1 axiom5

Relation R has properties:

$$
\cfrac{
  \cfrac{
    x : \Diamond A \supset \Box B; \vdash x : \Diamond A \supset \Box B \; (Hyp)
    \qquad
    \cfrac{
      \cfrac{y : A; \vdash y : A \;(Hyp) \qquad xRy; \vdash xRy \;(Hyp)}{y : A; xRy; \vdash x : \Diamond A} \;(\Diamond I)
    }{}
  }{
    \cfrac{x : \Diamond A \supset \Box B; y : A; xRy; \vdash x : \Box B \qquad xRy; \vdash xRy \;(Hyp)}{x : \Diamond A \supset \Box B; y : A; xRy; \vdash y : B} \;(\Box E)
  }
}{
}
$$

$$
\cfrac{
\cfrac{
\cfrac{
\cfrac{
x : \Diamond A \supset \Box B; y : A; xRy; \vdash y : B
}{
x : \Diamond A \supset \Box B; xRy; \vdash y : A \supset B
}\;(\supset I)
}{
x : \Diamond A \supset \Box B; \vdash x : \Box(A \supset B)
}\;(\Box I)
}{
\vdash x : (\Diamond A \supset \Box B) \supset \Box(A \supset B)
}\;(\supset I)
}{}
$$

2 axiom11

Relation R2 has properties: Directedness,

$$
\cfrac{
\cfrac{
x : \Diamond\Box A; \vdash x : \Diamond\Box A \;(Hyp)
\qquad
\cfrac{\cdots}{xR2y; xR2z; z : \Box A; \vdash y : \Diamond A}\;(R_2)
}{
\cfrac{x : \Diamond\Box A; xR2y; \vdash y : \Diamond A}{\cfrac{x : \Diamond\Box A; \vdash x : \Box\Diamond A \;(\Box I)}{\vdash x : \Diamond\Box A \supset \Box\Diamond A}\;(\supset I)}
}{}
}{}\;(\Diamond E)
$$

3 axiom4

Relation R has properties:

$$
\cfrac{
x : \Diamond(A \vee B); \vdash x : \Diamond(A \vee B)\;(Hyp)
\qquad
\cdots
}{
\cfrac{x : \Diamond(A \vee B); \vdash x : \Diamond A \vee \Diamond B}{\vdash x : \Diamond(A \vee B) \supset \Diamond A \vee \Diamond B}\;(\supset I)
}\;(\Diamond E)
$$

4 axiom2

Relation R has properties:

$$\cfrac{\cfrac{\cfrac{x : \Box(A \supset B); \vdash x : \Box(A \supset B)}{}^{(Hyp)} \quad \cfrac{xRy; \vdash xRy}{}^{(Hyp)}}{x : \Box(A \supset B); xRy; \vdash y : A \supset B}{}^{(\Box E)} \quad \cfrac{y : A; \vdash y : A}{}^{(Hyp)}}{\cfrac{\cfrac{x : \Box(A \supset B); xRy; y : A; \vdash y : B}{}^{(\supset E)} \quad \cfrac{xRy; \vdash xRy}{}^{(Hyp)}}{\cfrac{x : \Box(A \supset B); xRy; y : A; \vdash x : \Diamond B}{}^{(\Diamond I)}}}{\cfrac{\cfrac{x : \Diamond A; \vdash x : \Diamond A}{}^{(Hyp)} \quad \cfrac{x : \Diamond A; x : \Box(A \supset B); \vdash x : \Diamond B}{}^{(\Diamond E)}}{\cfrac{x : \Box(A \supset B); \vdash x : \Diamond A \supset \Diamond B}{}^{(\supset I)}}{\vdash x : \Box(A \supset B) \supset \Diamond A \supset \Diamond B}{}^{(\supset I)}}$$

5 axiom10

Relation R5 has properties: Euclideanness,

$$\cfrac{\cfrac{x : \Diamond A; \vdash x : \Diamond A}{}^{(Hyp)} \quad \cfrac{\cfrac{xR5y; \vdash xR5y}{}^{(Hyp)} \quad \cfrac{xR5z; \vdash xR5z}{}^{(Hyp)} \quad \cfrac{\cfrac{z : A; \vdash z : A}{}^{(Hyp)} \quad \cfrac{yR5z; \vdash yR5z}{}^{(Hyp)}}{z : A; yR5z; \vdash y : \Diamond A}{}^{(\Diamond I)}}{xR5y; xR5z; z : A; \vdash y : \Diamond A}{}^{(R_5)}}{\cfrac{x : \Diamond A; xR5y; \vdash y : \Diamond A}{}^{(\Diamond E)}}}{\cfrac{x : \Diamond A; \vdash x : \Box \Diamond A}{}^{(\Box I)}}{\vdash x : \Diamond A \supset \Box \Diamond A}{}^{(\supset I)}$$

6 axiom1

Relation R has properties:

$$\cfrac{\cfrac{\cfrac{x : \Box(A \supset B); \vdash x : \Box(A \supset B)}{}^{(Hyp)} \quad \cfrac{xRy; \vdash xRy}{}^{(Hyp)}}{x : \Box(A \supset B); xRy; \vdash y : A \supset B}{}^{(\Box E)} \quad \cfrac{\cfrac{x : \Box A; \vdash x : \Box A}{}^{(Hyp)} \quad \cfrac{xRy; \vdash xRy}{}^{(Hyp)}}{x : \Box A; xRy; \vdash y : A}{}^{(\Box E)}}{\cfrac{\cfrac{x : \Box(A \supset B); xRy; x : \Box A; \vdash y : B}{}^{(\supset E)}}{\cfrac{x : \Box(A \supset B); x : \Box A; \vdash x : \Box B}{}^{(\Box I)}}{\cfrac{x : \Box(A \supset B); \vdash x : \Box A \supset \Box B}{}^{(\supset I)}}{\vdash x : \Box(A \supset B) \supset \Box A \supset \Box B}{}^{(\supset I)}}$$

7 axiom7

Relation RT has properties: Reflexivity,

$$\cfrac{\cfrac{\cfrac{x : \Box A; \vdash x : \Box A}{}^{(Hyp)} \quad \cfrac{xRTx; \vdash xRTx}{}^{(Hyp)}}{\cfrac{x : \Box A; xRTx; \vdash x : A}{}^{(\Box E)}}{\cfrac{x : \Box A; \vdash x : A}{}^{(R_T)}}}{\vdash x : \Box A \supset A}{}^{(\supset I)}$$

8 axiom8

Relation RB has properties: Symmetry,

$$
\cfrac{
  \cfrac{
    \cfrac{}{xRBy;\vdash xRBy}\,(Hyp)
    \qquad
    \cfrac{
      \cfrac{}{x:A;\vdash x:A}\,(Hyp)
      \qquad
      \cfrac{}{yRBx;\vdash yRBx}\,(Hyp)
    }{x:A;yRBx;\vdash y:\diamond A}\,(\diamond I)
  }{
    \cfrac{xRBy;x:A;\vdash y:\diamond A}{
      \cfrac{x:A;\vdash x:\square\diamond A}{\vdash x:A\supset\square\diamond A}\,(\supset I)
    }\,(\square I)
  }\,(R_B)
}{}
$$

9 axiom9

Relation R4 has properties: Transitivity,

$$
\cfrac{
  \cfrac{}{xR4y;\vdash xR4y}\,(Hyp)
  \qquad
  \cfrac{}{yR4z;\vdash yR4z}\,(Hyp)
  \qquad
  \cfrac{
    \cfrac{}{x:\square A;\vdash x:\square A}\,(Hyp)
    \quad
    \cfrac{}{xR4z;\vdash xR4z}\,(Hyp)
  }{x:\square A;xR4z;\vdash z:A}\,(\square E)
}{
  \cfrac{xR4y;yR4z;x:\square A;\vdash z:A}{
    \cfrac{xR4y;x:\square A;\vdash y:\square A}{
      \cfrac{x:\square A;\vdash x:\square\square A}{\vdash x:\square A\supset\square\square A}\,(\supset I)
    }\,(\square I)
  }\,(\square I)
}\,(R_4)
$$

10 axiom6

Relation RD has properties: Seriality,

$$
\cfrac{
  \cfrac{
    \cfrac{}{x:\square A;\vdash x:\square A}\,(Hyp)
    \quad
    \cfrac{}{xRDy;\vdash xRDy}\,(Hyp)
  }{x:\square A;xRDy;\vdash y:A}\,(\square E)
  \qquad
  \cfrac{}{xRDy;\vdash xRDy}\,(Hyp)
}{
  \cfrac{x:\square A;xRDy;\vdash x:\diamond A}{
    \cfrac{x:\square A;\vdash x:\diamond A}{\vdash x:\square A\supset\diamond A}\,(\supset I)
  }\,(R_D)
}\,(\diamond I)
$$

11 axiom3

Relation R has properties:

$$
\cfrac{
  \cfrac{}{x:\diamond\bot;\vdash x:\diamond\bot}\,(Hyp)
  \qquad
  \cfrac{
    \cfrac{
      \cfrac{}{y:\bot;\vdash y:\bot}\,(Hyp)
    }{y:\bot;\vdash x:\bot}\,(\bot E)
  }{xRy;y:\bot;\vdash x:\bot}\,(W)
}{
  \cfrac{x:\diamond\bot;\vdash x:\bot}{\vdash x:\diamond\bot\supset\bot}\,(\supset I)
}\,(\diamond E)
$$