coding specifically ensures that the same number of code values exist across any 1-stop difference in luminance.

Courtesy of George Joblove

10-Bit Log Cineon/DPX File Format

The now standard 10-bit log Cineon file format was introduced by Eastman Kodak in the early 1990s for data exchange in the Cineon system. The Cineon image file format is very similar to the ANSI/SMPTE DPX file format, and they are for all intents and purposes used interchangeably. Both file formats have variable header lengths and share the same format for the image data. However, the format of the headers is different. The DPX file

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headers are flexible, allowing variable image headers to accommodate the needs of different industries. The Cineon file format is more directed to digital film. The SMPTE specification is recommended to all developers.

To obtain the SMPTE specification, contact SMPTE directly at +1.914.761.1100 and ask for the specification ANSI/SMPTE 268M-1994, *SMPTE Standard for File Format for Digital Moving-Picture Exchange (DPX)*, v 2.0, 2003.

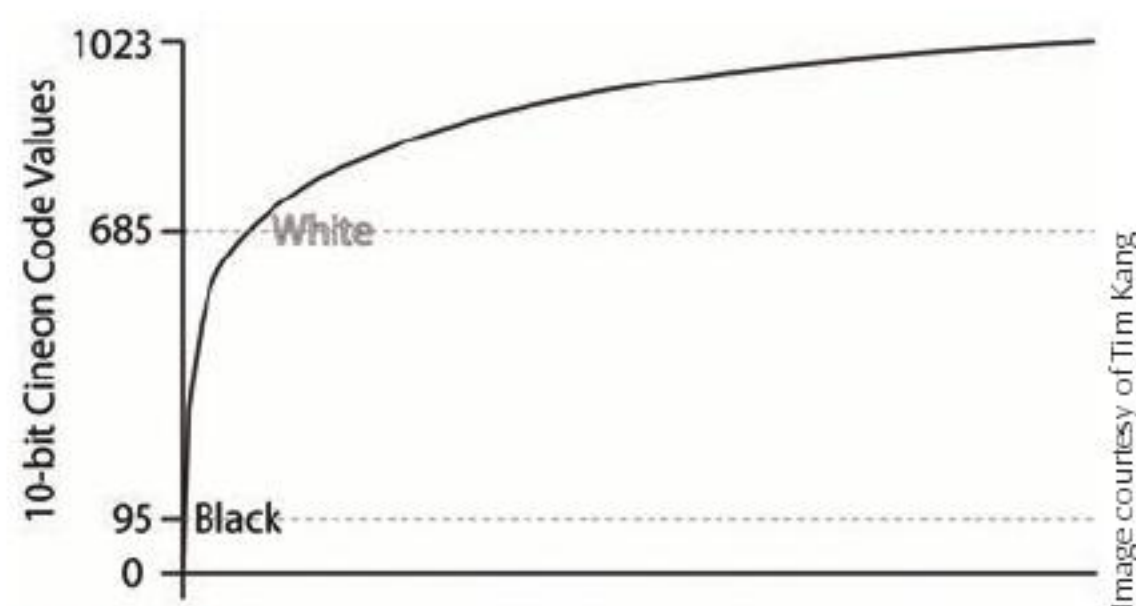


Figure 1.55 Cineon log encoding for film negative.

The enormous dynamic range of film could not be preserved through the post process without characterizing the response of film with its unique exposure shoulder and toe, and log encoding it, so the Cineon Log encoding curve was devised by Eastman Kodak to preserve the maximum latitude from film scans for digital manipulation into a 10-bit log file. Notice that black is mapped to Cineon code value 95 (of 1024) instead of 0. The Cineon white code value is 685, which represents 90% of a 100% reflector. In this way, Cineon files store both Dmin values and Dmax values in order to emulate film's latitude and response characteristics. When a film negative is printed up, information in blacks can reveal itself; when a film negative is printed down, it can reveal additional detail in overbright areas.

The code values in the Cineon file represent negative film densities, and a range of 2.046D is encoded into up to 1,024 values each of red, green, and blue. The RGB values of a pixel are contained in 32 bits, making a 10-bit log Cineon file 33% smaller than a 16-bit file, whereas 48 bits are used to store one RGB pixel. This compromise reduced the disk space that would be needed to store images and also reduced the bandwidth needed to record and play them.

Cineon/DPX Encoding Ranges

Table 1.1 Various Cineon/DPX Encoding Ranges

Cineon / DPX Encoding Ranges				
Encoding Type	Description	Black Point (out of 1024)	White Point (out of 1024)	Gamma
ITU Rec 709	Legal (Video) Range	64	940	2.2
DCI Gamma	Full DCI Range	0	1023	2.6
Print Density	For Use in Film Scans and Film Print Work	95	685	1.66
sRGB	Full Range	0	1023	2.2

In 1994, the Society of Motion Picture and Television Engineers standardized the DPX (Digital Moving-Picture Exchange) format incorporating the 10-bit encoding of the Cineon format, which served us well in the era of the adoption of digital cameras, but in the last few years, the dynamic range and quantization of the 10-bit log format has been found lacking in sufficient bit depth for the needs of modern production. Because black is mapped to code value 95, that leaves only a range of 928 code values for the remaining exposure range. A scene with very bright highlights can result in densities above the Cineon range. Those highlights will be lost in a standard DPX scan, and that effect is called *clipping*. See Table 1.1.

The Calculus of Color Sampling (Uh-Oh . . . Math Again!)

Did you sleep through calculus class or copy from your best friend's homework because you thought, "Where in the world does this stuff have any real world use?" If so, I have some slightly bad news for you: Digital sampling theory *is* the calculus—pure and simple. The analog-to-digital convertors in a digital camera sample the voltage produced from each photosite and turn that voltage into a numerical code value. The more code values that the waveforms coming off the chip are divided into, the more accurate the representation of the analog waveforms is. If we allow n (the width of a sample) to approach zero, and therefore the sample rate to approach infinity, in an

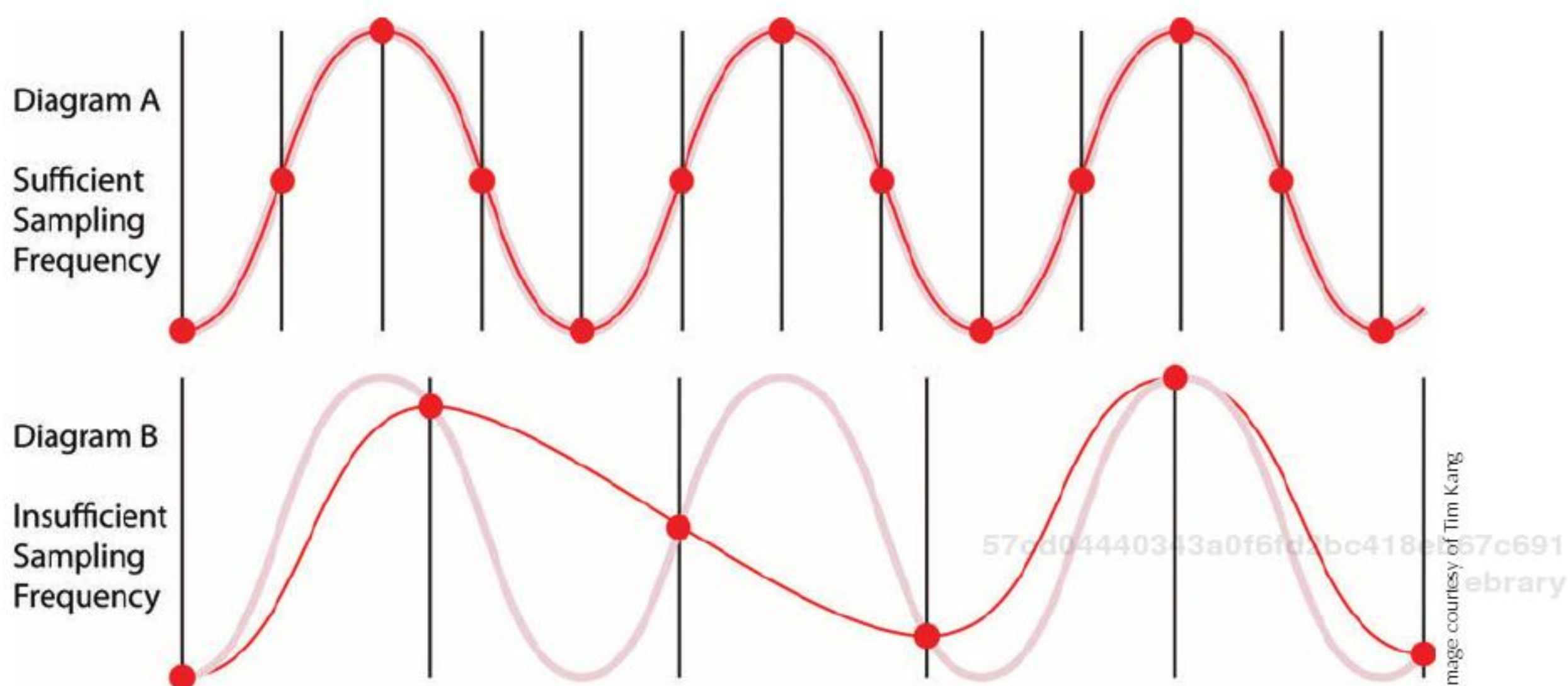


Figure 1.56 Sufficient sampling and insufficient sampling of waveforms.

integral model, we theoretically increase the accuracy of our approximation of the analog phenomenon being sampled to approaching 100% precision.

Nyquist Sampling Theory

Mathematician and electrical engineer Harry Nyquist stated in 1928 that an analog frequency that has been sampled can be accurately reconstructed from those samples if the sampling rate exceeds at least twice the highest frequency in the original signal frequency being sampled (see Figure 1.56).³ Nyquist's sampling theory was then mathematically proved by mathematician Claude Shannon in 1949, and some books use the term "Nyquist Sampling Theorem" interchangeably with "Shannon Sampling Theorem." They are in fact the same sampling theorem.

In Figure 1.56, the upper diagram (A) exhibits a sufficiently high sample rate to result in an acceptable reconstruction of the original waveform. The lower diagram (B) shows how a lower insufficient sample rate cannot accurately reproduce the original waveform being sampled, and the digital reconstruction is said to be an "alias" of the original waveform.

What in the world does that mean?

Nyquist Sampling Theory Simplified

Let's imagine that we discover a person living in a closed cave, someone who has lived his or her whole life in the darkness in that cave. Now let's also suppose



Figure 1.57 A man who has never been outside his cave.

that we have the key to the door to that cave. This affords us an opportunity to learn something about sampling theory from our cave dweller. Suppose that we decide to allow the person out of the cave for 1 minute a day to observe (sample) the world outside. The phenomenon he will be sampling is the Earth's rotation: the 24-hour cycle of day and night.

First, we allow the person out at midnight, once every night for a month.



Figure 1.58 The world outside the cave at night.

After a moment outside, our subject reenters the cave and we ask him or her what the world outside looks like. The answer is likely to be that the world looks quite like the inside of the cave, depending on the phase of the moon, but usually pretty dark.

Now let's suppose that, instead, we allow our subject to go outside at noon every day for a month. The answer upon his or her return to the darkness of the cave is quite different; the world outside is always blindingly bright! But neither sampling of the world is accurate, because the phenomenon being sampled is occurring at a *higher* frequency than our sampling rate.



Figure 1.59 The world outside the cave at midday.

So, now let's increase the sampling rate and send our bewildered cave person outside twice a day, once at noon and once at midnight.

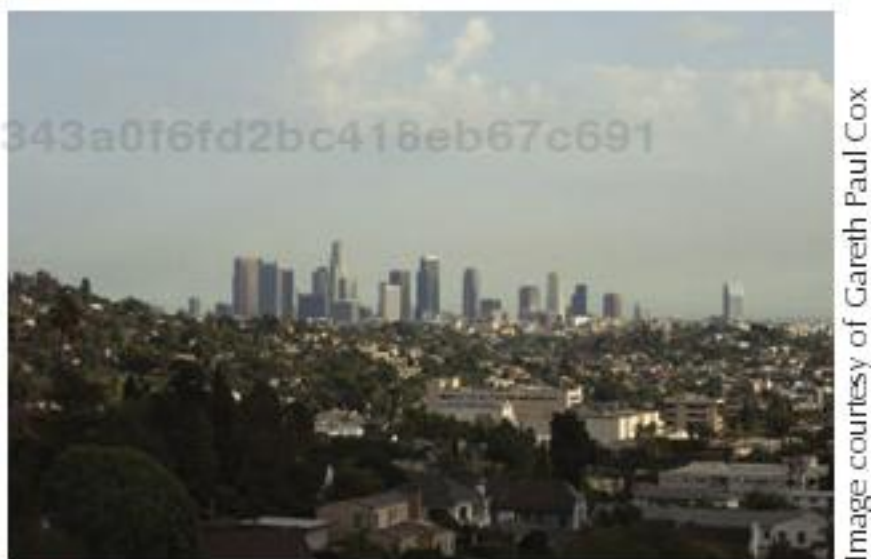


Figure 1.60 Midday view outside.

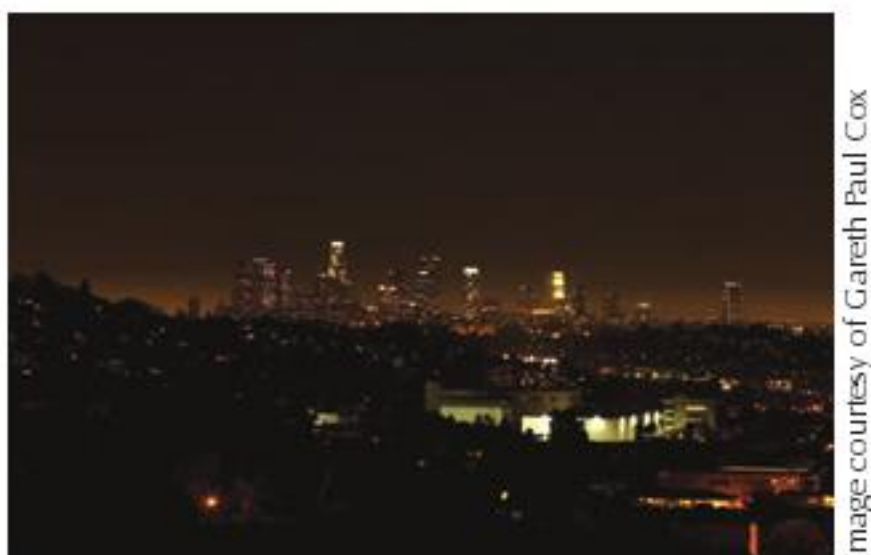


Figure 1.61 Midnight view of the same scene.

And if we increase our sampling rate even higher, let's say to 4, or 8, or 24 times a day, every day for a month, the cave dweller has much more sampling data on which to judge the world outside. So we can see that as we increase our sampling rate, our picture of the phenomenon being sampled becomes more accurate.



Figure 1.62 Midnight view.



Figure 1.63 Dawn view.



Figure 1.64 Midday view.



Figure 1.65 Sunset view.

In summary, Nyquist shows us that higher sampling rates yield greater accuracy in re-creation of the original analog phenomenon. By utilizing more bits to encode

color, digital cameras can more faithfully record and reproduce the original scene data. We will learn more on this topic in subsequent chapters of this book.

Notes

1. Billups, S. (2008). Digital moviemaking 3.0 (p. 19). Studio City, CA: Michael Wiese Productions.
2. The following section is taken from G. Joblove. (1997). Digital tone reproduction, scanning and recording of motion picture film. Paper presented at *24th International Conference on Computer Graphics and Interactive Techniques*. Los Angeles, CA.
3. H. Nyquist. (1928). Certain topics in telegraph transmission theory. *Transactions of the American Institute of Electrical Engineers*, 47(2), pp. 617–644.